

# WEAK LOCALIZATION IN $p$ -TYPE HETEROSTRUCTURES IN THE PRESENCE OF PARALLEL MAGNETIC FIELD

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Theory of weak localization is developed for two-dimensional holes in the presence of in-plane magnetic field. The Zeeman splitting even in the hole momentum results in the spin-dependent phase changing the quantum interference. The negative correction to the conductivity is shown to decrease by a factor of two by the in-plane magnetic field. The positive magnetoconductivity in a classically weak perpendicular field caused by the weak localization is calculated for both quadratic and quartic in momentum Zeeman hole splittings. Calculations show that the conductivity corrections are very close to each other in these two cases of low and high hole density.

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Spin-dependent phenomena attract a great attention due to spin-orbit interaction allowing spin manipulation by electrical or optical means. The first important move in this way was a discovery of the Rashba splitting of electron energy spectrum in bulk wurtzite-type semiconductors [1]. In two-dimensional (2D) systems, this splitting is present in heterostructures made of any material provided the structure inversion asymmetry is present [2]. Generally, the spin-orbit interaction is described by the term in the Hamiltonian which can be presented in the form

$$\mathcal{H}_{SO} = \hbar\boldsymbol{\sigma}\boldsymbol{\Omega}, \quad (1)$$

where  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$  is a vector of Pauli matrices, and the spin-orbit splitting equals  $2\hbar\Omega$ .

The spin-orbit splitting of the electron energy spectrum leads to many interesting optical and transport phenomena [3]. In transport, it leads to a remarkable beating patterns in the Shubnikov–de Haas oscillations, where it can be easily detected. However, a good mobility is needed for such kind of manifestation of the Rashba splitting, which should be much larger than the level broadening. Nevertheless, even

in low-mobility samples the Rashba splitting can be measured. This can be done in classically low magnetic fields, where the magnetoresistance is caused by the weak localization (WL) effect, see Ref. [4] for review. Developed theoretical expressions for the WL correction to the conductivity valid for arbitrary values of the Rashba splitting allow adequately extracting the splitting value and other electron kinetic and band parameters by fitting the experimental data.

2D holes in semiconductor heterostructures represent a system which is very different from electrons. This happens because the holes in the ground 2D subband have spin projection  $\pm 3/2$  on the structure main axis. In particular, they have a cubic in momentum Rashba splitting [5]. Due to the same reason, the Zeeman splitting of heavy-holes in the in-plane magnetic field at the bottom of the 2D subband is cubic in the field strength in the axial approximation. A small contribution for free holes is present due to cubic symmetry of the zinc-blende lattice forming the heterostructure [6] which, however, increases strongly for localized holes in quantum dots [7]. At finite wavevectors the situation changes, and the momentum-dependent in-plane Zeeman splitting arises. In the axial approximation, the Hamiltonian of heavy holes in the ground subband of a symmetrical quantum well in the presence

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of an in-plane magnetic field  $\mathbf{B}_{\parallel}$  is given by [8]

$$\mathcal{H}_{SO} = \hbar\sigma_{-}(\Delta_1 B_{+} k_{+}^2 + \Delta_2 B_{-} k_{-}^4) + \text{H. c.} \quad (2)$$

Here,  $\mathbf{k}$  is the in-plane wavevector,  $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$  with the operators  $\sigma_{x,y}$  coupling two Kramers-degenerate hole states,

$$B_{\pm} = B_x \pm iB_y, \quad k_{\pm} = k_x \pm ik_y,$$

and  $\Delta_{1,2}$  are constants. This expression coincides with Eq. (1) where  $\mathbf{\Omega}$  is given by

$$\Omega_x + i\Omega_y = B_{\parallel}(\Delta_1 k^2 e^{2i\varphi} + \Delta_2 k^4 e^{4i\varphi}) \quad (3)$$

with  $\varphi$  being an angle between  $\mathbf{k}$  and  $\mathbf{B}_{\parallel}$ .

According to estimates given in Ref. [9], the Zeeman splitting at  $B_{\parallel} = 1$  T is  $2\hbar\Omega \sim 0.1 \dots 1$  meV. This allows us to solve the WL problem by the method used in Refs. [4,10] assuming the ratio of the splitting and level broadening to be arbitrary but ignoring the difference in the Fermi wavevectors in spin subbands.

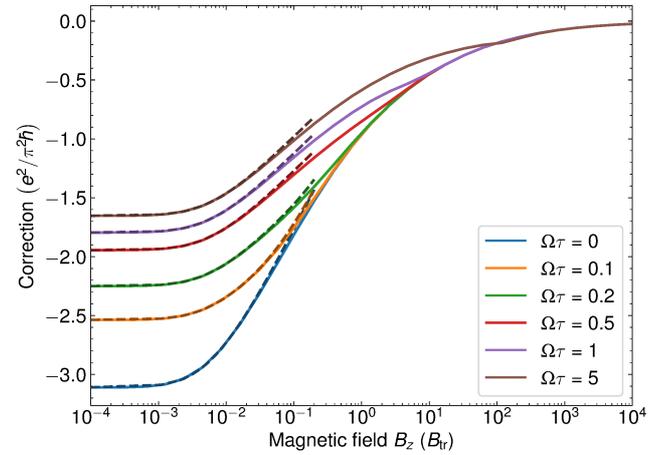
We study two limits of low and high hole densities where  $\Delta_1 k_F^2$  prevails over  $\Delta_2 k_F^4$  or vice versa. Here  $k_F$  is the Fermi wavevector. In both cases, the Zeeman splitting is isotropic in the  $\mathbf{k}$ -space, and the WL problem can be solved analytically. The hole Hamiltonian (2) is even in  $\mathbf{k}$ , and therefore the WL correction to conductivity and the anomalous magnetoresistance are negative. We consider diffusive and ballistic regimes of WL [11], where the interference contribution to the conductivity occurs on large and small trajectories, respectively.

In the low-density limit, the Hamiltonian (1) with  $\mathbf{\Omega}$  from Eq. (3) has the same form as that of exciton-polaritons in microcavities with  $\Omega = \Delta_1 k_F^2 B_{\parallel}$  instead of the longitudinal-transverse splitting, see Ref. [12]. Therefore the negative WL correction to conductivity in the diffusion approximation is given by the expression following from Ref. [12], see full paper [13] for details.

At high density the results for both zero-field correction and the magnetoconductivity are the same, the only difference is that  $\Omega = \Delta_2 k_F^4 B_{\parallel}$ . Differences in the functional forms of the WL contribution to the conductivity at low and high densities appear in stronger perpendicular fields  $B_z \sim B_{tr}$ . Here  $B_{tr} = \hbar/(2|e|l^2)$  is the ‘‘transport’’ magnetic field with  $l$  being the mean free path. In this case the ballistic trajectories with a few, three or more, impurities contribute to the conductivity, therefore this is called ballistic regime of WL.

In the ballistic regime, we take into account non-logarithmic corrections to the conductivity as well as

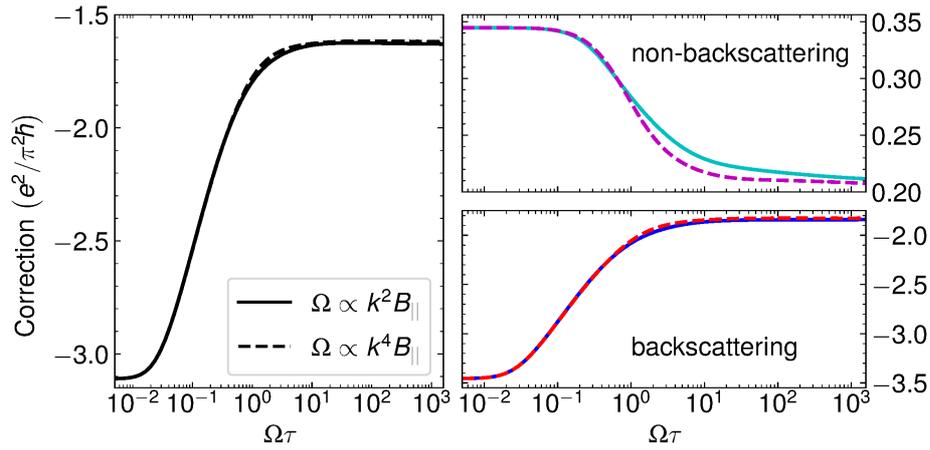
the non-backscattering contribution [14,15]. For numerical calculations of the conductivity corrections, we extend the approach used in Ref. [16], see full paper for details. In Fig. 1, the conductivity correction is shown as a function of  $B_z$  for various values of a product of the spin-orbit splitting  $\Omega \propto B_{\parallel} k^2$  and the momentum scattering time  $\tau$ . The zero- $B_z$  value at large  $\Omega\tau$  is twice smaller than at  $\Omega = 0$ . At large  $B_z \gg B_{tr}$  all curves tend to the same dependence because of absence of spin rotations at characteristic trajectories with the size  $\sim l_B \ll l$ . The conductivity at  $k^4$ -splitting is very close to these dependencies. Therefore the results of Fig. 1 are valid for the  $k^4$ -type of splitting as well.



**Fig. 1.** Conductivity correction at  $k^2$ -splitting as a function of  $B_z/B_{tr}$  for various  $\Omega\tau \propto B_{\parallel}$ . The dephasing time  $\tau_{\phi}/\tau = 10^3$ . Diffusion approximation results are shown by dashed lines

The WL correction to conductivity at  $B_z = 0$  is analyzed in Fig. 2. We see that the size of the WL correction is a little bit larger in the case of  $k^2$ -splitting. However, the difference is very small.

To summarize, the theory of WL of 2D holes in the presence of an in-plane magnetic field is developed. The momentum-dependent Zeeman splitting is taken into account which can be squared or quartic in  $k$ . The WL conductivity correction, which is negative, is derived for both cases. Calculations show that the results are very close to each other. The  $k$ -dependent Zeeman splitting suppresses the WL correction up to factor of two at large splitting. The positive magnetoconductivity in classically-weak perpendicular magnetic fields is calculated for arbitrary values of the Zeeman splitting. The developed theory is valid for arbitrary values of the product  $\Omega\tau$ , but with the spin splitting  $2\hbar\Omega$  assumed much smaller than the Fermi energy. For higher spin



**Fig. 2.** Conductivity correction at  $B_z = 0$  as a function of  $\Omega\tau \propto B_{\parallel}$  at  $\tau_{\phi}/\tau = 10^3$ . The total conductivity correction, backscattering and non-backscattering contributions are shown in the left, upper right and lower right panels, respectively. Solid and dashed curves correspond to the  $k^2$ - and  $k^4$ -splittings

splittings, when they are comparable, one should take into account the difference of the Fermi wavevectors in two spin-split subbands, as it has been done for large Rashba splittings in Ref. [17].

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