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PAIRING BY A DYNAMICAL INTERACTION IN A METAL

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The pairing near a quantum-critical point (QCP) in a metal and its interplay with non-Fermi-liquid behavior in the normal state is a fascinating subject, which attracted substantial attention in the correlated electron community after the discovery of superconductivity (SC) in the cuprates, Fe-based systems, heavy-fermion materials, organic materials, and, most recently, twisted bilayer graphene [1–13]. Itinerant QC models, analyzed in recent years, include models of fermions in spatial dimensions $D \leq 3$, various two-dimensional models near zero-momentum spin and charge nematic instabilities, and instabilities towards spin and charge density-wave order with either real or imaginary (current) order parameter, 2D fermions at a half-filled Landau level, Sachdev – Ye – Kitaev (SYK) and SYK – Yukawa models, strong coupling limit of electron-phonon superconductivity, and even color superconductivity of quarks, mediated by gluon exchange. These problems have been studied analytically and using various numerical techniques [14].

From theory perspective, pairing near a QCP is a fundamentally novel phenomenon, because an effective dynamic electron-electron interaction, $V(q,\Omega)$, mediated by a critical collective boson, which condenses at a QCP, provides a strong attraction in one or more pairing channels and, at the same time, gives rise to a non-Fermi liquid (NFL) behavior in the normal state. The two tendencies compete with each other: fermionic incoherence, associated with the NFL behavior, destroys the Cooper logarithm and by this reduces the tendency to pairing, while an opening of a SC gap eliminates the scattering at low energies and reduces the tendency to a NFL. To find the winner of this competition (SC or NFL), one needs to analyze the set of integral equations for the fermionic self-energy, $\bar{\Sigma}(\mathbf{k},\omega)$, and the gap function, $\Delta(\mathbf{k},\omega)$, for fermions with momentum/frequency $(\mathbf{k},\omega)$ and $(-\mathbf{k},-\omega)$.

We consider the subset of models, in which collective bosons are slow modes compared to dressed fermions, for one reason or the other. In this situation, which bears parallels with Eliashberg theory for electron-phonon interaction [15], the self-energy and the pairing vertex can be approximated by their values on the Fermi surface (FS) and computed within the one-loop approximation. The self-energy on the FS, $\bar{\Sigma}(\mathbf{k},\omega)$, is invariant under rotations from the point group of the underlying lattice. The rotational symmetry of the gap function $\Delta(\mathbf{k}_F,\omega)$ and the relation between the phases of $\Delta(\mathbf{k}_F,\omega)$ on different FS's in multi-band systems are model specific. E.g., near an antiferromagnetic QCP in a system with a single FS, the strongest attraction is in the d-wave channel. In each particular case, one has to project the pairing interaction into the irreducible channels $V(q,\Omega) \to V(\Omega)$, find the strongest one, and solve for the pairing vertex for a given pairing symmetry.

Away from a QCP, the effective $V(\Omega)$ tends to a finite value at $\Omega = 0$. In this situation, the fermionic self-energy has a FL form at the smallest frequencies, and the equation for $\Delta(\omega)$ is similar to that in a conventional Eliashberg theory for phonon-mediated superconductivity. At a QCP, the situation is qualitatively

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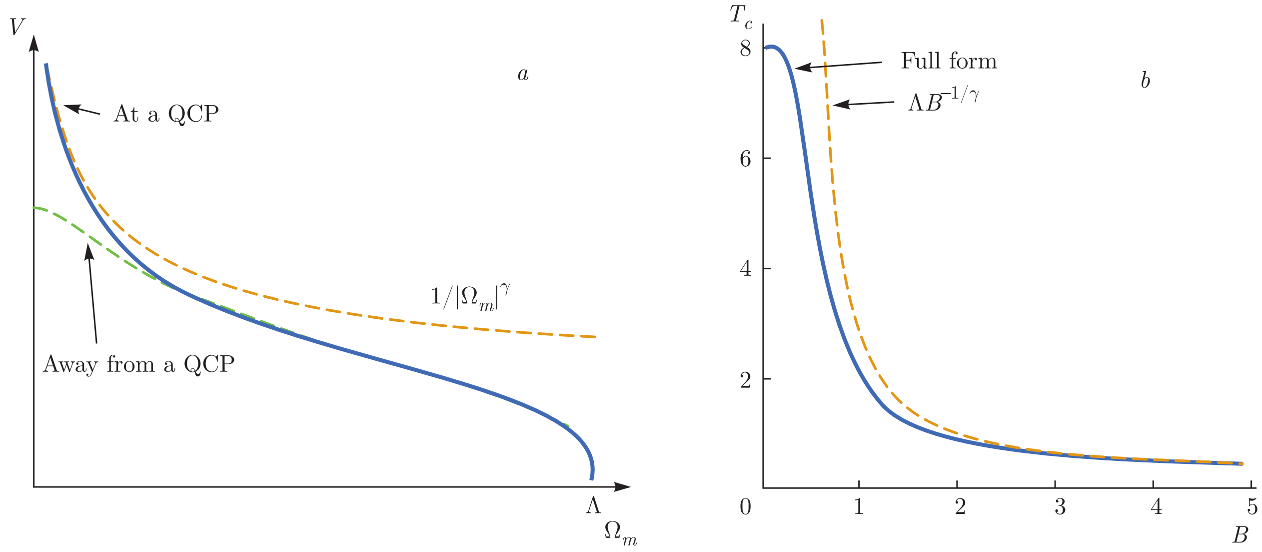

a) The frequency dependence of the effective interaction $V(\Omega_m)$, mediated by a soft boson, at $T = 0$. Away from a QCP, $V(\Omega_m)$ tends to a finite value at $\Omega_m = 0$. Right at a QCP, the boson becomes massless, and at frequencies below the upper cutoff Λ, the dimensionless $V(\Omega_m)$ behaves as $\log \Lambda/|\Omega_m|$ at $\gamma = 0+$ and as $(\tilde{g}/|\Omega_m|)^\gamma$ at a finite γ. *b*) T_c as a function of the parameter $B = \gamma(\Lambda/\tilde{g})^\gamma$, which determines the crossover between the behavior at a finite γ (the limit of large B) and at $\gamma = 0+$ (the limit of small B)

different, because the effective interaction $V(\Omega)$, mediated by a critical massless boson, is a singular function of frequency. Quite generally, the dimensionless interaction behaves at the smallest Ω_m on the Matsubara axis as $V(\Omega_m) = (\tilde{g}/|\Omega_m|)^\gamma$, where $\gamma > 0$ is some exponent (Figure *a*). This holds at frequencies below some upper cutoff Λ. At larger $\Omega_m > \Lambda$, the interaction drops even further, and can be safely neglected.

In this communication, we consider the pairing at small γ. This limit attracted a lot of attention in the last few years from various sub-communities of physicists [16–31]. We consider this limit analytically for $V(\Omega)$, which crosses over from $(\tilde{g}/|\Omega_m|)^\gamma$ behavior at a finite γ to the logarithmic behavior at $\gamma = 0+$ (the dimensionless $V(\Omega) = \lambda \log \Lambda/|\Omega_m|$). In the latter case, $T_c \sim \Lambda \exp(-\pi/(2\sqrt{\lambda}))$. This expression is similar to the one in the BCS case, but with $\sqrt{\lambda}$ instead of λ in the exponent, because the "Cooper" logarithm appears from the combination of the logarithms in fermion and boson propagators. At a finite γ, the transition temperature remains finite even if $\Lambda \to \infty$ and its dependence on γ is $T_c \sim \tilde{g}(1/\gamma)^{1/\gamma}$. This T_c rapidly increases as γ decreases.

When both Λ and γ are finite, one expects the crossover between the expressions for T_c at finite γ and $\Lambda \to \infty$ and at $\gamma = 0+$ and a finite Λ. This crossover is the main theme of our paper. We find the full crossover function for T_c and show that the two limiting cases correspond to small and large values of the single parameter $B = \gamma(\Lambda/\tilde{g})^\gamma$.

The structure of the paper is the following. Section 1 is a preface for the paper. Section 2 is the detailed Introduction. In Sec. 3 we present the set of coupled Eliashberg equations for the pairing vertex $\Phi(\omega_m)$ and the fermionic self-energy $\bar{\Sigma}(\omega_m)$ and combine them into the equation for the gap function $\Delta(\omega_m)$. In Sec. 4 we analyze the structure of the logarithmic perturbation theory for $\gamma = 0+$ and $\gamma > 0$, keeping a finite high frequency cutoff Λ. We show that for $\gamma = 0+$, the summation of the leading logarithms capture $T_c \sim \Lambda \exp(-\pi/(2\sqrt{\lambda}))$, although logarithmic series are not geometric, in distinction from the BCS theory. However, for a finite γ, summation of the logarithms does not give rise to a pairing instability — the pairing susceptibility does not diverge. In Sec. 5 we go beyond perturbation theory. We re-express the integral Eliashberg equation as an approximate differential equation for the pairing vertex and solve it. We show that for $\gamma = 0+$, the solution coincides with the result of summation of the logarithmic series. For $\gamma > 0$, we show that the absence of an instability within the logarithmic approximation implies that there is a threshold on the strength of the pairing interaction. We find the threshold and show explicitly that, once the interaction exceeds the threshold, the normal state becomes unstable against pairing at some finite T_c. We show that for a finite γ, the calculation of the pairing instability is ultra-violet convergent, hence T_c remains finite even when the cutoff Λ is set to infinity. We analyze the crossover between the forms of T_c at a finite γ and at $\gamma = 0+$ and show that the crossover is governed by the single parameter $B = \gamma(\Lambda/\tilde{g})^{\gamma}$.

In Sec. 6 we analyze the pairing at small γ from the renormalization group (RG) perspective — as the flow of the 4-fermion pairing vertex at a finite γ. We show that the solution of the RG equations describes the same crossover between T_c at a finite γ and at $\gamma = 0+$. We present our conclusions in Sec. 7.

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