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GRAVITATIONAL COLLAPSE OF DARK ENERGY FIELD CONFIGURATIONS AND SUPERMASSIVE BLACK HOLE FORMATION

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Dark energy is the dominant component of the total energy density of our Universe. The primary interaction of dark energy with the rest of the Universe is gravitational. It is therefore important to understand the gravitational dynamics of dark energy. Since dark energy is a low-energy phenomenon from the perspective of particle physics and field theory, a fundamental approach based on fields in curved space should be sufficient to understand the current dynamics of dark energy. Here, we take a field theory approach to dark energy. We discuss the evolution equations for a generic dark energy field in curved space–time and then discuss the gravitational collapse for dark energy field configurations. We describe the $3+1$ BSSN formalism to study the gravitational collapse of fields for any general potential for the fields and apply this formalism to models of dark energy motivated by particle physics considerations. We solve the resulting equations for the time evolution of field configurations and the dynamics of space-time. Our results show that gravitational collapse of dark energy field configurations occurs and must be considered in any complete picture of our Universe. We also demonstrate the black hole formation as a result of the gravitational collapse of the dark energy field configurations. The black holes produced by the collapse of dark energy fields are in the supermassive black hole category with the masses of these black holes being comparable to the masses of black holes at the centers of galaxies.

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1. INTRODUCTION

Increased accuracy in astronomical observations led us to an increased precision in the determination of cosmological parameters. This in turn led us to critically re-examine our cosmological models. In particular, the accurate determination of the Hubble constant and the independent determination of the age of the Universe forced us to critically re-examine the simplest cosmological model, a flat Universe with a zero cosmological constant [1, 2]. These observations forced one of the current authors [3] to consider the idea of a small non-vanishing vacuum energy due to fields as playing an important role in the Universe. Subsequently, there has been a large body of work on both observational and theoretical sides that has shaped our belief in what we now call dark energy.

Given that, we now believe that dark energy is the dominant component of the Universe, there is a pressing need to understand the dynamics of dark energy and in particular the gravitational dynamics of dark energy. In the discussion in what follows, we describe the gravitational dynamics of dark energy and show the gravitational collapse of dark energy field configurations. We also demonstrate black hole formation as a result of the collapse of dark energy field configurations.

Before we start our detailed look at the gravitational dynamics of dark energy field configurations, we briefly introduce field theory models of dark energy in order that all readers can readily relate to the discussion that follows.

Field theory models for dark energy and relations to particle physics were previously discussed in detail by one of the current authors [3]. It was noted in [3] that these fields must have very light mass scales in order to be cosmologically relevant today. Realistic particle-physics models with particles of small masses capable of generating interesting cosmological consequences have been discussed by several authors [4]. It has been pointed out that the most natural class of models for the realization of these ideas are models of

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neutrino masses with pseudo Nambu–Goldstone bosons (PNGBs). The reason for this is that the mass scales associated with such models can be related to the neutrino masses, while any tuning that needs to be done is protected from radiative corrections by the symmetry that gave rise to the Nambu–Goldstone modes (see, e. g., [5]).

Holman and Singh [6] studied the finite temperature behavior of the see-saw model of neutrino masses and found phase transitions in this model, resulting in the formation of topological defects. In fact, the critical temperature in this model is naturally linked to the neutrino masses.

In the next section, we describe the formalism and evolution equations for studying the gravitational dynamics of dark energy field configurations. We start by writing the equations for fields with any general potential. This keeps the initial discussion general. However, in light of the above discussion on realistic models with small masses, we then specialize to PNGB models with a potential of a simple and explicit form. For this purpose, we take the simplest PNGB potential [4].

2. EVOLUTION OF FIELDS IN THE PRESENCE OF GRAVITY

We now turn to the study of the gravitational dynamics of dark energy field configurations. The dynamics of fields in cosmological space–times has been extensively discussed elsewhere (see, e. g., [7]). Likewise, gravitational collapse in the context of general relativity has also been extensively discussed elsewhere (see, e. g., [8]). These ideas can be pulled together to write the evolution equations describing the coupled dynamics of the field configurations and space–time interacting with each other. In this section, we discuss the evolution equations. In the following sections, we discuss solutions of these evolution equations and describe the results we have obtained.

We use the $3+1$ BSSN formalism to numerically study the time evolution of scalar fields in the presence of gravity. The formalism for doing this has been previously described in [9].

The evolution equations

The action describing a self-gravitating complex scalar field in a curved space–time is

$$I = \int d^4x \sqrt{-g} \times \times \left(\frac{1}{16\pi} R - \frac{1}{2} [g^{\mu\nu} \partial_\mu \Phi^* \, \partial_\nu \Phi + V(|\Phi|^2)] \right), \quad (1)$$

where R is the Ricci scalar, $g_{\mu\nu}$ is the metric of the space–time, g is the determinant of the metric, Φ is the scalar field, and V is its potential. Varying this action leads to equations of motion for the entire system. Variation with respect to the scalar field leads to the Klein–Gordon equation for the scalar field

$$\Phi^{;\mu}{}_{;\mu} - \frac{dV}{d|\Phi|^2} \Phi = 0. \quad (2)$$

When Eq. (1) is varied with respect to the metric $g^{\mu\nu}$, the Einstein equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ follow. The resulting stress energy tensor is

$$T_{\mu\nu} = \frac{1}{2} [\partial_\mu \Phi^* \partial_\nu \Phi + \partial_\mu \Phi \partial_\nu \Phi^*] - - \frac{1}{2} g_{\mu\nu} [\Phi^{*,\eta} \Phi_{,\eta} + V(|\Phi|^2)]. \quad (3)$$

To obtain numerical solutions, it is convenient to use the 3+1 decomposition of the Einstein equations, for which the line element can be written as

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij} (dx^i + \beta^i dt)(dx^j + \beta^j dt), \quad (4)$$

where γ_{ij} is the 3-dimensional metric. The Latin indices label the three spatial coordinates. The functions α and β^i in Eq. (4) are gauge parameters, respectively known as the lapse function and the shift vector. The determinant of the 3-metric is γ. The Greek indices range from 0 to 3 and the Latin indices range from 1 to 3.

For the purpose of numerical evolution, the Klein–Gordon equation can be written as a first-order system. This is done by first splitting the scalar field into the real and imaginary parts as $\Phi = \phi_1 + i\phi_2$ and then defining variables in terms of combinations of their derivatives:

$$\Pi = \pi_1 + i\pi_2, \quad \psi_a = \psi_{1a} + i\psi_{2a}.$$

Here,

$$\pi_1 = (\sqrt{\gamma}/\alpha)(\partial_t \phi_1 - \beta^c \partial_c \phi_1), \quad \psi_{1a} = \partial_a \phi_1,$$

and we can similarly replace the subscript 1 with 2 to obtain the remaining quantities of interest. With this notation, the evolution equations become

$$\begin{gathered} \partial_t \phi_1 = \frac{\alpha}{\sqrt{\gamma}} \pi_1 + \beta^j \psi_{1j}, \\ \partial_t \psi_{1a} = \partial_a \left(\frac{\alpha}{\gamma^{\frac{1}{2}}} \pi_1 + \beta^j \psi_{1j} \right), \\ \partial_t \pi_1 = \partial_j (\alpha \sqrt{\gamma} \phi_1^j) - \frac{1}{2} \alpha \sqrt{\gamma} \frac{\partial V}{\partial |\Phi|^2} \phi_1. \end{gathered} \quad (5)$$

Again, we can replace the subscript 1 with 2 to obtain the remaining quantities of interest. On the other hand, the geometry of the space–time is evolved using the BSSN formulation of the $3+1$ decomposition. According to this formulation, the variables to be evolved are

$$\Psi = \ln(\gamma_{ij}\gamma^{ij})/12, \quad \tilde{\gamma}_{ij} = e^{-4\Psi}\gamma_{ij}, \quad K = \gamma^{ij}K_{ij},$$
$$\tilde{A}_{ij} = e^{-4\Psi}(K_{ij} - \gamma_{ij}K/3),$$

and the contracted Christoffel symbols $\tilde{\Gamma}^i = \tilde{\gamma}^{jk}\Gamma^i_{jk}$, instead of the ADM variables γ_{ij} and K_{ij}. The equations for the BSSN variables are described in Refs. [10, 11]:

$$\partial_t \Psi = -\frac{1}{6}\alpha K, \tag{6}$$

$$\partial_t \tilde{\gamma}_{ij} = -2\alpha \tilde{A}_{ij}, \tag{7}$$

$$\partial_t K = -\gamma^{ij} D_i D_j \alpha + \\ + \alpha \left[\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + \frac{1}{2}\left(-T^t{}_t + T\right)\right], \tag{8}$$

$$\partial_t \tilde{A}_{ij} = e^{-4\Psi}\left[-D_i D_j \alpha + \alpha\left(R_{ij} - T_{ij}\right)\right]^{TF} + \\ + \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j\right), \tag{9}$$

$$\frac{\partial}{\partial t}\tilde{\Gamma}^i = -2\tilde{A}^{ij}\alpha_{,j} + \\ + 2\alpha\left(\tilde{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\tilde{\gamma}^{ij}K_{,j} - \tilde{\gamma}^{ij}T_{jt} + 6\tilde{A}^{ij}\phi_{,j}\right) - \\ - \frac{\partial}{\partial x^j}\left(\beta^l\tilde{\gamma}^{ij}{}_{,l} - 2\tilde{\gamma}^{m(j}\beta^{i)}{}_{,m} + \frac{2}{3}\tilde{\gamma}^{ij}\beta^l{}_{,l}\right), \tag{10}$$

where D_i is the covariant derivative on the spatial hypersurface, T is the trace of stress–energy tensor (3), and the label TF denotes the trace-free part of the quantity in brackets.

The above equations are true for any general potential V. We can of course write the corresponding equations for PNGB fields. All we need to do is specify the appropriate potential. In our case, the field is a real scalar field. The simplest potential for the physically motivated PNGB fields [4] can be written in the form

$$V = m^4\left[K - \cos\frac{\Phi}{f}\right]. \tag{11}$$

As discussed in Ref. [4], m is of the order of the neutrino mass and K is of the order of unity. We consider such a potential for studying the dynamics in the next section.

3. SOLUTION FOR THE DYNAMICAL EVOLUTION OF THE DARK ENERGY FIELDS

The evolution equations described above can be solved numerically to study the gravitational collapse of field configurations.

To obtain the numerical solutions discussed below, we have used the publicly available Einstein Toolkit [12]. The code uses the Method of Lines to do the time evolution. In particular, we have used the Iterative Crank Nicholson method to do the time evolution.

Guided by the evolution equations given in the preceding section, we define dimensionless quantities such that the field is measured in units of f and time and space are measured in units of f/m^2. The energy density is measured in units of m^4.

One issue we want to address is the time scale on which the collapse happens. Clearly, if this timescale for collapse is equal to or larger than the age of the Universe, then the collapse has no practical significance or implications. On the other hand, if the collapse happens on a timescale shorter than the age of the Universe, then we must consider the collapse of dark energy field configurations.

The initial field configuration and the field configuration at $t = 1.8$ in terms of these dimensionless variables are displayed in Fig. 1. In addition to the time evolution of the dark energy field, we also track the time evolution of the energy density as a function of time and space. The evolution of the energy density is shown in Fig. 2.

It can be clearly seen from the figures that the field configuration and the energy density collapse and the timescale for collapse can be seen by studying the figures. Since the units of time are given by f/m^2, we note that the collapse happens on timescales of the order of f/m^2. This timescale is shorter than the age of the Universe.

To convert into physical units, we note the following.

The scale f is the high-energy symmetry-breaking scale in PNGB models. In the see-saw model of neutrino masses [3], this corresponds to the heavy scale of symmetry breaking. While f has a range of possible values, the typical value of f in the see-saw model of neutrino masses is $f \sim 10^{13}$ GeV. The typical value of m is given by $m \sim 10^{-3}$ eV. It should also be noted that so far we have been working in the particle physics and cosmology units in which $\hbar = c = k = 1$. It is straightforward to convert from these units into more familiar units using standard conversion factors [7]. Thus, $1\ \text{GeV}^{-1} = 1.98\cdot10^{-14}$ cm and $1\ \text{GeV}^{-1} = 6.58\cdot10^{-25}$ s.

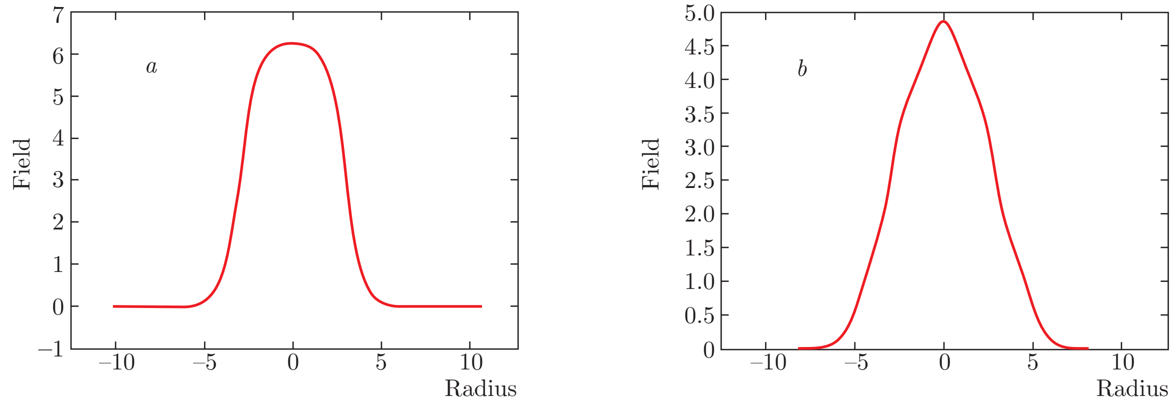

Fig. 1. Field configurations at (a) $t = 0$ and (b) $t = 1.8$

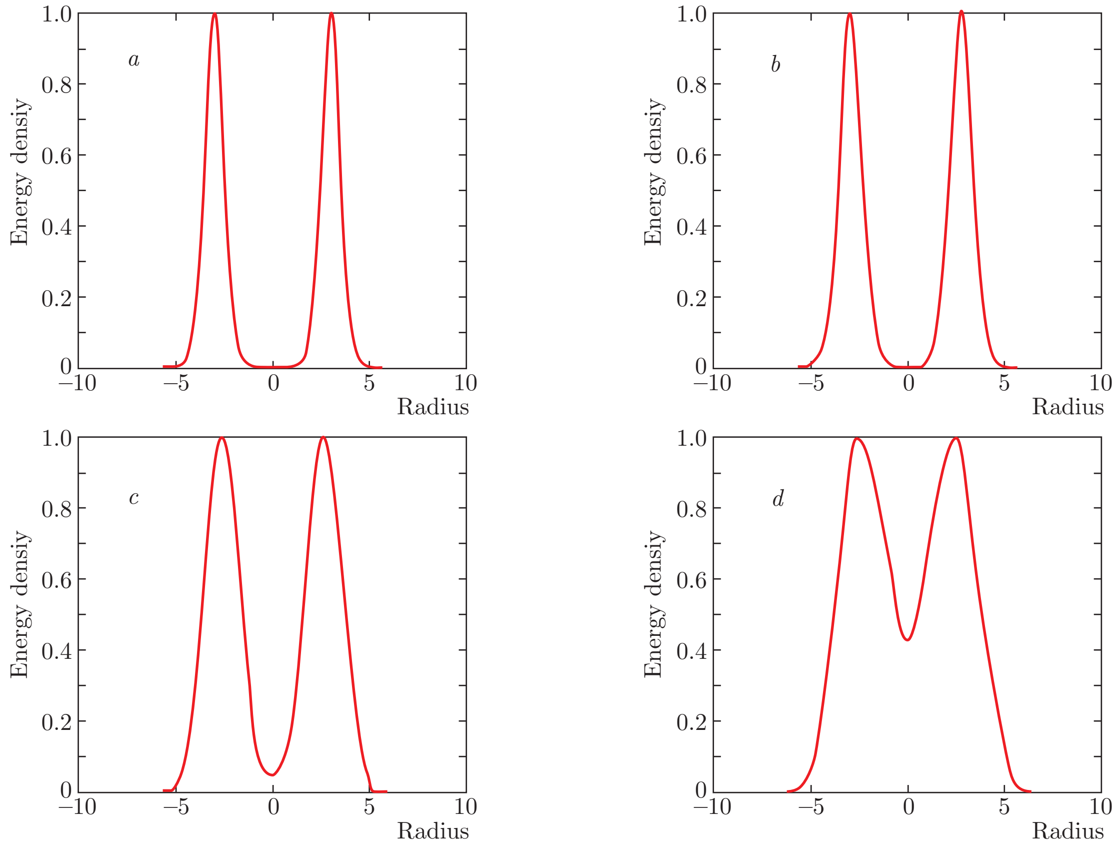

Fig. 2. Energy densities at (a) $t = 0$, (b) $t = 0.6$, (c) $t = 1.2$, and (d) $t = 1.8$

Using these conversion factors, we see that the fundamental time scale of the dynamics corresponding to f/m^2 is $2 \cdot 10^5$ years and that the fundamental spatial distance corresponding to f/m^2 is $50 \cdot 10^3$ pc or 50 kpc.

We were also able to demonstrate that the collapse of the dark energy field leads to black hole formation. For this, we used an additional component of the Einstein Toolkit [12, 13] which is also publicly available.

An apparent horizon satisfies the equation

$$H \equiv \nabla_i n^i + K_{ij} n^i n^j - K = 0, \qquad (12)$$

where n^i is the outward-pointing unit normal to the horizon and all the field variables are evaluated on the horizon surface.

As previously shown [13], Newton's method provides an excellent and efficient horizon-finding algorithm by numerically finding the roots of the above equation. By numerically executing the horizon-finding algorithm, we were able to demonstrate the formation of black holes resulting from the collapse of dark energy. Further, we computed the mass M_{BH} of the black hole formed as a result of the dark energy collapse, which in physical units turns out to be

$$M_{BH} = \frac{R_0^2}{(50\ \text{kpc})^2} \cdot 2.5 \cdot 10^6 M_{solar}, \qquad (13)$$

where R_0 is the physical size of the initial configuration that collapses.

4. SUMMARY AND CONCLUSIONS

It is now a well-established fact that the Universe is dominated by dark energy. The most promising candidate for dark energy is the energy density due to fields. Understanding the dynamics of these dark energy fields is of paramount importance. We described the formalism to study the gravitational collapse of fields given any general potential for the fields. We then applied this formalism to the simplest and most natural model of dark energy fields motivated by particle physics considerations. We discussed the results obtained by numerically solving the resulting evolution equations.

Finally, we wish to emphasize the important results discussed and reported in this article.

1) The dynamical length scale f/m^2 of the dark energy field was shown to be comparable to the galaxy length scale.

2) Dark energy fields collapse on a timescale of $2 \cdot 10^5$ years, which is much shorter than the age of the Universe and hence this dynamics cannot be neglected.

3) The collapse of dark energy fields results in black hole formation.

4) We computed the masses of these black holes and determined that these are supermassive black holes with masses comparable to the masses of the black holes at the centers of galaxies.

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