ЖЭТФ, 2015, том 147, вып. 3, стр. 438–443

EXPLORING VARIATIONS IN THE GAUGE SECTOR OF A SIX-DIMENSIONAL FLAVOUR MODEL

J.-M. Frère [a*], ***M. Libanov*** [b,c**], ***S. Mollet*** [a***], ***S. Troitsky*** [b****]

[a] *Service de Physique Théorique, Université Libre de Bruxelles*
1050, Brussels, Belgium

[b] *Institute for Nuclear Research of the Russian Academy of Sciences*
117312, Moscow, Russia

[c] *Moscow Institute of Physics and Technology*
141700, Dolgoprudny, Moscow Region, Russia

Received October 1, 2014

In the context of extra-dimensional models which describe three families of fermions, including their masses and mixings in terms of a single 6-dimensional family, we explore possible variations, including in the geometry of the extra dimensions, and argue that the apparent plethora of variants does not lead to drastic changes in the expected phenomenology.

Contribution for the JETP special issue in honor of V. A. Rubakov's 60th birthday

DOI: 10.7868/S0044451015030064

1. INTRODUCTION

The wonderful world of large and infinite extra dimensions (ED), where low-energy excitations of multidimensional fields ("zero modes") are bound to a (3+1)-dimensional manifold ("the brane") representing our world, was discovered for theoretical physicists in independent works of Rubakov and Shaposhnikov [1], Akama [2], and Visser [3] more than four decades ago. Since then, enlarged symmetries of multidimensional worlds have been exploited in field-theory frameworks to address various fine-tuning and hierarchy problems of the Standard Model (SM) of particle physics (see, e. g., reviews [4, 5] and references therein). One of the approaches transfers geometric symmetries of the ED into flavour symmetries of our world, explaining in an elegant way the hierarchy of masses and mixings of SM quarks and charged leptons [6–8] and leading to rich testable phenomenology [9–12]. The same model explains as well a very different pattern of neutrino masses and mixing, the difference with quarks being caused by the Majorana form of the neutrino mass term [13] (see Ref. [14] for a recent update). The purpose of the present work is to explore some ways beyond the simplest model and to sketch how robust its predictions are.

In ED models that hope to embed the SM, some vector fields must be introduced which will play the role of usual gauge fields at low energy. Their (almost) massless "zero" modes appear as the usual (3+1)-dimensional ($4D$) gauge bosons. The way of implementing a mechanism responsible for that is not always an easy task for there are further requirements to build a realistic model. Indeed, while we want the gauge zero mode to interact properly with the fermionic ones, we know that there will also exist a set of heavier (excited) modes which should not talk too much with this low energy sector, i. e., either there must exist a mass gap or these modes must only interact very weakly with the low-energy sector [15]. On the other hand, these new modes could manifest themselves at higher energy (in collider experiments for instance) or in (very) rare processes (e. g., flavour-changing neutral currents), thus providing hints for this kind of models.

In this note, we would like to provide with a short update of the constraints from these experiments for

* E-mail: frere@ulb.ac.be
** E-mail: ml@ms2.inr.ac.ru
*** E-mail: smollet@ulb.ac.be
**** E-mail: sergey.troitsky@gmail.com

various models of this kind. We will focus on a particular class in (4+2) dimensions where a Nielsen–Olesen vortex-like defect plays the role of our $4D$ world [6–8, 15, 16]. We know that, quite generally in this background, we can get several localized (chiral) fermion zero modes from a single spinor in $6D$ [17], each of them associated with a different winding in ED[1] $(e^{iw\varphi}, e^{i(w+1)\varphi}, e^{i(w+2)\varphi}, \ldots)$. They can acquire (small) masses through the vacuum expectation value (vev) of a Brout–Englert–Higgs (BEH) field H. In a certain range of parameters [12], the particular shape of this vev in ED (nonzero in the core, almost zero outside) leads to a hierarchical pattern of masses. This idea was exploited in different contexts to reproduce the three SM generations and their spectrum. Here however, we will only be interested in their interactions with gauge bosons (both zero and heavy modes).

In Sec. 2, we come back on some possible ways of introducing gauge bosons in the model and try to convince the reader that the expected phenomenology should not change drastically from one realization to the other. In particular, we will recall the existence of heavy localized modes whose mass scale is set by the geometry. Unlike the zero mode, the former possess nonzero windings and can therefore be responsible for flavour changing processes (even in the absence of mixing in the fermionic sector) [10, 11]. In Sec. 3, we comment on these processes and provide with some numerical results for the precise realization of [14]. Finally, we conclude in Sec. 4.

2. SOME GENERIC EXAMPLES

Let us here quickly remind some general results. We will focus on models with $4D$ Poincaré invariance and $4D$ flat space. The most general metrics of such a kind can be written as [18]

$$ds^2 = G_{AB}dx^A dx^B = \sigma(y)\eta_{\mu\nu}dx^\mu dx^\nu - \gamma_{ab}(y)dy^a dy^b. \quad (1)$$

With the following choice of gauge:

$$\partial_0 W_0 - \partial_i W_i = 0,$$
$$\frac{\partial_a\left(\sqrt{|G|}\sigma^{-1}\gamma^{ab}W_b\right)}{\sqrt{|G|}\sigma^{-1}} = 0,$$

we have the obvious separation of variables in the equation of motion for vector modes,

$$W_\mu(x,y) = \sum_n \omega_{\mu;n}(x)P_n(y),$$

with the modal wavefunctions P_n satisfying

$$\frac{\partial_a\left(\sqrt{|G|}\sigma^{-1}\gamma^{ab}\partial_b P\right)}{\sqrt{|G|}\sigma^{-1}} + \sigma^{-1}m^2 P = 0.$$

There always exists a zero mode ($m^2 = 0$) with a constant transverse wavefunction ($P(y) = \text{const}$), but we cannot conclude, at this level, if it is normalizable or not.

Two ways to ensure the normalizability are (i) to deal with compact ED whose finite volume renders the integral with the constant delocalized wavefunction bounded, or (ii) to make use of warp factors [19–22] which will sufficiently "dilute" the wavefunction, yet yield to a finite integral [23, 24]. Note that in the latter case, we can also consider effective wavefunctions in flat space which include warp factors and are thus localized from this point of view [18]. We will provide realizations of these two scenarios in the further simplified metrics, which is a particular case of (1):

$$ds^2 = \sigma(u)\eta_{\mu\nu}dx^\mu dx^\nu - du^2 - \gamma(u)dv^2.$$

A simple example of the first way (compact space) is the 2-sphere [8, 10, 11] of radius R which corresponds to $\sigma = 1$, $u = R\theta$, $v = R\varphi$, and $\gamma = \sin^2\theta$. The modal equation becomes then the equation for spherical harmonics with $R^2m^2 = \ell(\ell+1)$. As expected, we have a (normalizable) zero mode $\ell = 0$ with constant wavefunction $P = 1/\sqrt{4\pi}R$. Heavier modes appear to be normalizable, too. The mass scale is dictated by the size of ED. In particular, there is a mass gap of the order of $1/R$. For each value of ℓ, there are degenerate modes with windings $-\ell \le m \le \ell$. The wavefunctions oscillate on a scale of order of R for the lightest modes.

If we opt instead for the warped case, the warp metrics can be parametrized [15] as $u = r$, $v = a\theta$, $\sigma = e^{A(r)}$, and $\gamma = e^{B(r)}$. The precise behavior of the A and B functions are determined by the exact realization of the defect, but we can establish general features of their asymptotics by requiring (i) the metrics to be a regular solution of the $6D$ Einstein equations where a negative bulk cosmological constant balances a positive string tension (in the core)[2] and (ii) the gravity to be localized[3]. What we get is [15, 16] $A'(0) = 0$ and $B(r \to 0) \sim 2\ln(r/a)$ around the origin and

[1] The exact values of the windings are not important. What will really be relevant for us are the difference in windings between two modes.

[2] Note that at $4D$ level, we ask for a zero cosmological constant to have a flat space.

[3] i.e., ask for a normalizable zero mode for the graviton [25].

$A = B = -2rc$ outside the core (c is a dimensional constant related to the bulk cosmological constant) which correspond to an AdS_6 geometry. We still have the arbitrariness of normalization and choose $A(0) = 0$. The dimensionfull constant, which will play an important role later on, a is not a free parameter but is determined by an interplay between the gravity and the vortex scales. With these asymptotics it is easy to realize that the two ED are a warped plane in polar coordinates and it is then obvious to further develop the P wavefunctions on a Fourier basis:

$$P_n(r,\theta) = \sum_\ell \rho_{n\ell}(r) e^{i\ell\theta}.$$

With this, the equation for ρ becomes

$$\rho'' + \left(A' + \frac{B'}{2}\right)\rho' + \left(m^2 e^{-A} - \frac{\ell^2}{a^2} e^{-B}\right)\rho = 0.$$

Outside the core, the solutions are classified in terms of $\mu^2 = m^2 - \ell^2/a^2$. For $\mu = 0$, we have a constant solution, while for $\mu \neq 0$, it reads

$$\rho(r) = e^{3cr/2}\left[C_1 J_{3/2}\left(\frac{\mu}{c}e^{cr}\right) + C_2 Y_{3/2}\left(\frac{\mu}{c}e^{cr}\right)\right],$$

where J and Y are Bessel functions, and C_i are arbitrary constants. The boundary conditions (absence of the flux at infinity) lead to a continuous spectrum for $\mu > 0$ [26]. If we use the expression of J and Y in terms of elementary functions, it is easy to show that ρ behaves as ηe^{cr} at sufficiently large r, where η is some oscillating and bounded function. Now remember that, in the initial action, we have a factor $\sim \sqrt{|g|}(g^{00})^2 = a e^{B/2} \sim e^{-cr}$ for the kinetic term of $4D$ gauge component (and the integral over r fixes the normalization). As announced, we can define an effective wavefunction that takes this warp factor into account, then we can conclude if the associated mode is localized or not. With the definition $\zeta(r) = e^{-cr/2}\rho(r)$, we see that for the "constant" mode $\zeta_0(r) \sim e^{-cr/2}$ is localized[4], while the continuous spectrum $\zeta_c(r) \sim \eta e^{cr/2}$ is not. The "not localized" states have most of their weight at large distances (therefore reducing the overlap). Now near the origin, the regular solution is

$$\rho(r) \sim J_\ell(mr).$$

For $m = \ell/a$ (corresponding to localized mode $\mu = 0$ at infinity), we have (note that here, the metric factor is simply r)

$$\rho_0(r) \sim J_\ell\left(\ell\frac{r}{a}\right).$$

For $\ell = 0$, we get the usual constant solution (which matches with the constant solution at infinity, since we know that $\rho = \text{const}$ is an exact solution for the all range of r). For nonzero ℓ, we cannot get an exact solution, but we see that (at least for the first modes) we have oscillating functions with a scale of oscillation of order a.

In conclusion, we have a pattern which looks very much like the spherical case: discrete (localized) modes with mass scale $1/a$ and this same scale giving also an idea of the oscillation scale for the associated wavefunctions. On the other hand, there are (associated to each of these bounded modes) a continuum, starting just above, but the delocalization should kill the overlaps with localized profiles. Of course, this should be computed properly to be more quantitative.

3. FLAVOUR VIOLATING PROCESSES

Thanks to the separation of variables, the whole set of modal wavefunctions can be decomposed as a product of a radial part[5] and an angular one. For the fermion zero modes, the radial part is localized around the vortex[6], while for the bosonic modes these are oscillating functions spread in the bulk. In the compactification procedure (which reduces the complete $6D$ theory to an effective $4D$ one where all modes interact among themselves), the integration over the radial component controls the strength of the interaction through the overlaps of wavefunctions, while the one over angular component gives a selection rule which forbids interactions with non zero total winding (this can be interpreted as the angular momentum conservation in the ED).

If we neglect mixing between fermions, each family is associated with one and only one winding number i. Then the interaction

$$\kappa\bar\psi_i\gamma^\mu\omega_{\mu,m}\psi_{i'}$$

[4] Note that in the usual $5D$ Randall–Sundrum models, this zero mode is not normalizable because the e^B factor is not present. The presence of an extra warped dimension helps to "dilute" more efficiently the constant wavefunction.

[5] On the sphere the angle θ plays the role of the radial variable.

[6] Note nevertheless that the size of these functions must be larger than the size of the vortex in general if we want to produce a sufficiently strong hierarchy between families (see, e.g., [14]).

is allowed if and only if $m = i - i'$. The strength κ depends on the radial integral[7)]. Allowed effective four-fermion interactions,

$$\frac{\kappa\kappa'}{M_\omega^2}\left(\bar\psi_i O \psi_{i'}\right)\left(\bar\psi_j O' \psi_{j'}\right),$$

correspond to $(i' - i) = (j - j')$, or in other words $\Delta G = 0$, if G is some kind of family number. Thus, in first approximation (no mixing), only $\Delta G = 0$ interactions can be observed.

3.1. Forbidden kaon decays

The best experimental restriction on flavour violating processes with $\Delta G = 0$ comes from the decay $K_L^0 \to \mu^+ e^-$. In SM, this process is suppressed because it is forbidden at the tree level. In our model however, there is an excited gauge mode which can mediate this decay.

To be more precise, let us focus on the spherical compactification for which we have a specific realization [14]. There, we have presented a set of couplings which reproduce well the SM masses and mixings as well as satisfy all constraints for masses and mixings in the neutrino sector, giving some predictions for future experiments. This realization of the model has a fixed $R = (100\ \text{TeV})^{-1}$. Having all couplings fixed, we can perform quantitative calculations of all particular processes.

For any neutral gauge field W_A which interacts with the fermions, we get the following effective Lagrangian at $4D$ level (the scalar modes do not interact with SM fermions):

$$\mathcal{L}_{4D} \supset \sum_{\ell} \sum_{\substack{m,n \\ |n-m|\le\ell}} E_{mn}^{\ell,|n-m|} U_{mj}^* U_{nk} \times \times \left(\bar\psi_j \gamma^\mu Q \psi_k\right) \omega_{\mu;\ell,|n-m|}^{(*)}, \quad (2)$$

where $E_{mn}^{\ell,|n-m|}$ are the results of the overlaps (see [10] for details). For $\ell = 0$, we have $E_{nn}^{0,0} = 1$ (normalization) which permits to identify Q with SM charges. U is the unitary mixing matrix[8)]. If it disappears properly for $\ell = 0$, this is no more the case for higher ℓ's. Thus, in our model, it makes sense to talk about mixing in up quarks and down quarks separately, for instance. $\omega_\mu^{(*)}$ are the $4D$ fields for each modes. When $n - m \neq 0$, these are complex fields. In our notations, for $n - m > 0$, we have to use ω_μ, so it destroys a mode with winding $|n - m|$, while for $n - m < 0$ we have to use ω_μ^*, so it creates a mode with winding $|m - n|$.

K_L^0 is a combination of $\bar sd$ and $\bar ds$. The first one corresponds to indices $j = 2$ and $k = 1$ in (2). We can define matrices $\Omega_{mn}^\ell = U_{m2}^* U_{n1} E_{mn}^{\ell,|n-m|}$ which tell us about the strength of coupling with each mode $\omega_{\mu;\ell,0}$, $\omega_{\mu;\ell,1}$, and $\omega_{\mu;\ell,2}$. Note that mixings in left and right sectors are different in general. For the model of [14], we have

$$\Omega_L^\ell = \begin{pmatrix} 0.232E_{11}^{\ell,0} & -0.057E_{12}^{\ell,1} & 0.003E_{13}^{\ell,2} \\ 0.941E_{21}^{\ell,1} & -0.231E_{22}^{\ell,0} & 0.013E_{23}^{\ell,1} \\ -0.052E_{31}^{\ell,2} & 0.013E_{32}^{\ell,1} & -0.001E_{33}^{\ell,0} \end{pmatrix}$$

and

$$\Omega_R^\ell = \begin{pmatrix} 0.053E_{11}^{\ell,0} & -0.003E_{12}^{\ell,1} & 0 \\ 0.997E_{21}^{\ell,1} & -0.053E_{22}^{\ell,0} & 0 \\ -0.001E_{31}^{\ell,2} & 0 & 0 \end{pmatrix}.$$

For both matrices, the dominant elements are $E_{m,n}^{\ell,1}$ with $m = 2$ and $n = 1$. This means that the dominant process is the (virtual) creation of a $\omega_{\mu;\ell,1}$ (for all allowed ℓ). At first sight, it seems that the contribution to $\omega_{\mu;\ell,0}$ is significant too. But to be more precise, we have to evaluate the overlaps E^ℓ and sum over all contributions. In particular, the total contribution to $\omega_{\mu;\ell,0}$ is simply the trace (other can be obtained as sums over elements of lines parallel to the diagonal). It then is obvious (because of the unitarity of U) that this is negligible as long as $E_{11}^{\ell,0} \approx E_{22}^{\ell,0} (\approx E_{33}^{\ell,0})$. This result is exact for $\ell = 0$ by definition and is expected to be a good approximation for the first ℓ's which correspond to slowly oscillating modes (thus embracing all fermion wavefunctions in a very similar way). As an example, we compute the contributions of the first modes in Table (for left-handed quarks only).

We can perform the same procedure for the charged lepton sector, and our previous conclusions stay more or less valid. In particular, the fact that $\omega_{\mu;\ell,0}$ don't couple much with $\bar e\mu$ is expected to be robust, since it

[7)] In principle, κ could be infinitely reduced by localizing more and more the fermion wavefunctions (through stronger and stronger interactions with the vortex). However as mentioned above, we are technically limited because we require (high) hierarchies between generations. We could still hope to squeeze both fermion and H fields in such a way that the hierarchy is safe, but a detailed analysis (too technical to be put in here) of the scalar sector (in the spherical case only, up to now) showed that, once m_H is fixed, we do not have this freedom anymore. Nevertheless, it still is worth looking for smaller κ than imposed by the model, because we do not know what happens in a different geometry.

[8)] Note that U matrices are not unique. Indeed, we could as well use $U_L' = U_L \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ and $U_R' = U_R \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ (with the same phases) since it does not affect the masses, but these are obviously not physical.

Table. Overlaps between fermion pairs and first gauge modes for left down quarks. The rows $\omega_{\mu;\ell,0}$ and $\omega^*_{\mu;\ell,1}$ refer to the couplings (mixings taken into account) with these particular modes

ℓ	0	1	2	3	4	5
$E_{11}^{\ell,0}$	1	1.004	0.492	0.149	0.014	−0.020
$E_{22}^{\ell,0}$	1	1.073	0.496	0.027	−0.172	−0.206
$E_{33}^{\ell,0}$	1	1.419	1.268	0.923	0.603	0.374
$\omega_{\mu;\ell,0}$	0	−0.016	−0.017	0.027	0.042	0.043
$E_{12}^{\ell,1}$	–	0.780	0.872	0.621	0.359	0.186
$E_{23}^{\ell,1}$	–	0.638	0.908	0.844	0.640	0.440
$\omega^*_{\mu;\ell,1}$	–	0.742	0.832	0.595	0.346	0.181
$E_{13}^{\ell,2}$	–	–	0.051	0.027	0.018	0.013

depends mainly on the relative equality of all the $E_{nn}^{\ell,0}$ for low ℓ.

We now provide the results of exact numerical evaluation at the tree level for $\Gamma(K_L^0 \to \mu^+ e^-)$ with and without mixings taken into account. Recall about the chiral suppression of this decay (angular momentum conservation imposes cancellation of the amplitude for massless fermions). Thus, our result will be of the form $\Gamma \sim \beta m_\mu^2 m_K R^4 f_K^2$, where β is some dimensionless factor that accounts for the effective coupling constant which is of the order of $(g\kappa)^4$. For a SM coupling $g \sim 10^{-1}$ and an overlap $\kappa \sim 10^{-1} \div 1$ (see Table), we expect $\Gamma \sim R^4 10^{-10}$. This gives a bound on R, but we remind that R plays already a role in the size of the wavefunctions, so this is only a test *a posteriori* of the validity of our choice for this parameter[9]. We could be skeptical about this rough estimation for Γ because we have to sum over all heavy modes (all ℓ's), but remember that (in addition to overlaps reduction) we have a mass suppression $1/(\ell+1)^4\ell^4 \sim \ell^{-8}$ which makes the series rapidly converging. Indeed, with mixing we have $\Gamma \cdot 10^{10}/R^4 = 2.24, 3.78, 4.12, 4.18, 4.18$ for $\ell_{max} = 1$, 2, 3, 5, 10, respectively. Without mixing, we get, for the same ℓ_{max}, $\Gamma \cdot 10^{10}/R^4 = 3.31, 5.34, 5.72, 5.78$, 5.78. It gives the following limits on R:

$$\frac{1}{R} > 51(55)\ \text{TeV}, \tag{3}$$

with (without) mixing (for the experimental limit [27] on the branching ratio Br$< 4.7 \cdot 10^{-22}$), which is well below the value $R^{-1} = 100$ TeV assumed in this realization of the model. The model with parameters of Ref. [14] (the mass of the new family-changing vector boson there is ~ 142 TeV) is therefore self-consistent. To obtain a precise lower bound on R for all models, one needs to perform additional numerical work which is beyond the scope of the present note. Other rare processes may also be analyzed [28].

3.2. Collider processes

Let us briefly comment on the collider phenomenology. At the LHC, our massive bosons $\omega_{\mu;11}$ could mediate flavour violating processes if their scale is within the energy reach of the accelerator – which would assume an hypothetical geometry where $\kappa \approx 0.1$. The typical signature would be a pair, involving a lepton and an antilepton of different flavor with large and opposite transverse momenta. This is very similar to Drell–Yan pair production for which a typical feature is the suppression of the cross section with increasing of the resonance mass at a fixed center-of-mass energy. Note also that, since we are dealing here with proton-proton collisions, we expect a dominance of $(e^-\mu^+)$ and $(\mu^-\tau^+)$ over $(e^+\mu^-)$ and $(\mu^+\tau^-)$. Indeed, the former processes can use valence quarks (u and d) in the proton, while the latter involve only partons from the sea.

A detailed evaluation of the expected number of events at LHC requires numerical simulation to which we will return in a future note. At this point however, it is already possible to compute the width of the $\omega_{\mu;11}$ boson thanks to (2). Note that for these energies, it is more coherent to use b_μ and ω_μ^3 instead of z_μ and a_μ. If we neglect possible model-dependent scalar interactions, we have[10]

$$\Gamma(b_{\mu;11} \to \text{all}) = \frac{\sqrt{2}}{R}\frac{g'^2}{32\pi} \times$$
$$\times \left(y_e^2 A_e + 2y_L^2 A_L + y_u^2 A_u + y_d^2 A_d + 2y_Q^2 A_Q\right)$$

and

$$\Gamma(\omega^3_{\mu;11} \to \text{all}) = \frac{\sqrt{2}}{R}\frac{g^2}{64\pi}(A_L + A_Q)$$

for $A = (E_{12}^{1,1})^2 + (E_{23}^{1,1})^2$. According to [11], we expect $\Gamma/\text{GeV} \sim \kappa^2 M/M_Z$, thus $\Gamma \sim 10^{-1}$ TeV. The exact numerical values for our example are $\Gamma(b_\mu) = 0.44$ TeV and $\Gamma(\omega_\mu^3) = 0.67$ TeV.

[9] Nevertheless, if we consider free κ's, we can replace R by κR in (3).

[10] We also neglect masses of all fermions and therefore mixings are irrelevant.

4. CONCLUSIONS AND PERSPECTIVES

We have discussed the gauge sector of a successful extra-dimensional model for masses and mixing of quarks, charged leptons and neutrinos. It is important for quantitative experimental predictions of the model. Further details of the warped-geometry case will be discussed elsewhere.

We dedicate this paper to the birthday of Valery Rubakov who is not only an appreciated pioneer of large extra dimensions. He was a supervisor for two of us (M. L. and S. T.), but he is more than a teacher. He continuously sets a very high level in his studies and in the works of his school, but also in his everyday and social life. We are trying to use this level as a benchmark. Last but not least, it was Valery who initiated the first contact between J.-M. F., M. L. and S. T. in 1999, which resulted in the development of the branch discussed here.

This work is funded in part by IISN and by Belgian Science Policy (IAP "Fundamental Interactions"). The work of M. L. and S. T. (elaboration of the model of the origin and hierarchy of masses and mixings in the context of new experimental data) is supported by the Russian Science Foundation (grant 14-22-00161).

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