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# COHERENT COOLING OF ATOMS IN A FREQUENCY-MODULATED STANDING LASER WAVE: WAVE FUNCTION AND STOCHASTIC TRAJECTORY APPROACHES

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The wave function of a moderately cold atom in a stationary near-resonant standing light wave delocalizes very fast due to wave packet splitting. However, we show that frequency modulation of the field can suppress packet splitting for some atoms whose specific velocities are in a narrow range. These atoms remain localized in a small space for a long time. We demonstrate and explain this effect numerically and analytically. We also demonstrate that the modulated field can not only trap but also cool the atoms. We perform a numerical experiment with a large atomic ensemble having wide initial velocity and energy distributions. During the experiment, most of atoms leave the wave while the trapped atoms have a narrow energy distribution.

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## 1. INTRODUCTION

Laser cooling and trapping of atoms is a rapidly developing field of modern physics. Cold particles in a laser field are a common physical substrate used in numerous fundamental and applied issues such as Bose–Einstein condensates, quantum chaos, single-atom laser, quantum computer, etc.

In general, the idea of mechanical action of light on matter is rather old. As far as we know, it was first suggested by Kepler [1] in 1619 in order to explain a deviation of the comet tails nearby the Sun. In 1873, Maxwell first estimated the light pressure [2], and in 1899, Lebedev first measured it in experiment [3] with a macroscopic body. In the first half of the 20th century, analogous experiments with microscopic particles were carried out by Gerlach and Stern [4], by Kapitza and Dirac [5], and by Frisch [6].

The modern paradigm of mechanical manipulation of atomic motion by the laser began to emerge in the second half of the 20th century. The discovery of a gradient dipole force acting on neutral atoms in an intensive variable field by Gaponov, Miller, and Askaryan [7, 8] was a theoretical basis for further results. In 1968, Letokhov theoretically predicted the trapping of atoms in the nodes or antinodes of a standing wave [9]. Soon, in the 1970s, first experimental methods of laser acceleration [10, 11] and cooling (the Doppler cooling) [12–15] of atoms were proposed. The basic theory of dissipative atomic motion in a laser field was built by Kazantsev [16]. The theory considered the field in terms of the optical friction force acting on a moving atom. The friction force can be positive (atoms decelerate) or negative (atoms accelerate), and it nonlinearly depends on the atomic velocity.

In 1978, the Doppler cooling was first demonstrated in the experiment by Wineland and his collegaues [17]. In the 1980s, a series of other mechanical effects (predicted by early theoretical works) were also demonstrated experimentally: atomic monochromatization in the velocity space [18], collimation of an atomic beam [19], beam diffraction in a standing wave, beam reflection from a wave ("laser mirror"), and channeling of atoms [20]. New methods of atomic cooling in a laser field were proposed: the Sisyphus cooling [21] and the velocity selective coherent population trapping (VSCPT) [22]. Experimental realization of various cooling techniques in the 1980–1990s established a series of temperature records. While the early 1980s experiments provided the temperatures of the order of 0.1 K [23], in 1990s, Nobel laureates Chu, Cohen-Tannooji, and Phillips reached the temperatures of the order of 0.2 $\mu$K [24], and nowadays, sophisticated methods provide temperatures of the order of 0.2 nK [25].

In the 1990s, numerous new mechanical effects were

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discovered in the study of cold atoms in a nonstationary field with modulation and jumps. The groups of Raizen and Zoller [26–28] reported various effects related to dynamical chaos and the quantum–classical correspondence (having not only a pure physical but also methodological importance). In particular, they first experimentally demonstrated some manifestations of chaos in quantum systems and measured the difference between predictions of semiclassical models and real quantum behavior (in the study of so-called dynamical localization). In the framework of our study, it is important to note the special possibility of atomic localization in phase space: atoms with special values of the initial positions and momenta can be trapped in the resonance stability "islands" embedded in a chaotic "sea" [28] (in terms of the dynamical systems theory [29, 30]).

Although the basic theory of atomic motion in a laser field was formed in the 1970–1990s, it contained a number of approximations and considered a limited class of physical systems. In most of studies, atoms were treated either as plane waves in the coordinate space (approximation valid when the spatial extent of the atomic wave packet is substantially larger than the wavelength of the field) or as dot-like particles (approximation valid when the atomic velocity is sufficiently large). In recent years, the growth of computational power has provided tools for precise analysis of atomic motion beyond most of old approximations. Today, it is possible to model fully quantized atomic motion in terms of the wave function (atomic wave packets) or the density matrix. This helps study the regimes of small atomic momenta (of the order of the photon momentum), weak fields (of the order of few photons), small atom–field detunings (when intense Rabi oscillations occur and both resonant and nonresonant potentials [16] virtually coexist in a system), etc. In the quantum consideration, even comparatively simple systems (a standing wave or a two-level atom) demonstrate new effects (unknown in previous studies). For example, in [31], the splitting of traveling atomic wave packets on standing-wave nodes was discovered, and in [32], the anomalous atomic spatial concentration in the field (not fitting old semiclassical predictions) was demonstrated. In particular, it was shown that for some values of the field intensity, atoms can concentrate not only in the wave nodes or antinodes but also in intermediate positions. None of these effects could be demonstrated without precise quantum description of atomic motion (taking the mechanical photon recoil and finite atomic spatial and momentum uncertainity into account).

In our studies, we focus on the quantized atomic dynamics in the regime of small atom–field detuning. When an atom moves in a near-resonant standing light wave, two periodic optical potentials form in space [16]. When the atom crosses a standing wave node, it can undergo the Landau–Zener (LZ) transition between these two potentials. Such transitions cause splitting of atomic wave packets [31, 33] and rapid delocalization of the wave function [34]. However, under some additional conditions, manifestations of atomic localization also appear. In [35], we reported that in a stationary field, the interference between packet splitting products can break the symmetry of LZ transitions and cause localization of atoms in the momentum space. In this paper, we study a similar quantum system, but in a modulated field. We show that frequency modulation of the field can suppress the splitting of wave packets for atoms having velocities in a specific narrow range (determined by the field modulation parameters). These atoms stay trapped in the field for a long time (the effect of velocity-selective trapping of atoms). We provide additional simulations showing that in an experiment, this effect may significantly decrease the energy distribution of moderately cold atoms, and can therefore be used for coherent laser cooling.

In this paper, we pay much attention to methodological aspects of the study. The paper provides three different approaches to the analysis of atomic motion. First, we demonstrate the manifestations of the velocity-selective trapping numerically by solving quantum equations (describing the dynamics of atomic wave functions). Second, we explain the effect theoretically using semiclassical model (describing the dynamics of dot-like atoms with continuous trajectories). Third, we develop a stochastic-trajectory model (similar to the hybrid model used in [31], describing the dynamics of dot-like atoms with piecewise continuous trajectories accompanied by occasional quantum jumps) and use it in a numerical experiment demonstrating the cooling of large atomic ensemble. We also provide additional numerical experiments demonstrating the similarity of purely quantum and stochastic trajectory predictions.

## 2. EQUATIONS OF MOTION

We consider a two-level atom (with the transition frequency $\omega_a$ and mass $m_a$) moving in a strong standing laser wave with the modulated frequency $\omega_f[t]$. We assume that the depth of modulation is neglible in comparison with the average frequency value $\langle\omega_f[t]\rangle$ (but not with the detuning $\omega_f[t]-\omega_a$), and we can therefore

consider the corresponding wave vector $k_f$ a constant. In the absence of spontaneous emission (the atomic excited state must have a long lifetime, or some experimental methods must be used to suppress the decoherence), the atomic motion can be described by the Hamiltonian

$$\hat{H} = \frac{\hat{P}^2}{2m_a} + \frac{1}{2}\hbar(\omega_a - \omega_f[t])\hat{\sigma}_z - \\ - \hbar\Omega\,(\hat{\sigma}_- + \hat{\sigma}_+)\cos(k_f\hat{X}), \quad (1)$$

where $\hat{\sigma}_{\pm,z}$ are the operators of transitions between the atomic excited and ground states (the Pauli matrices), $\hat{X}$ and $\hat{P}$ are the operators of the atomic coordinate and momentum, and $\Omega$ is the Rabi frequency. This Hamiltonian was used in [33–35], albeit for a constant field without modulation.

We use the following dimensionless normalized quantities: the momentum $p \equiv P/\hbar k_f$, the time $\tau \equiv \equiv \Omega t$, the position $x \equiv k_f X$, the mass $m \equiv m_a\Omega/\hbar k_f^2$, and the detuning $\Delta[\tau] \equiv (\omega_f[\tau] - \omega_a)/\Omega$. We suppose that the field modulation is harmonic,

$$\Delta[\tau] = \Delta_0 + \Delta_1\cos[\zeta\tau + \phi], \quad (2)$$

and apply the following conditions: $\zeta \ll 1$, $\Delta_0 \lesssim \Delta_1 \ll \ll 1$. Using these approximations, we obtain the equations for the respective probability amplitudes to find an atom with a normalized momentum $p$ in the excited or ground state, $a[p,\tau]$ and $b[p,\tau]$:

$$i\dot{a}[p,\tau] = \left(\frac{p^2}{2m} - \frac{\Delta[\tau]}{2}\right)a[p] - \frac{1}{2}(b[p-1] + b[p+1]), \\ i\dot{b}[p,\tau] = \left(\frac{p^2}{2m} + \frac{\Delta[\tau]}{2}\right)b[p] - \frac{1}{2}(a[p-1] + a[p+1]). \quad (3)$$

Here, the dot denotes differentiation with respect to $\tau$. For each value of $p$, there is its own pair (3).

## 3. WAVEFUNCTION APPROACH: NUMERICAL MANIFESTATIONS OF VELOCITY-SELECTIVE TRAPPING

We choose the values of the parameters and initial conditions in order to perform a numerical simulation. The average initial atomic momentum $\langle p[0]\rangle$ is a variable condition for the purpose of this paper. All other conditions are fixed: the normalized mass $m = 10^5$ (by the order of magnitude, this corresponds to the experiments with Cs [36] and Rb [37] atoms, but for a stronger field $\Omega \sim 10^9$–$10^{10}$ Hz), the field parameters $\Delta_0 = -0.02$, $\Delta_1 = 0.047$, $\zeta = 0.00508$, and $\phi = 0$, and the initial form of the wave packet

$$a[p,0] = b[p,0] = \frac{1}{\sqrt{2\sigma_p[0]\sqrt{2\pi}}} \times \\ \times \exp\frac{-(p - \langle p[0]\rangle)^2}{4\sigma_p^2[0]}. \quad (4)$$

Therefore, the initial wave packet has the Gaussian form with $\langle x[0]\rangle = 0$ and the initial probability to find the atom in the excited state 0.5. Here, $\sigma_p$ is the standard deviation of the atomic momentum (equal to the half-width of the packet by an order of magnitude). At $\tau = 0$, we fix it by the value $\sigma_p[0] = 5\sqrt{2}$. Therefore, in accordance with the Heisenberg relation, the standard deviation of the initial coordinate is

$$\sigma_x[0] = \frac{1}{2\sigma_p[0]} = \frac{0.1}{\sqrt{2}}$$

(it is much less than the normalized optical wavelength $2\pi$).

In numerical experiments, we use these initial conditions to simulate the system of 8000 equations (3) with $-1000 \le p \le 1000$. For larger values of $|p|$, we set $a[p,\tau] = b[p,\tau] = 0$ due to the energy restrictions. Obtaining the solution in the momentum space, we perform the Fourier transform and obtain the wave function in the coordinate space in the range $-4\pi < x \le 4\pi$ (see Fig. 2).

We first study the effect of field modulation on atomic delocalization. In [34], we studied the atomic motion in the absence of modulation. The following basic modes of motion were reported.

At $\Delta = 0$ and $|\Delta| \gtrsim 1$, the atomic motion is simple. Atoms move in constant spatially periodic potentials. Slow atoms are trapped in potential wells and fast atoms move ballistically through the wave.

At $0 < |\Delta| \ll 1$, the atomic motion is more complex. The slowest atoms ($|\langle p[0]\rangle| < \sqrt{2m}$) are trapped in potential wells. Faster atoms ($\sqrt{2m} \le |\langle p[0]\rangle| < < 2\sqrt{m}$) perform a kind of random walk. Their wave packets split each time they cross standing-wave nodes (the effect shown in [31] and described in detail in [33]), and this causes fast delocalization of the wave functions. The fastest atoms ($|\langle p[0]\rangle| > 2\sqrt{m}$) move ballistically through the wave. Their wave packets split, but all products move in the same direction, and hence the overall delocalization is slow.

In Fig. 1, we calculate the variance of the atomic position $\sigma_x^2$ after a relatively long time span of coherent evolution $\tau = 5000$ as a function of the initial atomic momentum $\langle p[0]\rangle$. For a constant field (solid curve), this function shows fast delocalization of all atoms in the range $\sqrt{2m} \approx 440 \lesssim \langle p[0]\rangle \lesssim 2\sqrt{m} \approx 640$ (cold atoms with velocities of the order of 1 m/s). A local

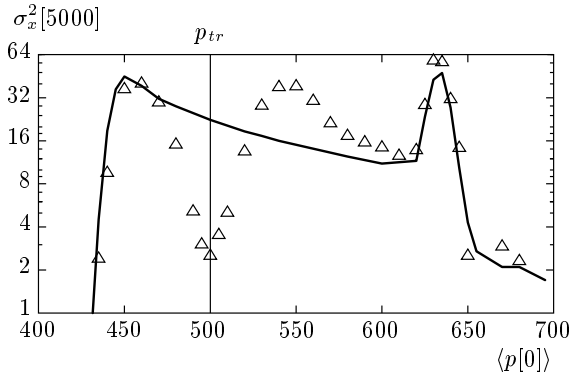


**Fig. 1.** The variance of atomic position $\sigma_x^2$ at $\tau = 5000$ as a function of the initial atomic momentum $\langle p[0]\rangle$. Curve, constant field $\Delta = -0.02$; triangles, modulated field $\Delta = -0.02 + 0.047\cos[0.00508\tau]$

peak at $\langle p[0]\rangle \approx 630$ is produced by moderately fast atoms having an uncertain scenario of either random walking or flying ballistically.

We now "switch on" the field modulation and see the changes. In Fig. 1, the analogous function of $\sigma_x^2$ is shown with triangles. This function has a more complex structure. In particular, it has a prominent additional minimum at $\langle p[0]\rangle = p_{tr} \approx 500$. These atoms are not trapped in potential wells in a strict sense (their energy is too high; see the theory in the next sections), but some mechanism significantly suppresses the delocalization of their wave functions (we note that both functions are shown in a logarithmic scale).

We consider the evolution of the corresponding wave packets in the coordinate space. In Fig. 2, we show the evolution of wave functions with $\langle p[0]\rangle = 600$ and 500 in a modulated field (the other parameters are the same as in Fig. 1). In both cases, the wave packets split. The first splitting occurs near the first node, $x \approx 1.57$ (the products overlap at $\tau = 400$, but become completely independent at $\tau = 800$). However, the proportion of splitting radically differs for $\langle p[0]\rangle = 600$ and 500. In Fig. 2*a*, fission products have similar "weights", while in Fig. 2*b*, they are radically different: a single large packet regularly oscillates in the range of $-2 \lesssim x \lesssim 2$, "emitting" very small packets in both directions.

We conclude that field modulation produces the velocity-selective trapping of atoms. It suppresses the splitting of wave packets of some atoms, and these atoms are almost completely trapped in the range $-2 \lesssim x \lesssim 2$ (the variance of their position $x$ is even smaller; see Fig. 1). This suppression is significant only for atoms having special initial momenta in a narrow range (in our case, $490 \lesssim \langle p[0]\rangle \lesssim 510$).

## 4. SEMICLASSICAL APPROACH: EXPLANATION OF THE EFFECT AND ESTIMATION OF THE TRAPPING CONDITIONS

In the preceding section, we used quantum equations to simulate atomic dynamics. In this section, in order to explain the effect of velocity selective trapping, we mention some semiclassical analytic results from [33, 34] (obtained for the stationary field).

In a stationary field with $|\Delta| \ll 1$, the atomic motion can be described in terms of two potentials

$$U^- = -\sqrt{\cos^2 x + \frac{\Delta^2}{4}}, \quad U^+ = \sqrt{\cos^2 x + \frac{\Delta^2}{4}} \qquad (5)$$

(Fig. 3*a*, dotted lines). An atom moves in one of these potentials when it is far from the standing wave nodes ($x = \pm 1.57, \pm 4.7, \ldots$). When the atom crosses a node, the potential can change sign (the atom undergoes Landau–Zener tunneling between potentials $U^{\pm}$) with the probability

$$W_{LZ} \approx \exp\frac{-\Delta^2 m\pi}{4\langle p_{node}\rangle}, \qquad (6)$$

where $\langle p_{node}\rangle$ is the average momentum of the atom when it crosses the node. At $0 < |\Delta| \ll 1$, the tunneling causes the wave packet splitting (observed in numerical experiments). At $\Delta = 0$, the potentials coincide at the nodes, and hence the tunneling probability is equal to 1 and wave packets do not split. The corresponding potential takes the simplest form $U = \pm\cos x$ (Fig. 3*a*, solid line).

What happens if we "switch on" the field modulation? When an atom moves far from the nodes, nothing radically changes. It moves in a constant potential that does not depend much on the value of $\Delta$. Far from the nodes, we can neglect the term $\Delta^2/4$ in (5) and set $U \approx \pm\cos x$ with good accuracy.

There are two possible scenarios when an atom crosses the node (at time $\tau$): 1) $\Delta[\tau] \neq 0$, then the packet splits significantly; and 2) $\Delta[\tau] \approx 0$, then the splitting is suppressed.

The first scenario is typical if the modulation is not synchronized with the atomic mechanical motion (because $\Delta[\tau] \neq 0$ most of the time). The second scenario can occur sometimes, but does not change the overall statistics of the atomic motion. The evolution of the wave function shown in Fig. 2*a* is typical for moderately small detunings $|\Delta| \sim 0.01$ (both for the stationary and modulated field).

The evolution radically changes if the field modulation is synchronized with the atomic mechanical mo-

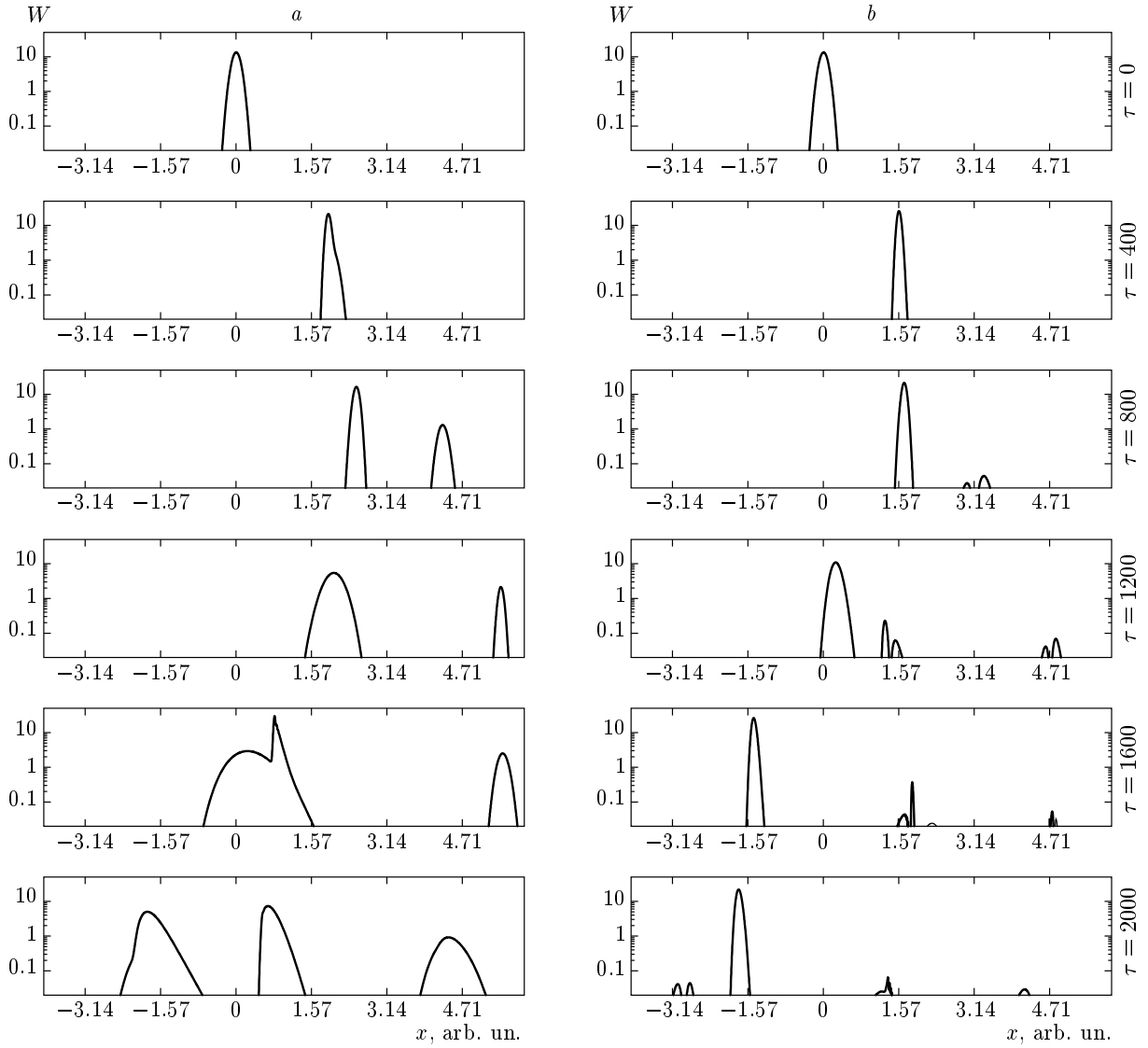


**Fig. 2.** Atomic wave packet splitting during quantum evolution (in the coordinate space): *a*) fast delocalization of a typical wave function ($\langle p[0]\rangle = 600$), *b*) slow delocalization of the wave function in the velocity-selective trapping mode ($\langle p[0]\rangle = 500$). $W[x]$ is the probability density to find an atom at a coordinate $x$

tion. In particular, it is possible to choose modulation parameters and the atomic momentum (see analytic estimates below) such that $\Delta[\tau]$ takes zero values each time an atom crosses the node. With our parameters, such synchronization occurs at $\langle p[0]\rangle = p_{tr} \approx 500$. Therefore, packet splitting is suppressed (Fig. 2*b*). We note that the suppression is not complete; slight splittings still exist. They are caused by the Landau–Zener transitions that occur not exactly at a standing wave node, but in its small vicinity (when $\Delta[\tau]$ is small but not equal to zero).

We obtain an analytic relation between the trapping momentum $p_{tr}$ and field parameters. When an atom moves between the nodes, its center-of-mass motion can be described by the semiclassical equations of motion [35]

$$\dot{x} = \frac{p}{m}, \quad \dot{p} = -\mathrm{grad}[U], \tag{7}$$

with the energy

$$E \equiv \frac{p^2}{2m} + U[x,\tau] \tag{8}$$

being an integral of motion. If the initial energy $E[0] \lesssim 0$ (for $x[0] = 0$, this corresponds to $|p[0]| \lesssim \sqrt{2m}$), then an atom cannot reach any standing-wave node.

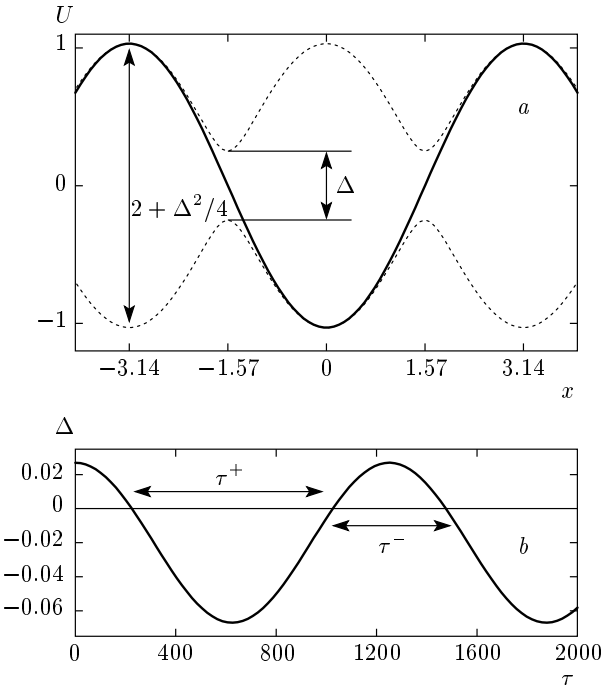


**Fig. 3.** *a*) Periodic potentials in space: dotted line, nonresonant potentials $U^{\pm}$; solid line, the resonant potential $-\cos x$. *b*) Illustration of the trapping condition: the modulation of detuning $\Delta[\tau]$ must be synchronized with atomic mechanical motion ($\Delta = 0$ each time a trapped wave packet crosses the standing wave node)

It is trapped in the bottom of the first potential well near $x = 0$ (Fig. 3*a*). If the initial energy is in the range of $0 \lesssim E \lesssim 1$ (for $x[0] = 0$, this corresponds to $\sqrt{2m} \lesssim |p[0]| \lesssim 2\sqrt{m}$), then an atom can either perform a random walk or be trapped (if $p[0] = p_{tr}$). Faster atoms with $E \gtrsim 1$ move ballistically through the wave in a constant direction.

For trapped atoms, equations (7) remain valid during the entire evolution (even during node crossings), and take a simpler form. Trapping occurs if an atom either does not cross nodes at all or node crossings occur when $\Delta[\tau] = 0$. Therefore, the term $\Delta^2/4$ in (5) is always negligible, and the trapped atom moves in the constant effective potential $U \approx -\cos x$ (we choose the negative sign of $U$ because atoms with the initial position $x[0] = 0$ start their motion from the potential well in this paper). Therefore, the atomic center-of-mass motion can be described by the simple equations

$$\dot{x} = \frac{p}{m}, \quad \dot{p} = -\sin x, \tag{9}$$

with the simplified energy

$$\tilde{E} \equiv \frac{p^2}{2m} - \cos x \tag{10}$$

being an integral of motion during entire evolution.

We calculate the atomic traveling time between two successive node crossings $\tau^{\mp}$ in the negative and positive segments of the potential $U = -\cos x$ in the regime of velocity-selective trapping (it can be either the traveling time from one node to another or the return time to the same node). We integrate (9) using the condition $0 < \tilde{E} < 1$:

$$\begin{aligned} \tau^- &= 2k\sqrt{m}, \quad k \equiv \sqrt{\frac{2}{1+\tilde{E}}}, \\ \tau^+ &= 2k\sqrt{m}\left(F\left[\frac{\pi - |\arccos\tilde{E}|}{2}, k\right] - 1\right), \end{aligned} \tag{11}$$

where $F$ is the elliptic integral of the first kind.

In order to trap atoms, the field modulation must be synchronized with the atomic mechanical motion. The time intervals $\tau^{\mp}$ must be equal to the time intervals between successive zeros of $\Delta[\tau]$ (Fig. 3*b*). Hence, using (2), we obtain the trapping condition

$$\zeta = \frac{2\pi}{\tau^- + \tau^+}, \quad \frac{\Delta_0}{\Delta_1} = -\cos\frac{\pi\tau^-}{\tau^- + \tau^+}, \tag{12}$$

where $\tau^{\mp}$ is given by (11). These formulas are valid for atoms with any initial positions (not only $x[0] = 0$ used in (4)). At any given value of the initial atom energy in the range $0 < E[0] < 1$ (and an appropriate initial momentum), the velocity-selective trapping of atoms can be achieved with the appropriate values of $\Delta_{0,1}$ and $\zeta$. For example, in order to observe trapping at $\langle p[0]\rangle = 500$, $x[0] = 0$ ($E[0] = 0.25$), the field must have the parameters $\zeta = 0.00508$ and $\Delta_0/\Delta_1 = -0.4248$. We use them in numerical experiments, additionally setting $\Delta_0 = -0.02$.

## 5. STOCHASTIC TRAJECTORY APPROACH: MODELING THE ATOMIC COOLING PROCESS

In preceding sections, we analyzed velocity selective trapping of atoms with a semiclassical analytic treatment and quantum numerical simulation of wave functions. In this section, we use a third approach: numerical simulation of stochastic atomic trajectories.

In order to show that the reported effect is not only trapping of atoms but also their cooling, we must simulate the dynamics of an atomic ensemble having a wide initial velocity (and energy) distribution and

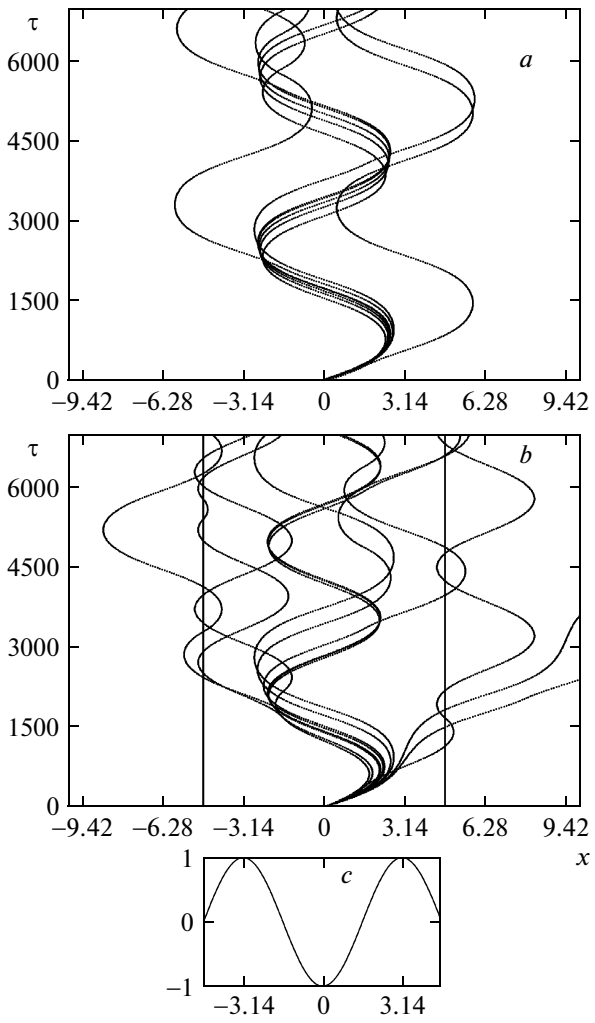


**Fig. 4.** *a*) Typical stochastic trajectories of the atomic ensemble with a narrow initial momentum distribution (of the size of wave packet (4)) with $\langle p[0]\rangle = 600$. *b*) Typical stochastic trajectories of the atomic ensemble with a wide initial momentum distribution (shown in Fig. 6*a*) with $\langle p[0]\rangle = 550$. *c*) Working part of a laser wave in a cooling experiment (when an atom leaves this area, it is excluded from statistics)

show that the distribution narrows during the evolution. Such simulation with quantum equations requires a huge computational time. Therefore, we develop an alternative simplified model of atomic motion based on the following principles.

1. An atom is a dot-like particle having a particular trajectory.

2. Between standing-wave nodes, an atom moves in the effective potential $\mp U$, Eq. (5), with a constant-sign but oscillating factor $\Delta(\tau)$. Such motion is governed by semiclassical equations (7).

3. At the initial time moment, the potential $\mp U$ has negative sign. Any time when an atom crosses a node, the potential changes its sign with probability (6).

In Fig. 4*a*,*b*, typical atomic stochastic trajectories are shown for narrow and wide initial momentum distributions. Most of them illustrate atomic random walk. However, in Fig. 4*b*, there are also two ballistic and two trapped trajectories.

In Fig. 5, we check the correctness of the stochastic trajectory model. We compare the evolution of atomic wave functions (computed with quantum equations) and the evolution of stochastic atomic ensembles (computed with the stochastic trajectory model) for $\langle p[0]\rangle = 600$ and $\langle p[0]\rangle = 500$. These ensembles of dot-like atoms have narrow Gaussian initial momentum and position distributions analogous to those used in quantum model (4) (typical stochastic trajectories for $\langle p[0]\rangle = 600$ are shown in Fig. 4*a*). In Fig. 5, both methods demonstrate similar probability functions to find an atom at a given position at $\tau = 2000$ and 3000.

In Fig. 6, we simulate the dynamics of an atomic ensemble (several thousand atoms) with a comparatively wide initial momentum distribution moving in the positive direction with the average velocity $\langle p[0]\rangle = 550$. This distribution is shown in Fig. 6*a*. The corresponding energy distribution is shown in Fig. 6*b* (we calculate simplified energy $\tilde{E}$, Eq. (10), but it is equal to the general energy $E$, Eq. (8), at the initial time moment).

To show that velocity selective trapping actually cools atoms, we consider a small part of a laser wave in the range

$$-\frac{3}{2}\pi < 0 < \frac{3}{2}\pi \qquad (13)$$

(Fig. 4*c*). At the beginning of the experiment, all atoms have $x \approx 0$. During the evolution, the trapped atoms ($p[0] \lesssim 440$, $E[0] \lesssim 0$, and $p[0] \approx p_{tr} = 500$, $E[0] \approx 0.25$) stay in range (13), while most of other atoms leave it (due to ballistic flight or random walk). Trapped atoms have a wide momentum distribution because their momena oscillate in a wide range. However, their energy distributuon is very narrow. In Figs. 6*c*,*d*, there is a prominent peak near $\tilde{E} = 0.25$, and it is very narrow in comparison with the initial energy distribution. This is because the majority of atoms with other initial values of energy leaved the wave. We note that the simplified energy $\tilde{E}$ is conserved only for trapped atoms. Other atoms can change it during the evolution (see, e. g., a spontaneous peak at $\tilde{E} \approx -0.6$, Fig. 6*c*). However, the number of such atoms in area (13) rapidly decays, and hence they do not change the overall picture.

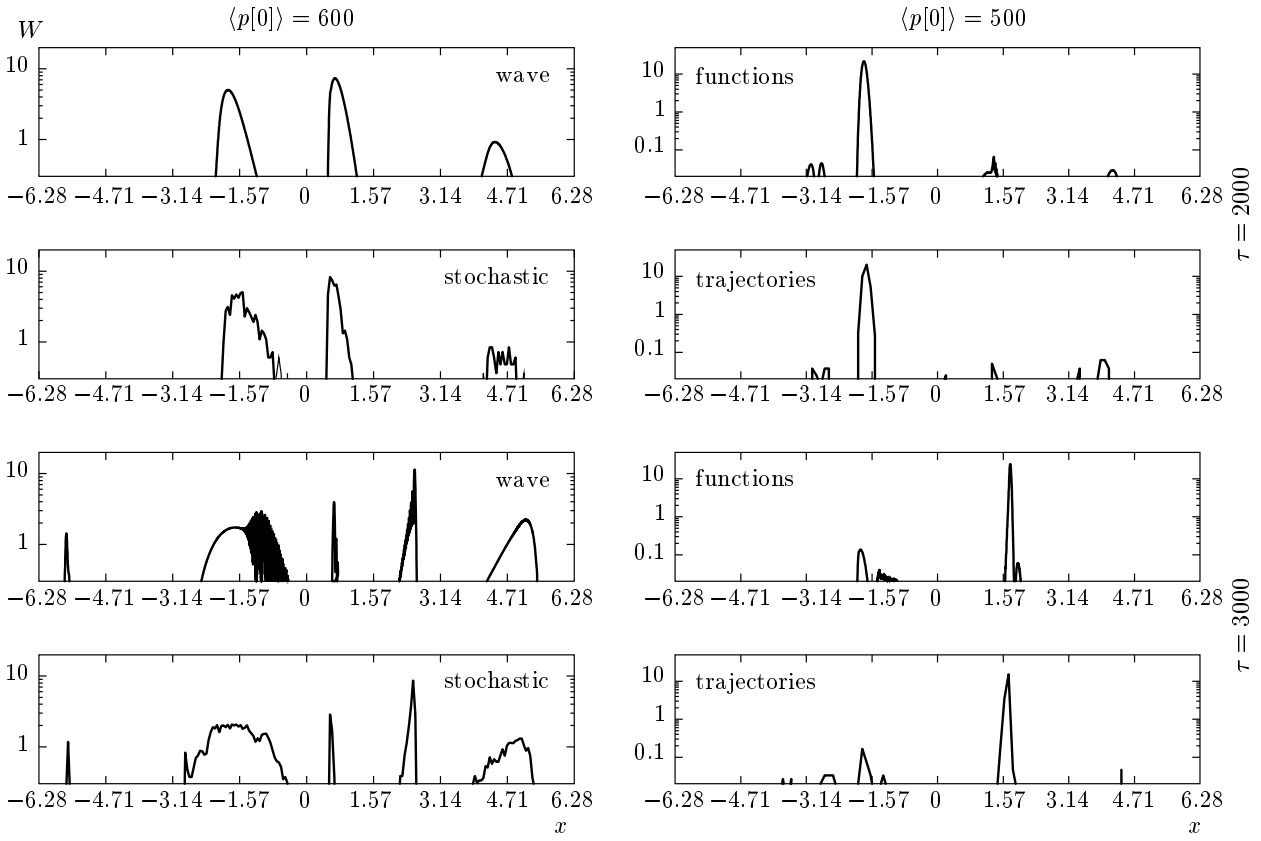


**Fig. 5.** Atomic wave functions (computed with quantum equations) and the corresponding stochastic atomic ensembles (computed with the stochastic trajectory model) for $\langle p[0]\rangle = 600$ and $\langle p[0]\rangle = 500$ at $\tau = 2000$ and $\tau = 3000$

## 6. CONCLUSIONS

In this paper, we report the effect of velocity-selective trapping and cooling of atoms in a frequency-modulated standing laser wave. Intensive coherent light produces significant mechanical action on cold atoms having velocities of the order of 1 m/s. There is a wide range of field parameters at which an atom performs a kind of random walk accompanied with wave packets splitting and fast delocalization of the wave function. In this paper, we report a specific field modulation mode that suppresses the wave packet splitting for atoms with precisely selected velocities. These atoms oscillate in a small space of the order of wavelength, and their wave functions are almost completely localized.

This effect cannot cool atoms in the sense of achieving zero velocity, but it can decrease their mechanical energy distribution (see Fig. 6). If a cloud of moderately cold atoms in a modulated wave has wide initial momentum end energy distributions, then most of these atoms leave the wave, while a small fraction is trapped. The trapped atoms have a narrow energy distribution. The ideology of our cooling method is similar to phase-space trapping of atoms in stable islands [28]. These islands are produced by nonlinear resonances [29, 30]. In our study, the resonance between field modulation and atomic mechanical oscillations plays similar stabilizing role. In both situations, only a small fraction of atoms is trapped due to special initial contidions. However, there is a significant difference between atomic trapping in a phase space far from the atom–field resonance and atomic trapping in a coordinate space near the atom–field resonance. In our study, there are two optical potentials and LZ tunnelings between them. The presence of LZ tunnelings is crucial in our cooling method, and its physical basis differs significantly from the effects shown in [28].

In this paper, the effect has been studied by three approaches: semiclassical analytic treatment, quantum numerical modeling, and stochastic trajectory modeling. All these approaches show similar results. Therefore, the effect of velocity selective trapping of atoms is not just an artefact of some particular method

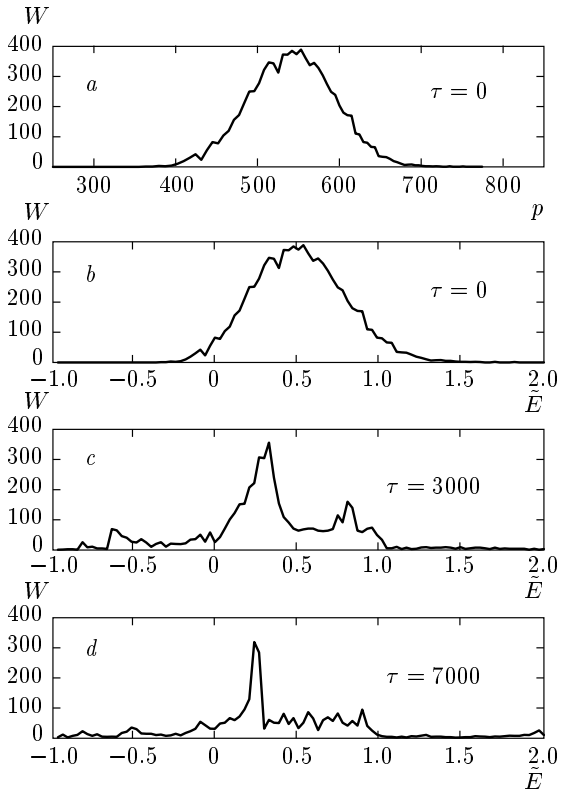


**Fig. 6.** Cooling of an atomic cloud due to velocity-selective trapping (statistics of atoms having positions in the range $-3\pi/2 < x < 3\pi/2$). The probability density $W$ to find an atom with a given momentum or energy is shown in arbitrary units

but a real possibility. The only significant drawback is that it occurs in the absence of dissipation. However, we believe that this is just a quantitative technical limitation that may be overcome by an appropriate choice of atoms and hi-Q cavities.

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