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BIANCHI TYPE-I COSMOLOGY IN $f(R,T)$ GRAVITY

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We investigate the exact solutions of a Bianchi type-I space-time in the context of $f(R,T)$ gravity [1], where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace of the energy–momentum tensor T. For this purpose, we find two exact solutions using the assumption of a constant deceleration parameter and the variation law of the Hubble parameter. The obtained solutions correspond to two different models of the Universe. The physical behavior of these models is also discussed.

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1. INTRODUCTION

The most popular issue in the modern-day cosmology is the current expansion of the Universe. It is now evident from observational and theoretical facts that our universe is in the phase of accelerated expansion [2–10]. The phenomenon of dark energy and dark matter is another topic of discussion [11–18]. It was Einstein who first proposed the concept of dark energy and introduced a small positive cosmological constant. But after some time, he referred to it as the biggest mistake in his life. However, it is now believed that the cosmological constant may be a suitable candidate for dark energy. Another proposal to justify the current expansion of the Universe comes from modified or alternative theories of gravity. The $f(T)$ theory of gravity is one such example that has been recently developed. This theory is a generalized version of teleparallel gravity in which the Weitzenböck connection is used instead of the Levi-Civita connection. The interesting feature of the theory is that it may explain the current acceleration without involving dark energy. A considerable amount of work has been done in this theory so far [19]. Another interesting modified theory is the $f(R)$ theory of gravity involving a general function of the Ricci scalar in the standard Einstein–Hilbert Lagrangian. Some review articles [20] can be helpful in understanding the theory.

Many authors have investigated $f(R)$ gravity in different contexts [21–34]. Spherically symmetric solutions are most commonly studied solutions due to their closeness to Nature. Vacuum and perfect fluid solutions of a spherically symmetric spacetime in the metric version of this theory were explored in [35]. They used the assumption of a constant scalar curvature and found that the solutions corresponded to the already existing solutions in general relativity (GR). Noether symmetries have been used in [36] to study spherically symmetric solutions in $f(R)$ gravity. Similarly, many interesting results have been found using spherical symmetry in $f(R)$ gravity [37]. Cylindrically symmetric vacuum and nonvacuum solutions have also been explored in this theory [38]. Plane symmetric solutions were found in [39]. The same authors [40] discussed the solutions of Bianchi type-I and V cosmologies for vacuum and nonvacuum cases. Conserved quantities in $f(R)$ gravity via the Noether symmetry approach were recently calculated in [41].

In a recent paper [1], a new generalized theory known as $f(R,T)$ gravity was proposed. In this theory, gravitational Lagrangian involves an arbitrary function of the scalar curvature R and the trace of the energy–momentum tensor T. In [42], $f(R,T)$ gravity was discussed with explicitly presented point-like Lagrangians. The laws of thermodynamics in this theory were studied in [43]. The same authors [44] investigated holographic and agegraphic $f(R,T)$ models. In [45], $f(R,T)$ gravity was reconstructed by taking

$$f(R,T) = f_1(R) + f_2(T),$$

and it was proved that $f(R,T)$ gravity allows transi-

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tion from matter-dominated phase to an acceleration phase. Thus, it is hoped that $f(R,T)$ gravity may explain the recent phase of cosmic acceleration of our Universe. This theory can be used to explore many issues and may provide some satisfactory results.

The isotropic models are considered to be most suitable to study the large-scale structure of the Universe. However, it is believed that the early Universe may not have been exactly uniform. This prediction motivates us to describe the early stages of the Universe with the models having an anisotropic background. Thus, the existence of anisotropy in early phases of the Universe is an interesting phenomenon to investigate. A Bianchi type-I cosmological model, being a generalization of the flat Friedmann–Robertson–Walker (FRW) model, is one of the simplest models of the anisotropic Universe. Therefore, it seems interesting to explore Bianchi-type models in the context of $f(R,T)$ gravity. Exact solutions of the $f(R,T)$ field equations for a locally rotationally symmetric Bianchi type-I spacetime were investigated in [46]. Solutions of a Bianchi type-III spacetime were explored in [47] using the law of variation of Hubble's parameter. Bianchi type-III dark energy model in the presence of a perfect fluid source has been reported [48]. Bianchi type-V cosmology in this theory was studied in [49] by involving the cosmological constant in the field equations. Solutions of the Bianchi type-V bulk viscous string cosmological model, were given in [50].

In this paper, we focus on investigating the exact solutions of a Bianchi type-I spacetime in the framework of $f(R,T)$ gravity. The plan of the paper is as follows. In Sec. 2, we give some basics of $f(R,T)$ gravity. Section 3 provides the exact solutions for a Bianchi type-I spacetime. Concluding remarks are given in the last section.

2. SOME BASICS OF $f(R,T)$ GRAVITY

The $f(R,T)$ theory of gravity is a generalization or modification of GR. The action for this theory is given by [1]

$$S=\int\sqrt{-g}\left(\frac{1}{16\pi G}f(R,T)+L_m\right)d^4x, \qquad (1)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy–momentum tensor $T_{\mu\nu}$, and L_m is the usual matter Lagrangian. It is worth mentioning that if we replace $f(R,T)$ with $f(R)$, we obtain the action for $f(R)$ gravity, and the replacement of $f(R,T)$ with R leads to the GR action. The energy–momentum tensor $T_{\mu\nu}$ is defined as [51]

$$T_{\mu\nu}=-\frac{2}{\sqrt{-g}}\frac{\delta\left(\sqrt{-g}L_m\right)}{\delta g^{\mu\nu}}. \qquad (2)$$

We assume that the dependence of the matter Lagrangian is merely on the metric tensor $g_{\mu\nu}$ rather than on its derivatives. In this case, we obtain

$$T_{\mu\nu}=L_m g_{\mu\nu}-2\frac{\delta L_m}{\delta g^{\mu\nu}}. \qquad (3)$$

The $f(R,T)$ gravity field equations are obtained by varying the action S in Eq. (1) with respect to the metric tensor $g_{\mu\nu}$:

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f(R,T)g_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)\times \times f_R(R,T)=\kappa T_{\mu\nu}-f_T(R,T)(T_{\mu\nu}+\Theta_{\mu\nu}), \qquad (4)$$

where ∇_μ denotes the covariant derivative and

$$\Box\equiv\nabla^\mu\nabla_\mu, \quad f_R(R,T)=\frac{\partial f_R(R,T)}{\partial R},$$

$$f_T(R,T)=\frac{\partial f_R(R,T)}{\partial T}, \quad \Theta_{\mu\nu}=g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{\mu\nu}}.$$

Contraction of (4) yields

$$f_R(R,T)R+3\Box f_R(R,T)-2f(R,T)= =\kappa T-f_T(R,T)(T+\Theta), \qquad (5)$$

where

$$\Theta=\Theta_\mu{}^\mu.$$

This is an important equation because it provides a relation between the Ricci scalar R and the trace T of the energy–momentum tensor. Using the matter Lagrangian L_m, the standard matter energy–momentum tensor is derived as

$$T_{\mu\nu}=(\rho+p)u_\mu u_\nu-pg_{\mu\nu}, \qquad (6)$$

where

$$u_\mu=\sqrt{g_{00}}(1,0,0,0)$$

is the four-velocity in comoving coordinates and ρ and p respectively denote the energy density and pressure of the fluid. Perfect-fluid problems involving energy density and pressure are not easy tasks. Moreover, there does not exist any unique definition for the matter Lagrangian. We can assume the matter Lagrangian $L_m=-p$, which gives

$$\Theta_{\mu\nu}=-pg_{\mu\nu}-2T_{\mu\nu}, \qquad (7)$$

and consequently field equations (4) take the form

$$f_R(R,T)R_{\mu\nu}-\frac{1}{2}f(R,T)g_{\mu\nu}-(\nabla_\mu\nabla_\nu-g_{\mu\nu}\Box)\times \times f_R(R,T)=\kappa T_{\mu\nu}+f_T(R,T)(T_{\mu\nu}+pg_{\mu\nu}). \qquad (8)$$

We note that these field equations depend on the physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, three classes of these models were given in [1]:

$$f(R,T) = \begin{cases} R + 2f(T), \\ f_1(R) + f_2(T), \\ f_1(R) + f_2(R)f_3(T). \end{cases}$$

In this paper, we focuse on the first class, i. e.,

$$f(R,T) = R + 2f(T).$$

For this model, the field equations become

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} + 2f'(T)T_{\mu\nu} + + \left[f(T) + 2pf'(T)\right]g_{\mu\nu}, \quad (9)$$

where the prime represents the derivative with respect to T.

3. EXACT SOLUTIONS OF THE BIANCHI TYPE-I UNIVERSE

In this section, we find exact solutions of a Bianchi-I spacetime in $f(R,T)$ gravity. For simplicity, we use the natural system of units ($G = c = 1$) and $f(T) = \lambda T$, where λ is an arbitrary constant. For a Bianchi type-I spacetime, the line element is given by

$$ds^2 = dt^2 - A^2(t)\,dx^2 - B^2(t)\,dy^2 - C^2(t)\,dz^2, \quad (10)$$

where A, B, and C are defined as cosmic scale factors. The Bianchi-I Ricci scalar turns out to be

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right], \quad (11)$$

where the dot denotes the derivative with respect to t.

Using Eq. (9), we obtain four independent field equations,

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} = (8\pi + 3\lambda)\rho - \lambda p, \quad (12)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = \lambda\rho - (8\pi + 3\lambda)p, \quad (13)$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{C}\dot{A}}{AC} = \lambda\rho - (8\pi + 3\lambda)p, \quad (14)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \lambda\rho - (8\pi + 3\lambda)p. \quad (15)$$

These are four nonlinear differential equations with five unknowns A, B, C, ρ, and p. Subtracting Eq. (14) from Eq. (13), Eq. (15) from Eq. (14), and Eq. (15) from Eq. (12) yields

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \quad (16)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \quad (17)$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = 0. \quad (18)$$

These equations imply that

$$\frac{B}{A} = d_1 \exp\left[c_1 \int \frac{dt}{a^3}\right], \quad (19)$$

$$\frac{C}{B} = d_2 \exp\left[c_2 \int \frac{dt}{a^3}\right], \quad (20)$$

$$\frac{A}{C} = d_3 \exp\left[c_3 \int \frac{dt}{a^3}\right], \quad (21)$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are integration constants that satisfy the relations

$$c_1 + c_2 + c_3 = 0, \quad d_1d_2d_3 = 1. \quad (22)$$

Using Eqs. (19)–(21), we can write the unknown metric functions in the explicit form

$$A = ap_1 \exp\left[q_1 \int \frac{dt}{a^3}\right], \quad (23)$$

$$B = ap_2 \exp\left[q_2 \int \frac{dt}{a^3}\right], \quad (24)$$

$$C = ap_3 \exp\left[q_3 \int \frac{dt}{a^3}\right], \quad (25)$$

where

$$p_1 = ({d_1}^{-2}{d_2}^{-1})^{1/3}, \quad p_2 = (d_1{d_2}^{-1})^{1/3}, \quad p_3 = (d_1{d_2}^{2})^{1/3} \quad (26)$$

and

$$q_1 = -\frac{2c_1 + c_2}{3}, \quad q_2 = \frac{c_1 - c_2}{3}, \quad q_3 = \frac{c_1 + 2c_2}{3}. \quad (27)$$

We note that p_1, p_2, p_3 and q_1, q_2, q_3 also satisfy the relation

$$p_1p_2p_3 = 1, \quad q_1 + q_2 + q_3 = 0. \quad (28)$$

3.1. Some important physical parameters

We now present some important definitions of physical parameters. The average scale factor a and the volume scale factor V are defined as

$$a = \sqrt[3]{ABC}, \quad V = a^3 = ABC. \quad (29)$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{30}$$

where

$$H_1 = \frac{\dot{A}}{A}, \quad H_2 = \frac{\dot{B}}{B}, \quad H_3 = \frac{\dot{C}}{C}$$

are defined as the directional Hubble parameters in the directions of x, y, and z axes. The mean anisotropy parameter A is

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{31}$$

The expansion scalar θ and the shear scalar σ^2 are defined as

$$\theta = u^{\mu}_{;\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{32}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right], \tag{33}$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}h^{\alpha}_{\nu} + u_{\nu;\alpha}h^{\alpha}_{\mu}) - \frac{1}{3}\theta h_{\mu\nu} \tag{34}$$

with

$$h_{\mu\nu} = g_{\mu\nu} - u_{\mu}u_{\nu}$$

defined as the projection tensor. The deceleration parameter q is the measure of the cosmic accelerated expansion of the Universe. It is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{35}$$

The behavior of the Universe models is determined by the sign of q. The positive value of the deceleration parameter suggests a decelerating model, while the negative value indicates inflation. Because there are four equations (12)–(15) and five unknowns, we need an additional constraint to solve them. Here, we use a well-known relation [52] between the average scale factor a and the Hubble parameter H to solve the equations,

$$H = la^{-n}, \tag{36}$$

where l and n are positive constants. This is an important relation because it yields a constant value of the deceleration parameter and we consequently obtain power-law and exponential models of the Universe. Using Eqs. (30) and (36), we obtain

$$\dot{a} = la^{1-n} \tag{37}$$

and the deceleration parameter becomes

$$q = n - 1. \tag{38}$$

Integrating Eq. (37) yields

$$a = (nlt + k_1)^{1/n}, \quad n \neq 0, \tag{39}$$

and

$$a = k_2 \exp(lt), \quad n = 0, \tag{40}$$

where k_1 and k_2 are integration constants.

3.2. Singular model of the Universe

Here, we investigate the model of the Universe when $n \neq 0$, i. e.,

$$a = (nlt + k_1)^{1/n}.$$

In this case, the metric coefficients A, B, and C take the form

$$A = p_1(nlt + k_1)^{1/n} \exp\left[\frac{q_1(nlt + k_1)^{(n-3)/n}}{l(n-3)}\right], \quad n \neq 3, \tag{41}$$

$$B = p_2(nlt + k_1)^{1/n} \exp\left[\frac{q_2(nlt + k_1)^{(n-3)/n}}{l(n-3)}\right], \quad n \neq 3, \tag{42}$$

$$C = p_3(nlt + k_1)^{1/n} \exp\left[\frac{q_3(nlt + k_1)^{(n-3)/n}}{l(n-3)}\right], \quad n \neq 3. \tag{43}$$

The directional Hubble parameters H_i $(i = 1, 2, 3)$ turn out to be

$$H_i = \frac{l}{nlt + k_1} + \frac{q_i}{(nlt + k_1)^{3/n}}. \tag{44}$$

The mean generalized Hubble parameter and the volume scale factor are

$$H = \frac{l}{nlt + k_1}, \quad V = (nlt + k_1)^{3/n}. \tag{45}$$

The mean anisotropy parameter becomes

$$A = \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{3l^2(nlt + k_1)^{(6-2n)/n}}. \tag{46}$$

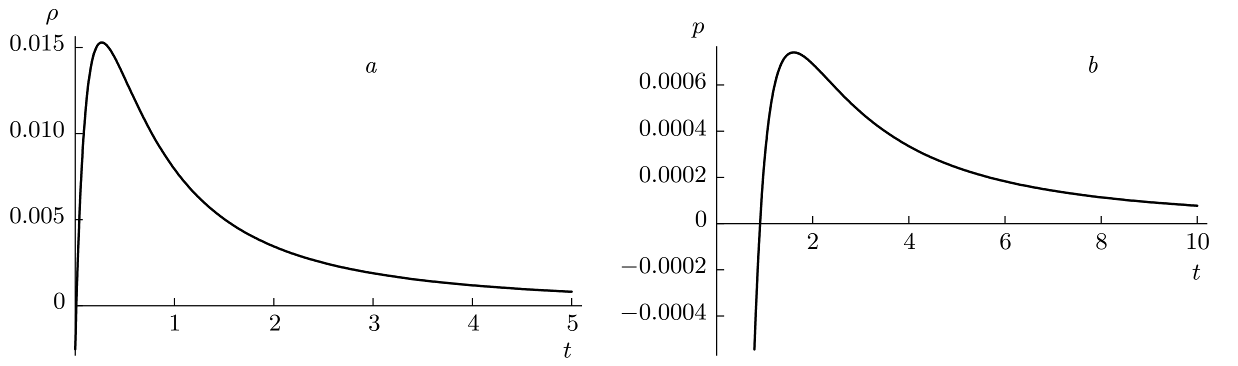

Fig. 1. Behavior of the energy density (a) and pressure (b) versus time for $t > 0$ with $n = 2$, $\lambda = 1$, $l = 1$, $k_1 = 0$, $q_1 = 1 = q_2$, and $q_3 = -2$

The expansion scalar and shear scalar for this model are given by

$$\theta = \frac{3l}{nlt + k_1}, \quad \sigma^2 = \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{2(nlt + k_1)^{6/n}}. \tag{47}$$

With Eqs. (12)–(15), the energy density of the Universe turns out to be

$$\rho = \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \times \\ \times \left[4(\lambda + 3\pi)\left\{\frac{3l^2}{(nlt + k_1)^2} + \frac{q_1q_2 + q_2q_3 + q_3q_1}{(nlt + k_1)^{6/n}}\right\} - \right. \\ \left. - \lambda\left\{\frac{3l^2(1-n)}{(nlt + k_1)^2} + \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{(nlt + k_1)^{6/n}}\right\}\right] \tag{48}$$

while the pressure of the Universe becomes

$$p = \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \times \\ \times \left[4\pi\left\{\frac{3l^2}{(nlt + k_1)^2} + \frac{q_1q_2 + q_2q_3 + q_3q_1}{(nlt + k_1)^{6/n}}\right\} + \right. \\ \left. + (3\lambda + 8\pi)\left\{\frac{3l^2(1-n)}{(nlt + k_1)^2} + \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{(nlt + k_1)^{6/n}}\right\}\right]. \tag{49}$$

The plots of ρ, p, and the equation-of-state parameter $w = p/\rho$ as functions of the time coordinate t are shown in Figs. 1 and 2. It is evident from Fig. 2 that $w \to 1/3$ as $t \to \infty$. Thus, the model corresponds to a radiation-dominated Universe as the time increases.

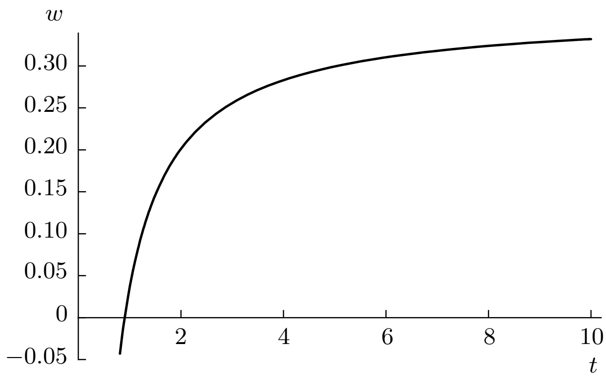

Fig. 2. Behavior of w versus time for $t > 0$ with $n = 2$, $\lambda = 1$, $l = 1$, $k_1 = 0$, $q_1 = 1 = q_2$, and $q_3 = -2$

3.3. Nonsingular model of the Universe

For this model, $n = 0$ and the average scale factor

$$a = k_2 \exp(lt)$$

turns the metric coefficients A, B, and C into

$$A = p_1 k_2 \exp(lt) \exp\left[-\frac{q_1 \exp(-3lt)}{3l{k_2}^3}\right], \tag{50}$$

$$B = p_2 k_2 \exp(lt) \exp\left[-\frac{q_2 \exp(-3lt)}{3l{k_2}^3}\right], \tag{51}$$

$$C = p_3 k_2 \exp(lt) \exp\left[-\frac{q_3 \exp(-3lt)}{3l{k_2}^3}\right]. \tag{52}$$

The directional Hubble parameters H_i become

$$H_i = l + \frac{q_i}{{k_2}^3}\exp(-3lt). \tag{53}$$

The mean generalized Hubble parameter and the volume scale factor turn out to be

$$H = l, \quad V = {k_2}^3 \exp(3lt). \tag{54}$$

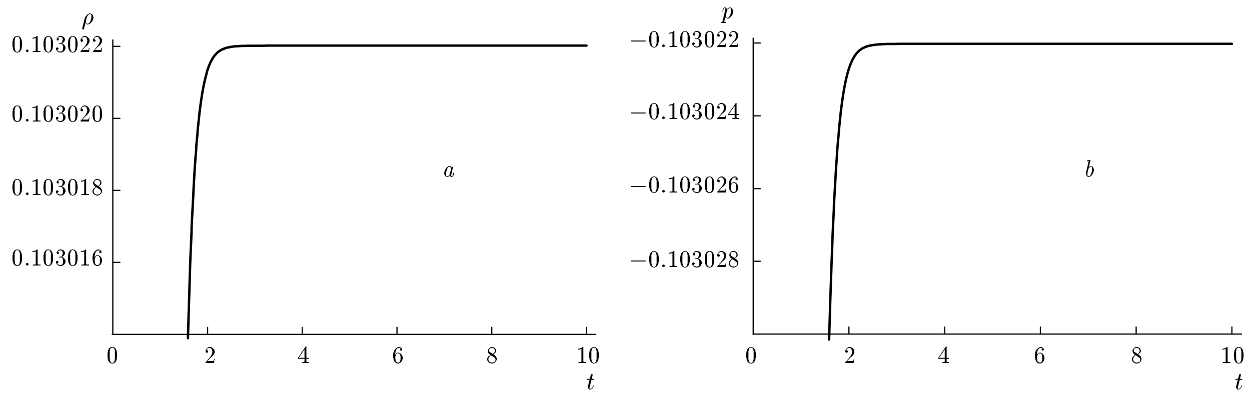

Fig. 3. Behavior of the energy density (a) and pressure (b) versus time for $t > 0$ with $n = 2$, $\lambda = 1$, $l = 1$, $k_2 = 1$, $q_1 = 1 = q_2$, and $q_3 = -2$

The mean anisotropy parameter, the expansion scalar, and the shear scalar are

$$A = \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{3l^2 {k_2}^6 \exp(6lt)}, \quad \theta = 3l,$$
$$\sigma^2 = \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{2{k_2}^6 \exp(6lt)}. \tag{55}$$

The energy density and pressure of the Universe take the form

$$\rho = \frac{1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \times \left[4(\lambda + 3\pi)\left\{3l^2 + \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{{k_2}^6 \exp(6lt)}\right\} - \lambda\left\{3l^2 + \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{{k_2}^6 \exp(6lt)}\right\}\right], \tag{56}$$

$$p = \frac{-1}{12(\lambda + 2\pi)(\lambda + 4\pi)} \times \left[4\pi\left\{3l^2 + \frac{q_1 q_2 + q_2 q_3 + q_3 q_1}{{k_2}^6 \exp(6lt)}\right\} + (3\lambda + 8\pi)\left\{3l^2 + \frac{{q_1}^2 + {q_2}^2 + {q_3}^2}{{k_2}^6 \exp(6lt)}\right\}\right]. \tag{57}$$

For this model, the plots of ρ, P, and w as functions of the time coordinate t are shown in Figs. 3 and 4. It can be seen from Fig. 4 that $w \to -1$ as $t \to \infty$, which indicates that the nonsingular model corresponds to a vacuum fluid-dominated Universe.

4. CONCLUDING REMARKS

This paper is devoted to a discussion of the current phenomenon of accelerated expansion of the Universe in the framework of the recently proposed $f(R,T)$ theory of gravity. For this purpose, we take

$$f(R,T) = R + 2\lambda T$$

and explore the exact solutions of the Bianchi type-I cosmological model. We obtain two exact solutions using the assumption of a constant value of the deceleration parameter and the law of variation of the Hubble parameter. The obtained solutions correspond to two different models of the Universe. The first solution forms a singular model with a power-law expansion, while the second solution gives a nonsingular model with exponential expansion of Universe. The physical parameters for both of these models are discussed below.

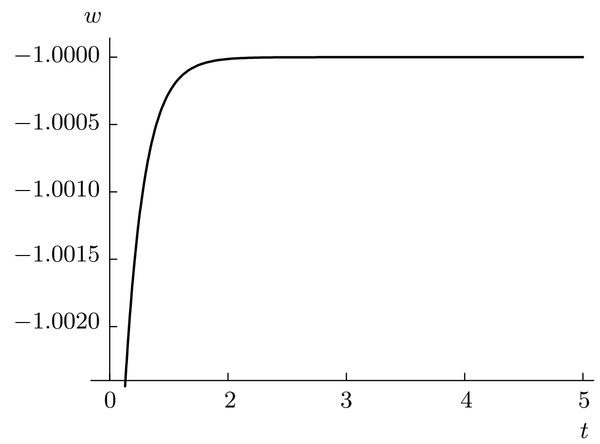

Fig. 4. Behavior of w versus time for $t > 0$ with $n = 2$, $\lambda = 1$, $l = 1$, $k_2 = 1$, $q_1 = 1 = q_2$, and $q_3 = -2$

The singular model of the universe corresponds to $n \neq 0$ with the average scale factor

$$a = (nlt + k_1)^{1/n}.$$

This model has a point singularity when

$$t \equiv t_s = -\frac{k_1}{nl}.$$

The volume scale factor vanishes and the metric coefficients A, B, and C vanish at this singularity point. The cosmological parameters H_1, H_2, H_3, H, θ, and σ^2 are all infinite at this point of singularity. If we choose $k_1 = 0$, Fig. 1 suggests that the energy density of the Universe is zero at this time. The pressure approaches negative infinity as $t \to 0$. This strong negative pressure is an indication of dark energy. For this model, $w \to 1/3$ as $t \to \infty$, which corresponds to a radiation-dominated Universe. The mean anisotropy parameter A also becomes infinite at this point for $0 < n < 3$ and vanishes for $n > 3$. Moreover, the isotropy condition $\sigma^2/\theta \to 0$ as $t \to \infty$ is verified for this model. All these conclusive observations suggest that the Universe starts its expansion with zero volume, strong negative pressure and energy density from $t = t_s$, and it will continue to expand for $0 < n < 3$.

We now discuss the nonsingular model of the Universe corresponding to $n = 0$. For this model, the average scale factor is $a = k_2 \exp(lt)$. The model is nonsingular due to its exponential behavior. The expansion scalar θ and the mean generalized Hubble parameter H are constant in this case. For finite values of t, the physical parameters H_1, H_2, H_3, σ^2, and A are all finite. The metric functions are defined for finite times and the isotropy condition is satisfied. There is an exponential increase in the volume as the time increases. However, the energy density is approximately zero initially and becomes constant after some time. Pressure of the Universe remains in the negative zone for this model, which may be an indication of the presence of dark energy in the Universe. Figure 4 suggests that $w \to -1$ as $t \to \infty$. Hence, the exponential model corresponds to a vacuum fluid-dominated Universe. According to the observations in [53], the expansion of the Universe is accelerating when $w \approx -1$.

Therefore, we can hope that the problematic issues such as dark energy and accelerated expansion of the Universe may be addressed using modified theories of gravity, especially the $f(R,T)$ gravity. It would be interesting to explore more Bianchi-type solutions in this context. Exact solutions of a Bianchi type-V cosmological model in this theory are in progress.

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