ЖЭТФ, 2013, том 143, вып. 3, стр. 476–485

DIRAC TENSOR WITH HEAVY PHOTON

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Received January 26, 2012

For the large-angle hard-photon emission by initial leptons in the process of high-energy annihilation of e^+e^- to hadrons, the Dirac tensor is obtained by taking the lowest-order radiative corrections into account. The case of large-angle emission of two hard photons by initial leptons is considered. In the final result, the kinematic case of collinear hard-photons emission and soft virtual and real photons is included; it can be used for the construction of Monte-Carlo generators.

DOI: 10.7868/S0044451013030073

1. INTRODUCTION

In the experiments with e^+e^- annihilation to hadrons, the important role is played by the so-called "returning to resonance" mechanism. It consists in the emission of a hard real photon by initial leptons [1].

The Born contribution and the one-loop correction are taken into account in the Dirac tensor (the cross-symmetry partner of the Compton tensor — a bilinear combination of the hard photon emission currents averaged over lepton spin states and summed over photon polarization states). Infrared divergences are parameterized by introducing the "photon mass" λ. In the final expression, it is removed in a usual way by adding the contribution from additional soft photon emission.

We do not consider photon emission by the final charged particles and the effects of charge-odd interference of virtual or real photon emission from leptons and hadrons. Therefore, the Dirac tensor obtained in this way is universal.

The paper is organized as follow. In Sec. 2, the relation of the Dirac tensor to the cross section of radiative annihilation of a lepton pair to hadrons is clarified. We give the Born-level expression for the Dirac tensor and derive the general form of the radiative correction to it using the symmetry relation. In Sec. 3, we obtain the contribution arising from the mass operator of the positron and vertex function in the case where a positron and a photon are on the mass shell. In Sec. 4, we consider the contribution from the vertex function to the case of an on-shell electron and the box-type Feynman amplitude with an electron, positron, and one of the photons on the mass shell. In Sec. 5, we analyze the total result for the Dirac tensor, adding the emission of additional soft photon contributions, which provide the final result that is free from the infrared divergences. The limit case of an almost collinear hard photon emission is considered and some numerical estimates are given.

We give the hadronic tensor for several final states:

$$\gamma^* \to \pi^+\pi^-, \mu^+\mu^-, \rho^+\rho^-.$$

In Appendices A and B, the details of the calculation are presented. In Appendix C, the contribution to the Dirac tensor in the case of the emission of two hard photons is given.

2. GENERAL ANALYSIS

The Born-level matrix element of hard-photon emission by initial leptons in the process of e^+e^- annihilation to hadrons via a single virtual photon intermediate state

$$e^+(p_+)+e^-(p_-) \to \gamma^*(q)+\gamma(p_1) \to \gamma(p_1)+h(q) \quad (1)$$

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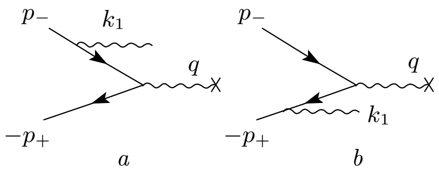

Fig. 1. Diagrams contributing at the Born level

has the form (see Fig. 1)

$$M = \frac{(4\pi\alpha)^{3/2}}{q^2}\bar{v}(p_+)O_\rho^{(B)}u(p_-)H_\rho(q),$$
$$O_\rho^{(B)} = \gamma_\rho\frac{\hat{p}_- - \hat{p}_1}{-\chi_-}\hat{e} + \hat{e}\frac{-\hat{p}_+ + \hat{p}_1}{-\chi_+}\gamma_\rho, \quad (2)$$

where $\hat{e}(p_1)$ is the polarization vector of the real photon and $H_\rho(q)$ is the current describing the conversion of a virtual photon with momentum q to a hadronic state. We restrict ourselves to the kinematic conditions of large-angle scattering,

$$s = 2p_+p_-, \quad \chi_\pm = 2p_1p_\pm, \quad p_1^2 = 0,$$
$$p_\pm^2 = m^2, \quad s - \chi_+ - \chi_- = q^2, \quad q^2 > 0, \quad (3)$$
$$s \sim q^2 \sim \chi_+ \sim \chi_- \gg m^2.$$

In the expressions below, we set $m = 0$ everywhere except the denominators of loop integrals.

The cross section can be expressed in terms of the modulus of a squared matrix element summed over spin states:

$$\sum_{spin}|M|^2 = (4\pi\alpha)^3\frac{4B_{\rho\rho_1}H_{\rho\rho_1}}{(q^2)^2},$$

$$B_{\rho\rho_1} = \frac{1}{4}\,\mathrm{Tr}\,\hat{p}_+O_\rho\hat{p}_-\bar{O}_{\rho_1}, \quad H_{\rho\rho_1} = \sum_{spin}H_\rho(q)H_{\rho_1}^*(q).$$

The differential cross section can be written as

$$d\sigma^{e^+e^-\to\gamma X} = \frac{1}{8s}\sum_{spin}|M|^2\frac{d^3p_1}{2\omega(2\pi)^3}d\Gamma_f,$$
$$d\Gamma_f = (2\pi)^4\delta^4\left(p_+ + p_- - p_1 - \sum_f q_i\right)\times \quad (4)$$
$$\times\prod_f\frac{d^3q_i}{2\varepsilon_i(2\pi)^3}.$$

For the differential hard-photon cross section, we obtain

$$\frac{\omega_1\,d\sigma^{e^+e^-\to\gamma X}}{d^3p_1d\Gamma_f} = \frac{2\alpha^3}{s(q^2)^2}H_{\rho\rho_1}B_{\rho\rho_1}, \quad (5)$$

where

$$B_{\rho\rho_1} = B_g\tilde{g}_{\rho\rho_1} + B_{++}\tilde{p}_{+\rho}\tilde{p}_{-\rho_1} + B_{--}\tilde{p}_{-\rho}\tilde{p}_{-\rho_1} +$$
$$+B_{+-}(\tilde{p_-}\tilde{p_+})_{\rho\rho_1}, \quad (6)$$
$$(p_+p_-)_{\rho\rho_1} = p_{+\rho}p_{-\rho_1} + p_{+\rho_1}p_{-\rho}.$$

The quantities with the "tilde" are defined as

$$\tilde{g}_{\rho\rho_1} = g_{\rho\rho_1} - \frac{1}{q^2}q_\rho q_{\rho_1}, \quad \tilde{p}_{\pm\rho} = p_{\pm\rho} - q_\rho\frac{p_\pm q}{q^2}. \quad (7)$$

In the Born approximation (see Fig. 1), we have

$$B_g^B = \frac{1}{\chi_+\chi_-}(2sq^2 + \chi_+^2 + \chi_-^2),$$
$$B_{++}^B = B_{--}^B = \frac{4q^2}{\chi_+\chi_-}, \quad B_\pm^B = 0. \quad (8)$$

For $q^2 = 0$, we reproduce the Dirac cross section of $e^-e^+ \to \gamma\gamma$:

$$\frac{d\Gamma}{dO_1} = \frac{2\alpha^2}{s}\frac{\chi_+^2 + \chi_-^2}{\chi_+\chi_-}. \quad (9)$$

Below, we concentrate on the calculation of the one-loop radiative correction to the Dirac tensor.

We show that in considering the corrections, only a half of the full set of Feynman diagrams for process (2) can be used. We set

$$O_\rho = O_\rho^- + O_\rho^+,$$

separating the contribution of emission from the electron leg O_ρ^- and the positron one O_ρ^+ (see Fig. 1 for the Born case and Fig. 2 for the one-loop corrections).

It can be shown that using the cyclic property of the trace and the mirror property

$$\mathrm{Tr}\,\hat{a}_1\hat{a}_2\ldots\hat{a}_{2n} = \mathrm{Tr}\,\hat{a}_{2n}\ldots\hat{a}_2\hat{a}_1,$$

the total contribution to the Dirac leptonic tensor can be written as

$$\mathrm{Tr}\,\hat{p}_+O_\rho^{(1)}\hat{p}_-\bar{O}_{\rho_1}^B + \mathrm{Tr}\,\hat{p}_+O_\rho^B\hat{p}_-\bar{O}_{\rho_1}^{(1)} =$$
$$= (1 + \Delta_{\rho\rho_1})(1 + \mathcal{P})\,\mathrm{Tr}\,\hat{p}_+O_\rho^+\hat{p}_-\bar{O}_{\rho_1}^B. \quad (10)$$

Here, the exchange operators act as

$$\Delta_{\rho\rho_1}F_{\rho\rho_1} = F_{\rho_1\rho},$$
$$\mathcal{P}F(p_+, p_-, p_1) = F(-p_-, -p_+, -p_1), \quad (11)$$
$$\mathcal{P}F(s, q^2, \chi_+, \chi_-) = F(s, q^2, \chi_-, \chi_+) \equiv \tilde{F}.$$

Here and hereafter, we imply only the real part of the leptonic tensor.

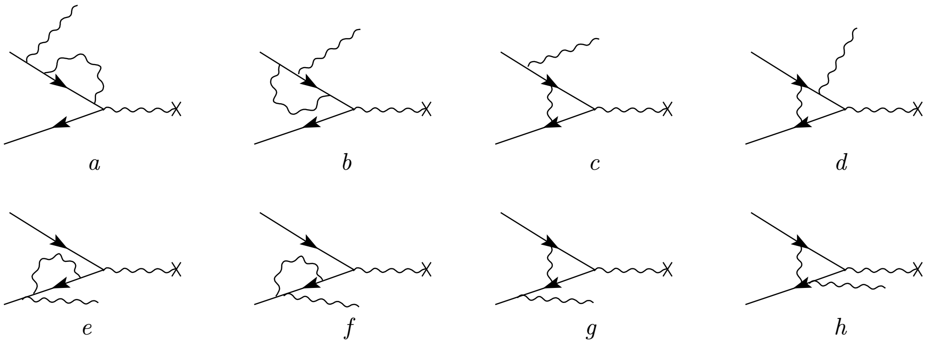

Fig. 2. Diagrams contributing at the one-loop level

3. ONE-LOOP CORRECTIONS. REAL PHOTON VERTEX AND SELF-ENERGY CONTRIBUTION

The virtual correction in the lowest order is described by 8 Feynman diagrams shown in Fig. 2.

We segregate the contribution of the Feynman diagrams in Fig. 2*e*–*h* to the three classes

$$\mathrm{Tr}\,\hat{p}_+ O^+_\rho p_- \bar{O}^B_{\rho_1} = T^{box}_{\rho\rho_1} + T^{vert}_{\rho\rho_1} + T^{\Sigma}_{\rho\rho_1}, \qquad (12)$$

with T^{box} and T^{vert} corresponding to Fig. 2*h*, *g* and T^Σ to Fig. 2*e*, *f*.

We first consider the contribution to the matrix element arising from the Feynman diagram in Fig. 2*e*, *f*. The matrix element of the Feynman diagram in Fig. 2*e* contains the mass operator of the electron $\Sigma(\hat{p})$. In the kinematics of our problem ($\chi_+ \gg m^2$), we obtain [4]

$$M_e = \frac{\alpha}{2\pi}\left(\frac{3}{2} + \frac{1}{2}l_+ - l_\lambda\right) \times \times \bar{v}(p_+)\hat{e}\left(\frac{-\hat{p}_+ + \hat{p}_1}{-\chi_+}\right)\gamma_\rho u(p_-), \qquad (13)$$
$$l_\pm = \ln\frac{\chi_\pm}{m^2}, \quad l_\lambda = \ln\frac{m^2}{\lambda^2},$$

where λ is the so-called "photon mass".

The matrix elements of the Feynman diagram in Fig. 2*f* contain the vertex function with a real photon [4],

$$M_f = \frac{\alpha}{4\pi}\bar{v}(p_+)\int\frac{d^4k}{i\pi^2} \times \times \frac{\gamma^\lambda(-\hat{p}_+ - \hat{k})(-\hat{p}_+ + \hat{p}_1 - \hat{k})\gamma^\lambda((-\hat{p}_+ + \hat{p}_1)\gamma_\rho}{(0)(\bar{2})(q)} \times \times \frac{1}{-\chi_+}u(p_-).$$

We here use the notation in (36).

Using the relevant loop integrals obtained in [3] (see the Appendix), we have matrix elements of the Feynman diagram in Fig. 2*f*, which contain the vertex function with a real photon [4]

$$M_f = \frac{\alpha}{2\pi}\bar{v}(p_+)\left[-\frac{1}{\chi_+}\left(l_+ - \frac{1}{2}\right)\hat{p}_1\hat{e} + + \hat{e}\left(l_\lambda - \frac{1}{2}l_+ - \frac{3}{2}\right)\right]\frac{-\hat{p}_+ + \hat{p}_1}{-\chi_+}\gamma_\rho u(p_-). \qquad (14)$$

As a result, we obtain the gauge-invariant expression free of infrared divergences:

$$M_e + M_f = \frac{\alpha}{\pi}\Phi_+\bar{v}(p_+)\hat{p}_1\hat{e}\gamma_\rho u(p_-),$$
$$\Phi_+ = \frac{1}{2\chi_+}\left(l_+ - \frac{1}{2}\right). \qquad (15)$$

Inserting this expression in the relevant part of O^+_ρ yields

$$T^\Sigma_{\rho\rho_1} = -\Phi_+\,\mathrm{Tr}\,\hat{p}_+\hat{p}_1\gamma_\lambda\gamma_\rho\hat{p}_- \times \times\left[\frac{1}{\chi_-}\gamma_\lambda(\hat{p}_- - \hat{p}_1)\gamma_{\rho_1} + \frac{1}{\chi_+}\gamma_{\rho_1}(-\hat{p}_+ + \hat{p}_1)\gamma_\lambda\right], \qquad (16)$$

where we use the relation

$$p_{1\rho} = (p_+ + p_-)_\rho$$

and the gauge invariance of the hadronic tensor,

$$q_\mu H_{\mu\nu} = 0.$$

Expression (16) containing the contributions of diagrams in Fig. 2*e*, *f* can be written as

$$T^\Sigma_{\rho\rho_1} = -4\Phi_+\left[2p_{-\rho}p_{-\rho_1}\left(\frac{q^2}{\chi_-} - 1\right) + + 2p_{-\rho}p_{+\rho_1}\left(\frac{s}{\chi_-} - 1\right) - (s - \chi_-)g_{\rho\rho_1}\right]. \qquad (17)$$

Applying the operation $1+\Delta_{\rho\rho_1}$ and $1+\mathcal{P}$, we obtain the full result

$$(1+\Delta_{\rho\rho_1})(1+\mathcal{P})T^{\Sigma}_{\rho\rho_1} = B^{\Sigma}_g \tilde{g}_{\rho\rho_1} + \\ + B^{\Sigma}_- \tilde{p}_{-\rho}\tilde{p}_{-\rho_1} + B^{\Sigma}_+ \tilde{p}_{+\rho}\tilde{p}_{+\rho_1} + B^{\Sigma}_{+-}(\tilde{p}_+\tilde{p}_-)_{\rho\rho_1}, \quad (18)$$

where

$$B^{\Sigma}_g = \frac{4}{\chi_-\chi_+}[sc-\chi_-^2-\chi_+^2]l_s + T^{\Sigma}_g,$$
$$B^{\Sigma}_- = -\frac{8}{\chi_-\chi_+}[q^2-\chi_-]l_s + T^{\Sigma}_-,$$
$$B^{\Sigma}_+ = -\frac{8}{\chi_-\chi_+}[q^2-\chi_+]l_s + T^{\Sigma}_+, \quad (19)$$
$$B^{\Sigma}_{+-} = -\frac{4}{\chi_-\chi_+}[q^2+s]l_s + T^{\Sigma}_{+-},$$
$$c = \chi_+ + \chi_-$$

and we use the notation

$$T^{\Sigma}_g = -2\frac{s-\chi_-}{\chi_+}[1+2l_{sp}] - \\ -2\frac{s-\chi_+}{\chi_-}[1+2l_{sm}],$$
$$T^{\Sigma}_- = \frac{4}{\chi_+\chi_-}[q^2-\chi_-][1+2l_{sp}],$$
$$T^{\Sigma}_+ = \frac{4}{\chi_+\chi_-}[q^2-\chi_+][1+2l_{sm}], \quad (20)$$
$$T^{\Sigma}_{+-} = \frac{2}{\chi_+\chi_-}[s-\chi_-][1+2l_{sp}] + \\ + \frac{2}{\chi_+\chi_-}[s-\chi_+][1+2l_{sm}],$$
$$l_{sp} = l_s - l_+, \quad l_{sm} = l_s - l_-.$$

4. VERTEX AND BOX-TYPE DIAGRAM CONTRIBUTIONS

Contribution of the diagram in Fig. $2g,h$ can be written as

$$T^{box}_{\rho\rho_1} + T^{vert}_{\rho\rho_1} = \frac{S_1}{\chi_-} + \frac{S_2}{\chi_+} - \frac{C_1}{\chi_-\chi_+} - \frac{C_2}{\chi_+^2}, \quad (21)$$

where

$$S_1 = \int \frac{1}{4}\frac{d^4k}{i\pi^2}\frac{\mathrm{Tr}\,\hat{B}_\rho \hat{p}_-\gamma_\eta(\hat{p}_- - \hat{p}_1)\gamma_{\rho_1}}{(0)(2)(\bar{2})(q)},$$
$$S_2 = \int \frac{1}{4}\frac{d^4k}{i\pi^2}\frac{\mathrm{Tr}\,\hat{B}_\rho \hat{p}_-\gamma_{\rho_1}(-\hat{p}_+ + \hat{p}_1)\gamma_\eta}{(0)(2)(\bar{2})(q)},$$
$$C_1 = \int \frac{1}{4}\frac{d^4k}{i\pi^2}\frac{\mathrm{Tr}\,\hat{V}_\rho \hat{p}_-\gamma_\eta(\hat{p}_- - \hat{p}_1)\gamma_{\rho_1}}{(0)(2)(q)},$$
$$C_2 = \int \frac{1}{4}\frac{d^4k}{i\pi^2}\frac{\mathrm{Tr}\,\hat{V}_\rho \hat{p}_-\gamma_{\rho_1}(-\hat{p}_+ + \hat{p}_1)\gamma_\eta}{(0)(2)(q)}, \quad (22)$$
$$\hat{B}_\rho = \hat{p}_+\gamma_\lambda(-\hat{p}_+ - \hat{k})\gamma_\eta(-\hat{p}_+ + \hat{p}_1 - \hat{k}) \times \\ \times\gamma_\rho(\hat{p}_- - \hat{k})\gamma_\lambda,$$
$$\hat{V}_\rho = \hat{p}_+\gamma_\eta(-\hat{p}_+ + \hat{p}_1)\gamma_\lambda(-\hat{p}_+ + \hat{p}_1 - \hat{k}) \times \\ \times\gamma_\rho(\hat{p}_- - \hat{k})\gamma_\lambda.$$

Using the loop integrals listed in Appendix A, we obtain

$$T^{box}_{\rho\rho_1} + T^{vert}_{\rho\rho_1} = D_g g_{\rho\rho_1} + D_- p_{-\rho}p_{-\rho_1} + D_+ p_{+\rho}p_{+\rho_1} + \\ + D_{+-}p_{+\rho}p_{-\rho_1} + D_{-+}p_{-\rho}p_{+\rho_1}. \quad (23)$$

Applying the interchange operator $1+\Delta_{\rho\rho_1}$ and $1+\mathcal{P}$,

$$\tilde{D}(\chi_+,\chi_-) = \mathcal{P}D(\chi_-,\chi_+),$$

and rearranging the gauge invariance then leads to

$$(1+\Delta_{\rho\rho_1})(1+\mathcal{P})(T^{box}_{\rho\rho_1} + T^{vert}_{\rho\rho_1}) = B^{VB}_g \tilde{g}_{\rho\rho_1} + \\ + B^{VB}_- \tilde{p}_{-\rho}\tilde{p}_{-\rho_1} + B^{VB}_+ \tilde{p}_{+\rho}\tilde{p}_{+\rho_1} + B^{VB}_{+-}(\tilde{p}_+\tilde{p}_-)_{\rho\rho_1},$$

where

$$B^{VB}_g = 2(\tilde{D}_g + D_g), \quad B^{VB}_- = 2(D_- + \tilde{D}_+), \\ B^{VB}_+ = 2(D_+ + \tilde{D}_-), \quad (24) \\ B^{VB}_{+-} = D_{+-} + D_{-+} + \tilde{D}_{+-} + \tilde{D}_{-+}.$$

Here, by construction,

$$B^{VB}_g = \tilde{B}^{VB}_g, \quad B^{VB}_- = \tilde{B}^{VB}_+, \quad B^{VB}_{+-} = \tilde{B}^{VB}_{-+} \quad (25)$$

and the B^{VB}_i are given by

$$B^{VB}_g = \frac{4sc-8s^2}{\chi_-\chi_+}l_s + \frac{2\chi_-^2+2\chi_+^2-4sc+4s^2}{\chi_-\chi_+} \times \\ \times [l_s^2 + 2(l_s-1)l_\lambda - l_s] + T^{VB}_g,$$
$$B^{VB}_- = \frac{8\chi_+-8s}{\chi_-\chi_+}l_s + \frac{8q^2}{\chi_-\chi_+}[l_s^2+2(l_s-1)l_\lambda-l_s] + T^{VB}_-,$$
$$B^{VB}_+ = \frac{8\chi_--8s}{\chi_-\chi_+}l_s + \frac{8q^2}{\chi_-\chi_+}[l_s^2+2(l_s-1)l_\lambda-l_s] + T^{VB}_+,$$
$$B^{VB}_{+-} = \frac{4(s+q^2)}{\chi_-\chi_+}l_s + T^{VB}_{+-},$$

where the expressions T_i^{VB} contain nonleading terms. These quantities contain the ultraviolet cut-off logarithm

$$L = \ln \frac{\Lambda^2}{m^2},$$

which is eliminated by the standard regularization procedure [4] $L \to 2l_\lambda - 9/2$.

Collecting the leading terms that contain the large logarithm l_s and the infrared one l_λ, we obtain

$$\begin{gathered}(B_g^{VB}+B_g^{\Sigma})_{leading} = 2B_g^B(l_s^2+2(l_s-1)L_\lambda-3l_s),\\ (B_+^{VB}+B_+^{\Sigma})_{leading} = (B_-^{VB}+B_-^{\Sigma})_{leading} = \\ = 2B_+^B(ls^2+2(l_s-1)L_\lambda-3l_s),\\ (B_{+-}^{VB}+B_{+-}^{\Sigma})_{leading} = 0.\end{gathered} \tag{26}$$

5. DISCUSSION: EXPLICIT FORM OF TENSOR STRUCTURES

The infrared divergences contained in the contribution of virtual photon emission are canceled when the emission of an additional soft photon (center-of-mass of e^+e^- is implied) is taken into account,

$$\begin{gathered}d\sigma_{soft}^{\gamma} = \delta_{soft} d\sigma_B,\\ \delta_{soft} = -\frac{4\pi\alpha}{16\pi^3}\int \frac{d^3k}{w}\left(-\frac{p_-}{p_-k}+\frac{p_+}{p_+k}\right)^2,\\ w < \Delta\varepsilon \ll \frac{\sqrt{s}}{2},\end{gathered} \tag{27}$$

where

$$w = \sqrt{k^2+\lambda^2}.$$

Using the standard integrals, we obtain

$$\delta_{soft} = \frac{\alpha}{\pi}\left[(l_s-1)\left(l_\lambda+2\ln\frac{\Delta E}{E}\right)+\frac{1}{2}l_s^2-\frac{\pi^2}{3}\right]. \tag{28}$$

Summing all contributions, we find the Dirac tensor in the form

$$\begin{gathered}B_{\rho\rho_1} = (B_g^B\tilde{g}_{\rho\rho_1}+B_{++}^B\tilde{p}_{+\rho}\tilde{p}_{-\rho_1}+B_{--}^B\tilde{p}_{-\rho}\tilde{p}_{-\rho_1})\times\\ \times\left[1+\frac{\alpha}{\pi}(l_s-1)\left(\frac{3}{2}+2\ln\frac{\Delta E}{E}\right)+\frac{\alpha}{\pi}\left(-\frac{\pi^2}{3}+\frac{3}{2}\right)\right]-\\ -\frac{\alpha}{4\pi}[T_g\tilde{g}_{\rho\rho_1}+T_-\tilde{p}_{-\rho}\tilde{p}_{-\rho_1}+\\ +T_+\tilde{p}_{+\rho}\tilde{p}_{+\rho_1}+T_{+-}(\tilde{p}_+\tilde{p}_-)_{\rho\rho_1}].\end{gathered} \tag{29}$$

The quantities

$$T_i = T_i^{\Sigma}+T_i^{VB}$$

are free from infrared singularities and do not contain large logarithms. The T_i^{Σ} are given in (20) and the T_i^{VB} are given in Appendix B.

Expressions for T_i contain additional nonphysical singularities $\chi_\pm^{-2}$ and $\chi_\pm^{-3}$. Nevertheless, we can verify the cancelation of the terms proportional to $\chi_\pm^{-3}$ with the structure G (see Eq. (54)) and terms $\chi_\pm^{-2}$ in the contraction of T_i. For definiteness, we consider the case of small values of χ_-,

$$m^2 \ll \chi_- \ll s \sim q^2 \sim \chi_+.$$

It corresponds to the kinematics

$$p_1 = yp_-.$$

In this case, the nonleading terms containing poles can be put in the form

$$\begin{gathered}T_g\tilde{g}_{\rho\rho_1}+(T_-+\bar{y}^2T_+-2\bar{y}T_{+-})\tilde{p}_{-\rho}\tilde{p}_{-\rho_1},\\ \bar{y}=1-y,\quad y=\frac{\chi_+}{s},\end{gathered} \tag{30}$$

and this combination contains only the lowest-order pole χ_-^{-1}. The Dirac tensor in the limit

$$m^2 \ll \chi_- \ll s$$

has the form

$$\begin{gathered}B_{\rho\rho_1}^{lim} = B_{\rho\rho_1}^{B,lim}\left[1+\frac{\alpha}{\pi}(l_s-1)\left(\frac{3}{2}+2\ln\frac{\Delta E}{E}\right)+\right.\\ \left.+\frac{\alpha}{\pi}\left(\frac{3}{2}-\frac{\pi^2}{3}\right)\right]-\\ -\frac{\alpha}{4\pi}T_g^{lim}\left(\tilde{g}_{\rho\rho_1}+\frac{4\bar{y}}{s}\tilde{p}_{-\rho}\tilde{p}_{-\rho_1}\right),\end{gathered} \tag{31}$$

where

$$\begin{gathered}B_{\rho\rho_1}^{B,lim} = \frac{1+\bar{y}^2}{xy}\left(\tilde{g}_{\rho\rho_1}+\frac{4\bar{y}}{s}\tilde{p}_{-\rho}\tilde{p}_{-\rho_1}\right),\\ \bar{x}=1-x,\quad x=\frac{\chi_-}{s},\\ T_g^{lim} = \frac{1}{xy}\left(16-18y+10y^2+4(1+\bar{y}^2)\times\right.\\ \times\left(\ln\bar{y}\ln\frac{y}{x}-\mathrm{Li}_2\left(\frac{1}{1-y}\right)-\frac{\pi^2}{2}\right)-\\ \left.-8y\bar{y}\ln y+(-12+20y-14y^2)\ln\bar{y}\right).\end{gathered} \tag{32}$$

In Fig. 3, xyT_g^{lim} is presented as a function of y at $x = 0.1$. The obtained formula has a power accuracy, and we systematically omit the terms of the order of m^2/s compared to terms of the order of unity.

A cross-channel Compton tensor with one real and another virtual (space-like) photon with terms of the order of m^2/s were taken into account, has similar properties [5].

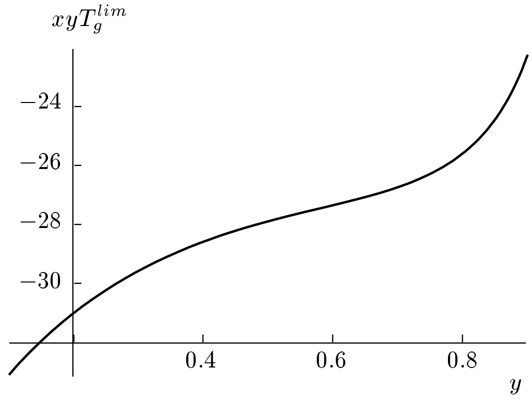

Fig. 3. xyT_g^{lim} as a function of y at $x = 0.1$

6. SAMPLES OF HADRONIC TENSORS

The hadronic tensor is a bilinear combination of matrix elements $M_\rho M^*_{\rho_1}$ summed over spin states, where the current M_ρ describes the conversion of a heavy time-like photon to some set of hadrons. In the case of creation of a pair of charged pseudoscalar mesons ($\pi^+\pi^-$, $K^+K^-, \ldots$), we have

$$H^{p^s_+ p^s_-}_{\rho\rho_1} = (p^s_+ - p^s_-)_\rho (p^s_+ - p^s_-)_{\rho_1}, \quad q = p^s_+ + p^s_-.$$

For the conversion to a pair of charged spin-1/2 fermions,

$$\gamma \to \mu^+(p_+) + \mu^-(p_-),$$

we have

$$H^{\mu_+\mu_-}_{\rho\rho_1} = 4\left[p^m_{+\rho}p^m_{-\rho_1} + p^m_{+\rho_1}p^m_{-\rho} - \frac{q^2}{2}g_{\rho\rho_1}\right]. \quad (33)$$

For the creation of a pair of charged vector mesons $\rho^+\rho^-$, $K^{*+}K^{*-}$, we obtain

$$H^{q_+q_-}_{\rho\rho_1} \approx q^2(8-2\eta)\left(g_{\rho\rho_1} - \frac{q_\rho q_{\rho_1}}{q^2}\right) + \\ + (q_+ - q_-)_\rho (q_+ - q_-)_{\rho_1}\left(3 - 5\eta + \frac{9}{4}\eta^2\right), \quad (34)$$

$$\eta = \frac{q^2}{m^2_\rho}, \quad q = q_{\rho+} + q_{\rho-}. \quad (35)$$

The gauge invariance requirement

$$H_{\rho\rho_1} q_\rho = H_{\rho\rho_1} q_{\rho_1} = 0$$

is fulfilled.

E. A. K. is grateful to Hamburg University, II Institute for Theoretical Physics, where the most part of this work was done, for the warm hospitality in November 2011. We are grateful to Yu. M. Bystritskiy for help and to V. Tayursky and V. Druzhinin for the interest in this problem. Also E. A. K. acknowledges the support from the RFBR (grants №№ 12-02-31703 and 11-02-00112), and Belorussian 2011 and Heisenberg–Landau 2011 grants.

APPENDIX A

One-loop Feynman integrals

Here, we present the result of calculation of 4-fold integrals associated with a one-loop Feynman diagram. Here and below, we imply only the real part of integrals. The denominators of the integrals are defined as

$$\begin{aligned} (0) &= k^2 - \lambda^2, \\ (2) &= (p_- - k)^2 - m^2 + i0 = k^2 - 2p_-k + i0, \\ (\bar{2}) &= (-p_+ - k)^2 - m^2 + i0, \\ (q) &= (p_1 - p_+ - k)^2 - m^2 + i0. \end{aligned} \quad (36)$$

The four-denominator scalar integral

$$I_{02\bar{2}q} = \int dk \frac{1}{(0)(2)(\bar{2})(q)}, \quad dk = \frac{d^4k}{i\pi^2}, \quad (37)$$

has the form

$$I_{02\bar{2}q} = \frac{1}{s\chi_+} \times \\ \times \left[l_q^2 - 2l_+l_s - l_sl_l + 2\,\mathrm{Li}_2\left(1 - \frac{q^2}{s}\right) - \frac{5\pi^2}{6}\right], \quad (38)$$

where the logarithms were introduced in (2) and

$$l_q = \ln\frac{q^2}{m^2}, \quad l_s = \ln\frac{s}{m^2}. \quad (39)$$

For the three- and two-denominator scalar integrals

$$I_{ijk} = \int dk \frac{1}{r}, \quad (40)$$

where

$$r = (ij), (ijk), (ijkl)$$

with

$$i, j, k, l = (0), (2), (\bar{2}), (q),$$

we have the expressions

$$\begin{aligned}
I_{0\bar{2}q} &= -\frac{1}{2\chi_+}\left[l_+^2 + \frac{2\pi^2}{3}\right], \\
I_{02\bar{2}} &= \frac{1}{2s}\left[l_s^2 + 2l_s l_l - \frac{4\pi^2}{3}\right], \\
I_{2\bar{2}q} &= -\frac{1}{2(s-q^2)}\left[l_q^2 - l_s^2\right], \\
I_{02q} &= \frac{1}{\chi_+ + q^2}\left[l_q(l_q - l_+) + \frac{1}{2}(l_q - l_+)^2 + \right. \\
&\left. + 2\,\mathrm{Li}_2\left(1 + \frac{\chi_+}{q^2}\right) - \frac{3\pi^2}{2}\right].
\end{aligned} \tag{41}$$

The two-denominator scalar integrals are

$$I_{02} = L + 1, \quad I_{2q} = L - l_q + 1, \quad I_{0q} = L - l_+ + 1,$$

$$I_{0\bar{2}} = L + 1, \quad I_{2\bar{2}} = L - L_s + 1, \quad I_{\bar{2}q} = L - 1.$$

The vector integrals can be defined as

$$I_r^\mu = \int \frac{d^4 k k^\mu}{r} = a_r^+ q_+^\mu + a_r^- q_-^\mu + a_r^1 p_1^\mu. \tag{42}$$

For the vector integrals with two denominators, we have (the imaginary part is neglected):

$$\begin{aligned}
a_{2q}^- &= a_{2q}^1 = -a_{2q}^+ = \frac{1}{2}\left(L - l_q + \frac{1}{2}\right), \\
a_{0q}^1 &= -a_{0q}^+ = \frac{1}{2}\left(L - l_+ + \frac{1}{2}\right), \\
a_{2\bar{2}}^- &= -a_{2\bar{2}}^+ = \frac{1}{2}\left(L - l_s + \frac{1}{2}\right), \\
a_{\bar{2}q}^1 &= -\frac{1}{2}a_{\bar{2}q}^+ = \frac{1}{2}\left(L - \frac{3}{2}\right), \\
a_{02}^- &= \frac{1}{2}L - \frac{1}{4}, \quad a_{0\bar{2}}^+ = -\frac{1}{2}L + \frac{1}{4}
\end{aligned} \tag{43}$$

and the coefficients for the vector integrals with three denominators are

$$\begin{aligned}
a_{02q}^- &= \frac{1}{a}\left(\chi_+ I_{02q} + \frac{2\chi_+}{a}l_+ + \frac{q^2 - \chi_+}{a}l_q\right), \\
a_{02q}^+ &= -a_{02q}^1 = \frac{1}{a}(l_+ - l_q), \\
a_{0\bar{2}q}^1 &= \frac{1}{\chi_+}(-l_+ + 2), \quad a = \chi_+ + q^2, \\
a_{0\bar{2}q}^+ &= -I_{0\bar{2}q} - \frac{1}{\chi_+}l_+, \quad a_{02\bar{2}}^- = -a_{02\bar{2}}^+ = \frac{1}{s}l_s, \\
a_{2\bar{2}q}^- &= \frac{1}{c}(l_s - l_q), \quad a_{2\bar{2}q}^+ = -I_{2\bar{2}q} + \frac{1}{c}(l_s - l_q), \\
a_{2\bar{2}q}^1 &= \frac{s}{c}I_{2\bar{2}q} + \frac{1}{c}(-l_q + 2) - \frac{2s}{c^2}(l_s - l_q), \\
&c = s - q^2 = \chi_+ + \chi_-.
\end{aligned} \tag{44}$$

Finally, the coefficient of the vector integral with four denominators has the form

$$\begin{aligned}
a^1 &= \frac{s}{d}(\chi_+ A + \chi_- B - sC), \\
a^+ &= \frac{\chi_-}{d}(\chi_+ A - \chi_- B + sC), \\
a^- &= \frac{\chi_+}{d}(-\chi_+ A + \chi_- B + sC), \quad d = 2s\chi_+\chi_-, \\
A &= I_{2\bar{2}q} - I_{0\bar{2}q}, \quad B = I_{02q} - I_{2\bar{2}q}, \\
C &= I_{02q} - I_{02\bar{2}} - \chi_+ I_{02\bar{2}q}.
\end{aligned} \tag{45}$$

The second-rank tensor integrals can be parameterized in the form

$$\begin{aligned}
I_r^{\mu\nu} &= \int dk \frac{k_\mu k_\nu}{r} = \Big[a_r^g g + a_r^{11} p_1 p_1 + a_r^{++} q_+ q_+ + \\
&+ a_r^{--} q_- q_- + a_r^{1+}(p_1 q_+ + q_+ p_1) + \\
&+ a_r^{1-}(p_1 q_- + q_- p_1) + a_r^{+-}(q_+ q_- + q_- q_+)\Big]_{\mu\nu}.
\end{aligned} \tag{46}$$

The coefficients for the tensor integral with four denominators are (we omit the index $02\bar{2}q$)

$$\begin{aligned}
a^{1+} &= \frac{1}{\chi_+}(A_6 + A_7 - A_{10}), \\
a^{+-} &= \frac{1}{s}(A_2 + A_6 - A_{10}), \\
a^{1-} &= \frac{1}{\chi_-}(A_2 + A_7 - A_{10}), \\
a^{11} &= \frac{1}{\chi_-}\left(A_1 - s a^{1+}\right), \\
a^{--} &= \frac{1}{s}\left(A_5 - \chi_+ a^{1-}\right), \\
a^{++} &= \frac{1}{s}\left(A_3 - \chi_- a^{1+}\right), \\
a^g &= \frac{1}{2}\left(A_{10} - A_2 - \chi_+ a^{1+}\right),
\end{aligned} \tag{47}$$

with

$$\begin{aligned}
A_1 &= a_{2\bar{2}q}^1 - a_{0\bar{2}q}^1, \quad A_6 = a_{02q}^+ - a_{2\bar{2}q}^+, \\
A_2 &= a_{2\bar{2}q}^-, \quad A_7 = a_{02q}^1 - \chi_+ a^1, \\
A_3 &= a_{2\bar{2}q}^+ - a_{0\bar{2}q}^+, \quad A_8 = a_{02q}^- - a_{02\bar{2}}^- - \chi_+ a^-, \\
A_4 &= a_{02q}^1 - a_{2\bar{2}q}^1, \quad A_9 = a_{02q}^+ - a_{02\bar{2}}^+ - \chi_+ a^+, \\
A_5 &= a_{02q}^- - a_{2\bar{2}q}^-, \quad A_{10} = I_{2\bar{2}q}.
\end{aligned} \tag{48}$$

For the tensor integrals with three denominators $I_{02q}^{\mu\nu}$, we have the coefficients[1)]

[1)] Formula (2.15) in [3] contains a misprint.

$$
\begin{aligned}
a^{g}_{02q} &= \frac{1}{4}L + \frac{3}{8} - \frac{q^2}{4a}l_q - \frac{\chi_+}{4a}l_+, \\
a^{+-}_{02q} &= -a^{1-}_{02q} = \frac{1}{2a}\left[\frac{\chi_+}{a}(l_+ - l_q) - 1\right], \\
a^{++}_{02q} &= a^{11}_{02q} = -a^{1+}_{02q} = \frac{1}{2a}(l_q - l_+), \\
a^{--}_{02q} &= \frac{1}{a^2}\left[\chi_+^2 I_{02q} + \frac{3\chi_+^2}{a}l_+ - \right. \\
&\left. - \frac{(q^2)^2 + 4q^2\chi_+ - 3\chi_+^2}{2a}l_q - \frac{q^2 + 3\chi_+}{2}\right].
\end{aligned}
\tag{49}
$$

The coefficients entering the tensor integral $I^{\mu\nu}_{02\bar{2}}$ are

$$
\begin{aligned}
a^{g}_{02\bar{2}} &= \frac{1}{4}(L - l_s) + \frac{3}{8}, \\
a^{++}_{02\bar{2}} = a^{--}_{02\bar{2}} &= \frac{1}{2s}(l_s - 1), \quad a^{+-}_{02\bar{2}} = -\frac{1}{2s},
\end{aligned}
\tag{50}
$$

and the coefficients for the tensor integral $I^{\mu\nu}_{0\bar{2}q}$ are

$$
\begin{aligned}
a^{g}_{0\bar{2}q} &= \frac{1}{4}(L - l_+) + \frac{3}{8}, \quad a^{1+}_{0\bar{2}q} = \frac{1}{\chi_+}\left(l_+ - \frac{5}{2}\right), \\
a^{11}_{0\bar{2}q} &= \frac{1}{2\chi_+}(-l_+ + 2), \\
a^{++}_{0\bar{2}q} &= I_{0\bar{2}q} + \frac{1}{2\chi_+}(3l_+ - 1).
\end{aligned}
\tag{51}
$$

In the case of the tensor integral $I^{\mu\nu}_{2\bar{2}q}$, they have the form

$$
\begin{aligned}
a^{g}_{2\bar{2}q} &= \frac{1}{2}\left[\frac{1}{2}L + \frac{3}{4} - \frac{s}{2c}l_s + \frac{q^2}{2c}l_q\right], \\
a^{--}_{2\bar{2}q} &= -\frac{1}{2c}(l_q - l_s), \\
a^{++}_{2\bar{2}q} &= I_{2\bar{2}q} + \frac{3}{2c}(l_q - l_s), \quad a^{+-}_{2\bar{2}q} = \frac{1}{2c}(l_q - l_s), \\
a^{1-}_{2\bar{2}q} &= \frac{1}{c}\left[-\frac{1}{2} + \frac{s}{2c}l_s - \frac{s}{2c}l_q\right], \\
a^{1+}_{2\bar{2}q} &= \frac{1}{c}\left[-\frac{5}{2} - sI_{2\bar{2}q} + \frac{5s}{2c}l_s - \frac{2q^2 + 3s}{2c}l_q\right], \\
a^{11}_{2\bar{2}q} &= \frac{1}{c^2}\left[4s - q^2 + s^2 I_{2\bar{2}q} - \frac{3s^2}{c}l_s + \right. \\
&\left. + \frac{3s^2 - (q^2)^2 + 4sq^2}{2c}l_q\right].
\end{aligned}
\tag{52}
$$

APPENDIX B

Explicit form of coefficients of nonleading tensor structures

We have

$$
\begin{aligned}
T^{VB}_g = (1 + \mathcal{P})&\left[a_0 + a_1 l_{sq} + a_2 l_{qp} + a_4 l_{sp} + \right. \\
&+ a_6 l_{sq}l_{sp} + a_8 l^2_{sq} + a_9 \operatorname{Li}_2\left(1 - \frac{q^2}{s}\right) + \\
&\left. + a_{10}\operatorname{Li}_2\left(1 + \frac{\chi_-}{q^2}\right) - 4\frac{sq^2}{\chi_-^2}G\right],
\end{aligned}
\tag{53}
$$

where

$$
\begin{aligned}
a_0 &= \frac{\pi^2}{3}\left[-10\frac{\chi_+}{\chi_-} + 22\frac{s}{\chi_-} - 4\frac{s^2}{\chi_+\chi_-}\right] + \\
&+ 8\frac{\chi_+}{\chi_-} - 16\frac{s}{\chi_-} + 8\frac{s^2}{\chi_+\chi_-}, \\
a_1 &= \frac{4s}{c} + 10\frac{\chi_+}{\chi_-} - 20\frac{s}{\chi_-} + \frac{8s^2}{\chi_+\chi_-}, \\
a_2 &= \frac{6(s - \chi_+)}{q^2 + \chi_+}, \\
a_4 &= -\frac{4\chi_-}{\chi_+} - \frac{4\chi_+}{\chi_-} + \frac{4s}{\chi_+} + \frac{4s}{\chi_-}, \\
a_6 &= -\frac{4\chi_-}{\chi_+} + \frac{8s}{\chi_+} + \frac{4s}{\chi_-} - \frac{8s^2}{\chi_+\chi_-}, \\
a_8 &= \frac{2\chi_+}{\chi_-} - \frac{6s}{\chi_-} + \frac{4s^2}{\chi_+\chi_-}, \\
a_9 &= -4\frac{\chi_+}{\chi_-} + \frac{12s}{\chi_-} - \frac{8s^2}{\chi_+\chi_-}, \\
a_{10} &= 4\frac{\chi_+}{\chi_-} - \frac{4s}{\chi_+} - \frac{8s}{\chi_-} + \frac{8s^2}{\chi_+\chi_-}, \\
G &= \operatorname{Li}_2\left(1 - \frac{q^2}{s}\right) + \operatorname{Li}_2\left(1 + \frac{\chi_+}{q^2}\right) + \\
&+ l_{sq}l_{sp} - \frac{1}{2}l^2_{sq} + \frac{\pi^2}{6}.
\end{aligned}
\tag{54}
$$

We note that in the limit $\chi_- \to 0$, the quantity G vanishes. Next,

$$
\begin{aligned}
T^{VB}_+ &= \mathcal{P}T^{VB}_-, \\
T^{VB}_- &= b_0 + b_1 l_{sq} + b_2 l_{qp} + b_3 l_{sm} +
\end{aligned}
\tag{55}
$$

$$
\begin{aligned}
&+ b_4 l_{sp} + b_5 l_{sq}l_{sm} + b_6 l_{sq}l_{sp} + b_7 l^2_{sq} + \\
&+ b_8 \operatorname{Li}_2\left(1 - \frac{q^2}{s}\right) + b_9 \operatorname{Li}_2\left(1 + \frac{\chi_-}{q^2}\right) + \\
&+ b_{10}\operatorname{Li}_2\left(1 + \frac{\chi_+}{q^2}\right) - \\
&- 8\frac{(s - \chi_+)^3}{\chi_+\chi_-^3}G + 8\frac{s^2}{\chi_+^3}\left(1 - \frac{s}{\chi_-}\right)\mathcal{P}G,
\end{aligned}
$$

where

$$
\begin{aligned}
b_0 &= \frac{16}{c}\left(1-\frac{s}{\chi_+}+\frac{s^2}{\chi_+^2}\right)-\frac{32}{\chi_+}-\frac{48}{\chi_-}+ \\
&\quad +\frac{60s}{\chi_+\chi_-}-\frac{16s^2}{\chi_+^2\chi_-}+\frac{\pi^2}{3}\times \\
&\quad \times\left[\frac{44}{\chi_+}+\frac{44}{\chi_-}+\frac{4\chi_+}{\chi_-^2}-44\frac{s}{\chi_+\chi_-}-8\frac{s}{\chi_-^2}+\right. \\
&\quad \left.+4\frac{s^2}{\chi_+^2\chi_-}+4\frac{s^2}{\chi_+\chi_-^2}\right]+ \\
&\quad +\frac{1}{\chi_++q^2}\left(-8+\frac{4\chi_+}{\chi_-}+\frac{4s}{\chi_+}\right), \\
b_1 &= \frac{16s}{c^2}\left(-1+\frac{s}{\chi_+}-\frac{s^2}{\chi_+^2}\right)+ \\
&\quad +\frac{16}{c}\left(1-\frac{s}{\chi_+}+\frac{2s^2}{\chi_+^2}+\frac{s^3}{\chi_+^2\chi_-}\right)- \\
&\quad -\frac{20}{\chi_+}-\frac{32}{\chi_-}-\frac{32\chi_+}{\chi_-^2}+\frac{8s}{\chi_+^2}+\frac{64s}{\chi_+\chi_-}+ \\
&\quad +\frac{48s}{\chi_-^2}-\frac{48s^2}{\chi_+^2\chi_-}-\frac{32s^2}{\chi_+\chi_-^2}+\frac{8s^3}{\chi_+^2\chi_-^2}, \\
b_2 &= \frac{1}{(q^2+\chi_+)^2}\left(-8\chi_++4\frac{\chi_+^2}{\chi_-}+4s\right)+ \\
&\quad +\frac{1}{q^2+\chi_+}\left(4+4\frac{\chi_+}{\chi_-}-\frac{8\chi_+^2}{\chi_-^2}-\frac{4s}{\chi_+}\right), \\
b_3 &= -\frac{4}{\chi_+}-\frac{4}{\chi_-}-\frac{8s}{\chi_+^2}-\frac{4s}{\chi_+\chi_-}+\frac{8s^2}{\chi_+^2\chi_-}, \\
b_4 &= \frac{16}{\chi_+}+\frac{12}{\chi_-}+\frac{24\chi_+}{\chi_-^2}-\frac{12s}{\chi_+\chi_-}-\frac{24s}{\chi_-^2}+\frac{8s^2}{\chi_+\chi_-^2}, \\
b_5 &= \frac{8}{\chi_+}+\frac{8}{\chi_-}-\frac{8s}{\chi_+\chi_-}+\frac{8s^2}{\chi_+^2\chi_-}, \\
b_6 &= \frac{16}{\chi_+}+\frac{16}{\chi_-}+\frac{8\chi_+}{\chi_-^2}-\frac{16s}{\chi_+\chi_-}-\frac{16s}{\chi_-^2}+\frac{8s^2}{\chi_+\chi_-^2}, \\
b_7 &= -\frac{12}{\chi_+}-\frac{12}{\chi_-}-\frac{4\chi_+}{\chi_-^2}+\frac{12s}{\chi_+\chi_-}+\frac{8s}{\chi_-^2}- \\
&\quad -\frac{4s^2}{\chi_+^2\chi_-}-\frac{4s^2}{\chi_+\chi_-^2}, \\
b_8 &= \frac{24}{\chi_+}+\frac{24}{\chi_-}+\frac{8}{\chi_+\chi_-^2}-\frac{24s}{\chi_+\chi_-}-\frac{16s}{\chi_-^2}+ \\
&\quad +\frac{8s^2}{\chi_+^2\chi_-}+\frac{8s^2}{\chi_+\chi_-^2}, \\
b_9 &= -\frac{8}{\chi_+}-\frac{8}{\chi_-}+\frac{8s}{\chi_+\chi_-}-\frac{8s^2}{\chi_+^2\chi_-}, \\
b_{10} &= -\frac{16}{\chi_+}-\frac{16}{\chi_-}-\frac{8\chi_+}{\chi_-^2}+ \\
&\quad +\frac{16s}{\chi_+\chi_-}+\frac{16s}{\chi_-^2}-\frac{8s^2}{\chi_+\chi_-^2}.
\end{aligned}
\tag{56}
$$

Finally,

$$
\begin{aligned}
T_{+-}^{VB} &= (1+\mathcal{P})\Bigg(c_0+c_1l_{sq}+c_3l_{qm}+c_4l_{sm}+c_6l_{sq}l_{sm}+ \\
&\quad +c_8l_{sq}^2+c_9\,\mathrm{Li}_2\left(1-\frac{q^2}{s}\right)+ \\
&\quad +c_{10}\,\mathrm{Li}_2\left(1+\frac{\chi_-}{q^2}\right)-\frac{8s(s-\chi_+)^2}{\chi_+\chi_-^3}G\Bigg),
\end{aligned}
\tag{57}
$$

where

$$
\begin{aligned}
c_0 &= \frac{8}{c}\left(1-\frac{s^2}{\chi_+\chi_-}\right)-\frac{4}{q^2+\chi_-}-\frac{6}{\chi_-}+\frac{10s}{\chi_+\chi_-}+ \\
&\quad +\frac{\pi^2}{3}\left[-\frac{4}{\chi_-}-\frac{4\chi_+}{\chi_-^2}+\frac{4s^2}{\chi_+\chi_-^2}\right], \\
c_1 &= \frac{-8s}{c^2}\left(1-\frac{s^2}{\chi_+\chi_-}\right)+\frac{1}{c}\left(8-\frac{12s^2}{\chi_+\chi_-}\right)+ \\
&\quad +\frac{8}{\chi_-}-\frac{8\chi_+}{\chi_-^2}+\frac{16s}{\chi_+\chi_-}+\frac{24s}{\chi_-^2}-\frac{24s^2}{\chi_+\chi_-^2}+\frac{4s^3}{\chi_+^2\chi_-^2}, \\
c_3 &= \frac{-4\chi_-}{(q^2+\chi_-)^2}+\frac{12\chi_-}{\chi_+(q^2+\chi_-)}, \\
c_4 &= \frac{8\chi_-}{\chi_+^2}-\frac{8}{\chi_+}+\frac{4}{\chi_-}-\frac{16s}{\chi_+^2}-\frac{8s}{\chi_+\chi_-}+\frac{8s^2}{\chi_+^2\chi_-}, \\
c_6 &= -\frac{8\chi_-}{\chi_+^2}-\frac{8}{\chi_+}+\frac{8s^2}{\chi_+^2\chi_-}, \\
c_8 &= \frac{4}{\chi_-}+\frac{4\chi_+}{\chi_-^2}-\frac{4s^2}{\chi_+\chi_-^2}, \\
c_9 &= -\frac{8}{\chi_-}-\frac{8\chi_+}{\chi_-^2}+\frac{8s^2}{\chi_+\chi_-^2}, \\
c_{10} &= \frac{8\chi_-}{\chi_+^2}+\frac{8}{\chi_+}-\frac{8s^2}{\chi_+^2\chi_-}.
\end{aligned}
$$

APPENDIX C

Two-hard-photon large-angle emission by the initial leptons

The cross section of two-photon emission by the initial leptons

$$
{}^+(p_+)+{}^-(p_-)\to\gamma(p_1)+\gamma(p_2)+\mathrm{hadr}(q) \tag{58}
$$

has the form

$$
\begin{aligned}
\frac{d\sigma^{2\gamma}}{d\Gamma_h} &= \frac{1}{2!}\,\frac{\alpha^4}{2\pi^2 s}\,\frac{H_{\rho\rho_1}O^{(2)}_{\rho\rho_1}}{(q^2)^2}\times \\
&\quad \times\frac{d^2p_1}{\omega_1}\,\frac{d^2p_2}{\omega_2},\quad \omega_1,\omega_2<\Delta\varepsilon,
\end{aligned}
\tag{59}
$$

where the factor $(1/(2!))$ takes the identity of final-state hard photons into account. The relevant contribution to the lepton tensor is

$$
\begin{aligned}
Q^{(2)}_{\rho\rho_1} &= \frac{1}{4}\,\mathrm{Tr}\,p_+ O^{\sigma\eta}_{12\rho} p_- \bar{O}^{\sigma\eta}_{12\rho}, \\
O^{\sigma\eta}_{12\rho} &= \gamma_\rho \frac{\hat{p}_- - \hat{p}_1 - \hat{p}_2}{d_{-12}} \left(\gamma^\eta \frac{\hat{p}_- - \hat{p}_1}{d_{-1}} \gamma^\sigma + \right.
\end{aligned} \tag{60}
$$

$$
\begin{aligned}
&\left. + \gamma^\sigma \frac{\hat{p}_- - \hat{p}_2}{d_{-2}} \gamma^\eta \right) + \left(\gamma^\eta \frac{-\hat{p}_+ + \hat{p}_2}{d_{+2}} \gamma^\sigma + \right. \\
&\quad \left. + \gamma^\sigma \frac{-\hat{p}_+ + \hat{p}_1}{d_{+1}} \gamma^\eta \right) \frac{-\hat{p}_+ + \hat{p}_1 + \hat{p}_2}{d_{+12}} \gamma_\rho + \\
&\quad + \frac{1}{d_{-1} d_{+2}} \gamma^\sigma (-\hat{p}_+ + \hat{p}_2) \gamma_\rho (\hat{p}_- - \hat{p}_1) \gamma^\eta + \\
&\qquad + \frac{1}{d_{-2} d_{+1}} \gamma^\eta (-\hat{p}_+ + \hat{p}_1) \gamma_\rho (\hat{p}_- - \hat{p}_2) \gamma^\sigma ,
\end{aligned}
$$

and

$$
\begin{aligned}
d_{-12} &= (p_- - p_1 - p_2)^2 - m^2, \\
d_{-1} &= (p_- - p_1)^2 - m^2, \\
d_{-2} &= (p_- - p_2)^2 - m^2, \\
d_{+12} &= (-p_+ + p_1 + p_2)^2 - m^2, \\
d_{+1} &= (-p_+ + p_1)^2 - m^2, \\
d_{+2} &= (p_+ + p_2)^2 - m^2.
\end{aligned} \tag{61}
$$

The tensor $Q^{(2)}_{\rho\rho_1}$ obeys the gauge invariance

$$
Q^{(2)}_{\rho\rho_1} q_\rho = Q^{(2)}_{\rho\rho_1} q_{\rho_1} = 0
$$

and can be put in the form

$$
\begin{aligned}
Q^{(2)}_{\rho\rho_1} &= A_g \tilde{g}_{\rho\rho_1} + [A_- \tilde{p}_- \tilde{p}_- + A_+ \tilde{p}_+ \tilde{p}_+ + \\
&\quad + A_{11} \tilde{k}_1 \tilde{k}_1 + A_{+-} (\tilde{p}_+ \tilde{p}_- + \tilde{p}_- \tilde{p}_+) + \\
&+ A_{+1} (\tilde{p}_+ \tilde{p}_1 + \tilde{p}_1 \tilde{p}_+) + A_{-1} (\tilde{p}_- \tilde{p}_1 + \tilde{p}_1 \tilde{p}_-)]_{\rho\rho_1}.
\end{aligned} \tag{62}
$$

The coefficients A_i can be obtained standardly, by constructing the values

$$
\begin{aligned}
&B_g, B_{11}, B_{++}, B_{--}, \ldots = \\
&\qquad = Q_{\rho\rho_1} [g_{\rho\rho_1}, p_{1\rho} p_{1\rho_1}, p_{+\rho} p_{+\rho_1}, \ldots]
\end{aligned}
$$

and solving the set of seven linear equations.

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