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COMMENT ON THE PAPER "FINITE-SIZE SCALING FROM THE SELF-CONSISTENT THEORY OF LOCALIZATION" BY I. M. SUSLOV

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In the recent paper [1], a new scaling theory of electron localization was proposed. We show that numerical data for the quasi-one-dimensional Anderson model do not support predictions of this theory.

1. INTRODUCTION

In the recent paper [1], the scaling theory of electron localization is discussed. It is argued that the standard interpretation of numerical data based on the finite-size scaling analysis [2–4] is not correct. For the quasi-one-dimensional Anderson model, a new formulation of the scaling, based on the analytic self-consistent theory, is presented. The theory gives the critical exponent $\nu = 1$ for the three dimensional (3D) Anderson model, in agreement with the original self-consistent theory of Anderson localization [5]. New scaling relations have been proposed for higher dimension $d > 4$.

In this comment, we show that the theory in [1] is not consistent with the present numerical data for the 3D and 5D Anderson model.

We consider the Anderson model [6] with diagonal disorder W defined on the quasi-one-dimensional system of the size

$$L^{d-1} \times L_z, \quad L_z \gg L \tag{1}$$

(d is the dimension of the model) and calculate the smallest Lyapunov exponent $z_1(W, L)$. It is related to the localization length ξ_{1D} as

$$z_1 = 2L/\xi_{1D} \tag{2}$$

and determines the exponential decrease of the wave function, $|\Psi|^2 \propto \exp[-z_1 L_z/L]$ [4]. For the 3D model, $L_z = 2L/\varepsilon^2$ is sufficient to achieve the relative numerical accuracy ε [7]. The size L varies from $L = 8$ to $L = 34$ for $d = 3$ and is $L \le 8$ for $d = 5$.

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2. THE 3D SYSTEM

Suslov's theory predicts [1] that in the vicinity of the critical point ($\tau = W - W_c \ll 1$), the localization length follows the scaling behavior

$$\xi_{1D}/L = y^* + A\tau(L + L_0) \tag{3}$$

with a new additional length scale L_0 not considered in the standard scaling analysis (y^* is the size-independent critical value). This prediction is in variance with the standard scaling formula

$$z_1 = 2L/\xi_{1D} = z_{1c} + A\tau L^{1/\nu}, \tag{4}$$

used in the finite-size scaling analysis of numerical data [2, 3].

To support the result (3), Suslov used numerical data for the parameter z_1 published in [4] and found that $L_0 \approx 5$ (Fig. 6, left in [1]). In Fig. 1, we show the same figure with additional data for $24 \le L \le 34$. The power fit $z_1(L) = a + bL^\alpha$ calculated for $W = 16$ and $W = 17$ supports the validity of the relation (4).

Before testing the validity of Eq. (3), we have to notice the relation (2) between the localization length expressed in Eq. (3) and the parameter z_1 shown in Fig. 1. We fit our data for z_1 to the function

$$\zeta = \frac{1}{a_0 + a_1 L} \tag{5}$$

shown by dotted lines in Fig. 1. Comparing with Eq. (3) and using $y^* = z_{1c}^{-1} = 3.48^{-1}$ (Fig. 2), we obtain $L_0 \approx 8.6$ from the $W = 16$ data, but a significantly different value $L_0 \approx 17$ for $W = 17$.

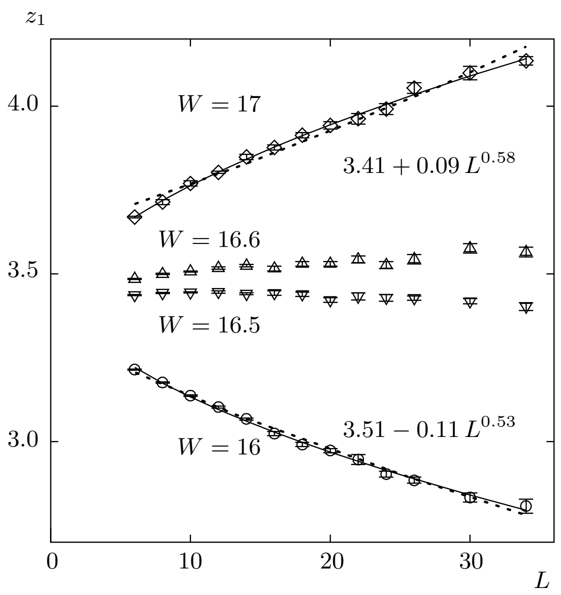

Fig. 1. The 3D Anderson model: the parameter $z_1(L)$ for different disorder. Solid lines are power fits for $W = 16$ and $W = 17$. Contrary to [1], fits are not linear in L. Note that z_1 decreases for $W = 16.5$ and increases for $W = 16.6$. Therefore, we expect that $16.5 < W_c < 16.6$. Scaling analysis gives $W_c \approx$ ≈ 16.55. Dotted lines are fits (5) with $a_0 = 0.302$ and $a_1 = 0.0017$ ($W = 16$) and with $a_0 = 0.267$ and $a_1 = -0.00108$ ($W = 17$)

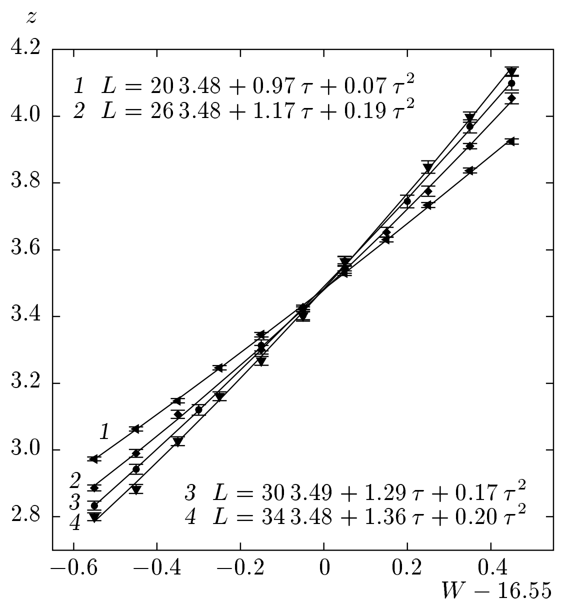

Fig. 2. Quadratic fit (6) of $z_1(W - W_c)$ for four values of the size L. The cross section determines $z_{1c} \approx 3.48$

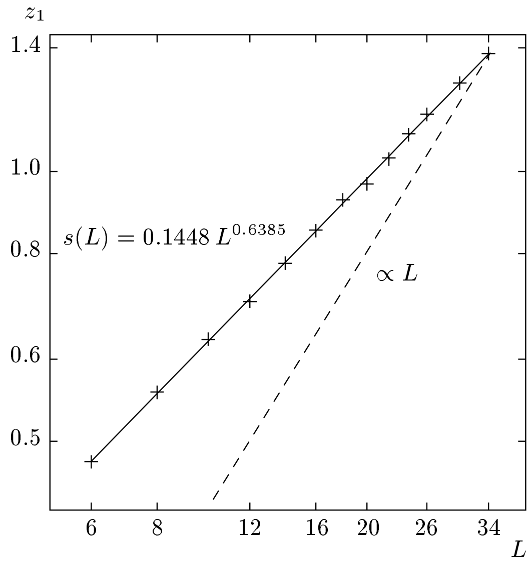

Fig. 3. The L-dependence of the slope $s(L) \propto L^{1/\nu}$. The critical exponent is $\nu = 1.566$. The dashed line shows the linear L-dependence predicted by Eq. (3)

Although the power fit in (4) is clearly better than the one in (5), Fig. 1 shows that the estimation of true scaling behavior might be difficult since various analytic functions seem to fit numerical data with sufficient accuracy. In the present case, the problem lies in the nonzero critical value z_{1c}. To avoid the ambiguity in the choice of the fitting function, we have to extract the critical value from numerical data [8]. When the data for z_1 are plotted as a function of the disorder (Fig. 2), we can fit them by a quadratic polynomial

$$z_1(W, L) = z_{1c} + \tau s(L) + \tau^2 t(L) \tag{6}$$

and calculate the L-dependence of the slope $s(L)$. From Eq. (3), we see that $s(L)$ should be a linear function of L, while Eq. (4) predicts a power-law behavior $s(L) \propto L^{1/\nu}$. Figure 3 shows $s(L)$ as a function of L. The fit confirms the power-law dependence $s(L) \propto L^{1/\nu}$ with the critical exponent $\nu \approx 1.56$, as obtained by other methods [2].

3. THE 5D MODEL

For higher dimensions, the following size dependence of the localization length at the critical point ($\tau = 0$) was derived:

$$\xi_{1D}/L = \left(\frac{L}{a}\right)^{(d-4)/3}. \tag{7}$$

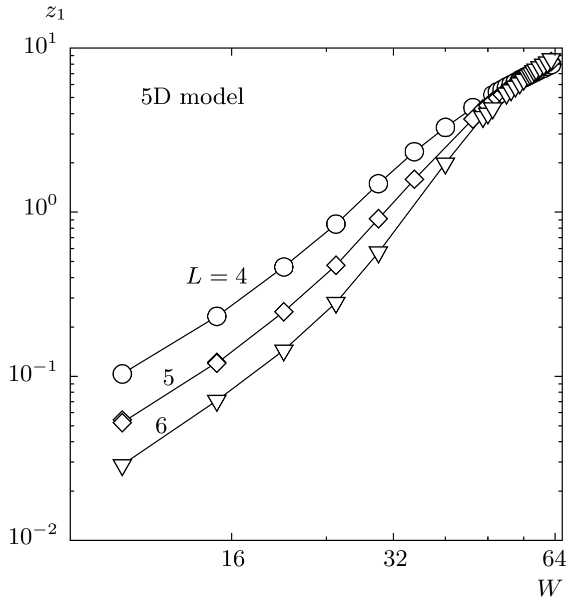

Fig. 4. The parameter $z_1(W)$ for different system sizes. For $W < 57.5$, z_1 decreases as L increases. There is no indication of the critical behavior described by Eq. (9)

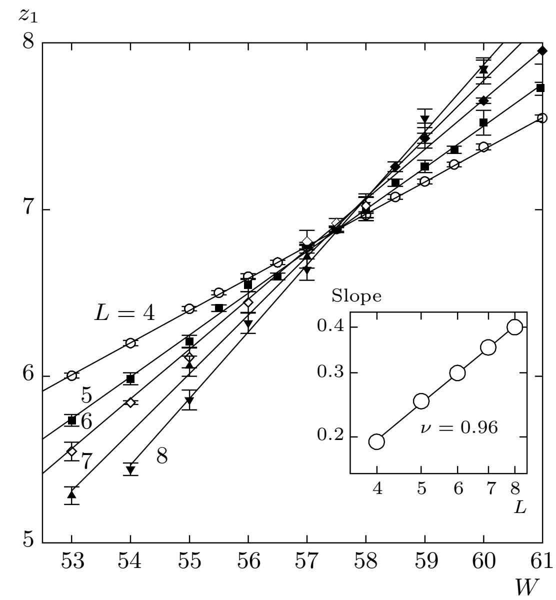

Fig. 5. The 5D Anderson model: the parameter z_1 as a function of disorder W for $L = 4, 5, 6, 7$ and $L = 8$. The data indicate that z_1 does not depend on the size L when $W \approx 57.5$. This value is considered as a critical disorder W_c in the "standard" finite-size scaling theory. Solid lines are fits $z_1(L) = z_1 + s(L)(W - W_c)$. The inset shows the L-dependence of the slope $s(L) \propto L^{1.0413}$. The original figure was published in [4] but new data for $L = 8$ are added

In particular, for $d = 5$, Eq. (7) gives

$$z_1(\tau = 0) \propto L^{-1/3}, \tag{8}$$

which means that the critical value of z_1 is not size independent but decreases to zero as $L \to \infty$. Since the localization length is finite for $\tau > 0$, the τ-dependence of $z_1(L, \tau)$ for a fixed L must exhibit an infinite discontinuity at $\tau = 0$:

$$z_1(\tau) \propto \begin{cases} L^{-1/3}, & \tau = 0 \\ L, & \tau > 0 \end{cases}. \tag{9}$$

We test the size and disorder dependence of z_1 numerically. We show in Figs. 4 and 5 the disorder dependence of z_1 for fixed L. Our data in Fig. 4 do not indicate any discontinuity in the L dependence. On the contrary, z_1 is a smooth analytic function of both parameters, W and L.

For smaller disorder, z_1 is always a decreasing function of L. This is typical for the metallic regime. However, z_1 does not depend on the size L when $W = 57.5$. This is consistent with scaling equation (4). The insulating regime, where z_1 increases with the size L, is observed only when $W > 57.5$ (Fig. 5).

We note that $z_1 \approx 7$ for disorder $W \approx 57.5$. Therefore, the localization length

$$\xi_{1D} = \frac{2}{z_1} L \tag{10}$$

is much smaller than the size of the system and we do not expect that finite-size effects play a significant role, although the size L is much smaller than in 3D system.

A scaling analysis similar to that for the 3D model allows finding the critical exponent, $\nu_{5D} \approx 0.96$.

4. CONCLUSION

We showed that numerical data for the parameter z_1 do not agree with the predictions of the theory in [1]. Both z_1 and the localization length are analytic continuous functions of the disorder W and the size of the system L.

For the 3D system, we presented additional numerical data for larger system size $L \geq 24$ up to $L = 34$. These new data confirm the previous estimation of the critical exponent $\nu = 1.56$ [3, 8]. It is worth mentioning that the same value of the critical exponent was obtained already 20 years ago with the use of numerical data for $L \leq 12$ only [9]. We also note that the same value of the critical exponent was obtained from numerical analysis of other physical quantities: mean conductance, conductance distribution, inverse participation ratio [4] and also for critical points outside the band center [4, 10]. This value of the critical exponent was recently verified experimentally [11] and calculated analytically [12].

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