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CONTROLLING CHAOS IN THE BOSE–EINSTEIN CONDENSATE SYSTEM OF A DOUBLE LATTICE

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We study the chaotic dynamics in the Bose–Einstein condensate (BEC) system of a double lattice. Chaotic space–time evolution is investigated for the particle number density in a BEC. By changing of the s-wave scattering length with a Feshbach resonance, the chaotic behavior can be well controlled to enter into periodicity. Numerical calculation shows that there is periodic orbit according to the s-wave scattering length only if the maximal Lyapunov exponent of the system is negative.

1. INTRODUCTION

Creation of the Bose–Einstein condensate (BEC) has provided a platform for investigating many important phenomena in atomic physics, condensed-matter physics, and quantum optics. BEC has attracted much more attention for its potentially great application. Apart from being a marriage of two very recent disciplines within atomic and laser physics, BEC in optical lattices have relatives in many other fields of physics. The dynamics of the system is described by a Schrödinger equation combined with a nonlinear term, which represents the many-body interactions in the mean field approximation. This nonlinearity allows bringing chaos into the quantum system. The existence of the BEC chaos has been proved and the chaotic properties have also been extensively researched in many previous works [1–9]. Naturally, chaos, which plays a role in the regularity of the system, causes instability of the condensate wave function [10].

Chaos in a collapsing BEC has also been discussed in [6] and [11]. Chaos is also relevant to the phenomenon of macroscopic quantum self-trapping in a BEC [12]. Therefore, it is important to investigate the chaotic characteristics in the BEC system. For the purpose of applications, control of chaos is anticipated in practical investigations [13–21].

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Chaos control has always been a widely attractive field since the pioneering work of Ott, Grebogi, and Yorker in 1990 [22]. Controlling chaos can be separated into two categories: feedback control (active control) and nonfeedback control (passive control). The basic characteristics for nonfeedback control is that the controlling signal is not affected by system variation changes.

The main purpose of this present paper is to control the chaos in the stable states in the BEC by means of changing the s-wave scattering length by using the Feshbach resonance. We can force the system to a stable periodic orbit.

2. ANALYSIS OF THE CHAOTIC DYNAMICS

In the mean field approximation of the two-mode Gross–Pitaevskii (GP) equation, the BEC system is governed by the GP equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\psi_{xx} + g_{1d}|\psi|^2\psi + \left(V_1\sin^2 k_1x - V_2\sin^2 k_2x\right)\psi, \quad (1)$$

where m is the atomic mass and g_{1d} denotes the interatomic interaction. The value ψ is the macroscopic quantum wave function. The parameters V_1 and V_2 are optical intensities. The parameters k_1 and k_2 are laser wave vectors.

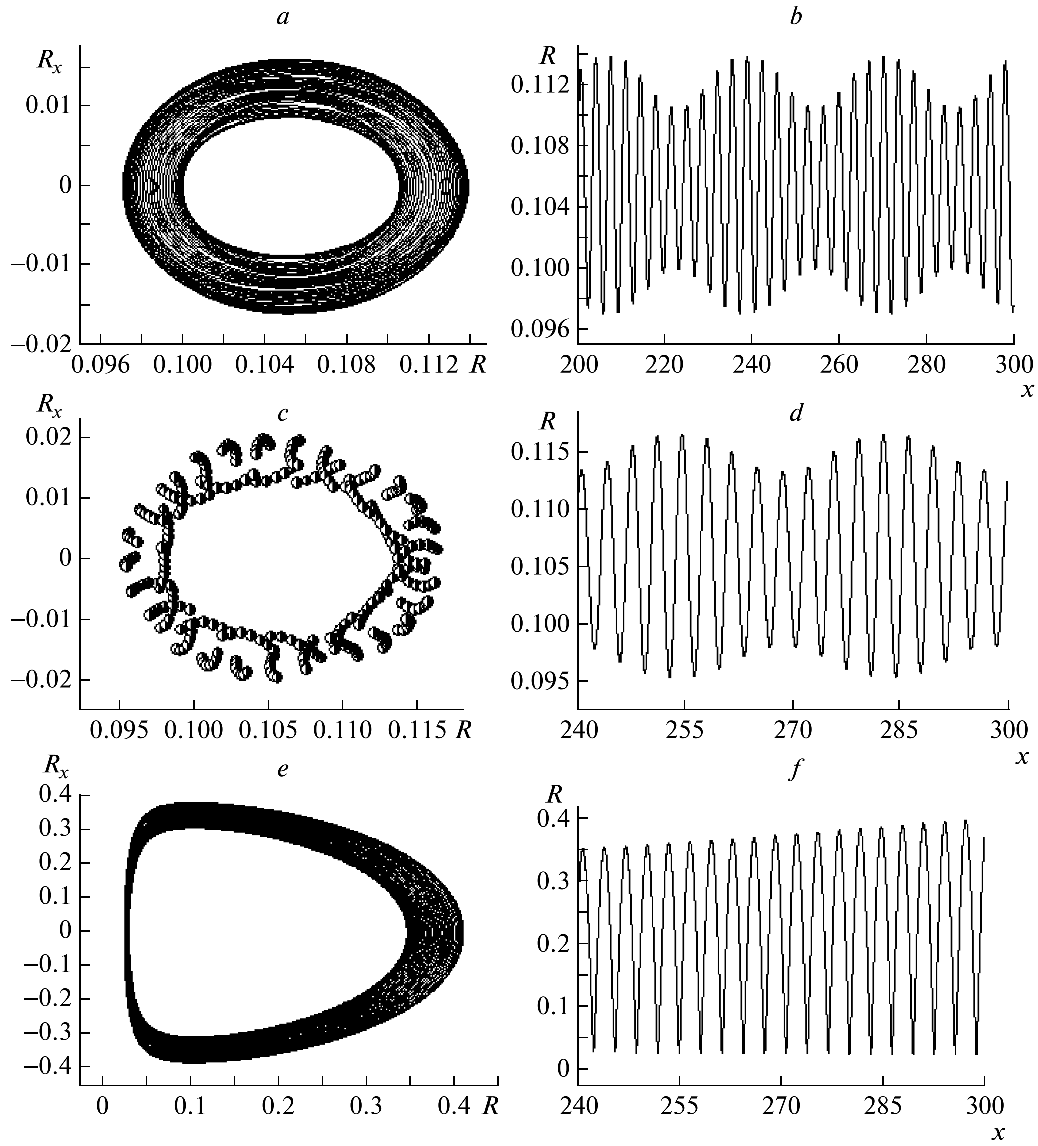

Fig. 1. The chaotic attractors projection on the RR_x plane and the time series with $V_1 = 0.0116698$, $k_1 = 2\pi/85$, $V_2 = 0.02$, $\mu = 0.410278$, $k_2 = 6\pi/85$, $c_1 = 0.01$. $g = 0.5$ (a–d) and -0.82 (e,f)

We set

$$0 < V_1, \quad V_2 \ll E_r, \quad E_r = \frac{\hbar^2 k_1^2}{2m}.$$

The parameter E_r is the recoil energy. The GP equation is

$$-\frac{1}{2}\psi_{xx} + g|\psi|^2\psi + \left(V_1 \sin^2 x - V_2 \sin^2 \frac{k_2}{k_1} x\right)\psi = \mu\psi, \quad (2)$$

where all variables and parameters are dimensionless. The parameter $g = 4\pi a_s k_1$ denotes the interatomic interaction with a being the s-wave scattering length. The variable x is the spatial coordinate.

We set

$$\psi = R(x)\exp\left[i\theta(x)\right]. \quad (3)$$

Substituting Eq. (3) in Eq. (2) yields

$$R_{xx} = R\theta_x^2 + 2gR^3 + 2\left(V_1 \sin^2 x - V_2 \sin^2 \frac{k_2}{k_1} x - \mu\right) R, \quad (4)$$

$$\theta_{xx} + \frac{2\theta_x R_x}{R} = 0, \quad (5)$$

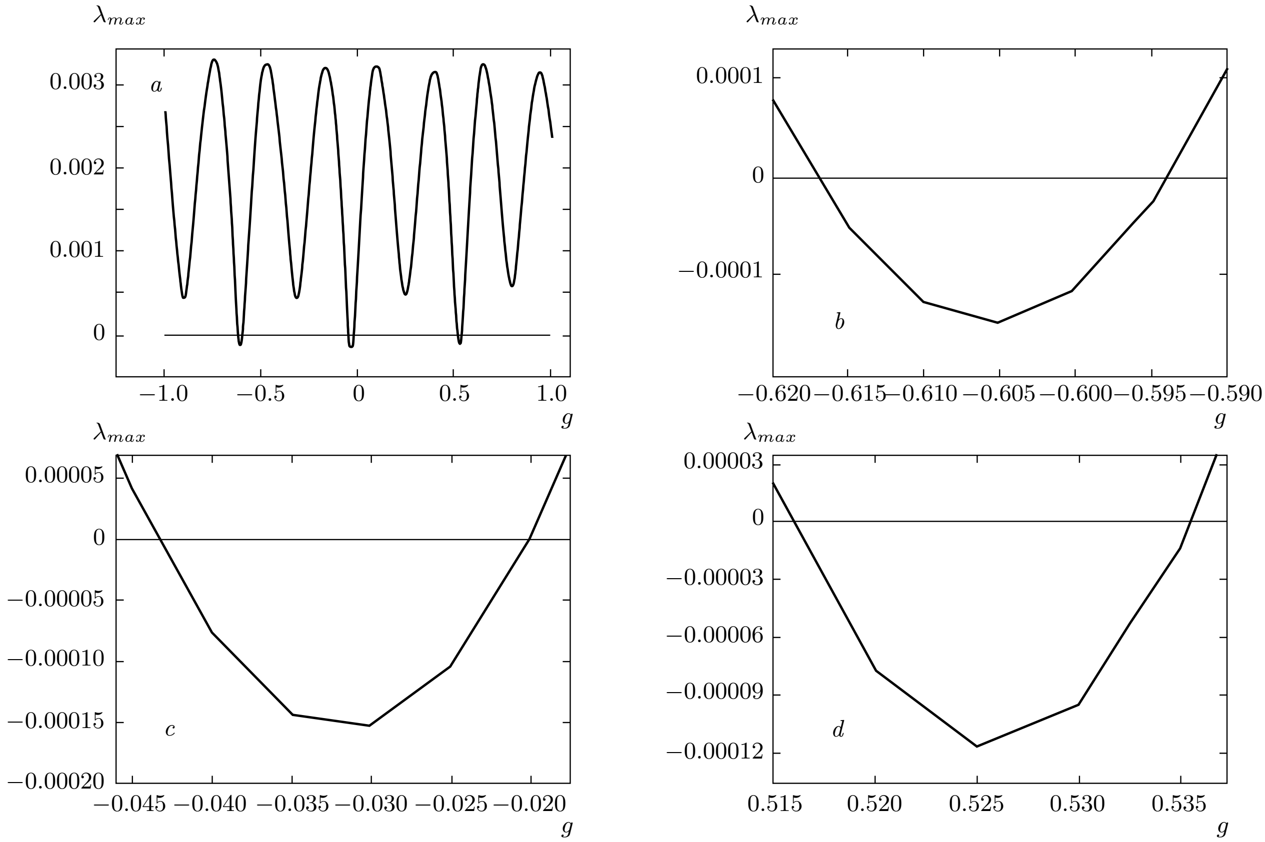

Fig. 2. The maximal Lyapunov exponent λ_{max} as a function of the s-wave scattering length g with $V_1 = 0.0116698$, $k_1 = 2\pi/85$, $V_2 = 0.02$, $\mu = 0.410278$, $k_2 = 6\pi/85$, and $c_1 = 0.01$

where $R(x)$ is the amplitude and $\theta(x)$ is the phase of the state.

We integrate Eq. (4):

$$V(x) = \frac{\hbar k_1 \theta_x}{m} = \frac{c_1 \hbar k_1}{m R^2}, \tag{6}$$

where

$$c_1 = \theta_x(x_0) R^2(x_0)$$

is a constant. Substituting Eq. (6) in Eq. (4), we obtain

$$R_{xx} = \frac{c_1^2}{R^3} + 2gR^3 + + 2\left(V_1 \sin^2 x - V_2 \sin^2 \frac{k_2}{k_1} x - \mu\right) R. \tag{7}$$

Equation (7) is the Duffing equation [23].

Using the fourth Runge–Kutta (RK) algorithm, we solve Eq. (7) numerically, and illustrate the attractors in the equivalent phase RR_x in Fig. 1. To avoid transient chaos, the values of R and R_x in the initial values of 10000 steps are eliminated. Only the values of R and R_x in the final 20000 steps are retained. Figure 1 shows the final attractors and the time series. The parameters in Fig. 1 are as follows: $V_1 = 0.0116698$, $k_1 = 2\pi/85$, $V_2 = 0.02$, $\mu = 0.410278$, $k_2 = 6\pi/85$, and $c_1 = 0.01$. In Figs. 1*a*,*b*, the initial condition is $(R, R_x) = (0.1, 0.0)$ and $g = 0.5$. The three Lyapunov exponents are $\lambda_1 = 3.3149 \cdot 10^{-4}$, $\lambda_2 = 0.0$, and $\lambda_3 = -3.51109 \cdot 10^{-4}$. The BEC system is in a chaotic state because the maximal Lyapunov exponent is positive. The chaotic orbit in the equivalent phase space RR_x is localized in a finite region and shows a confused structure.

In Fig. 1*c*,*d*, the initial condition is $(R, R_x) = (0.01, 0.0)$ and $g = 0.5$. The three Lyapunov exponents are $\lambda_1 = 3.87209 \cdot 10^{-4}$, $\lambda_2 = 0.0$, and $\lambda_3 = -4.04412 \cdot 10^{-4}$. The BEC system is in a chaotic state because the maximal Lyapunov exponent is positive.

In Fig. 1*e*,*f*, the initial condition is $(R, R_x) = (0.03, 0.01)$ and $g = -0.82$. The three Lyapunov

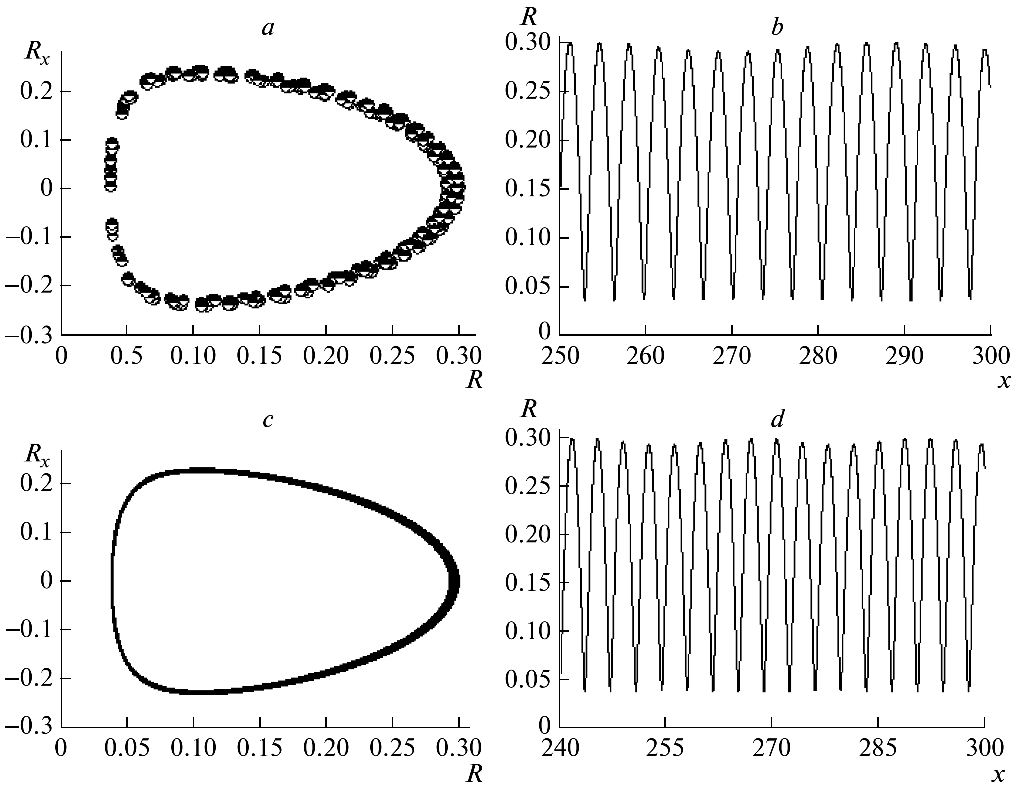

Fig. 3. The attractors projection on the RR_x and the time series of R at different s-wave scattering length with $V_1 = 0.0116698$, $k_1 = 2\pi/85$, $V_2 = 0.02$, $\mu = 0.410278$, $k_2 = 6\pi/85$, and $c_1 = 0.01$. $g = -0.033$ (a,b), $g = 0.525$ (c,d)

exponents are $\lambda_1 = 0.01526$, $\lambda_2 = 0.0$, and $\lambda_3 = -0.01586$. The BEC system is in a chaotic state because the maximal Lyapunov exponent is positive. The chaotic orbit in the equivalent phase space RR_x is localized in a finite region and shows a confused structure.

3. NUMERICAL RESULTS

To control the chaos in a BEC, we adjust the interaction by changing the s-wave scattering length, that is, changing the value of g. In this paper, we only consider the effect of the s-wave.

Figure 2 shows the maximal Lyapunov exponent as a function of the s-wave scattering length g. The horizontal line shows the value of zero. We find that in many ranges, for example, $-0.617 < g < -0.593$, $-0.043 < g < -0.02$, and $0.514 < g < 0.536$, the maximal Lyapunov exponent is negative. If g takes a value in these ranges, then the BEC is in a periodic state. The BEC is in a periodic state when g takes values -0.033 and 0.525.

We solve Eq. (7) numerically by using the fourth RK algorithm. The values of R and R_x in the initial 10000 steps are eliminated. The last 20000 steps of R and R_x are retained. The initial conditions are $(R, R_x) = (0.3, 0.01)$.

Figure 3 shows the attractor projected onto the RR_x plane, and the time series of R. The parameters are the same as in Fig. 2, the other parameters being $g = -0.033$ and 0.525. In Fig. 3a,c, the period is 1 when $g = -0.033$ and 0.525. Figures 3b,d show the respective time series. We can therefore transform the chaotic state into a periodic regular state by modulating the s-wave length g.

4. CONCLUSIONS

In summary, we have investigated the chaotic features in the spatial distributions of the BEC. We present a method to control chaos via modulating the s-wave scattering length. Numerical calculation shows that there is a periodic orbit depending on the s-wave scattering length only if the maximal Lyapunov exponent of the system is negative.

It is well known that the periodic lattice systems in a BEC exhibit many fantastic properties. For example, quantum computation with BEC atoms in a Mott insulating state is an interesting advancement in the application of the BEC. On the other hand, chaos is associated with quantum entanglement and quantum error correcting, which are both the fundamental subjects in quantum computations. It is therefore valuable to apply or control chaos in the system.

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