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ANALYSIS OF THE LOGARITHMIC SLOPE OF F_2 FROM THE REGGE GLUON DENSITY BEHAVIOR AT SMALL x

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We study the accuracy of the Regge behavior of the gluon distribution function for an approximate relation that is frequently used to extract the logarithmic slopes of the structure function from the gluon distribution at small x. We show that the Regge behavior analysis results are comparable with HERA data and are also better than other methods that expand the gluon density at distinct points of expansion. We also show that for $Q^2 = 22.4$ GeV^2, the x dependence of the data is well described by gluon shadowing corrections to the GLRMQ equation. The resulting analytic expression allows us to predict the logarithmic derivative $\partial F_2(x,Q^2)/\partial \ln Q^2$ and to compare the results with the H1 data and a QCD analysis fit with the MRST parameterization input.

Several methods of relating the F_2 scaling violations to the gluon density at low x have been suggested previously [1–3]. All the methods rely on an approximate relation based on the assumption that quark densities can be neglected and the nonsinglet contribution F_2^{Ns} can safely be ignored. To investigate this, we have used the DGLAP evolution equations [4] for four flavors,

$$\frac{dF_2}{d\ln Q^2} = \frac{5\alpha_s}{9\pi}\int\limits_x^1 dz\, G\left(\frac{x}{z}, Q^2\right) P_{qg}(z), \qquad (1)$$

where

$$P_{qg}(z) = (1-z)^2 + z^2.$$

In the LO (leading order), an approximate relation between the gluon density at the point $2x$ and the logarithmic slopes F_2 at the point x was given in [1] in the form

$$\frac{dF_2}{d\ln Q^2} = \frac{5\alpha_s}{9\pi}\frac{2}{3}G(2x, Q^2). \qquad (2)$$

A similar relation based on the expansion of the gluon distribution around $z = 0$ was found in [2],

$$\frac{dF_2}{d\ln Q^2} = \frac{5\alpha_s}{9\pi}\frac{3}{4}G\left(\frac{4}{3}x, Q^2\right). \qquad (3)$$

In [3], this expansion was derived at an arbitrary point of expansion. In the limit $x \to 0$, the equation becomes

$$\frac{dF_2}{d\ln Q^2} = \frac{5\alpha_s}{9\pi}\frac{2}{3}G\left(\frac{x}{1-a}\left(\frac{3}{2}-a\right), Q^2\right). \qquad (4)$$

The better choice $a = 0.75$ was suggested in [3], with the result

$$\frac{dF_2}{d\ln Q^2} = \frac{5\alpha_s}{9\pi}\frac{2}{3}G(3x, Q^2). \qquad (5)$$

All relations (2), (3), and (5) estimate the logarithmic slopes F_2 with respect to the gluon distribution function at the points $2x$, $4x/3$ and $3x$. In this paper, we extend the method using the Regge technique. We first introduce the Regge behavior of the gluon distribution, which can be expressed as

$$G(x,t) = A_g x^{-\lambda_g(t)}, \qquad (6)$$

where A_g is a constant and λ_g is the intercept ($t = \ln(Q^2/\Lambda^2)$). Using this behavior, after integrating and somewhat rearranging, we find an approximate relation between $dF_2(x,Q^2)/d\ln Q^2$ and $G(x,Q^2)$ at the same point x:

$$\frac{dF_2}{d\ln Q^2} = \frac{5\alpha_s}{9\pi}T(\lambda_g)G(x, Q^2), \qquad (7)$$

where

$$T(\lambda_g) = \int\limits_x^1 dz\, z^{\lambda_g}(1 - 2z + 2z^2).$$

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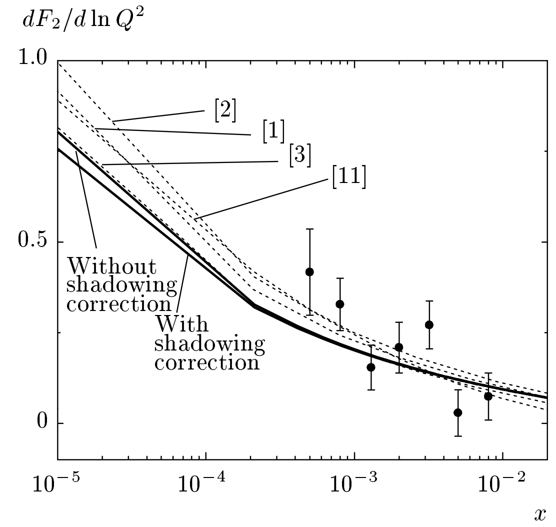
Relation (7) [5] helps estimate the logarithmic slopes F_2 in the leading logarithmic approximation. We also note that if we wish to evolve shadowing corrections to the gluon density, we can simply show these recombinations using the Gribov–Levin–Ryskin–Mueller–Qiu (GLRMQ) equations [6, 7]. These nonlinear terms reduce the growth of the gluon distribution in the kinematic region where α_s is still small but the density of partons becomes large. According to the fusion of two-gluon corrections, the evolution of the shadowing structure function with respect to $\ln Q^2$ corresponds to the modified DGLAP evolution equation. We therefore have [8]

$$\frac{\partial F_2^s(x,Q^2)}{\partial \ln Q^2} = \frac{5\alpha_s}{9\pi}T(\lambda_g)G^s - \frac{5}{18}\frac{27\alpha_s^2}{160R^2Q^2}[G^s]^2, \quad (8)$$

where R is the size of the target populated by the gluons. The value of R depends on how the gluon ladders couple to the proton, or on how the gluons are distributed within the proton. The value of R is of the order of the proton radius ($R \approx 5$ GeV^{-1}) if the gluons are spread throughout the entire nucleon, or much smaller ($R \approx 2$ GeV^{-1}) if gluons are concentrated in hot spots [9] within the proton. We show a plot of $\partial F_2(x,Q^2)/\partial \ln Q^2$ in the Figure for a set of values of x at Q^2 constant at a hot spot point $R = 2$ GeV^{-1}, compared to the values measured by the H1 collaboration [10] and a fit to the ZEUS data inspired by the Froissart bound [11] based on the MRST input parameterization [12].

In Figure, our results for $dF_2/d\ln Q^2$ obtained from the Regge behavior of the gluon density are compared with other models based on the expansion of the gluon density. For these results, the input gluon was taken from MRST parameterizations. It is clear that our results based on this behavior are the lowest among all the models. It also follows from Figure that the GLRMQ equation results in a tame behavior with respect to gluon saturation as x decreases. This shadowing correction suppresses the rate of growth in comparison with the DGLAP approach.

To conclude, the recombination of gluons becomes dominant at high density, and must be included in the calculations. When the shadowing term is combined with the DGLAP evolution in the double leading logarithmic approximation, we obtain the GLRMQ equation for the integrated gluon. We have therefore solved the DGLAP equation with the nonlinear shadowing term included in order to determine the behavior of the gluon distribution $G(x,Q^2)$ of the proton at very small x. In this way, we were able to study the interplay of the singular behavior generated by the linear



A plot of the derivative of the structure function with respect to $\ln Q^2$ vs. x for $Q^2 = 22.4$ GeV^2 with the MRST parameterization [12], compared to the data from H1 Collaboration [10] (circles) with total error, and also a QCD fit [11] and other models [1–3] (dotted curves). Solid curves are our results with and without the shadowing correction with respect to the Regge behavior of the gluon density

term in the equation with the taming of this behavior by the nonlinear shadowing term. With decreasing x, we find that an $\sim x^{-\lambda_g}$ behavior of the gluon function emerges from the GLRMQ equation. Based on our present calculations, we conclude that the behavior of $\partial F_2(x,Q^2)/\partial \ln Q^2$ as measured by HERA is tamed based on the gluon saturation at low x. Our results show that the data can be described in PQCD taking shadowing corrections into account.

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