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RADIATION OF QUANTIZED BLACK HOLE

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The maximum entropy of a quantized surface is demonstrated to be proportional to the surface area in the classical limit. The general structure of the horizon spectrum and the value of the Barbero–Immirzi parameter are found. The discrete spectrum of thermal radiation of a black hole naturally fits the Wien profile. The natural widths of the lines are very small as compared to the distances between them. The total intensity of the thermal radiation is calculated.

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1. INTRODUCTION

The idea of quantizing the horizon area of black holes was put forward many years ago by Bekenstein in the pioneering article [1]. He pointed out that reversible transformations of the horizon area of a nonextremal black hole found by Christodoulou and Ruffini [2, 3] have an adiabatic nature. Of course, the quantization of an adiabatic invariant is perfectly natural, in accordance with the correspondence principle.

Once this hypothesis is accepted, the general structure of the quantization condition for large quantum numbers becomes obvious, up to an overall numerical constant β. The quantization condition for the horizon area A should be

$$A = \beta\, l_p^2 N, \tag{1}$$

where N is some large quantum number [4]. Indeed, the presence of the Planck length squared

$$l_p^2 = \frac{k\hbar}{c^3}$$

is only natural in this quantization rule. Then, for the horizon area A to be finite in the classical limit, the power of N must be the same as that of $\hbar$ in l_p^2. This argument can be checked by considering any expectation value in quantum mechanics, nonvanishing in the classical limit. It is worth mentioning that there are no compelling reasons to believe that N is an integer. Neither are there compelling reasons to believe that spectrum (1) is equidistant [5, 6].

On the other hand, formula (1) can be interpreted as follows. The entire horizon area A is split into elements of typical size $\sim l_p^2$, each of them giving a contribution to the large quantum number N. This scheme arises, in particular, in the framework of loop quantum gravity (LQG) [7–11].

A quantized surface in LQG looks as follows. The surface is assigned a set of edges. Each edge is supplied with an integer or half-integer «angular momentum» j:

$$j = 1/2, 1, 3/2, \ldots \tag{2}$$

The projections m of these «angular momenta» range as usual from $-j$ to j. The area of the surface is

$$A = 8\pi\gamma\, l_p^2 \sum_i \sqrt{j_i(j_i+1)}\,. \tag{3}$$

The numerical factor γ in (3) cannot be determined without an additional physical input. This free (so-called Barbero–Immirzi) parameter [12, 13] corresponds to a family of inequivalent quantum theories, all of them being viable without such an input.

We mention that although spectrum (3) is not equidistant, it is not far away from it. Indeed, even for the smallest quantum number $j = 1/2$, the quantity $\sqrt{j(j+1)}$ can be approximated by $j + 1/2$ with the accuracy 13 %. As j grows,

$$\sqrt{j(j+1)} \approx j + 1/2$$

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becomes better and better, i.e., spectrum (3) approaches an equidistant one. This feature of spectrum (3) is of interest in connection with the observation by Bekenstein: quantum effects result in the following lower bound on the change of the horizon area ΔA under an adiabatic process:

$$(\Delta A)_{min} = \xi l_p^2 ; \qquad (4)$$

here, ξ is a numerical factor reflecting «the inherent fuzziness of the uncertainty relation» [14]. Of course, the right-hand side of formula (4) is proportional to $\hbar$, together with the Planck length squared l_p^2 .

Due to the uncertainty of the numerical factor ξ itself, one cannot see any reason why ξ should not slightly change from one act of capture to another. Therefore, the discussed quasiequidistant spectrum (3) agrees with the bound (4), practically as well as the equidistant one. We return to relation (4) below.

As regards the unknown parameter γ in (3), the first attempts to fix its value, based on the analysis of the black hole entropy, were made in [15, 16]. However, these attempts did not lead to concrete quantitative results.

Then it was argued in [17] that for the black hole horizon, all quantum numbers j are equal to $1/2$ (as is the case in the so-called «it from bit» model formulated earlier by Wheeler [18]). With these quantum numbers, one arrives easily at the equidistant area spectrum and at the value

$$\gamma = \frac{\ln 2}{\pi\sqrt{3}}$$

for the Barbero – Immirzi parameter. However, the result in [17] was demonstrated in [5] to be certainly incorrect[1)] because it violates the so-called holographic bound formulated in [22–24]. According to this bound, among the spherical surfaces of a given area, the surface of the black hole horizon has the largest entropy.

2. MICROCANONICAL ENTROPY OF BLACK HOLE

On the other hand, the requirement of maximum entropy allows one to find the correct structure of the horizon area [25], which in particular is of crucial importance for the problem of radiation of a quantized black hole.

We actually consider the «microcanonical» entropy S of a quantized surface defined as the logarithm of the number of states of this surface for a fixed area A (instead of a fixed energy in common problems). Obviously, this number of states K depends on the assumptions concerning the distinguishability of edges.

To analyze the problem, it is convenient to rewrite formula (3) as

$$A = 8\pi\gamma\, l_p^2 \sum_{jm} \sqrt{j(j+1)}\, \nu_{jm}, \qquad (5)$$

where ν_{jm} is the number of edges with given j and m. It can be demonstrated [5, 6] that the only reasonable assumption on the distinguishability of edges that may result in acceptable physical predictions (i.e., may comply both with the Bekenstein – Hawking relation and with the holographic bound) is as follows:

$$\begin{array}{llll} \text{nonequal } j, & \text{any } m & \longrightarrow & \text{distinguishable;} \\ \text{equal } j, & \text{nonequal } m & \longrightarrow & \text{distinguishable;} \\ \text{equal } j, & \text{equal } m & \longrightarrow & \text{indistinguishable.} \end{array}$$

Under this assumption, the number of states of the horizon surface for a given number ν_{jm} of edges with momenta j and their projections $j_z = m$, is obviously given by

$$K = \nu\,! \prod_{jm} \frac{1}{\nu_{jm}\,!}, \qquad (6)$$

where

$$\nu = \sum_j \nu_j\,, \quad \nu_j = \sum_m \nu_{jm}\,,$$

and the corresponding entropy equals

$$S = \ln K = \ln(\nu\,!) - \sum_{jm} \ln(\nu_{jm}\,!)\,. \qquad (7)$$

The structures of the last expression and of formula (5) are so different that the entropy certainly cannot be proportional to the area in the general case. However, this is the case for the maximum entropy in the classical limit.

In this limit, with all the effective «occupation numbers» large, $\nu_{jm} \gg 1$, we use the Stirling approximation, and hence the entropy is

$$S = \nu \ln \nu - \sum_{jm} \nu_{jm} \ln \nu_{jm}\,. \qquad (8)$$

We calculate its maximum for a fixed area A, i.e., for a fixed sum

$$N = \sum_{jm}^{\infty} \sqrt{j(j+1)}\, \nu_{jm} = \text{const}\,. \qquad (9)$$

[1)] Later, the result in [17] was also criticized in [19, 20]. Then an error made in [17] was acknowledged [21]. We demonstrate below that the result in [19, 20] is also incorrect.

The problem reduces to the solution of the system of equations

$$\ln \nu - \ln \nu_{jm} = \mu \sqrt{j(j+1)}\,, \tag{10}$$

where μ is the Lagrange multiplier for constraint (9). These equations can be rewritten as

$$\nu_{jm} = \nu \exp\left(-\mu\sqrt{j(j+1)}\right), \tag{11}$$

or

$$\nu_j = (2j+1)\exp\left(-\mu\sqrt{j(j+1)}\right)\nu. \tag{12}$$

We now sum expressions (12) over j, and with

$$\sum_j \nu_j = \nu,$$

arrive at the equation for μ:

$$\sum_{j=1/2}^{\infty} (2j+1)\exp\left(-\mu\sqrt{j(j+1)}\right) = 1. \tag{13}$$

Its solution is

$$\mu = 1.722. \tag{14}$$

Strictly speaking, the summation in formula (14) extends not to infinity but to some j_{max}. Its value follows from the obvious condition: none of the ν_{jm} should be less than unity. Then, for $\nu \gg 1$, Eq. (11) gives

$$j_{max} = \frac{\ln \nu}{\mu}\,. \tag{15}$$

It is well-known that the Stirling approximation for $n!$ has reasonably good numerical accuracy even for $n = 1$. Therefore, formula (15) for j_{max} is not just an estimate but has reasonably good numerical accuracy. The relative magnitude of the corresponding correction to (14) can be easily estimated as $\sim \ln \nu/\nu$.

We now return to Eq. (10). Multiplying it by ν_{jm} and summing over jm, we arrive, with constraint (9), at the following result for the maximum entropy for a given value of N:

$$S_{max} = 1.722\,N. \tag{16}$$

Therefore, with the Bekenstein–Hawking relation and formula (5), we find the value of the Barbero–Immirzi parameter

$$\gamma = 0.274. \tag{17}$$

Quite recently, this calculation with the same result, although with somewhat different motivation, was reproduced in [26].

We emphasize that the above calculation is not special for LQG only, but applies (with obvious modifications) to a more general class of approaches to the quantization of surfaces. The following assumption is actually necessary here: the surface should consist of sites of different sorts, such that there are ν_i sites of each sort i, with a generalized effective quantum number r_i (here, $\sqrt{j(j+1)}$) and a statistical weight g_i (here, $2j+1$). Then in the classical limit, with given functions r_i and g_i, the maximum entropy of a surface can be found, at least numerically, and is certainly proportional to the area of the surface.

As regards the previous attempts to calculate γ, one should indicate an apparent error in state counting made in [19, 20]. It can be easily checked that the transition from formula (25) to formulas (29) and (36) in [19] performed therein and then used in [20], is certainly valid under the assumption that only two maximum projections $\pm j$ are allowed for each quantum number j. But it cannot then hold for the correct number $2j+1$ of the projections. Therefore, it is not surprising that the equation for the Barbero–Immirzi parameter in [20] is

$$2\sum_{j=1/2}^{\infty} \exp\left(-\mu\sqrt{j(j+1)}\right) = 1\,, \tag{18}$$

instead of ours in (13) (see also the discussion of (18) in [26]).

The conclusion is obvious. Any restriction on the number of admissible states for the horizon, as compared to a generic quantized surface, be it the restriction to

$$j = 1/2\,, \quad m = \pm 1/2\,,$$

made in [17], or the restriction to

$$\text{any}\, j\,, \quad m = \pm j\,,$$

made in [19, 20], results in a conflict with the holographic bound.

3. QUANTIZATION OF ROTATING BLACK HOLE

In discussing the radiation spectrum of quantized black holes, one should take the angular momentum selection rules into account. Obviously, radiation of any particle with a non-vanishing spin is impossible if both initial and final states of the black hole are spherically symmetric. Therefore, to find the radiation spectrum, the quantization rule for the mass of a Schwarzschild black hole must be generalized to that of a rotating Kerr black hole.

To derive the quantization rule for a Kerr black hole, we return to the thought experiment analyzed in [2, 3]. Therein, under adiabatic capture of a particle with an angular momentum j, the angular momentum J of a rotating black hole changes by a finite amount j, but the horizon area A does not change. Of course, under some other variation of parameters, it is the angular momentum J that remains constant. In other words, we have here two independent adiabatic invariants, A and J, for a Kerr black hole with mass M.

Such a situation is quite common in ordinary mechanics. For example, the energy of a particle with mass m bound in the Coulomb field

$$U(r) = -\frac{\alpha}{r}$$

is

$$E = -\frac{m\alpha^2}{2\,(I_r + I_\phi)^2}\,, \tag{19}$$

where I_r and I_ϕ are the respective adiabatic invariants for the radial and angular degree of freedom. Of course, the energy E is in a sense also an adiabatic invariant, but it is invariant only under those variations of parameters that preserve both I_r and I_ϕ. In quantum mechanics, formula (19) becomes

$$E = -\frac{m\alpha^2}{2\,\hbar^2\,(n_r + 1 + l)^2}\,, \tag{20}$$

where n_r and l are the radial and orbital quantum numbers, respectively.

This example prompts the solution of the quantization problem for a Kerr black hole. It is conveniently formulated in terms of the so-called irreducible mass M_{ir} of a black hole, related by definition to its horizon radius r_h and area A as

$$r_h = 2kM_{ir}\,, \quad A = 16\pi k^2 M_{ir}^2\,. \tag{21}$$

Together with the horizon area A, the irreducible mass is an adiabatic invariant. In accordance with (3) and (9), it is quantized as

$$M_{ir}^2 = \frac{1}{2}\,m_p^2\,N\,, \tag{22}$$

where

$$m_p^2 = \hbar c/k$$

is the Planck mass squared.

For a Schwarzschild black hole, M_{ir} coincides with its ordinary mass M. But for a Kerr black hole, the situation is more interesting. Here,

$$M^2 = M_{ir}^2 + \frac{J^2}{r_h^2} = M_{ir}^2 + \frac{J^2}{4k^2 M_{ir}^2}\,, \tag{23}$$

where J is the internal angular momentum of a rotating black hole.

Now, taking (22) into account, we arrive at the following quantization rule for the mass squared M^2 of a rotating black hole:

$$M^2 = \frac{1}{2}\,m_p^2\left[\gamma N + \frac{J(J+1)}{\gamma N}\right]. \tag{24}$$

Obviously, as long as the black hole is far from an extremal one, i.e., while $\gamma N \gg J$, we can neglect the dependence of M^2 on J, and the angular momentum selection rules have practically no effect on the black hole radiation spectrum.

As regards the mass and irreducible mass of a charged black hole, they are related by

$$M = M_{ir} + \frac{q^2}{2r_h}, \tag{25}$$

where q is the black hole charge. This formula has a simple physical interpretation: the total mass (or total energy) M of a charged black hole consists of its irreducible mass M_{ir} and of the energy $q^2/2r_h$ of its electric field in the outer space $r > r_h$.

With $r_h = 2kM_{ir}$, relation (25) can be rewritten as

$$M^2 = M_{ir}^2 + \frac{q^4}{16k^2 M_{ir}^2} + \frac{q^2}{2k}\,. \tag{26}$$

Thus, for a charged black hole, M^2 is quantized as

$$M^2 = \frac{1}{2}\,m_p^2\left[\gamma N + \frac{q^4}{4\gamma N} + q^2\right]. \tag{27}$$

In fact, relations of this type (even in a more general form, for Kerr – Newman black holes, both charged and rotating) were already presented in the pioneering article [1], although with the equidistant quantization rule for M_{ir}^2, i.e., for the horizon area (see also [14]). More recently, the conclusion that the mass of a quantized black hole must be expressed via its quantized area and angular momentum, was made in the approach based on the notion of the so-called isolated horizons [27, 28].

Here, we do not mention the attempts to quantize rotating and charged black holes that resulted in weird quantization rules for $\hat{J}^2$ and $e^2/\hbar c$.

4. RADIATION SPECTRUM OF QUANTIZED BLACK HOLE

It follows from expression (24) that for a rotating black hole, the radiation frequency ω, which coincides with the loss ΔM of the black hole mass, is

$$\omega = \Delta M = T\mu\Delta N + \frac{1}{4kM}\,\frac{2J+1}{\gamma N}\Delta J, \qquad (28)$$

where ΔN and ΔJ are the respective losses of the area quantum number N and the angular momentum J. Here, in line with (24), we have used the identity

$$T = \frac{\partial M}{\partial S} = \frac{1}{8\pi kM}\,\frac{\partial M^2}{\partial M_{ir}^2} \qquad (29)$$

for the Hawking temperature T as well as formula (23).

In the same way, for a charged black hole, with formula (27), we obtain the radiation frequency

$$\omega = \Delta M = T\mu\Delta N + \frac{1}{4kM}\left(2 + \frac{q^2}{\gamma N}\right) q\,\Delta q\,, \qquad (30)$$

where Δq is the loss of the charge.

We are mainly interested in the first, temperature terms in (28) and (30), dominating everywhere except the vicinity of the extremal regime, where $J \to \gamma N$, or $q^2 \to 2\gamma N$, and $T \to 0$. The natural assumption is that the thermal radiation occurs when an edge with a given value of j disappears, which means that

$$\Delta N_j = r_j\,, \quad \omega_j = T\mu\, r_j\,. \qquad (31)$$

Thus we arrive at the discrete spectrum with a finite number of lines. Their frequencies start at

$$\omega_{min} = T\mu\sqrt{3}/2$$

and terminate at

$$\omega_{max} = T\ln\nu.$$

We recall that

$$j \le j_{max} = \ln\nu/\mu,$$

and hence the number of lines is not very large, $\sim 10^2$, if the black hole mass is comparable to the mass of the Sun. But because of the exponential decrease of the radiation intensity with ω or j (see below), the existence of ω_{max} and a finite number of lines are not of great importance.

To substantiate the assumption made, we return to the lower bound (4) on the change of the horizon area under an adiabatic capture of a particle. The presence of the gap (4) in this process means that this threshold capture effectively consists in the increase by unity of the occupation number ν_{jm} with the smallest j, equal to 1/2. If the capture were accompanied by a reshuffle of few occupation numbers, the change of the area could be easily made arbitrarily small. For instance, one could delete two edges with quantum numbers j_1 and j_2, and add an edge with the quantum number $j_1 + j_2$. Obviously, with $j_{1,2} \gg 1$, the area increase could be made arbitrarily small.

It is only natural to assume that in the radiation process as well, changing several occupation numbers instead of one is at least strongly suppressed. We thus arrive at Eqs. (31).

Our next assumption, at least as natural as this one, is that the probability of radiation of a quantum with the frequency ω_j is proportional to the occupation number ν_j. Correspondingly, the radiation intensity I_j at this frequency ω_j is proportional to $\nu_j\,\omega_j$:

$$I_j \sim \nu_j\omega_j \sim \nu(2j+1)\omega_j\exp(-\omega_j/T)\,. \qquad (32)$$

We compare this expression with the intensity of the black-body radiation in the Wien limit $\omega/T \gg 1$,

$$I(\omega) = A\frac{\omega^3}{4\pi^2}\exp(-\omega/T)\,d\omega\,, \qquad (33)$$

where A is the area of a spherical black body. First of all, our relation (32) for I_j directly reproduces the exponential factor of the Wien spectrum. Next, $d\omega$ in (33) goes over into $(1/2)\mu T$ because the limit $\omega/T \gg 1$ corresponds in our problem to $\sqrt{j(j+1)} \gg 1$, i.e., to

$$\sqrt{j(j+1)} \approx j + 1/2,$$

and the minimum increment of j is 1/2. Now, to reproduce the Wien profile, we supplement relation (32) with the following factors: some "oscillator strength" proportional to ω_j, obvious powers of μT, the Newton constant k (necessary to transform ν into A), and obvious numerical ones. We thus arrive at the final formula for the discrete radiation spectrum of a black hole:

$$I_j = AT^4\frac{\mu^4}{8\pi^2}j\left(j+\frac{1}{2}\right)(j+1)\times \\ \times\exp\left(-\mu\sqrt{j(j+1)}\right). \qquad (34)$$

Of course, because Wien spectrum (33) corresponds to $j \gg 1$, we cannot guarantee the exact structure of the j-dependence in formula (34), especially in the preexponential factor. For instance, it would perhaps be equally legitimate to write

$$j^{3/2}(j+1)^{3/2}$$

instead of

$$j(j+1/2)(j+1)$$

there. However, this ambiguity is not very essential, at least numerically.

We note that because the black hole temperature T is less than the minimum allowed frequency ω_{min}, this spectrum has no Rayleigh–Jeans region at all.

Now, the emission probability for a quantum of frequency $\omega_j = T\mu r_j$, i.e., the width of the corresponding line, is

$$\Gamma_j = \frac{I_j}{\omega_j} = AT^3\,\frac{\mu^3}{8\pi^2}\,(j+1/2)\sqrt{j(j+1)}\times \\ \times\exp\left(-\mu\sqrt{j(j+1)}\right). \quad (35)$$

The ratio of this natural line width to the distance

$$\Delta\omega_j = \omega_{j+1} - \omega_j \approx \frac{1}{2}\,\mu T$$

between the lines is very small numerically:

$$\frac{\Gamma_j}{\Delta\omega_j} \approx \frac{\mu^2}{16\pi^3}\,(j+1/2)\sqrt{j(j+1)}\times \\ \times\exp\left(-\mu\sqrt{j(j+1)}\right) \lesssim 10^{-3}. \quad (36)$$

Thus, the radiation spectrum of an isolated black hole is really discrete.

Finally, the total radiation intensity of a black hole is

$$I = \sum_j I_j = 0.150AT^4. \quad (37)$$

The numerical coefficient in this expression is close to that in the total intensity of the common thermal radiation, i.e., to the Stefan–Boltzmann constant

$$\pi^2/60 = 0.164.$$

The point is that the Rayleigh–Jeans contribution to the total intensity, which is completely absent in the present spectrum, would be small anyway.

Formulas (34) and (37) describe not only the thermal radiation of bosons, photons, and gravitons, but also the thermal radiation of fermions, massless neutrinos. However, in the last case, a proper account for the number of polarization states is necessary: for a two-component Dirac neutrino, the numerical factors in formulas (34) and (37) are two times smaller.

In fact, it was argued long ago [29] that the discrete thermal radiation spectrum of a black hole, with the equidistant quantization rule for the horizon area, should fit the Wien profile.

On the other hand, our conclusion of the discrete radiation spectrum of a black hole in LQG differs drastically from that of [30] according to which the black hole spectrum in LQG is dense.

As regards the nonthermal radiation of extremal black holes, described by the terms with ΔJ and Δq in Eqs. (28) and (30), these effects are due to tunneling (see a relatively recent discussion of the subject and a detailed list of relevant references in [31, 32]). The loss of charge by a charged black hole is in fact caused by the Coulomb repulsion between the black hole and the emitted particles with the same sign of charge. For a rotating black hole, the reason is the interaction of angular momenta: particles (mainly massless) whose total angular momentum is parallel to that of the black hole are repelled from it.

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