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FLAVORED EXOTIC MULTIBARYONS AND HYPERNUCLEI IN TOPOLOGICAL SOLITON MODELS

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The energies of baryon states with positive strangeness, or anticharm (antibeauty) are estimated in the chiral soliton approach, in the «rigid oscillator» version of the bound-state soliton model proposed by Klebanov and Westerberg. Positive strangeness states can appear as relatively narrow nuclear levels (Θ-hypernuclei), the states with heavy antiflavors can be bound with respect to strong interactions in the original Skyrme variant of the model (SK4 variant). The binding energies of antiflavored states are also estimated in the variant of the model with a 6-th order term in chiral derivatives added to the Lagrangian to stabilize solitons (SK6 variant). This variant is less attractive, and nuclear states with anticharm and antibeauty can be unstable relative to strong interactions. The chances to obtain bound hypernuclei with heavy antiflavors increase within the «nuclear variant» of the model with a rescaled model parameter (the Skyrme constant e or e' decreased by about 30 %), which is expected to be valid for baryon numbers greater than $B \sim 10$. The rational map approximation is used to describe multiskyrmions with the baryon number up to about 30 and to calculate the quantities necessary for their quantization (moments of inertia, sigma-term, etc.).

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1. INTRODUCTION

The remarkable recent discovery of the positive-strangeness pentaquark state [1] and its confirmation by several experiments [2] provided strong motivation for searches of other exotic states and revision of the existing ideas on the structure of hadrons and the role of the valence-quark picture in their description [3–8]. Subsequently, the discovery of the strangeness $S = -2$ state with charge -2, also manifestly exotic [9] (see [10] for a review of the previously existing data), and evidence for a narrow anticharmed baryon state [11] have been reported. Some experiments, however, did not confirm these results, see, e.g., [12] and [13], where some negative results were summarized and a pessimistic point of view was formulated. The high-energy physics community is now waiting for the results of high-statistics experiments; some plans for future pentaquark searches are presented, e.g., in [14].

The possible existence of such states has been forseen theoretically within the quark models [15–17][1)], as well as in chiral soliton models. The prediction of exotic states in chiral soliton models has not simple and instructive history, from the papers where the exotic antidecuplet and 27-plet of baryons were mentioned [18], a resonant behavior of the kaon – nucleon phase shift in the Θ channel was obtained in some version of the Skyrme model [19], first estimates of the antidecuplet mass were made [20, 21], and the masses of exotic baryon states were roughly estimated for arbitrary Baryon numbers B [22], to papers where more detailed calculations of the antidecuplet spectrum were performed [23–25], see also [26] for a recent discussion. The mass of the dibaryon with $S = +1$, $I = 1/2$ was determined to be only 590 MeV above nucleon–nucleon threshold within the soft rotator quantization

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[1)] The parity of lowest exotic states considered here is negative (see [7], however), in contrast with the chiral soliton model predictions, where it is positive. Spin and parity of exotic baryons are not yet measured.

scheme [27]. We note that paper [24], which predicted narrow width and low mass of the positive-strangeness state called[2)] Θ^+, stimulated experimental searches for such states, in particular, experiments [1] have been arranged specially to check the prediction of [24].

Theoretical ideas and methods that led to the prediction of such states within the chiral soliton models [23–25] have been criticized with quite sound reasoning in [4] and, in the large-N_c limit, in [29, 30]. In the absence of the complete theory of strong interactions, it was impossible in principle to provide firm predictions for the masses of states with the accuracy better than about several tens of MeV, and similarly for the widths of such states. One can agree with [29]: the fact that in some cases predictions coincided with the observed mass of the Θ^+ hyperon can be considered as «accidental», see also [28].

On the other hand, from the practical standpoint, the chiral soliton approach is useful and has a remarkable predictive power when at least one of the exotic baryon masses is fitted. The masses of exotic baryons with strangeness $S = -2$ and isospin $I = 3/2$ predicted in this way [31], 1.79 GeV for the antidecuplet component and 1.85 GeV for the 27-plet component, are close to the value 1.86 GeV measured later [9]. Calculations of the baryons spectra within the chiral soliton approach were more recently made in papers [32–36], not in contradiction with [31]; recent paper [37], where the interplay of rotational and vibrational modes has been investigated, should be mentioned specially. Some reviews and comparison of the chiral soliton approach with other models can be found, e.g., in [38].

The particular case of strangeness is in a certain respect more complicated in comparison with the case of other flavors: the rigid rotator quantization scheme is not quite valid in this case [29], whereas the bound-state approach is not quite good either [30]. In case of heavy flavors, the rotator quantization is not valid at all, but the bound-state approach becomes more adequate a compared to strangeness [30].

Baryons with heavy antiflavors are certainly not a new issue: they have been discussed in the literature long ago, with various results obtained for the energies of such states. The strange anticharmed pentaquark was obtained bound [39] in a quark model with the (u, d, s) $SU(3)$ flavor symmetry and in the limit of a very heavy c-quark. Long ago, there were already statements and suggestions in the literature that anticharm or antibeauty can be bound by chiral solitons in case of the baryon number $B = 1$ [40, 41] (the so called P-baryons). In [42], the mass differences of exotic baryons (Θ^+ and its analogies for anticharm and antibeauty) and nucleons were estimated in the flavor-symmetric limit for decay constants, $F_D = F_\pi$, in the chiral quark meson model. In [43], the antiflavor excitation energies were calculated in the rigid oscillator version [44] of the bound-state soliton model [45], for baryon numbers between 1 and 8. The rational map ansatz for multiskyrmions [46] was used as the starting configuration in the three-dimensional minimization $SU(3)$ program [47]. These energies were found to be close to 0.59 GeV for antistrangeness, 1.75 GeV for anticharm and 4.95 GeV for antibeauty, in the last two cases these energies are smaller than the masses of D- and B-mesons entering the Lagrangian [43]. The flavor symmetry breaking in flavor decay constants ($F_D/F_\pi > 1$) plays an important role for these estimates. This was therefore clear hint that such baryonic systems can be bound relative to strong interactions.

Similar results, in principle, follow from recent analysis within the bound-state soliton model [30] and within the diquark model [4]. The spectra of exotic states with heavy flavors have been estimated in different models, already after the discovery of the positive-strangeness pentaquark [48] (any baryon number), [49–53], and others. The possibility of the existence of nuclear matter fragments with positive strangeness was recently discussed in [54].

In this paper, we estimate the energies of ground states of multibaryons with baryon numbers up to approximately 30 with different (anti)flavors using a very transparent «rigid oscillator» model [44]. In the next section, the properties of multiskyrmions are considered that are required in calculating the energies of flavor excitations using the rational map approximation for $B > 1$ [46]. It is shown that the Θ^+ baryon is bound by nuclear systems, providing positive-strangeness multibaryons (Θ-hypernuclei), whose binding energy can reach several tens of MeV. The multiskyrmion configurations have some remarkable scaling properties, and as a result, the flavor and antiflavor excitation energies are close to those for $B = 1$. The quantization scheme (a slightly modified rigid oscillator version [44]) is described in Sec. 3, where the flavor and antiflavor excitation energies are also calculated. The masses (binding energies) of ground states of positive-strangeness states — Θ-hypernuclei — are presented in Sec. 4, followed by those for anticharmed or antibeautiful states. The last section contains some conclusions and prospects.

[2)] As was admitted recently in [28], the prediction of the low value of the mass $M_\Theta \approx 1530$ MeV «was to some extent a luck».

2. PROPERTIES OF MULTISKYRMIONS

Here, we calculate the properties of multiskyrmion configurations necessary for calculation of the flavor excitation energies and hyperfine splitting constants that govern the $1/N_c$-corrections to the energies of the quantized states. As already noted, the details of baryon–baryon interactions do not enter the calculations explicitly, although their effect is implicit via the integral characteristics of the bound states of skyrmions shown in Tables 1 and 2.

The Lagrangian of the Skyrme model in its well-known form depends on parameters F_π, F_D, and e and can be written as [55, 56]

$$\begin{aligned}\mathcal{L} = -\frac{F_\pi^2}{16}\,\mathrm{Tr}(l_\mu l^\mu) + \frac{1}{32e^2}\,\mathrm{Tr}[l_\mu, l_\nu]^2 + \\ + \frac{F_\pi^2 m_\pi^2}{16}\,\mathrm{Tr}(U+U^\dagger-2) + \\ + \frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24}\,\mathrm{Tr}\left(1-\sqrt{3}\lambda_8\right)(U+U^\dagger-2) + \\ + \frac{F_D^2 - F_\pi^2}{48}\,\mathrm{Tr}\left(1-\sqrt{3}\lambda_8\right)(Ul_\mu l^\mu + l_\mu l^\mu U^\dagger), \quad (1)\end{aligned}$$

where $U \in SU(3)$ is a unitary matrix incorporating chiral (meson) fields and $l_\mu = \partial_\mu U U^\dagger$. In this model, F_π is fixed at the physical value $F_\pi = 186$ MeV and m_D is the mass of the K-, D- or B-meson. The ratios F_D/F_π are known to be 1.22 and $2.28^{+1.4}_{-1.1}$ for kaons and D-mesons respectively. The Skyrme parameter e is close to 4 in numerical fits of the hyperons spectra (see the discussion at the end of this section). In the variant of the model with a 6-th order term added to stabilize solitons, the contribution added to the lagrangian density is [57–59]

$$L_6 = -\frac{c_6}{48}\,\mathrm{Tr}\left([l_\mu, l^\nu][l_\nu, l^\alpha][l_\alpha, l^\mu]\right), \quad (2)$$

where we introduce the coefficient 1/48 in the definition of the constant c_6 for further convenience. It is known that this term can be considered as an approximation to the exchange of ω-meson in the limit as $m_\omega \to \infty$ [57][3]. The flavor symmetry breaking (FSB) in the Lagrangian is of the usual form and is sufficient to describe the mass splittings of the octet and decuplet of baryons within the collective coordinate quantization approach [60]. A nice and useful feature of the Lagrangian in (1) and (2) is that it contains only the second power of the time derivative, which allows quantization to be performed without problems (see the next section).

The Wess–Zumino term, which is to be added to the action and which can be written as a five-dimensional differential form [56], plays an important role in the quantization procedure. It is given by

$$S^{WZ} = \frac{-iN_c}{240\pi^2}\int_\Omega d^5x\,\epsilon^{\mu\nu\lambda\rho\sigma}\,\mathrm{Tr}(l_\mu l_\nu l_\lambda l_\rho l_\sigma), \quad (3)$$

where Ω is a five-dimensional domain whose boundary is the four-dimensional space–time. Action (3) determines important topological properties of skyrmions, but it does not contribute to the static masses of classical configurations [21, 61]. The variation of this action can be represented as a well-defined contribution to the Lagrangian (an integral over the four-dimensional space–time).

We begin our calculations with $U \in SU(2)$. The classical mass of $SU(2)$ solitons, in the most general case, depends on three profile functions: f, α, and β and is given by

$$\begin{aligned}M_{cl} = \int \Bigg\{ \frac{F_\pi^2}{8}\left[\mathbf{l}_1^2+\mathbf{l}_2^2+\mathbf{l}_3^2\right] + \frac{1}{2e^2}\left[[\mathbf{l}_1\mathbf{l}_2]^2+[\mathbf{l}_2\mathbf{l}_3]^2+\right.\\ \left.+[\mathbf{l}_3\mathbf{l}_1]^2\right] + \frac{1}{4}F_\pi^2 m_\pi^2(1-c_f) + 2c_6(\mathbf{l}_1\mathbf{l}_2\mathbf{l}_3)^2\Bigg\}\,d^3r, \quad (4)\end{aligned}$$

where $\mathbf{l}_k$ are the $SU(2)$ chiral derivatives defined by $\vec{\partial}UU^\dagger = i\mathbf{l}_k\tau_k$, $k=1,2,3$. The general parameterization of U_0 for an $SU(2)$ soliton used here is given by

$$U_0 = c_f + s_f\boldsymbol{\tau}\cdot\mathbf{n}$$

with

$$n_z = c_\alpha, \quad n_x = s_\alpha c_\beta, \quad n_y = s_\alpha s_\beta,$$

$$s_f = \sin f, \quad c_f = \cos f.$$

For the rational map ansatz, we here use as the starting configurations [46]

$$\begin{aligned}n_x = \frac{2\,\mathrm{Re}\,R(\xi)}{1+|R(\xi)|^2}, \quad n_y = \frac{2\,\mathrm{Im}\,R(\xi)}{1+|R(\xi)|^2},\\ n_z = \frac{1-|R(\xi)|^2}{1+|R(\xi)|^2},\end{aligned} \quad (5)$$

where $R(\xi)$ is a ratio of polynomials of the maximal power B in the variable

$$\xi = \mathrm{tg}(\theta/2)\exp(i\phi),$$

[3] In (2), we use one of several possible forms of 6-th order term, all of which give the same contribution to the static mass of the $SU(2)$ solitons, see also a discussion in [57]. General consideration of higher-order terms and the discussion of their role in establishing skyrmion properties can be found in [58].

Table 1. Static characteristics of multiskyrmions: moments of inertia and the Σ-term Γ, $\tilde{\Gamma}$ in the SK4 variant of the model with $e = 4.12$, and for the SK6 variant of the model with $e' = 4.11$, in GeV^{-1}

B	Θ_I^{SK4}	$\Theta_F^{(0)SK4}$	Γ^{SK4}	$\tilde{\Gamma}^{SK4}$	Θ_I^{SK6}	$\Theta_F^{(0)SK6}$	Γ^{SK6}	$\tilde{\Gamma}^{SK6}$
1	5.56	2.05	4.80	14.9	5.13	2.28	6.08	15.8
2	11.5	4.18	9.35	22.0	9.26	4.94	14.0	24.7
3	14.4	6.34	14.0	27.0	12.7	7.35	20.7	30.4
4	16.8	8.27	18.0	31.0	15.2	8.93	24.5	33.7
5	23.5	10.8	23.8	35.0	18.7	11.8	32.8	38.3
6	25.4	13.1	29.0	38.0	21.7	14.1	39.3	41.6
7	28.9	14.7	32.3	44.0	23.9	15.4	42.5	43.4
8	33.4	17.4	38.9	47.0	27.2	18.5	51.6	46.9
9	37.8	20.6	46.3	47.5	30.2	21.1	59.1	49.7
10	41.4	23.0	52.0	50.0	32.9	23.5	65.8	51.9
11	45.2	25.6	58.5	52.4	35.8	26.1	73.6	54.3
12	48.5	28.0	64.1	54.6	38.4	28.3	79.9	56.2
13	52.1	30.5	70.2	56.8	41.2	30.8	87.1	58.1
14	56.1	33.6	78.2	58.9	44.3	34.0	96.9	60.5
15	59.8	36.3	85.1	60.9	47.1	36.7	105	62.4
16	63.2	38.9	91.5	62.8	49.7	39.3	112	64.1
17	66.2	41.2	96.8	64.6	52.1	41.3	118	65.4
18	70.3	44.5	106	66.4	55.2	44.8	129	67.5
19	73.9	47.4	113	68.2	58.0	47.8	138	69.2
20	77.5	50.4	121	69.9	60.8	50.8	147	70.8
21	80.9	53.2	128	71.5	63.5	53.6	156	72.4
22	84.3	56.0	136	73.1	66.1	56.4	164	73.8
23	88.0	59.2	144	74.7	69.0	59.7	174	75.4
24	91.3	62.0	151	76.2	71.6	62.5	183	76.7
25	94.7	64.9	159	77.6	74.2	65.4	192	78.0
26	98.2	68.1	168	79.1	77.0	68.7	202	79.4
27	102	71.1	176	80.5	79.7	71.7	211	80.8
28	105	74.3	185	81.9	82.5	75.1	222	82.2
32	118	86.4	217	87.2	93.0	87.4	260	86.9

with θ and ϕ being polar and azimuthal angles defining the direction of the radius vector $\mathbf{r}$. An important assumption is that the vector $\mathbf{n}$ depends on angular variables but is independent of r, whereas the profile $f(r)$ depends on the distance from the soliton center only. The explicit form of $R(\xi)$ is given in [46, 62] for different values of B. Within the rational map approximation, all characteristics of multiskyrmions that we need (including the mass and moments of inertia) depend on two quantities given by integrals over angular variables,

$$\mathcal{N} = \frac{1}{8\pi}\int r^2(\partial_i n_k)^2 d\Omega, \qquad \mathcal{I} = \frac{1}{8\pi}\int r^4[\vec{\partial}n_i\vec{\partial}n_k]^2 d\Omega, \tag{6}$$

which satisfy the inequality $\mathcal{I} \geq \mathcal{N}^2$ [46]. For the lowest-energy configuration, $\mathcal{N} = B$, $f(0) - f(\infty) = \pi$, and the value of $\mathcal{I}$ should be found by minimization of the map $S^2 \to S^2$ [46]. The classical mass of the

Table 2. Static characteristics of multiskyrmions: moments of inertia and Σ-term, Γ, $\tilde{\Gamma}$ for rescaled, or nuclear variants of the model: $e = 3.00$ in the SK4 and $e' = 2.84$ in the SK6 variants, in GeV^{-1}

B	$\Theta_I^{SK4^*}$	$\Theta_F^{(0)SK4^*}$	Γ^{SK4^*}	$\tilde{\Gamma}^{SK4^*}$	$\Theta_I^{SK6^*}$	$\Theta_F^{(0)SK6^*}$	Γ^{SK6^*}	$\tilde{\Gamma}^{SK6^*}$
1	12.8	4.66	10.1	19.6	14.2	6.21	15.3	22.3
2	24.3	9.87	20.9	28.8	25.7	13.6	35.9	34.7
3	34.7	15.1	31.7	35.6	35.5	20.4	53.9	42.5
4	42.9	19.4	40.1	41.1	43.2	25.0	64.6	46.9
5	53.5	25.4	53.2	46.2	52.9	32.9	86.2	53.1
6	62.6	30.7	64.7	50.6	61.4	39.4	103	57.4
7	69.6	34.9	72.5	54.4	68.0	43.3	112	59.8
8	79.9	41.3	87.4	58.2	77.3	51.7	135	64.4
9	88.9	47.1	101	61.7	85.7	58.9	154	67.9
10	97.4	52.6	113	64.9	93.5	65.3	171	70.8
11	106	58.5	126	67.9	102	72.5	191	73.8
12	114	63.8	138	70.8	109	78.7	207	76.1
13	122	69.5	151	73.6	117	85.4	225	78.6
14	132	76.3	168	76.3	125	94.0	249	81.5
15	140	82.3	182	78.8	133	101	269	83.9
16	148	88.1	196	81.2	141	108	287	86.0
17	155	93.2	207	83.5	148	114	302	87.6
18	164	100	225	85.9	156	123	328	90.1
19	173	107	241	88.1	164	131	350	92.2
20	181	113	257	90.3	172	139	372	94.1
24	213	138	320	98.2	202	170	457	101
28	245	165	387	105	232	202	550	107
32	275	191	454	112	261	234	640	113

multiskyrmion then simplifies to

$$M_{cl} = 4\pi \int \left[\frac{F_\pi^2}{8}\left(f'^2 + 2B\frac{s_f^2}{r^2}\right) + \frac{s_f^2}{2e^2r^2} \times \right.$$
$$\left. \times \left(2f'^2 B + s_f^2 \frac{\mathcal{I}}{r^2}\right) + 4c_6 \mathcal{I} f'^2 \frac{s_f^4}{r^4} + \rho_{M.t.} \right] r^2\, dr, \qquad (7)$$

which should and can be easily minimized for definite B and $\mathcal{I}$. The mass term density is simple for the starting $SU(2)$ skyrmion,

$$\rho_{M.t.} = F_\pi^2 m_\pi^2 (1 - c_f)/4.$$

The quantity λ can be introduced [59] that characterizes the relative weight of the 6-th order term as

$$\frac{\lambda}{(1-\lambda)^2} = c_6 F_\pi^2 e^4,$$

or

$$c_6 = \frac{\lambda}{F_\pi^2 e'^4}.$$

For the pure SK6 variant ($\lambda = 1$, $e \to \infty$, and $e' = e\sqrt{1-\lambda}$ is fixed), there is the relation

$$c_6 = \frac{1}{F_\pi^2 e'^4}.$$

The «flavor» moment of inertia plays a very important role in the procedure of $SU(3)$ quantization [61, 23], see formulas (16), (17), and (23) below. It defines the $SU(3)$ rotational energy

$$E_{rot}(SU_3) = \Theta_F(\Omega_4^2 + \Omega_5^2 + \Omega_6^2 + \Omega_7^2)/2$$

with Ω_a, $a = 4, \ldots, 7$, being the angular velocities of rotation in the $SU(3)$ configuration space. For $SU(2)$ skyrmions as starting configurations and the rational map ansatz describing the classical field configurations, Θ_F is given by [63, 64]

$$\Theta_F = \frac{1}{8}\int (1-c_f)\left[F_D^2 + \frac{1}{e^2}\left(f'^2 + 2B\frac{s_f^2}{r^2}\right) + \right. \\ \left. +2c_6\frac{s_f^2}{r^2}\left(2Bf'^2 + \mathcal{I}\frac{s_f^2}{r^2}\right)\right] r^2\,dr. \quad (8)$$

It is simply related with $\Theta_F^{(0)}$ of the flavor symmetric case ($F_D = F_\pi$),

$$\Theta_F = \Theta_F^{(0)} + (F_D^2/F_\pi^2 - 1)\Gamma/4, \quad (9)$$

with Γ defined in Eq. (11) below.

The isotopic momenta of inertia are the components of the corresponding tensor of inertia presented and discussed in many papers, see, e.g., [23, 61, 63]. For majority of multiskyrmions that we discuss, this tensor of inertia is close to the unit matrix multiplied by the isotopic moment of inertia:

$$\Theta_{ab} \approx \Theta_I \delta_{ab}, \quad \Theta_I = \Theta_{I,aa}/3.$$

This is exactly the case for $B = 1$ and, to within a good accuracy, for $B = 3$ and 7. Considerable deviations take place for the torus with $B = 2$, smaller ones for $B = 4$, 5, and 6, and, generally, deviations decrease with increasing the B-number. In our estimates, we use a very simple expression obtained within the rational map approximation [63, 64]:

$$\Theta_I = \frac{4\pi}{3}\int s_f^2\left[\frac{F_\pi^2}{2} + \frac{2}{e^2}\left(f'^2 + B\frac{s_f^2}{r^2}\right) + \right. \\ \left. + 8c_6 B s_f^4 \frac{f'^2}{r^2}\right] r^2\,dr. \quad (10)$$

At large enough baryon numbers, isotopic inertia (10) receives the leading contribution from the spherical envelope of the multiskyrmion where its mass is concentrated. The dimensions of this spherical bubble grow as $R_B \sim \sqrt{B}$ [63], and moments of inertia are roughly proportional to the baryon number.

The quantity Γ (or the Σ-term) determines the contribution of the mass term to the classical mass of solitons, and $\tilde{\Gamma}$ enters due to the presence of the FSB term proportional to the difference $F_D^2 - F_\pi^2$ in (1), the last term in (1). They define the potential in which the rigid oscillator moves and are given by

$$\begin{aligned} \Gamma &= \frac{F_\pi^2}{2}\int (1-c_f)\,d^3r, \\ \tilde{\Gamma} &= \frac{1}{4}\int c_f\left[(\vec{\partial} f)^2 + s_f^2(\vec{\partial} n_i)^2\right]. \end{aligned} \quad (11)$$

The relation

$$\tilde{\Gamma} = 2(M_{cl}^{(2)}/F_\pi^2 - e^2\Theta_F^{\mathrm{SK4}})$$

can also be established, where $M_{cl}^{(2)}$ is the second-order term contribution to the classical mass of the soliton and Θ_F^{SK4} is the Skyrme term contribution to the flavor moment of inertia. The calculated momenta of inertia Θ_F, Θ_I, Γ (or Σ-term), and $\tilde{\Gamma}$ for solitons with the baryon numbers up to 32 are presented in Tables 1 and 2. The Σ-term Γ receives contribution from the bulk of the multiskyrmion, where $c_f \sim -1$, and therefore grows faster than the moment of inertia Θ_I. The flavor inertia Θ_F receives contribution from the surface and the bulk of the multiskyrmion, and its behavior is intermediate between that of Γ and Θ_I.

For both variants of the model, SK4 and SK6, we calculated static characteristics of multiskyrmions for two values of the only parameter of the model, the constant e (or e') for the SK6 variant, related to c_6 via

$$e' = \frac{1}{(F_\pi^2 c_6)^{1/4}}.$$

For the SK4 variant of the model and $e = 4.12$, the numbers given in Table 1 for $B = 1$–8 are obtained as a result of direct numerical energy minimization in three dimensions performed using the calculation algorythm developed in [47]. Therefore, they differ slightly from those obtained in the pure rational-map approximation. This difference is maximum for $B = 2$ and decreases with increasing B. In all other cases, we used the rational map approximation with values of the Morse function $\mathcal{I}$ given in [46, 62].

The second value of the constants, $e = 3.00$ and $e' = 2.84$, leads to the «nuclear variant» of the model, which allows a quite successful description of the mass splittings of nuclear isotopes for atomic (baryon) numbers between approximately 10 and 30 [65]. The static characteristics of multiskyrmions change considerably when the constants e or e' change by about 30 %, see Table 2, because the dimensions of solitons scale as $1/F_\pi e$ and the isotopic mass splittings scale as $F_\pi e^3$. However, the flavor excitation energies change not crucially, even slightly for charm and beauty, according to the scale invariance of these quantities [63], as described in the next section.

3. FLAVOR AND ANTIFLAVOR EXCITATION ENERGIES

The $SU(3)$ effective action defined by (1), (3) leads to the collective Lagrangian obtained in [61]. To quantize the solitons in their $SU(3)$ configuration space, in the spirit of the bound-state approach to the description of strangeness proposed in [45, 44] and used in [63, 43], we consider the collective coordinate motion of the meson fields incorporated into the matrix U:

$$U(r,t) = R(t)U_0(O(t)\mathbf{r})R^\dagger(t), \quad R(t) = A(t)S(t), \tag{12}$$

where U_0 is the $SU(2)$ soliton embedded into $SU(3)$ in the usual way (into the upper-left corner), $A(t) \in SU(2)$ describes $SU(2)$ rotations, and $S(t) \in SU(3)$ describes rotations in the «strange», «charm» or «beauty» directions and $O(t)$ describes rigid rotations in real space. In the quantization procedure of the rotator with the help of $SU(3)$ collective coordinates, the following definition of angular velocities in the $SU(3)$ configuration space is accepted [61]:

$$R^\dagger(t)\dot{R}(t) = -\frac{i}{2}\Omega_\alpha\lambda_\alpha. \tag{13}$$

Here, λ_α, $\alpha = 1, \ldots, 8$, are the $SU(3)$ Gell-Mann matrices. For the quantization method proposed in [44] and used here, parameterization (12) is more convenient, the components Ω_α can be expressed via collective coordinates introduced in (12).

For definiteness, we consider the extension of the (u,d) $SU(2)$ Skyrme model in the (u,d,s) direction, with D being the field of K-mesons, but it is clear that quite similar extensions can also be made in the directions of charm or bottom. Therefore,

$$S(t) = \exp(i\mathcal{D}(t)), \quad \mathcal{D}(t) = \sum_{a=4,\ldots 7} D_a(t)\lambda_a, \tag{14}$$

where λ_a are the Gell-Mann matrices of the (u,d,s), (u,d,c) or (u,d,b) $SU(3)$ groups. The (u,d,c) and (u,d,b) $SU(3)$ groups are quite analogous to the (u,d,s) one. For the (u,d,c) group, a simple redefinition of hypercharge should be made. For the (u,d,s) group,

$$D_4 = \frac{K^+ + K^-}{\sqrt{2}}, \quad D_5 = \frac{i(K^+ - K^-)}{\sqrt{2}}, \quad \text{etc.}$$

For the (u,d,c) group,

$$D_4 = \frac{D^0 + \bar{D}^0}{\sqrt{2}}, \quad \text{etc.}$$

The angular velocities of the isospin rotations $\boldsymbol{\omega}$ are defined in the standard way [61]:

$$A^\dagger\dot{A} = -i\boldsymbol{\omega}\cdot\boldsymbol{\tau}/2.$$

Here, we do not consider the usual space rotations in detail because the corresponding momenta of inertia for baryonic systems are much greater than the isospin momenta of inertia, and for the lowest possible values of the angular momentum J, the corresponding quantum correction is either exactly zero (for even B) or small. The field D is small in magnitude. In fact, it is of the order $1/\sqrt{N_c}$ at least, where N_c is the number of colors in QCD, see Eq. (22). Therefore, the expansion of the matrix S in D can be made safely.

The mass term of Lagrangian (1) can be calculated exactly, without expansion in the powers of the field D, because the matrix S is given by [44]

$$S = 1 + i\mathcal{D}\sin d/d - \mathcal{D}^2(1-\cos d)/d^2$$

with

$$\mathrm{Tr}\,\mathcal{D}^2 = 2d^2.$$

We find that

$$\Delta\mathcal{L}_M = -\frac{F_D^2m_D^2 - F_\pi^2m_\pi^2}{4}(1-c_f)s_d^2. \tag{15}$$

This term can easily be expanded up to any order in d. The comparison of this expression with ΔL_M, within the collective coordinate approach of the quantization of $SU(2)$ solitons in the $SU(3)$ configuration space [23, 61], allows us to establish the relation

$$\sin^2 d = \sin^2\nu,$$

where ν is the angle of the λ_4 rotation or the rotation into the «strange» («charm», «beauty») direction. After some calculations, we find that the Lagrangian of the model, to the lowest order in the field D, can be written as

$$\begin{aligned} L = -M_{cl,B} + 4\Theta_{F,B}\dot{D}^\dagger\dot{D} - \\ - \left[\Gamma_B\left(\frac{F_D^2}{F_\pi^2}m_D^2 - m_\pi^2\right) + \tilde{\Gamma}_B(F_D^2 - F_\pi^2)\right]D^\dagger D - \\ - i\frac{N_cB}{2}(D^\dagger\dot{D} - \dot{D}^\dagger D). \end{aligned} \tag{16}$$

Here and below, D is the doublet K^+, K^0 (D^0, D^-, or B^+, B^0):

$$d^2 = \mathrm{Tr}\,\mathcal{D}^2/2 = 2D^\dagger D.$$

We keep the standard notation for the moment of inertia of the rotation into the «flavor» direction Θ_F for Θ_c, Θ_b or Θ_s [60, 61]; different notation is used in [44] (the index c denotes the charm quantum number, except in N_c). The contribution proportional to $\tilde{\Gamma}_B$ is suppressed by a small factor proportional to $(F_D^2 - F_\pi^2)/m_D^2$ in comparison with the term of the order of Γ, and is more important for strangeness. The term proportional to N_cB in (1) arises from the Wess–Zumino term in the action and is responsible for the difference of the excitation energies of strangeness and antistrangeness (flavor and antiflavor in the general case) [44, 45].

Following the canonical quantization procedure, the Hamiltonian of the system, including the terms of the order of N_c^0, can be written as [44]

$$H_B = M_{cl,B} + \frac{1}{4\Theta_{F,B}}\Pi^\dagger\Pi + \\ + \left[\Gamma_B \bar{m}_D^2 + \tilde{\Gamma}_B(F_D^2 - F_\pi^2) + \frac{N_c^2B^2}{16\Theta_{F,B}}\right] D^\dagger D + \\ + i\frac{N_cB}{8\Theta_{F,B}}(D^\dagger\Pi - \Pi^\dagger D), \quad (17)$$

where

$$\bar{m}_D^2 = (F_D^2/F_\pi^2)m_D^2 - m_\pi^2.$$

The momentum Π is canonically conjugate to variable D (see Eq. (18) below). Equation (17) describes an oscillator-type motion of the field D in the background formed by the (u,d) $SU(2)$ soliton. After diagonalization, which can be done explicitly following [44], the normal-ordered Hamiltonian can be written as

$$H_B = M_{cl,B} + \omega_{F,B}a^\dagger a + \bar{\omega}_{F,B}b^\dagger b + O(1/N_c), \quad (18)$$

where $a^\dagger$, $b^\dagger$ being the operators of creation of the strangeness (i.e., antikaons) and antistrangeness (flavor and antiflavor) quantum number, and $\omega_{F,B}$ and $\bar{\omega}_{F,B}$ are the frequencies of flavor (antiflavor) excitations. D and Π are related with a and b as [44]

$$D^i = \frac{1}{\sqrt{N_cB\mu_{F,B}}}(b^i + a^{\dagger i}), \\ \Pi^i = \frac{\sqrt{N_cB\mu_{F,B}}}{2i}(b^i - a^{\dagger i}), \quad (19)$$

where

$$\mu_{F,B} = \left[1 + \frac{16[\bar{m}_D^2\Gamma_B + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B]\Theta_{F,B}}{(N_cB)^2}\right]^{1/2} \quad (20)$$

is slowly varying quantity. For large mass m_D, it simplifies as

$$\mu_{F,B} \to 4\bar{m}_D\frac{\sqrt{\Gamma_B\Theta_{F,B}}}{N_cB}. \quad (21)$$

Obviously, at large N_c, $\mu \sim N_c^0 \sim 1$, and the dependence on the B-number is also weak, because both Γ_B, $\Theta_{F,B} \sim N_cB$[4]. For the lowest states, the values of D are small,

$$|D| \sim \left[16\Gamma_B\Theta_{F,B}\bar{m}_D^2 + N_c^2B^2\right]^{-1/4}, \quad (22)$$

and increase as $(2|F|+1)^{1/2}$ with increasing the flavor number $|F|$. As follows from (22) [44, 43], deviations of the field D from the vacuum one decrease with increasing the mass m_D, as well as with increasing the number of colors N_c; this explains why the method works for any m_D, including charm and beauty quantum numbers.

The excitation frequencies ω and $\bar{\omega}$ are

$$\omega_{F,B} = \frac{N_cB}{8\Theta_{F,B}}(\mu_{F,B} - 1), \\ \bar{\omega}_{F,B} = \frac{N_cB}{8\Theta_{F,B}}(\mu_{F,B} + 1). \quad (23)$$

The oscillation time can be estimated as

$$\tau_{osc} \sim \frac{\pi}{\omega_{F,B}} \sim \frac{2\pi(\Theta_B/\Gamma_B)^{1/2}}{m_D},$$

and hence it decreases with increasing m_D. As was observed in [63, 43], the difference

$$\bar{\omega}_{F,B} - \omega_{F,B} = \frac{N_cB}{4\Theta_{F,B}}$$

coincides, to the leading order in N_c, with the expression obtained in the collective coordinate approach [60, 61], see the Appendix. At large m_D, using (21) for the difference $\omega_{F,1} - \omega_{F,B}$, we obtain ($N_c = 3$)

$$\bar{\omega}_{F,1} - \bar{\omega}_{F,B} \approx \frac{\bar{m}_D}{2}\left[\left(\frac{\Gamma_1}{\Theta_{F,1}}\right)^{1/2} - \left(\frac{\Gamma_B}{\Theta_{F,B}}\right)^{1/2}\right] + \\ + \frac{3}{8}\left(\frac{B}{\Theta_{F,B}} - \frac{1}{\Theta_{F,1}}\right). \quad (24)$$

Obviously, at large m_D, the first term in (24) dominates and is positive if

$$\frac{\Gamma_1}{\Theta_{F,1}} \geq \frac{\Gamma_B}{\Theta_{F,B}}.$$

This is confirmed by the data in Table 1. We also note that the bracket in the first term in (24) is independent of the parameters of the model if the background $SU(2)$ soliton is calculated in the chirally symmetrical limit:

[4] Strictly, at large B, $\Gamma_B \sim B^{3/2}$, as explained above. But numerically at $B < 30$, $\Gamma_B \sim B$, as can be seen from Tables 1 and 2.

both Γ and Θ scale as $1/F_\pi e^3$. In a realistic case where the physical pion mass is included in (1), there is some weak dependence on the parameters of the model.

The FSB in the flavor decay constants, i.e., the fact that $F_K/F_\pi \approx 1.22$ and $F_D/F_\pi = 2.28^{+1.4}_{-1.1}$, should be taken into account. In the Skyrme model, this fact leads to the increase of the flavor excitation frequencies, which changes the spectra of flavored (c, b) baryons and puts them in a better agreement with the data [40]. It also leads to some changes of the total binding energies of baryonic system [43]. This is partly due to the large contribution of the Skyrme term to the flavor moment of inertia Θ_F. We note that in [44], the FSB in strangeness decay constant was not taken into account, and this led to underestimation of the strangeness excitation energies. Heavy flavors (c, b) have not been considered in these papers.

The addition of the term L_6 into starting Lagrangian (1) leads to modification of the flavored moment of inertia, according to the simple relation

$$\Theta_F = \Theta_F^{kin} + \Theta_F^{\mathrm{SK4}} + \Theta_F^{\mathrm{SK6}}.$$

But in order to adequately take the symmetry breaking terms into account, we have to express (in some order of N_c^{-1}) first set of coordinates (13) in terms of the collective coordinates $A(t)$ and $S(t)$ and substitute the result into L_{rot}.

The terms of the order of N_c^{-1} in the Hamiltonian, which also depend on the angular velocities of rotations in the isospin and the usual space, are not crucial but important for the numerical estimates of the spectra of baryonic systems. To calculate them, we should first obtain the Lagrangian of baryonic system including all the terms up to $O(1/N_c)$. The Lagrangian can be written in a compact form, slightly different from that in [44], as [42]

$$\begin{aligned} L \approx -M_{cl} + 2\Theta_{F,B}\Bigg[2\dot{D}^\dagger\dot{D}\left(1-\frac{d^2}{3}\right) - \\ -\frac{4}{3}\left(D^\dagger\dot{D}\dot{D}^\dagger D - (D^\dagger\dot{D})^2 - (\dot{D}^\dagger D)^2\right) + (\boldsymbol{\omega}\cdot\boldsymbol{\beta})\Bigg] + \\ +\frac{\Theta_{I,B}}{2}(\boldsymbol{\omega}-\boldsymbol{\beta})^2 - \left[\Gamma_B\tilde{m}_D^2 + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B\right]D^\dagger D\times \\ \times\left(1-\frac{d^2}{3}\right) + i\frac{N_cB}{2}\left(1-\frac{d^2}{3}\right)(\dot{D}^\dagger D - D^\dagger\dot{D}) - \\ -\frac{N_cB}{2}\boldsymbol{\omega}D^\dagger\boldsymbol{\tau}D, \end{aligned} \quad (25)$$

where

$$d^2 = 2D^\dagger D$$

and

$$\boldsymbol{\beta} = -i(\dot{D}^\dagger\boldsymbol{\tau}D - D^\dagger\boldsymbol{\tau}\dot{D}). \quad (26)$$

As we mentioned already, the role of the term L_6 reduces to the modification of the flavored inertia Θ_F in (25). It is a remarkable property of the starting Lagrangian including L_6 that only quadratic terms in Ω_a enter (25). To obtain this expression, we used the connection between components Ω_a and D, $\dot{D}$, ω_i:

$$\begin{aligned} \Omega_4^2 + \ldots + \Omega_7^2 = 8\dot{D}^\dagger\dot{D}\left(1-\frac{d^2}{3}\right) - \\ -\frac{16}{3}\left(D^\dagger\dot{D}\dot{D}^\dagger D - (D^\dagger\dot{D})^2 - (\dot{D}^\dagger D)^2\right) + 4(\boldsymbol{\omega}\cdot\boldsymbol{\beta}), \end{aligned}$$

and the component Ω_8 that determines the WZW term contribution,

$$\Omega_8 = \sqrt{3}\left[i(1-d^2/3)(D^\dagger\dot{D} - \dot{D}^\dagger D) + \boldsymbol{\omega}D^\dagger\boldsymbol{\tau}D\right].$$

Taking the terms proportional to $1/N_c$ into account, we find that the canonical variable Π conjugate to D is

$$\begin{aligned} \Pi = \frac{\partial L}{\partial\dot{D}^\dagger} = \\ = 4\Theta_{F,B}\left[\dot{D}\left(1-\frac{d^2}{3}\right) - \frac{2}{3}D^\dagger\dot{D}\,D + \frac{4}{3}\dot{D}^\dagger D\,D\right] + \\ + i(\Theta_{I,B} - 2\Theta_{F,B})\boldsymbol{\omega}\cdot\boldsymbol{\tau}D - i\Theta_{I,B}\boldsymbol{\beta}\cdot\boldsymbol{\tau}D + \\ + i\frac{N_cB}{2}\left(1-\frac{d^2}{3}\right)D. \end{aligned} \quad (27)$$

From (25), the body-fixed isospin operator is

$$\begin{aligned} \mathbf{I}^{bf} = \frac{\partial L}{\partial\boldsymbol{\omega}} = \Theta_{I,B}\boldsymbol{\omega} + (2\Theta_{F,B} - \Theta_{I,B})\boldsymbol{\beta} - \\ -\frac{N_cB}{2}D^\dagger\boldsymbol{\tau}D, \end{aligned} \quad (28)$$

which can also be written as

$$\mathbf{I}^{bf} = \Theta_I\boldsymbol{\omega} + \left(1-\frac{\Theta_I}{2\Theta_F}\right)\mathbf{I}_F - \frac{N_cB\Theta_I}{4\Theta_F}D^\dagger\boldsymbol{\tau}D \quad (29)$$

with the operator

$$\hat{\mathbf{I}}_F = \frac{i}{2}(D^\dagger\boldsymbol{\tau}\Pi - \Pi^\dagger\boldsymbol{\tau}D) = \frac{1}{2}(b^\dagger\boldsymbol{\tau}b - a^T\boldsymbol{\tau}a^{\dagger T}). \quad (30)$$

Using the connection between Π, $\dot{D}$, and D given by (27) in the leading order, we obtain

$$\boldsymbol{\beta} \approx \frac{1}{2\Theta_F}\left(\mathbf{I}_F + \frac{N_cB}{2}D^\dagger\boldsymbol{\tau}D\right). \quad (31)$$

For the states with a definite flavor quantum number, we should make the substitution

$$D^\dagger \boldsymbol{\tau} D \to -\frac{2\mathbf{I}_F}{N_c B \mu_F}$$

for flavor or

$$D^\dagger \boldsymbol{\tau} D \to \frac{2\mathbf{I}_F}{N_c B \mu_F}$$

for antiflavor; for matrix elements of states with a definite flavor, we can then write

$$\mathbf{I}^{bf} = \Theta_{I,B}\boldsymbol{\omega} + c_{F,B}\mathbf{I}_F \tag{32}$$

with

$$c_{F,B} = 1 - \frac{\Theta_{I,B}}{2\Theta_{F,B}\mu_{F,B}}(\mu_{F,B} - 1). \tag{33}$$

We also used that within our approximation,

$$\Theta_{I,B}\boldsymbol{\beta} \approx (1 - c_{F,B})\mathbf{I}_F. \tag{34}$$

A relation similar to (32) also holds for antiflavor with

$$c_{\bar{F},B} = 1 - \frac{\Theta_{I,B}}{2\Theta_{F,B}\mu_{F,B}}(\mu_{F,B} + 1), \tag{35}$$

and it therefore differs from (33) by the change $\mu \to -\mu$. Using the identities

$$-i\boldsymbol{\beta}\cdot\boldsymbol{\tau} D = 2D^\dagger D\dot{D} - (\dot{D}^\dagger D + D^\dagger \dot{D})D \tag{36}$$

and

$$\boldsymbol{\beta}^2 = 4D^\dagger D\dot{D}^\dagger \dot{D} - (\dot{D}^\dagger D + D^\dagger \dot{D})^2, \tag{37}$$

we find that the proportional to $1/N_c$ zero mode quantum corrections to the energies of skyrmions can be estimated [44] as

$$\Delta E_{1/N_c} = \\ = \frac{1}{2\Theta_{I,B}}\left[c_{F,B}I_r(I_r+1) + (1 - c_{F,B})I(I+1) + \right. \\ \left. + (\bar{c}_{F,B} - c_{F,B})I_F(I_F+1)\right], \tag{38}$$

where $I = I^{bf}$ is the value of the isospin of the baryon or baryon system, I_r is the quantity analogous to the «right» isospin I_r in the collective coordinate approach [61], and

$$\mathbf{I}_r = \mathbf{I}^{bf} - \mathbf{I}_F.$$

The hyperfine structure constants $c_{F,B}$ are given in (33) and $\bar{c}_{F,B}$ are defined by the relations

$$1 - \bar{c}_{F,B} = \frac{\Theta_{I,B}}{\Theta_{F,B}(\mu_{F,B})^2}(\mu_{F,B} - 1), \\ 1 - \bar{c}_{\bar{F},B} = -\frac{\Theta_{I,B}}{\Theta_{F,B}(\mu_{F,B})^2}(\mu_{F,B} + 1). \tag{39}$$

For nucleons,

$$I = I_r = 1/2, \quad I_F = 0$$

and

$$\Delta E_{1/N_c}(N) = \frac{3}{8\Theta_{I,1}},$$

for the Δ-isobar,

$$I = I_r = 3/2, \quad I_F = 0,$$

and

$$\Delta E_{1/N_c}(\Delta) = \frac{15}{8\Theta_{I,1}},$$

as in the $SU(2)$ quantization scheme. The Δ-N mass splitting is described satisfactorily according to the values of Θ_I presented in Table 1.

As can be seen in Table 3, the flavor excitation energies somewhat decrease in the SK4 variant as the B-number increases from 1 to 7, but further these energies increase again and exceed the $B = 1$ value for $B \geq 20$. The last property can be connected, however, with specific charachter of the rational-map approximation (the quantity Γ increases faster than the flavored inertia Θ_F, see (24)), which becomes less realistic for larger values of B. Such a behavior of the frequencies is important for conclusions about the possible existence of hypernuclei [66]. The Table 3 is presented here for comparison with antiflavor excitation energies presented in Table 4. Generally, the rigid oscillator version of the bound state model that we use here overestimates the flavor excitation energies. But phenomenological consequences derived in [63, 66] for the binding energies of strange $S = -1$ hypernuclei are based mainly on the differences of these energies. The qualitative and in some cases quantitative agreement takes place between the data for binding energies of ground states of hypernuclei with atomic numbers between 5 and 20 and the results of calculations within the SK4 variant of the chiral soliton model, with the collective motion of solitons in the $SU(3)$ configuration space taken into account [66].

Another peculiarity of interest is that for the rescaled variant of the model, the charm and beauty excitation energies are very close to those of the original variant (scaling property), but differ more substantially for strangeness, being greater by approximately 30 – 40 MeV. This somewhat unexpected behavior is related with the fact that flavor excitation energies appear as a result of subtraction of two quantities that behave differently under rescaling, see (23).

Similar to flavor energies, there is remarkable universality of antiflavor excitation energies for different baryon numbers, especially for anticharm and an-

Table 3. Flavor excitation energies for strangeness, charm, and beauty, in GeV. $e = 4.12$ for the SK4 variant and $e' = 4.11$ for the SK6 variant. For rescaled variants (the numbers marked with *), $e = 3.00$ and $e' = 2.84$ for SK4 and SK6 variants, correspondingly. The ratio $F_D/F_\pi = 1.5$ for charm and $F_B/F_\pi = 2$ for beauty

B	ω_s^{SK4}	ω_c^{SK4}	ω_b^{SK4}	ω_s^{SK6}	ω_c^{SK6}	ω_b^{SK6}	$\omega_s^{SK4^*}$	$\omega_c^{SK4^*}$	$\omega_b^{SK4^*}$	$\omega_s^{SK6^*}$	$\omega_c^{SK6^*}$	$\omega_b^{SK6^*}$
1	0.307	1.54	4.80	0.336	1.61	4.93	0.345	1.55	4.77	0.375	1.62	4.89
2	0.298	1.52	4.77	0.346	1.64	4.98	0.339	1.54	4.75	0.386	1.66	4.95
3	0.293	1.51	4.76	0.342	1.64	4.98	0.336	1.54	4.74	0.385	1.66	4.95
4	0.285	1.50	4.74	0.328	1.62	4.95	0.330	1.52	4.72	0.377	1.64	4.93
5	0.290	1.51	4.75	0.334	1.63	4.96	0.334	1.53	4.74	0.380	1.65	4.94
6	0.290	1.51	4.76	0.332	1.63	4.96	0.334	1.54	4.74	0.379	1.65	4.94
7	0.285	1.50	4.74	0.324	1.62	4.95	0.331	1.53	4.73	0.374	1.64	4.93
8	0.290	1.51	4.76	0.329	1.63	4.96	0.335	1.54	4.75	0.377	1.65	4.94
9	0.292	1.52	4.77	0.331	1.63	4.97	0.336	1.54	4.76	0.378	1.65	4.94
10	0.293	1.52	4.78	0.331	1.63	4.97	0.337	1.55	4.76	0.378	1.65	4.94
11	0.295	1.53	4.79	0.332	1.63	4.97	0.338	1.55	4.77	0.378	1.65	4.95
12	0.295	1.53	4.79	0.331	1.63	4.97	0.338	1.55	4.77	0.378	1.65	4.95
13	0.296	1.53	4.79	0.332	1.63	4.98	0.339	1.55	4.77	0.378	1.65	4.95
14	0.300	1.54	4.80	0.335	1.64	4.98	0.342	1.56	4.79	0.379	1.65	4.95
15	0.301	1.54	4.81	0.336	1.64	4.99	0.343	1.56	4.79	0.380	1.66	4.95
16	0.302	1.54	4.81	0.336	1.64	4.99	0.343	1.56	4.79	0.380	1.66	4.96
17	0.302	1.54	4.81	0.335	1.64	4.99	0.343	1.56	4.79	0.379	1.66	4.95
20	0.308	1.56	4.84	0.340	1.65	5.00	0.347	1.58	4.81	0.382	1.66	4.96
24	0.312	1.57	4.85	0.343	1.66	5.01	0.351	1.58	4.83	0.384	1.66	4.97
28	0.316	1.58	4.87	0.347	1.66	5.02	0.354	1.59	4.85	0.385	1.67	4.98
32	0.319	1.59	4.88	0.349	1.67	5.02	0.356	1.60	4.86	0.386	1.67	4.98

tibeauty: variations do not exceed few percent. It follows from Table 4 that there is some decrease of the antiflavor excitation energies as B increases from 1; this effect is striking for the SK4 variant and especially for strangeness. Within the SK6 variant, the $B = 1$ energies for anticharm and antibeauty are slightly smaller than for $B \geq 2$.

For strangeness, $\bar{\omega}_s$ decreases with increasing the B-number in most cases, as can be seen in Table 4 (except in the rescaled SK6 variant, where the $B = 1$ energy is slightly smaller than the $B = 2$ one), but it is always greater than the kaon mass, and therefore the state with positive strangeness can decay strongly into kaon and some final nucleus or nuclear fragments.

The heavy antiflavor excitation energies also reveal a notable scale independence, i.e., the values obtained with constant $e = 4.12$ and 3.00 (SK4 variant) shown in Tables 3 and 4, are close to each other within several percent, as well as the values for $e' = 4.11$ and 2.84 for the SK6 variant. This was actually expected from general arguments for large values of the FSB meson mass [43]. The change of numerical values of these energies is, however, important for conclusions concerning the binding energies of nuclear states with antiflavors. All excitation energies of antiflavors are smaller for rescaled variants, i.e., when the constants e or e' are decreased by about 30 %. This seems natural because dimensions of multiskyrmions, which scale as $1/F_\pi e$, increase due to this change, and all energies become «softer». Such a behavior occurs because antiflavor energies are the sum of two terms (see above (23)) that behave (roughly!) similarly under rescaling. Remarkably, the decrease of energies due to the rescaling is of the order of 100 MeV in all cases (e.g., for antistrangeness and $B = 1$, it is 119 MeV in the SK4 variant and 116 MeV in the SK6 variant), and slightly smaller for $\bar{c}$ (decrease due to rescaling about 100 MeV) and $\bar{b}$ (decrease by 110 MeV).

Table 4. Antiflavor excitation energies for strangeness, charm and beauty, as in Table 3. In the original variants of the model, $e = 4.12$ for the SK4 variant and $e' = 4.11$ for the SK6 variant. The numbers with * are for the rescaled variants of the model, $e = 3.0$ for the SK4 variant and $e' = 2.84$ for the SK6 variant. The ratio $F_D/F_\pi = 1.5$ for charm and $F_B/F_\pi = 2$ for beauty

B	$\bar\omega_s^{SK4}$	$\bar\omega_c^{SK4}$	$\bar\omega_b^{SK4}$	$\bar\omega_s^{SK6}$	$\bar\omega_c^{SK6}$	$\bar\omega_b^{SK6}$	$\bar\omega_s^{SK4^*}$	$\bar\omega_c^{SK4^*}$	$\bar\omega_b^{SK4^*}$	$\bar\omega_s^{SK6^*}$	$\bar\omega_c^{SK6^*}$	$\bar\omega_b^{SK6^*}$
1	0.591	1.75	4.94	0.584	1.79	5.04	0.472	1.65	4.83	0.468	1.69	4.93
2	0.571	1.72	4.90	0.571	1.80	5.08	0.459	1.63	4.81	0.470	1.72	4.99
3	0.564	1.71	4.89	0.569	1.80	5.07	0.455	1.63	4.80	0.468	1.72	4.99
4	0.567	1.71	4.87	0.580	1.80	5.06	0.454	1.62	4.78	0.468	1.71	4.97
5	0.558	1.71	4.88	0.571	1.80	5.07	0.452	1.62	4.80	0.466	1.71	4.98
6	0.555	1.71	4.88	0.571	1.80	5.07	0.451	1.62	4.80	0.465	1.71	4.98
7	0.559	1.71	4.88	0.578	1.80	5.06	0.451	1.62	4.79	0.466	1.71	4.97
8	0.553	1.71	4.89	0.571	1.80	5.07	0.450	1.63	4.80	0.465	1.71	4.98
9	0.550	1.71	4.90	0.569	1.80	5.07	0.450	1.63	4.81	0.465	1.71	4.98
10	0.549	1.71	4.90	0.569	1.80	5.07	0.450	1.63	4.82	0.465	1.71	4.98
11	0.547	1.71	4.90	0.567	1.80	5.08	0.450	1.63	4.82	0.464	1.71	4.98
12	0.547	1.72	4.91	0.568	1.80	5.08	0.450	1.63	4.82	0.464	1.71	4.98
13	0.546	1.72	4.91	0.567	1.80	5.08	0.450	1.64	4.83	0.464	1.71	4.99
14	0.543	1.72	4.92	0.564	1.80	5.08	0.450	1.64	4.84	0.464	1.72	4.99
15	0.542	1.72	4.92	0.563	1.80	5.08	0.450	1.64	4.84	0.464	1.72	4.99
16	0.541	1.72	4.93	0.562	1.80	5.08	0.450	1.64	4.85	0.464	1.72	4.99
17	0.542	1.72	4.93	0.564	1.80	5.09	0.450	1.64	4.85	0.464	1.72	4.99
18	0.540	1.72	4.93	0.561	1.81	5.09	0.451	1.65	4.85	0.464	1.72	5.00
19	0.539	1.73	4.94	0.559	1.81	5.09	0.451	1.65	4.86	0.464	1.72	5.00
20	0.538	1.73	4.94	0.558	1.81	5.09	0.451	1.65	4.86	0.464	1.72	5.00
24	0.536	1.73	4.96	0.555	1.81	5.10	0.452	1.66	4.88	0.463	1.72	5.00
28	0.533	1.74	4.97	0.552	1.81	5.10	0.453	1.67	4.89	0.463	1.72	5.01
32	0.532	1.74	4.98	0.550	1.81	5.11	0.453	1.67	4.90	0.463	1.73	5.01

4. THE BINDING ENERGIES OF Θ^+-HYPERNUCLEI AND ANTICHARMED (ANTIBEAUTIFUL) HYPERNUCLEI

In view of large enough values of the antistrangeness excitation energies, one cannot speak about positive-strangeness hypernuclei that decay weakly, similarly to ordinary $S = -1$ hypernuclei. But one can speak about Θ-hypernuclei where the Θ-hyperon is bound by several nucleons. A puzzling property of pentaquarks is their small width, $\Gamma_\Theta <\sim 10$ MeV according to experiments where Θ^+ has been observed [1, 2], and probably even smaller, according to analyses of kaon–nucleon interaction data [67]. Possible explanations, from some numerical cancellation [24] and cancellation in a large-N_c expansion [68] to qualitative one in terms of the quark-model wave function [3, 4] and calculation using operator product expansion [69] have been proposed[5]. The width of nuclear bound states of Θ should be of the same order of magnitude as the width of Θ^+ itself or smaller: besides the smaller energy release, some suppression due to the Pauli blocking for the final nucleon from Θ decay can occur for heavier nuclei.

For anticharm and antibeauty, the excitation energies are smaller than the masses of the D- or B-meson, and it makes sense to consider the possibility of the ex-

[5] In most of variants of the explanation, it is difficult to expect the width of the Θ-hyperon of the order 1 MeV, as obtained in [67]. Therefore, verification of the data analyzed in [67] seems to be of first priority.

istence of anticharmed or antibeauty hypernuclei that have the life time characteristic of weak interactions.

In the bound-state soliton model, and in its rigid oscillator version as well, the states predicted do not correspond a priory to definite $SU(3)$ or $SU(4)$ representations. They can be ascribed to definite irreducible $SU(3)$ representations as was shown in [44, 43]. Due to configuration mixing caused by the flavor symmetry breaking, each state with a definite value of flavor, s, c or b, is some mixture of the components of several irreducible $SU(3)$ representations with a given value of F and isospin I, which is strictly conserved in our approach (unless manifestly isospin-violating terms are included into the Lagrangian). In case of strangeness, as calculations show (see, e.g., [27]), this mixture is usually dominated by the lowest irreducible $SU(3)$ representation, and admixtures do not exceed several percents. The situation changes for charm or beauty quantum numbers, where admixtures can have the weight comparable with the weight of the lowest configuration. However, we here consider the simplest possibility of one lowest irreducible representation, for rough estimates.

Let (p, q) characterize the irreducible $SU(3)$ representation to which baryon system belongs. The quantization condition [61]

$$p + 2q = N_c B,$$

for arbitrary N_c, then changes to

$$p + 2q = N_c B + 3m,$$

where m is related to the number of additional quark–antiquark pairs $n_{q\bar{q}}$ present in the quantized states, $n_{q\bar{q}} \geq m$ [22, 70]. The nonexotic states with $m = 0$, or minimal states, have

$$p + 2q = 3B,$$

($N_c = 3$ in what follows), and states with the lowest «right» isospin $I_r = p/2$ have

$$(p, q) = (0, 3B/2)$$

for even B and

$$(p, q) = (1, (3B - 1)/2)$$

for odd B [22, 27]. For example, the state with $B = 1$, $|F| = 1$, $I = 0$ and $n_{q\bar{q}} = 0$ should belong to the octet of the (u, d, s) or (u, d, c) $SU(3)$ group, if $N_c = 3$; see also [44]. For the first exotic states, the lowest possible irreducible $SU(3)$ representations (p, q) for each value of the baryon number B are defined by the relations

$$p + 2q = 3(B + 1),$$

$$p = 1, \quad q = (3B + 2)/2$$

for even B, and

$$p = 0, \quad q = (3B + 3)/2$$

for odd B. For example, we have $\overline{35}$, $\overline{80}$, $\overline{143}$, and $\overline{224}$-plets for $B = 2$, 4, 6, and 8, and $\overline{28}$, $\overline{55}$ and $\overline{91}$-plets for $B = 3$, 5, and 7.

Because we are interested in the lowest energy states, we here discuss the baryonic systems with the lowest allowed angular momentum, i.e., $J = 0$, for $B = 4$, 6, etc. and $J = 1/2$ for odd values of the B-number. There are some deviations from this simple law for the ground states of real nuclei, but anyway, the correction to the energy of quantized states due to collective rotation of solitons is small and decreases with increasing B because the corresponding moment of inertia increases proportionally to B^2 [63, 64]. Moreover, the J-dependent correction to the energy may cancel in the differences of energies of flavored and flavorless states, and we therefore neglect these contributions in our rough estimates.

For the nonexotic states, we previously considered the energy difference between the state with flavor F belonging to the (p, q) irreducible representation and the ground state with $F = 0$ and the same angular momentum and (p, q) [66]. The situation is different for exotic states, because exotic and nonexotic states have different values of (p, q). The difference between $\bar{\omega}$ and ω,

$$\bar{\omega} - \omega = \frac{N_c}{4\Theta_F}$$

takes this distinction into account in the values of (p, q), as shown explicitly in Appendix.

For $B = 1, 3, 5, \ldots$, we have $I = I_r = 1/2$, $I_F = 0$ for the ground state, and therefore the correction

$$\Delta E_{1/N_c} = \frac{I(I+1)}{2\Theta_{I,B}} = \frac{3}{8\Theta_{I,B}}.$$

For exotic antiflavored state, we have $I = 0$, $I_r = I_F = 1/2$, and the corrections equal to

$$\Delta E_{1/N_c} = \frac{3\bar{c}_{F,B}}{8\Theta_{I,B}}.$$

For the difference of energies between exotic and nonexotic ground states, we obtain

$$\Delta E_{B,F} = \bar{\omega}_{F,B} + \frac{3(\bar{c}_{\bar{F},B} - 1)}{8\Theta_{F,B}} = \bar{\omega}_{F,B} + \frac{3(\mu_{F,B} + 1)}{8\mu_{F,B}^2 \Theta_{F,B}}. \quad (40)$$

We note that the moment of inertia Θ_I does not enter the difference of energies (40).

For $B = 4, 6, \ldots$ the ground state has $I = I_r = I_F = 0$ (as for nucleus ^{4}He) and

$$\Delta E_{1/N_c} = 0.$$

For the first exotic states, $I = I_F = 1/2$, and we have a choice for I_r, $I_r = 0$ or 1. If

$$c_{\bar{F},B} = 1 - \frac{\Theta_{I,B}(\mu_{F,B}+1)}{2\Theta_{F,B}\mu_{F,B}} > 0,$$

we have $I_r = 0$, and if $c_{\bar{F},B} < 0$, we should take $I_r = 1$. In the first case, the correction to the energy of the state

$$\Delta E_{1/N_c} = \frac{3(1+\bar{c}_{\bar{F},B} - 2c_{\bar{F},B})}{8\Theta_{I,B}} = \frac{3(\mu_{F,B}+1)^2}{8\Theta_{F,B}\mu_{F,B}^2)}.$$

For $B = 1$, the difference of the Θ_F and nucleon masses is

$$\Delta M_{\Theta_F N} = \overline{\omega}_{F,1} - \frac{3(1-\bar{c}_{\bar{F},1})}{8\Theta_{I,1}} = \overline{\omega}_{F,1} + \frac{3(\mu_{F,1}+1)}{8\mu_{F,1}^2\Theta_{F,1}}. \quad (41)$$

The difference of masses of the Θ and Λ-hyperons also is of interest and can be represented in the simple form

$$\Delta M_{\Theta_F \Lambda_F} = \overline{\omega}_{F,1} - \omega_{F,1} + \frac{3(\bar{c}_{\bar{F},1} - \bar{c}_{F,1})}{8\Theta_{I,1}} = \frac{3(\mu_{F,1}+1)}{4\mu_{F,1}\Theta_{F,1}}. \quad (42)$$

The binding energy differences $\Delta\epsilon_{\bar{s},\bar{c},\bar{b}}$ are the changes of binding energies of the lowest baryon system with flavor $\bar{s}$, $\bar{c}$ or $\bar{b}$ and isospin $I = 0$ (for odd B) and $I = 1/2$ (for even B) in comparison with the usual u, d nuclei (when one nucleon is replaced by the Θ-hyperon). The classical masses of skyrmions are cancelled in such differences:

$$\Delta\epsilon_{B,F} = \Delta E_{gr.st.}(B) - \Delta E(B,F) + \Delta M_{\Theta_F N}. \quad (43)$$

It follows from (40) that for an odd B-number, this change of the binding energy of the system is

$$\Delta\epsilon_{B,F} = \bar{\omega}_{F,1} - \bar{\omega}_{F,B} + \frac{3(\mu_{F,1}+1)}{8\mu_{F,1}^2\Theta_{F,1}} - \frac{3(\mu_{F,B}+1)}{8\mu_{F,B}^2\Theta_{F,B}}. \quad (44)$$

Evidently, in the limit of very heavy flavor, $\mu_F \to \infty$,

$$\Delta\epsilon_{B,F} \to \bar{\omega}_{F,1} - \bar{\omega}_{F,B}. \quad (45)$$

For B-numbers $4, 6, \ldots$, we obtain

$$\Delta\epsilon_{B,F} = \bar{\omega}_{F,1} - \bar{\omega}_{F,B} + \frac{3(\mu_{F,1}+1)}{8\mu_{F,1}^2\Theta_{F,1}} - \frac{3(\mu_{F,B}+1)^2}{8\mu_{F,B}^2\Theta_{F,B}}. \quad (46)$$

In the limit of very heavy flavor, it follows from (46) that

$$\Delta\epsilon_{B,F} = \bar{\omega}_{F,1} - \bar{\omega}_{F,B} - \frac{3}{8\Theta_{F,B}}, \quad (47)$$

and hence, in comparison with the case of odd B-numbers, there is an additional contribution decreasing with the increase of the B-number (because inertia increases with B) from approximately 25 MeV for $B = 3$.

The formulas (44) and (46) allow us to perform numerical estimates for the binding energies of antiflavored states, using the results for frequencies and moments of inertia presented in previous Tables.

For a special case of $B = 2$, we present in Tables 5–7 the binding energies of flavored $J = 0$ states relative to NN scattering state ($I = 1$, $J = 0$), which differ from (46) by adding $1/\Theta_{I,B=2}$.

One should keep in mind that for the SK4 model, the value of the Θ^+ mass is equal to 1588 MeV, which is approximately 150 MeV above the kaon–nucleon threshold. Therefore, the states with the largest binding energy shown in Table 5 are unstable relative to strong interactions. For the SK6 variant, $M_\Theta = 1566$ MeV and the binding energies are considerably smaller, by approximately 40–50 MeV in some cases (this is the main feature of the SK6 variant). For the rescaled variants, the difference between both variants is reduced considerably, but the binding energies are then underestimated.

From the phenomenological standpoint, we should describe the $B = 1$ states with the original variants of models, i.e., $e = 4.12$, $e' = 4.11$ and states with $10 < B = A < 30$, using rescaled variants, as is suggested by results of [65]. This procedure gives most optimistic values of $\Delta\epsilon_{S=+1}$, about 145 MeV for the SK4 variant and approximately 140 MeV for the SK6 variant. However, uncertainty of this prediction is few tens of MeV, at least.

For anticharm and antibeauty, there is considerable difference between the SK4 and SK6 variants (Tables 6 and 7). The mass of the Θ_c-hyperon in the SK4 model is equal to 2700 MeV and the mass of Θ_b is 5880 MeV, both well below the threshold for strong decay. For the SK6 variant, these masses are by 40 and 100 MeV greater, but also below the threshold. The SK6 variant is less attractive than the SK4 variant, mainly because

Table 5. The binding energy differences and total binding energies of positive strangeness Θ^+-hypernuclei (in MeV) for the SK4 and SK6 variants of the model in rational-map approximation

B	$\Delta\epsilon^{SK4}$	ϵ^{SK4}	$\Delta\epsilon^{SK6}$	ϵ^{SK6}	$\Delta\epsilon^{SK4^*}$	ϵ^{SK4^*}	$\Delta\epsilon^{SK6^*}$	ϵ^{SK6^*}
2	47	47	75	75	25	25	17	17
3	67	76	45	53	26	34	4	12
4	20	49	−4	24	9	38	−8	20
5	81	108	47	74	30	57	6	33
6	56	88	24	56	20	52	−1	31
7	83	121	41	80	32	70	7	45
8	69	126	31	87	24	81	2	58
9	94	152	53	110	33	90	8	66
10	79	144	39	103	27	92	4	68
11	99	173	56	130	33	108	9	84
12	86	178	43	135	28	120	5	97
13	101	196	56	152	33	129	9	104
14	93	197	50	154	29	133	6	111
15	105	219	61	175	33	147	9	123
16	96	224	53	181	29	157	7	134
17	105	235	61	191	33	163	9	139
18	100	237	56	194	29	167	7	144
19	109	255	65	211	33	178	10	156
20	103	263	60	220	29	190	8	168
21	111	276	67	232	32	197	10	175
22	105	279	62	236	29	203	8	182
23	113	297	69	253	32	216	10	194
24	107	305	64	263	29	228	8	206
25	113	316	70	273	31	235	10	213
26	109	321	66	278	29	241	8	220
27	115	337	72	294	31	253	10	232
28	111	347	69	305	29	265	9	245
29	116	358	73	315	31	273	10	252
30	112	363	70	321	29	279	9	259
31	117	376	75	335	30	290	10	270
32	113	385	71	343	29	300	9	281

the antiflavor excitation energies for $B = 1$ in the SK6 variant are smaller than for $B \geq 2$, which leads to repulsion for $B > 1$, in comparison with the more familiar SK4 model. Considerable decrease of binding energies for large B, greater than $B \sim 20$, may be connected with fact that the rational-map approximation becomes unrealistic for such large baryon numbers. The beauty decay constant F_b is not measured yet, which introduces additional uncertainty in our predictions. Probably, the value $F_b/F_\pi = 1.8$ is the best one for the description of the Λ_b mass.

In Table 7, we present the binding energies of hypernuclei with anticharm and antibeauty quantum numbers for the rescaled SK4 and SK6 variants of the model.

Table 6. The total binding energy differences and binding energies themselves (in MeV) for the antiflavored states, SK4 variant (first 4 columns), and SK6 variant (last 4 columns). $F_D/F_\pi = 1.5$, $F_B/F_\pi = 2$.

B	$\Delta\epsilon_{\bar{c}}^{SK4}$	$\epsilon_{\bar{c}}$	$\Delta\epsilon_{\bar{b}}^{SK4}$	$\epsilon_{\bar{b}}$	$\Delta\epsilon_{\bar{c}}^{SK6}$	$\epsilon_{\bar{c}}$	$\Delta\epsilon_{\bar{b}}^{SK6}$	$\epsilon_{\bar{b}}$
2	61	61	90	90	56	56	44	44
3	38	46	49	57	−8	0	−36	−28
4	15	44	48	76	−29	−1	−36	−7
5	44	71	55	82	−5	22	−30	−3
6	27	59	43	75	−20	12	−39	−7
7	47	85	62	101	−5	34	−23	16
8	31	87	41	98	−17	40	−37	19
9	42	100	43	100	−6	51	−33	24
10	31	96	33	98	−15	50	−40	25
11	40	114	34	108	−7	68	−37	37
12	31	123	27	119	−15	78	−42	50
16	27	154	8	136	−15	113	−50	78
17	32	162	11	141	−10	120	−47	83
20	22	183	−7	154	−15	145	−57	104
24	19	217	−19	179	−16	182	−62	136
28	15	251	−31	205	−17	220	−68	169
32	12	283	−40	232	−18	254	−72	200

Table 7. The same as in Table 6, for the rescaled SK4 and SK6 variants of the model

B	$\Delta\epsilon_{\bar{c}}^{SK4^*}$	$\epsilon_{\bar{c}}$	$\Delta\epsilon_{\bar{b}}^{SK4^*}$	$\epsilon_{\bar{b}}$	$\Delta\epsilon_{\bar{c}}^{SK6^*}$	$\epsilon_{\bar{c}}$	$\Delta\epsilon_{\bar{b}}^{SK6^*}$	$\epsilon_{\bar{b}}$
2	36	36	54	54	−5	−5	−30	−30
3	24	32	35	43	−27	−19	−59	−51
4	19	48	44	72	−26	2	−45	−16
5	27	54	39	66	−22	5	−50	−23
6	18	50	31	63	−27	5	−52	−20
7	30	69	46	84	−17	22	−38	1
8	19	75	27	84	−24	32	−49	7
9	21	78	23	80	−21	36	−49	8
10	15	80	17	82	−25	40	−52	13
11	17	91	13	88	−22	52	−52	23
12	12	104	9	101	−25	67	−53	39
16	3	131	−12	115	−28	100	−61	66
17	6	136	−10	120	−26	104	−60	70
20	−4	156	−30	131	−31	130	−68	93
24	−10	188	−43	155	−33	166	−73	125
28	−17	220	−57	179	−35	202	−78	158
32	−21	251	−67	205	−37	235	−82	190

Several peculiarities should be emphasized. The binding energies for the rescaled variants are in general smaller than those for the original variants (Table 6), mainly due to the decrease of excitation energies for the $B = 1$ configuration (by approximately 100 MeV for the anticharm and 110 MeV for antibeauty). For greater B-numbers, this decrease is smaller. However, because the rescaled or nuclear variant is valid for large enough baryon numbers, the binding energies can be greater than the values given in Tables 6 and 7, at least for B-numbers greater than approximately 10. This is similar to the situation with the strangeness quantum number (see Table 5 and its discussion).

5. CONCLUSIONS

The excitation energies of antiflavors are estimated within the bound state version of the chiral soliton model in two different variants of the model, SK4 and SK6, and for two values of the model parameter (e or e', see Tables 3 and 4). The bounds for the expected binding energies of hypernuclei are obtained in this way. These bounds are wide: variations of the total binding energy for the SK4 and SK6 models can reach 40–50 MeV. The difference between the original (baryon) variant and the rescaled (nuclear) variant is greater for strangeness and smaller for anticharm and antibeauty, where it is not greater than approximately 20–30 MeV for baryon numbers between 3 and approximately 20. If the logic is correct that the rescaled or nuclear variant of the model should be applied for large enough B-numbers, beginning with $B \sim 10$, then we should expect the existence of weakly decaying hypernuclei with anticharm and antibeauty.

The properties of multiskyrmion configurations that are necessary for these numerical estimates have been calculated within the rational-map approximation [46], which provides remarkable scaling laws for the excitation energies of heavy antiflavors. This scaling property of heavy flavors (antiflavors) excitation energies, noted previously [43, 63] and confirmed in the present paper by numerical calculations, is fulfilled with good accuracy. The relative role of the quantum correction of the order $1/N_c$ (hyperfine splitting) decreases with increasing the baryon number as $1/B$, and therefore, besides the $1/N_c$-expansion widely used and discussed in the literature, the $1/B$-expansion and arguments can be used to justify the chiral soliton approach at large enough values of the baryon number.

Positive strangeness nuclear states are mostly bound relative to the decay into Θ^+ and nuclear fragments, and one can therefore speak about Θ^+ hypernuclei [54, 71]. The particular value of the binding energy depends on the variant of the model and is greater for the original SK4 variant (Table 5). The existence of deeply bound states is not excluded by our treatment, although the energy of the state is in most cases sufficient for the strong decay into kaon and residual nucleus or nuclear fragments.

The binding energies of the ground states of hypernuclei with heavy antiflavors ($\bar{c}$ or $\bar{b}$) shown in Tables 6 and 7 are more stable relative to variations of the model parameters (e or e'), but more sensitive to the model itself. Similarly to the case of antistrangeness, the binding energies for the SK6 variant of the model are systematically smaller than for the SK4 variant.

Within our approach, it is possible to obtain the spectra of excited states with greater values of the isospin and angular momentum. The energy scale in the first case is given by $1/\Theta_I$ and in the second by $1/\Theta_J$, which is much smaller for large baryon numbers. Because $1/\Theta_I = 1/\Theta_J \approx 180$ MeV for $B = 1$ (see Table 1), it seems difficult, within the chiral soliton approach, to obtain such small spacing between the ground state and excited levels as derived, e.g., in [53] within the quark models.

Although we performed considerable numerical work, we feel that further refinements, improvements, and more precise calculations are necessary. For example, possible contributions of the order $1/N_c$ to the flavor excitation energies mentioned, e.g., in [44], might change our conclusions. When calculations for the present paper have been finished, we became aware of papers [71] and [72], where the possibility of the existence of antistrange Θ hypernuclei is discussed within more conventional approaches. The results obtained in [71] and [72] qualitatively agree with ours.

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APPENDIX

Comparison of the flavor and antiflavor excitation energy difference in the rigid rotator and bound state models

Here, we show that the difference between flavor and antiflavor excitation energies given by (23) coincides with the difference of the $SU(3)$ rotation energies between exotic and nonexotic multiplets within the rigid rotator approach, in the leading-N_c approximation. The method used here is similar to that in [22] applied for arbitrary B-numbers and $N_c = 3$. The generalization to arbitrary N_c and N_F was recently made in [70]. For nonexotic multiplets, we have the quantization condition $p+2q = N_cB$ [61]; we take $p = 1$ for odd B-numbers, and $p = 0$ for even B. The contribution to the $SU(3)$ rotation energy depending on the «flavor» moment of inertia, which is of interest here, is equal to [61]

$$E^{rot}(SU_3) = \frac{1}{2\Theta_F} \times \left[C_2(SU_3)(p,q) - I_r(I_r+1) - N_c^2B^2/12\right] \quad (48)$$

with

$$C_2(SU_3) = \frac{p^2+q^2+pq}{3} + p + q = \frac{(p+2q)^2+3p^2}{12} + \frac{p+2q}{2} + \frac{p}{2}.$$

The «right» isospin for the lowest nonexotic states is $I_r = p/2 = 0$ for even B (as for the nuclei ^{4}He, ^{12}C, ^{16}O, etc.) and $I_r = p/2 = 1/2$ for odd B (as for the isodoublets ^{3}H–^{3}He, ^{5}He–^{5}Li, etc.).

The lowest possible exotic irreducible $SU(3)$ representation (p,q) for each value of the baryon number B is defined by the relations

$$p' + 2q' = N_cB + 3m;$$

m coincides with the number of additional quark–antiquark pairs for several lowest values of p'. The difference of the $SU(3)$ rotation energies for exotic and nonexotic multiplets is given by

$$\Delta E^{rot} = \frac{1}{2\Theta_{F,B}}\left[C_2(SU_3)' - C_2(SU_3) - I_r'(I_r'+1) + I_r(I_r+1)\right]. \quad (49)$$

After simple transformations, it can be written as

$$\Delta E^{rot} = \frac{1}{2\Theta_{F,B}}\left[\frac{m(2N_cB+3m)+p'^2-p^2)}{4} + \frac{3m}{2} + \frac{p'-p}{2} + (I_r - I_r')(I_r+I_r'+1)\right]. \quad (50)$$

If $m = 1$, for the lowest irreducible $SU(3)$ representations, we have

$$p' = 1 \quad \text{and} \quad q' = (N_cB+2)/2$$

for even B, and

$$p' = 0 \quad \text{and} \quad q' = (N_cB+3)/2$$

for odd B. We should keep in mind that the right isospin is given by

$$I_r' = \frac{p'+1}{2} = I_r + 1$$

for $B = 2, 4, \ldots$ and

$$I_r' = \frac{p'+1}{2} = I_r$$

for $B = 1, 3, 5 \ldots$ For charm or beauty, due to the large configuration mixing caused by large values of D- or B-meson masses present in the Lagrangian, such lowest irreducible representations are often not the main component of the mixed state (papers [51] may be of interest in this relation), but for strangeness they are.

For even B ($m = 1, p = 0, p' = 1$), we have

$$\Delta E^{rot} = \frac{1}{4\Theta_{F,B}}\left[N_cB + 2\right]. \quad (51)$$

For odd B ($p = 1, p' = 0$), we obtain

$$\Delta E^{rot} = \frac{1}{4\Theta_{F,B}}\left[N_cB + 3\right]. \quad (52)$$

For $N_c = 3$ and $B = 1$, this coincides with well-known expression for the mass difference between the antidecuplet and the octet of baryons.

The leading-N_c contribution is the same as given by (23). For arbitrary m, the leading contribution is

$$\Delta E^{rot} = \frac{mN_cB}{4\Theta_{F,B}}$$

for any multiplets with the final values of p' and I_r, including the values not considered here. It is worth noting that the correction to the leading contribution decreases not only with increasing N_c but also with increasing B (we recall that $\Theta_{F,B} \sim N_cB$). Therefore, convergence of both methods is better for larger values of B.

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