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RESONANT TRANSMITTANCE THROUGH PERIODICALLY MODULATED FILMS

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We analyze the optical transmittance at normal incidence for an electron gas without losses. The electron gas is supposed to have a plane parallel slab geometry and its dielectric permittivity is assumed periodically modulated in one direction parallel to the interfaces. Due to the surface plasmon polariton mode exitation, there exist resonance frequencies where the transmittance equals to unity. The number and positions of peaks are investigated analytically and a comparison with the analytic theory by Dykhne et al. [6] is made.

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1. INTRODUCTION

In the past, it has been thought that subwavelength apertures have a very low transmission efficiency of light [1]. Recently, however, high transmission efficiencies from arrays of subwavelength structures in metal films have been reported. Since the publication of [2], many experimental and theoretical studies were carried out in order to determine the physical origin of the extraordinarily enhanced transmission. They focused on the description of complicated electromagnetic modes of the metal, originating from the interaction between photons and surface electrons, considering disordered arrays of holes in a metal film [3], organized nanoparticles [4] or periodic rough surfaces [5].

In this paper, we restrict ourselves to the case where the metal film occupyng the space

$$|z| < \frac{d}{2}, \quad -\infty < x < \infty, \quad \frac{\partial}{\partial y} = 0$$

is in a vacuum enviroinment ($|z| > d/2$, $-\infty < x < \infty$) and the dielectric permittivity has the simple form

$$\varepsilon(x) = \tilde{\varepsilon}_0 + \tilde{\varepsilon}_1 \cos(qx),$$

with some prescribed periodicity $a = 2\pi/q$ in the $\hat{x}$ direction. Only transverse magnetic waves (TM-mode)

$$(H_y(x,z), E_x(x,z), E_z(x,z)) \exp(-i\omega t)$$

are considered in the two-wave approximation

$$F(x,z) = F_0(z) + F_1(z)\cos(qx),$$

where a full analytic treatment of the complicated boundary value problem can be easily done. We follow the notation and the method of solution outlined in [6] in order to obtain a clear physical understanding of the phenomenon of enhanced transmission. We also derive concrete results for the dissipationless free-electron gas with

$$\tilde{\varepsilon}_0 = 1 - \frac{\omega_p^2}{\omega^2}$$

(ω_p is the electron plasma frequency). In the future work, these results will be extended to more realistic optical characteristics of metal films, including the experimentally available data for optical constants [7].

2. GENERAL ANALYTIC FORMULATION OF THE PROBLEM

We consider the two-dimensional electromagnetic problem shown schematically in Fig. 1. The magnetic permeability in the whole space is denoted by μ_0 and the dielectric permittivity of the free space is denoted by ε_0. The physical system considered in this work consists of a vacuum (the relative dielectric permittivity is $\varepsilon(\omega) = 1$) in two regions $|z| > d/2$ and a metal slab

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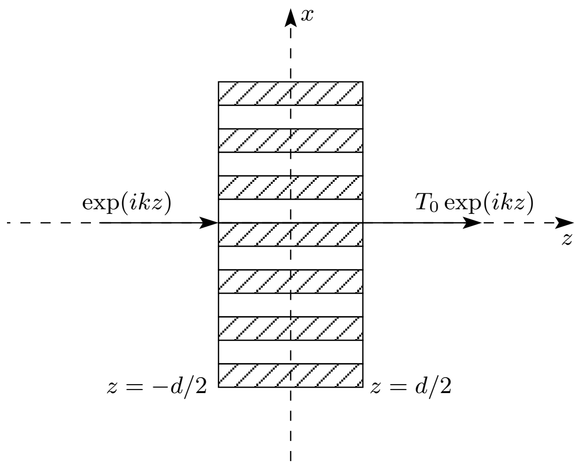

Fig. 1. A plane wave is incident normally on a modulated film $|z| < d/2$. The two arrows show the direction of propagation of the beam incident from $z=-\infty$ and the zero-order transmitted beam ($T_0\exp(ikz)$) at $z=\infty$. The transmittance is defined as $T=|T_0|^2$

(in the region $|z|<d/2$) characterized by the relative dielectric function

$$\varepsilon(\omega,x) = -n^2\left(1-g\cos(qx)\right)^{-1}. \qquad (1)$$

If the modulation factor $g=0$, the dielectric function of the slab (Eq. (1)) is assumed to be real and to satisfy the condition $n^2>1$ in some frequency range. It is within this frequency range that surface-plasmon polaritons exist. The particular periodic x-dependence in (1) facilitates the comparison with the analytic results given in [6]. The Maxwell equations in the linear harmonic approximation ($\exp(-i\omega t)$),

$$\operatorname{rot}\mathbf{E}(\omega,\mathbf{r}) = i\omega\mu_0\mathbf{H}(\omega,\mathbf{r}), \qquad (2)$$

$$\operatorname{rot}\mathbf{H}(\omega,\mathbf{r}) = -i\omega\varepsilon_0\varepsilon(\omega,x,z)\mathbf{E}(\omega,\mathbf{r}), \qquad (3)$$

are treated for transverse magnetic waves (p-polarization) $\mathbf{E}(E_x,0,E_z)$, $\mathbf{H}(0,H_y,0)$ under the assumption

$$\frac{\partial}{\partial y}=0.$$

In the region $|z|<d/2$, we have

$$\frac{\partial^2 H_y}{\partial z^2} + \varepsilon(x)\frac{\partial}{\partial x}\left[\frac{1}{\varepsilon(x)}\frac{\partial H_y}{\partial x}\right] + k^2\varepsilon(x)H_y(x,z) = 0, \qquad (4)$$

$$E_x = -\frac{i}{\omega\varepsilon_0\varepsilon(x)}\frac{\partial H_y}{\partial z}, \quad E_z = \frac{i}{\omega\varepsilon_0\varepsilon(x)}\frac{\partial H_y}{\partial x}, \qquad (5)$$

where

$$k=\omega(\varepsilon_0\mu_0)^{1/2}=\frac{\omega}{c}.$$

Equations (1) and (4) can be written as

$$[1-g\cos(qx)]\frac{\partial^2 H_y}{\partial z^2} + \frac{\partial}{\partial x}\left[(1-g\cos(qx))\frac{\partial H_y}{\partial x}\right] - k^2n^2H_y(x,z)=0. \qquad (6)$$

Neglecting the generation of the $\cos(lqx)$ harmonics with l higher than one and recalling the Floquet theorem, we find

$$H_y\left(x,|z|<\frac{d}{2}\right) = [A_1+2A_2\cos(qx)]\,[X] + [B_1+2B_2\cos(qx)]\,[Y], \qquad (7)$$

where

$$[X] = \operatorname{sech}\left(\frac{dk\,n}{2}\lambda_1\right)\times \left[X_1\operatorname{ch}(knz\lambda_1) - X_2\operatorname{sh}(knz\lambda_1)\right], \qquad (8)$$

$$[Y] = \operatorname{sech}\left(\frac{dk\,n}{2}\lambda_2\right)\times \left[Y_1\operatorname{ch}(knz\lambda_2) - Y_2\operatorname{sh}(knz\lambda_2)\right]. \qquad (9)$$

In formulas (8) and (9), X_1, X_2, Y_1, and Y_2 are arbitrary constants and $\lambda_{1,2}^2$ are dimensionless eigenvalues given by [6]

$$\lambda_1^2 = \frac{2-Q+q_1^2}{2-g^2}, \quad \lambda_2^2 = \frac{2+Q+q_1^2}{2-g^2}, \qquad (10)$$

where

$$Q^2 = q_1^4 + 2g^2(1-q_1^2), \quad q_1=\frac{q}{kn}; \qquad (11)$$

A_1, A_2 and B_1, B_2 are eigenvectors that satisfy the four relations

$$A_1(\lambda_1^2-1) - gA_2\lambda_1^2 = 0, \qquad (12)$$

$$-g\lambda_1^2A_1 + 2(\lambda_1^2-1-q_1^2)A_2 = 0, \qquad (13)$$

$$B_1(\lambda_2^2-1) - gB_2\lambda_2^2 = 0, \qquad (14)$$

$$-g\lambda_2^2B_1 + 2(\lambda_2^2-1-q_1^2)B_2 = 0. \qquad (15)$$

If the modulation amplitude is small ($q \ll 1$), it is straightforward to obtain the following expansions up to the order $O(g^4)$:

$$A_1 = 1 + \frac{g^2}{4q_1^2} + \frac{g^4}{4q_1^2}(2F+q_1^{-2}), \qquad (16)$$

$$A_2 = -\frac{g}{2q_1^2}\left[1 - \frac{g^2}{2q_1^2} - \frac{g^2}{4}(2F + q_1^{-2})\right], \qquad (17)$$

$$B_1 = \frac{g}{2q_1^2}\left[1 + q_1^2 - \frac{g^2}{2}(q_1 + q_1^{-1})^2 + \right. \\ \left. + \frac{g^4}{4}(2F + q_1^2 + q_1^{-2})\right], \qquad (18)$$

$$B_2 = \frac{1}{2}\left\{1 + \frac{g^2}{2q_1^2}(q_1 + q_1^{-1})^2 + \right. \\ \left. + \frac{g^4}{4}(2F + 2 + q_1^2 + q_1^{-2})\right\}, \qquad (19)$$

$$\lambda_1^2 = 1 - \frac{g^2}{2q_1^2} - \frac{g^4}{4}(2F + q_1^{-2}), \qquad (20)$$

$$\lambda_2^2 = 1 + q_1^2 + \frac{g^2}{2}(2 + q_1^2 + q_1^{-2}) + \\ + \frac{g^4}{4}(2F + 2 + q_1^2 + q_1^{-2}), \qquad (21)$$

where

$$F = -\frac{(1 + q_1^{-2})^2}{2q_1^2}. \qquad (22)$$

Because of a misprint or error (reversed signs in A_2^D, B_1^D), the coefficients in [6] (formula (12)), denoted with the superscript D here, must be corrected according to the relations

$$A_1 = A_1^D = \frac{Q + q_1^2 - g^2(1 + q_1^2)}{q_1^2(2 - g^2)}, \\ A_2^D = 2A_2 = -\frac{g[2 + q_1^2 - Q]}{q_1^2[2 - g^2]}, \qquad (23)$$

$$B_1 = B_1^D = -\frac{g[2 + q_1^2 + Q]}{2q_1^2[2 - g^2]}, \\ B_2^D = 2B_2 = \frac{q_1^2 + Q + g^2}{q_1^2(2 - g^2)}. \qquad (24)$$

Obviously, Eqs. (5), (7), (8), and (9) imply that the tangential electric field in the slab is given by

$$\frac{\omega\varepsilon_0}{k}E_x = e_x\left(x, |z| < \frac{d}{2}\right) = \frac{i}{n} \times \\ \times \{[X'](A_1 - gA_2 + \cos(qx)(2A_2 - gA_1)) + \\ + [Y'](B_1 - gB_2 + \cos(qx)(2B_2 - gB_1))\}, \qquad (25)$$

where analagously to (8) and (9), we have defined the z-dependent functions

$$[X'] = \lambda_1 \operatorname{sech}\left(\frac{dkn\lambda_1}{2}\right) \times \\ \times [X_1 \operatorname{sh}(knz\lambda_1) - X_2 \operatorname{ch}(knz\lambda_1)], \qquad (26)$$

$$[Y'] = \lambda_2 \operatorname{sech}\left(\frac{dkn\lambda_2}{2}\right) \times \\ \times [Y_1 \operatorname{sh}(knz\lambda_2) - Y_2 \operatorname{ch}(knz\lambda_2)]. \qquad (27)$$

In the vacuum regions, we have the following fields: in the left half-space in Fig. 1,

$$H_y\left(x, z + \frac{d}{2} < 0\right) = \exp(ik\zeta_+) + \\ + \sum_{p=0,\pm1} R_p \exp[ik(\gamma_p x - \beta_p \zeta_+)], \qquad (28)$$

$$\zeta_+ = z + \frac{d}{2}, \quad \gamma_p = p\frac{q}{k}, \quad R_1 = R_{-1}, \\ \beta_p = [1 - \gamma_p^2]^{1/2} = iV_p, \quad \operatorname{Im}\beta_p = \operatorname{Re}V_p \ge 0, \qquad (29)$$

and in the right half-space in Fig. 1,

$$H_y\left(x, z - \frac{d}{2} > 0\right) = \\ = \sum_{p=0,\mp1} T_p \exp[ik(\gamma_p x + \beta_p \zeta_-)], \qquad (30)$$

where

$$\zeta_- \equiv z - \frac{d}{2} > 0, \quad T_1 = T_{-1}.$$

The continuity condition for the tangential electromagnetic field on the interfaces $z = \mp d/2$ leads to the following four equations containing eight unknown quantities X_1, X_2, Y_1, Y_2, R_0, R_1, T_0, T_1:

$$A_1[X_1 + X_2t_1] + B_1[Y_1 + Y_2t_2] + 2\cos(qx) \times \\ \times [A_2[X_1 + X_2t_1] + B_2[Y_1 + Y_2t_2]] = \\ = 1 + R_0 + 2\cos(qx)R_1, \qquad (31)$$

$$A_1[X_1 - X_2t_1] + B_1[Y_1 - Y_2t_2] + 2\cos(qx) \times \\ \times [A_2[X_1 - X_2t_1] + B_2[Y_1 - Y_2t_2]] = \\ = T_0 + 2\cos(qx)T_1, \qquad (32)$$

$$\lambda_1(A_1 - gA_2)[X_1t_1 - X_2] + \lambda_2(B_1 - gB_2)[Y_1t_1 - Y_2] + \\ + \cos(qx)[\lambda_1(2A_2 - gA_1)[X_1t_1 - X_2] + \\ + \lambda_2(2B_2 - gB_1)[Y_1t_2 - Y_2]] = \\ = -inT_0 + 2nvT_1\cos(qx), \qquad (33)$$

$$\lambda_1(A_1 - gA_2)[X_1t_1 + X_2] + \lambda_2(B_1 - gB_2)[Y_1t_2 + Y_2] + \\ + \cos(qx)[\lambda_1(2A_2 - gA_1)[X_1t_1 + X_2] + \\ + \lambda_2(2B_2 - gB_1)[Y_1t_2 + Y_2]] = \\ = in(1 - R_0) + 2nvR_1\cos(qx). \qquad (34)$$

Here, we use the notation

$$t_{1,2} \equiv \mathrm{th}\left(kn\frac{d}{2}\lambda_{1,2}\right), \qquad v = \sqrt{\left(\frac{q}{k}\right)^2 - 1}, \quad \mathrm{Re}\, v \geq 0. \tag{35}$$

The introduction of the $\mathrm{sech}((dk\,n/2)\lambda_{1,2})$ coefficients in (8), (9) and in (26), (27) is not obligatory, but it simplifies the calculations because only tanh-terms defined in (35) then simultaneously appear in all four equations (31)–(34).

3. CALCULATION OF THE RESONANT TRANSMITANCE THROUGH A MODULATED SLAB

It is convenient to first equate the terms proportional to $\cos(qx)$ in (31)–(34) and to eliminate the unknowns R_1 and T_1 that are not interesting in this study. Thus we derive the following two relations between the constants (X_1, X_2) corresponding to the fundamental beam and the constants (Y_1, Y_2) describing the $\cos(qx)$ mode:

$$Y_1 = k_1 X_1 = \frac{\lambda_1(2A_2 - gA_1)t_1 - 2nvA_2}{2nB_2 v - \lambda_2(2B_2 - gB_1)t_2} X_1, \tag{36}$$

$$Y_2 = k_2 X_2 = \frac{\lambda_1(2A_2 - gA_1) - 2nvA_2 t_1}{2nB_2 t_2 v - \lambda_2(2B_2 - gB_1)} X_2. \tag{37}$$

We note that these expressions are exact in the accepted two-mode $(F_0(z) + 2F_1(z)\cos(qx))$ approximation. We now equate the zero-order terms in boundary conditions (31)–(34) (the fundamental x-independent mode); eliminating R_0 and T_0 from these four equations, we then have

$$X_1 = \frac{in}{inA_1 + inB_1k_1 + \lambda_1(A_1 - gA_2)t_1 + \lambda_2 t_2 k_1(B_1 - gB_2)}, \tag{38}$$

$$X_2 = \frac{in}{inA_1t_1 + inB_1k_1t_2 + \lambda_1(A_1 - gA_2) + \lambda_2 k_2(B_1 - gB_2)}. \tag{39}$$

The transmission coefficient is given by

$$T_0 = X_1(A_1 + k_1B_1) - X_2(t_1A_1 + k_2t_2B_1), \tag{40}$$

which can also be written as

$$T_0 = \frac{\beta(q_1) - \alpha(q_1)}{(1 + \alpha(q_1))\,(1 + \beta(q_1))}, \tag{41}$$

where $\alpha(q_1)$ and $\beta(q_1)$ can be written as simple functions of q_1, n, t_1, t_2, k_1, k_2 using formulas (16)–(21) for A_1, B_1, A_2, B_2, λ_1, λ_2 with $O(g^4)$ terms neglected,

$$\alpha(q_1) = \frac{t_1\left[1 + \frac{g^2}{4}(q_1^{-2} + 2q_1^{-4})\right] + t_2k_1g\frac{\sqrt{1+q_1^2}}{2}q_1^{-2}}{in\left\{1 + \frac{g^2}{2}q_1^{-4} + \frac{k_1g}{2}(1 + q_1^{-2})\right\}}, \tag{42}$$

$$\beta(q_1) = \frac{1 + \frac{g^2}{4}(q_1^{-2} + q_1^{-4}) + k_2g\frac{q_1^{-2}}{2}\sqrt{1+q_1^2}}{in\left\{t_1\left(1 + \frac{g^2}{2}q_1^{-4}\right) + t_2k_2g\frac{1+q_1^{-2}}{2}\right\}}. \tag{43}$$

We first consider two trivial consequences of formulas (42) and (43). If the film thickness vanishes $(t_1 = t_2 = 0)$, we have

$$\alpha = 0, \quad \beta = \infty,$$

and therefore

$$T_0 = 1.$$

If there is no modulation, then

$$g = 0, \quad \alpha = \frac{t_1}{in}, \quad \beta = \frac{1}{int_1},$$

and we have the well-known result

$$T_0(g=0) = \frac{2n}{2n\,\mathrm{ch}(knd) + im^2\,\mathrm{sh}(knd)}, \qquad m^2 = n^2 - 1. \tag{44}$$

We next consider the most interesting case of a thick metal film with thickness d greater than the skin depth, that is,

$$t_1 = 1 - 2\zeta_1, \quad t_2 = 1 - 2\zeta_2, \tag{45}$$

where

$$\zeta_1 = \exp(-knd), \quad \zeta_2 = \exp\left(-knd\sqrt{1+q_1^2}\right), \tag{46}$$

and $\zeta_1 \ll 1$, $\zeta_2 \ll 1$. In writting Eqs. (46), we approximate $\lambda_{1,2}$ from (20) and (21) as

$$\lambda_1 = 1, \quad \lambda_2 = \sqrt{1+q_1^2}.$$

Moreover, for an SPP resonance,

$$\frac{k}{q} = \frac{m}{n}, \quad m = \sqrt{n^2 - 1},$$

as we see in what follows, and therefore

$$q_1^{-1} = m$$

and

$$\zeta_2 = \exp\left(-knd\sqrt{1+\frac{1}{m^2}}\right) = \exp\left(-\frac{kdn^2}{m}\right). \quad (47)$$

In this regime, we derive from the definitions of $k_{1,2}$ in (36) and (37) that

$$k_1 = \frac{2gm(n-m)}{\frac{2m(1+n^2)}{n^2}(q_1^{-1}-m) - 4\zeta_2 - \frac{n^2g^2}{2}}, \quad (48)$$

$$k_2 = \frac{2gm(n-m)}{\frac{2m(1+n^2)}{n^2}(q_1^{-1}-m) + 4\zeta_2 - \frac{n^2g^2}{2}}. \quad (49)$$

It is important to note that the general formula (41) considered in the complex wave-number plane $(\mathrm{Re}\, q_1, \mathrm{Im}\, q_1)$ has two poles at the points where

$$\alpha(Q_1^+) = \beta(Q_1^-) = 1.$$

With the aid of (48) and (49), we can show that these complex wave numbers are given by

$$(Q_1^+)^{-1} = m + \frac{2n^2\zeta_2}{m(1+n^2)} + \frac{g^2n^3}{4m(1+n^2)^2} \times \times \left[n(1+n^2) - 2m(n-m)(n+m^3)\right] - - ig^2\frac{n^5(n-m)^2}{2(1+n^2)^2}, \quad (50)$$

$$Q_1^-(\zeta_2) = Q_1^+(-\zeta_2). \quad (51)$$

Two remarks are appropriate to formulas (50) and (51). The first remark concerns the absence of terms proportional to ζ_1, that is, the limit $t_1 = 1$ is appropriate, but the finite penetratation depth for the $\cos(qx)$ mode is crucial because there is no resonant enhancement of the transmission at $\zeta_2 = 0$. The second remark is that we neglect terms of the order $O(g^4)$ in (50) and (51). It is now clear that if we set

$$\xi = (q_1^{-1} - m)\frac{2m(1+n^2)}{n^2} = = \left(\frac{k}{q} - \frac{m}{n}\right)\frac{2m(1+n^2)}{n}, \quad (52)$$

then for small values of ξ such that terms of the order $\xi\zeta_2$, ξg^2 can be neglected, we have

$$\alpha = \frac{\xi - 4\zeta_2 + g^2M_1}{in[\xi - 4\zeta_2 + g^2M_2]}, \quad \beta = \frac{\xi + 4\zeta_2 + g^2M_1}{in[\xi + 4\zeta_2 + g^2M_2]}, \quad (53)$$

where

$$M_1 = m^2n(n-m) - \frac{n^2}{2}, \quad M_2 = mn^2(n-m) - \frac{n^2}{2}. \quad (54)$$

From (41), (53), and (54), we derive the transmittance of a dissipationless film in the form

$$T = |T_0|^2 = \frac{4\tilde{g}^4}{\left[\left(\tilde{\Delta}-1\right)^2 + \tilde{g}^4\right]\left[\left(\tilde{\Delta}^2+1\right)^2 + \tilde{g}^4\right]}, \quad (55)$$

where we have introduced the renormalized modulation

$$\tilde{g}^2 = \frac{g^2n^2m(n-m)^2}{4\zeta_2(n^2+1)} \quad (56)$$

and the detuning from the surface-plasmon polariton frequency

$$\tilde{\Delta} = -\frac{m(1+n^2)}{2n\zeta_2}\left(\frac{k}{q} - \frac{m}{n}\right) + + \frac{g^2n}{8\zeta_2(1+n^2)}\left[n^3 + n - 2m(n-m)(n^3+m)\right]. \quad (57)$$

Due to the equality

$$(n-m)^2(n+n^3+2m) \equiv n^3+n-2m(n-m)(n^3+m),$$

our formulas (56) and (57) are analogous to formulas (33) and (34) in [6], but ζ_2 is given by (47) and not by $\zeta = \zeta_1$ as defined in [6]. Only in the limit $n \to \infty$ both formulations coincide,

$$\lim_{n\to\infty}\left(n - \frac{n^2}{m}\right) = 0.$$

The physical effects associated with the two small parameters ζ_1 and ζ_2 were not discussed in [6]. Although this was not written explicitly, these authors assumed that $n \gg 1$ in order to consider the influence of a single small parameter $\zeta = \zeta_1 \approx \zeta_2$. Our treatment of the strong skin effect in the modulated slab (summarized in formulas (55)–(57)) is free of the restriction $n \gg 1$, that is, the formulas are valid for all $1 < n < \infty$ provided of course that the less restrictive conditions written after formula (46) are fulfilled. Our new and (as we believe) more correct analytic formulation (55)–(57) leads to appreciable differencies from the previously proposed analytic formulation [6] for a concrete plasma parameterization given in Sec. 4.

4. TRANSMISSION OF ELECTROMAGNETIC WAVES THROUGH A SLAB OF COLLISIONLESS PLASMA

As a specific example, we consider the case where

$$\begin{aligned} n^2 &= \frac{\omega_p^2}{\omega^2} - 1 = \frac{2-x^2}{x^2}, \\ \omega &= \frac{\omega_p}{\sqrt{2}}\, x, \quad 0 < x < 1. \end{aligned} \tag{58}$$

If we introduce the dimensionless parameters

$$D = \frac{\omega_p d}{c}, \quad p = \frac{\omega_p}{cq}, \tag{59}$$

the zero-order resonance frequency that follows from the condition

$$\frac{k}{q} = \frac{m}{n}$$

is equal to the following value of x:

$$x_0 = \sqrt{1 + \frac{2}{p^2} - \sqrt{1 + \frac{4}{p^4}}}\,, \quad 0 < x_0 < 1. \tag{60}$$

The value $x_0(p)$ is defined for all $0 < p < \infty$. In the special case where $p \gg 1$, $x_0(p)$ is very small, i.e., $x_0 \approx \sqrt{2}/p$. On the other hand, if $p \ll 1$, x_0 is very close to one,

$$x_0 \approx 1 - \frac{p^2}{8}.$$

It is instructive to note that

$$\zeta_2(x) = \exp\left(-D\frac{1 - x^2/2}{\sqrt{1-x^2}}\right) \tag{61}$$

tends to the constant value $\exp(-D)$ for small x, but if x is close to one, then

$$\zeta_2 \approx \exp\left(-\frac{D}{p}\right), \tag{62}$$

whereas

$$\zeta_1(x=1) = \exp\left(-\frac{D}{\sqrt{2}}\right),$$

and therefore using the result in [6] for $p \ll 1$ gives substantial deviations from the present theory. We consider the number and exact positions of points where the transmittance T is equal to one. We first note that formula (55) can be represented in the form

$$2\sqrt{T^{-1} - 1} = \left(\frac{\tilde{\Delta}}{\tilde{g}}\right)^2 - \tilde{g}^{-2} + \tilde{g}^2 = A(n, p, D, g). \tag{63}$$

In writting Eq. (63), we have fixed

$$\omega = \omega_p/\sqrt{1+n^2}\,,$$

also having definitions (59) in mind. The transcendental equation

$$A(n, p, D, g) = 0 \tag{64}$$

can be solved numerically or approximately by analytic treatment using the fact that $g \ll 1$ and D is of the order of one, and hence

$$\zeta_2(n) = \exp\left[-\frac{Dn^2}{\sqrt{n^4-1}}\right] \ll 1 \tag{65}$$

for every $n > 1$. An analysis of Eq. (64) for the model in [6] must be based on

$$\zeta_1(n) = \exp\left[-\frac{Dn}{\sqrt{n^2+1}}\right] \ll 1 \tag{66}$$

instead of Eq. (65). Using formulas (56) and (57), we rewrite Eq. (64) as

$$\begin{aligned} a^2(n) = \exp\left[-\frac{2Dn^2}{\sqrt{n^4-1}}\right] + 2g^2 a(n) b(n) - \\ - g^4\left[b^2(n) + c^2(n)\right] = B(n), \end{aligned} \tag{67}$$

where

$$\begin{aligned} a(n) &= \frac{1+n^2}{2n} m \left[\frac{p}{(1+n^2)^{1/2}} - \frac{m}{n}\right], \\ m^2 &\equiv n^2 - 1, \end{aligned} \tag{68}$$

$$b(n) = \frac{n(n-m)^2(n^3+n+2m)}{8(1+n^2)}, \tag{69}$$

$$c(n) = \frac{n^2 m (n-m)^2}{4(1+n^2)}. \tag{70}$$

If we neglect the right-hand side of Eq. (67), we derive the zero-order solution n_0, given by formula (60), that is,

$$n_0 = \sqrt{\frac{p^2 + \sqrt{p^4+4}}{2}}\,. \tag{71}$$

If $B(n_0) > 0$, we find two formal maxima of the transmittance ($T_{max} = 1$) at points $n_\pm$, where

$$n_\pm = n_0 \pm \frac{2n_0^3}{1+n_0^4}\sqrt{B(n_0)}\,, \tag{72}$$

within the first-order perturbation theory. The minus sign in Eq. (72) can lead to a nonphysical solution $n_- < 1$ if the correction term in (72) is sufficiently large. In the limiting case where $B(n) < 0$ for every n, the transmittance never attains a maximum value of one. Nevertheless, the transmittance can have maximum values that are smaller than one (Fig. 2*c*). This quantitative analysis was confirmed by numerical calculations shown in Fig. 2. Here, $D = 1$, $g = 0.2$, and

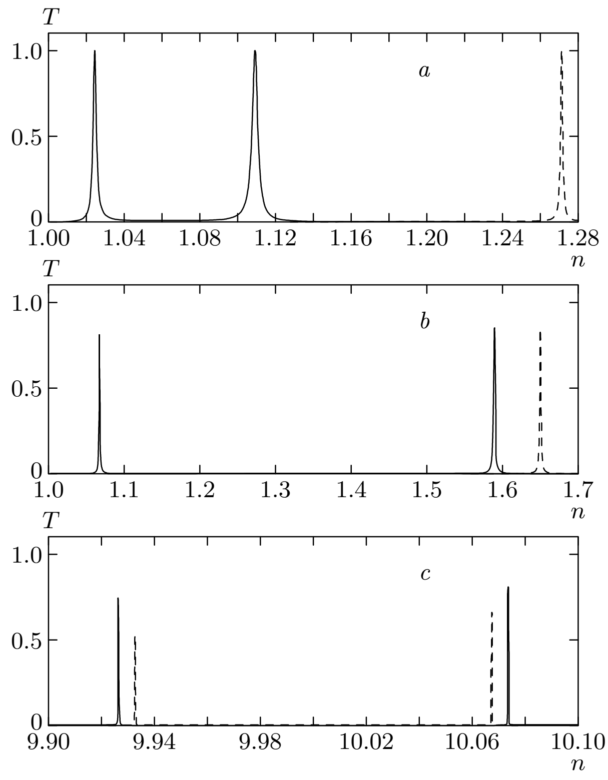

Fig. 2. Transmittance as a function of n at $D = 1$, $g = 0.2$, and $p = 0.1$ (a), 1 (b), 10 (c). Our results — continuous lines, model [6] — dashed lines

$p = 0.1$, 1 and 10 in the respective Figures 2*a, b, c*. The numerical results based on formula (66), that is, the Dykhne model [6], are shown by dashed lines. We see not more than two maxima in all cases. The Dykhne model predicts only one peak in the cases $p = 0.1$ and $p = 1$, whereas our model leads to two maxima in these two cases.

5. CONCLUSION

We have presented a method to analytically describe the resonant transmittance of electromagnetic waves through periodically modulated films. The phenomenological description of the medium $|z| < d/2$ through Eq. (1) allows complex values of the parameters

$$n = n_1 - in_2, \quad g = g_1 + ig_2,$$

but in this paper, we analyze in detail only the dissipationless case (real values of n and g). In the framework of the same physical model, it is not difficult to analyze the more general parameterization

$$\varepsilon(x) = \tilde{\varepsilon}_0 + \tilde{\varepsilon}_1 \cos(qx)$$

(where both numbers $\tilde{\varepsilon}_0$, $\tilde{\varepsilon}_1$ are complex) and to consider oblique incidence of the primary field. The investigation of the interaction of incident light with surface plasmon modes complements the study in [6] as well as the analytic results in [8].

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REFERENCES

1. H. A. Bethe, Phys. Rev. **66**, 163 (1944).
2. T. W. Ebbesen, H. J. Lezec, H. F. Ghaemi et al., Nature (London) **391**, 667 (1998).
3. A. K. Sarychev, V. A. Podolsky, A. M. Dykhne, and V. M. Shalaev, IEEE J. Quant. Electron. **38**, 956 (2002).
4. F. J. Garcia-Vidal and J. B. Pendry, Phys. Rev. Lett. **77**, 1163 (1996).
5. A. Barbara, P. Quemerais, E. Busfaret, T. Lopez-Rios, and T. Fourier, Europ. Phys. J. D **23**, 143 (2003).
6. A. M. Dykhne, A. K. Sarychev, and V. M. Shalaev, Phys. Rev. B **67**, 195402 (2003).
7. P. B. Johnson and R. W. Cristy, Phys. Rev. B **6**, 4370 (1972).
8. S. A. Darmanyan and A. V. Zayats, Phys. Rev. B **67**, 035424 (2003).