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DIPOLE–DIPOLE INTERACTIONS BETWEEN DUST GRAINS IN PLASMAS

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Complete screening of the negative dust grain charge by a cloud of trapped ions in plasmas is investigated. In the plasma electric field, the compound dust particle — «dust grain + ion cloud» — acquires a dipole moment due to displacement of the centers of positive and negative charges in the opposite directions within the compound particle. By analogy to the Van der Waals attractive interaction potential, the dipole–dipole interactions of the compound dust particle can have an attractive behavior. It is shown that for the electric field strengths typically observed in experiments, the dipole–dipole attractive force exceeds the shadowing force that is connected with the reciprocal interception of ions by the neighboring dust grains.

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1. INTRODUCTION

A cloud of dust particles in plasmas, confined by the walls (electrodes), is characterized by a self-organizing property that reveals itself by as capability of dust grains to form ordered spatial structures in the vicinity of electrodes [1–12]. The dust grains in a cloud usually have the electric charge of the same sign (negative); according to the general consideration, at large intergrain distances, such a capability of self-organization implies the existence of an attractive force between the dust grains that have the same polarity. In the past, different mechanisms have been proposed for the dust grain attraction in dusty plasmas. These are:

(i) The attraction of dust grains in the wake potential [13–16]: the ions are focused in the negative potential region of the wake field behind a moving dust grain and provide a possibility for attracting the following negatively charged grain in a linear chain [17, 18];

(ii) The shadowing force [19, 20]: the reciprocal shadowing of a pair of dust grains in a nonstreaming plasma and, as a result, reciprocal interception of ions moving from the outside of the system of the grain pair, leading to a net momentum transfer that pushes the grains to meet each other. This, in effect, represents an attractive force between two dust grains.

(iii) Placed into an external electric field (for instance, in the field of another charged dust particle), the dust grain, considered as a conductor, is polarized. The excess of charges with definite sign on one side leads to an anisotropy of the plasma particle flows to the dust particle surface. Even at the equality of the ion and electron currents to the dust particle surface, the momentum transferred to the dust particle by the ions incident on the surface considerably exceeds the momentum brought by electrons. Therefore, an additional force exerted by the plasma flow acts on a dust particle in the electric field. This additional force has the same direction as the electric field and can exceed

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the electrostatic force acting on the dust particle in the electric field [21]. The force is proportional to the electric field strength. If the given grain is placed into the electric field of another grain and the distance between the grains is much larger than the Debye radius (which is the most interesting case for investigating the grain–grain interactions), then the electric field and the force become very small because of the Debye shielding.

Furthermore, at the volume distribution of dust particles, the neighboring dust grains surrounding the given grain from every side can intercept the ions flowing towards the grain, and the attractive forces described in items (ii) and (iii) must be substantially reduced.

It looks more consistent to relate the creation of the attractive force to the screening of the dust charge by a cloud of trapped ions [22]. Below, we assume that the dust electric charge is completely screened by an ion cloud. Such a possibility is investigated and predicted in Refs. [23, 24]. Considering large distances between the grains instead of bare dust grains, we can operate with the grains «dressed» in the jacket of an ion cloud. The system «grain + ion cloud» is said to be a compound (dust) particle in what follows. In an external electric field, the centers of the negative and positive charges within the compound particle are displaced from each other and the compound particle acquires a dipole moment. The dipole–dipole interactions of the compound dust particle can have an attractive nature by analogy with the Van der Waals interaction in solid state physics.

The present paper is devoted to a quantitative analysis of the attractive force acting between the compound dust particles. It is shown below that the attractive force connected with dipole–dipole interactions of compound particles can exceed the shadowing force [19, 20]. Hence, a special feature of the interaction potential of dust particles in plasmas must be the existence of some equilibrium distance between dust grains at which the forces of attraction and repulsion balance each other. This paper is organized as follows. In Sec. 2, we discuss the theory of ion trapping in the potential well and calculate the induced dipole moment in a self-consistent electric field in plasmas. An expression for the attractive force associated with dipole–dipole interactions is obtained. For typical laboratory conditions, the newly found dipole–dipole attractive force dominates over the shadowing force. Section 3 contains a summary and approximations required for developing the present theory.

2. THEORY

We assume plasma to be collisionless, which means that the ion mean free path is much larger than the plasma Debye length, $\lambda_{mfp} \gg \lambda_D$. In Ref. [25], the capture of particles by a nonstationary potential well in a collisionless plasma was proposed. A brief description of this nonstationary capture is given in Ref. [26]. The nonstationarity of the potential well means that the height of the walls forming the well increases in time and is saturated at some stationary value. Therefore, initially free particles, passing a distance of the order of magnitude of the well's extent, can collide with the growing wall. After reflection, a particle can meet the analogous obstacle at the movement in the opposite direction. At the time of establishment of a stationary well, a definite number of particles is captured by the well. The distribution function of trapped particles can be found from the continuity condition for the distribution function at the limiting level of the trapped particle energy. At this level, the distribution function of trapped particles must be equal to the distribution function of free particles. In our case, capturing of ions by the potential well occurs during the process of dust grain charging. The adiabaticity condition [25, 26], (i.e., the condition that the creation of the well goes slowly), which is necessary for the analytic description of nonstationary particle trapping, is fulfilled: if the Debye radius λ_D exceeds the dust grain size a, then the characteristic time $\tau \approx \lambda_D / a\omega_{pi}$ of the dust grain charging (which is the same as the characteristic time of creation of the potential well) is much larger than the time τ_i necessary for an ion to pass the width of the well (here, ω_{pi} is the ion plasma frequency). As shown below, the width of the well is of the order of λ_D. The time τ_i can be estimated as follows: the potential of the grain is usually given by $|\varphi_0| \approx T_e/e$, where T_e is the electron temperature and $e > 0$ is the ion charge. For the average velocity of ions in the well, we then have

$$v_i \approx \sqrt{\frac{e\,|\varphi_0|}{m}} \approx \sqrt{\frac{T_e}{m}}$$

(where m is the ion mass), whence

$$\tau \approx \frac{\lambda_D}{v_i} \approx \frac{1}{\omega_{pi}}.$$

Therefore, the condition of the adiabaticity of ion capturing, $\tau \gg \tau_i$, is fulfilled at $\lambda_D \gg a$.

According to Refs. [25] and [26], at the adiabatic creation of the well, the distribution function of trapped particles (ions) is constant and equals the value of the

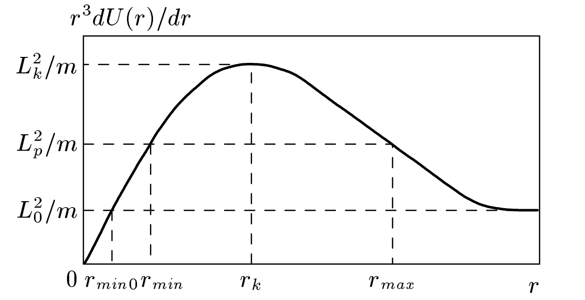

Fig. 1. A qualitative plot of the auxiliary function r^3dU/dr vs. r

distribution function of free particles (ions) at the limiting energy level of the trapped particles. The physical reason of this result is as follows: in a collisionless plasma, the trapped particles do not leave the well and the probability of finding them in the well is one.

To analyze the dust cloud formation, we consider the motion of ions in the field of a negatively charged dust grain. Dust grains are assumed to be absolutely absorbed and are considered spherical with the radius a much smaller than the Debye radius, $a \ll \lambda_D$ [27–30]. At the spherical symmetry of the grain field, the dependence of the ion effective potential energy on the distance r to the center of force, $r = 0$, is

$$U_{eff}(r;L) = \frac{L^2}{2mr^2} + U(r), \tag{1}$$

where

$$U(r) = e\varphi(r) = -e\,|\varphi(r)|$$

is the ion potential energy and $\varphi(r)$ is the electric potential. The angular momentum L is an integral of motion. From the equality

$$\frac{L^2}{m} = r^3\frac{dU(r)}{dr}, \tag{2}$$

we can find the extremum values of $U_{eff}(r;L)$. The qualitative dependence of r^3dU/dr on r is depicted in Fig. 1. It is due to the specific dependence of the potential energy $U(r)$ on r (see pp. 255–266 in Ref. [31]). At short distances (for r smaller than the Debye radius λ_D), the potential energy $U(r)$ decreases as $1/r$, i.e., slower than $1/r^2$. For $r \gtrsim \lambda_D$, $U(r)$ decreases exponentially due to the Debye screening, i.e., faster than $1/r^2$. The behavior of $U(r)$ at large distances ($r \gg \lambda_D$) significantly depends on the conditions at the dust grain surface. If the dust grain surface absorbs electrons and ions, the potential energy $U(r)$ decays as $1/r^2$ at $r \to \infty$ (see [32, 33], and pp. 140–141 in Ref. [31]). In Fig. 1, the intersection points of the curve with the dashed horizontal lines indicate the extremum points of $U_{eff}(r;L)$. The characteristic values L_0, L_p, and L_k are determined as follows.

1) Far from the grain, $r \gg \lambda_D$, the potential energy $U(r)$ can be written as [31, 32]

$$U(r) = U_\infty\left(\frac{a}{r}\right)^2, \tag{3}$$

where U_∞ is a constant. We then have

$$L_0^2 = m\left(r^3\frac{dU}{dr}\right)_{r\to\infty} = 2ma^2\,|U_\infty|\,. \tag{4}$$

At $L \le L_0$, the effective potential energy U_{eff} has only one extremum point, which corresponds to a minimum.

2) The characteristic angular momentum L_p is determined from the condition that the maximum value of the effective potential energy is equal to the value of the effective potential energy on the grain surface, $r = a$ [33, 34]. L_p and the corresponding point r_{max} of the maximum of the effective potential energy can be found from the system of equations

$$U_{eff}(a;L_p) = U_{eff}(r_{max}(L_p);L_p), \tag{5}$$

$$\frac{L_p^2}{m} = \left(r^3\frac{dU(r)}{dr}\right)_{r=r_{max}(L_p)}. \tag{6}$$

Equation (5) yields

$$L_p^2 \approx 2ma^2\,|U(a)|\,\frac{1-|U(r_{max})|/|U(a)|}{1-a^2/r_{max}^2} \approx \approx 2ma^2\,|U(a)|\,. \tag{7}$$

Usually, $r_{max} > \lambda_D$. As the angular momentum L increases, the minimum point of $U_{eff}(r;L)$ moves away from the center and the distance between the extremum points decreases. We stress that only those trapped ions that have the angular momentum $L < L_p$ can reach the grain surface and can be absorbed. At $L > L_p$, the edge of the well is far from the grain surface.

3) The maximum value of r^3dU/dr is reached at a certain point r_k ($> \lambda_D$) where

$$\left[\frac{\partial}{\partial r}\left(r^3\frac{\partial U(r)}{\partial r}\right)\right]_{r_k} = 0, \tag{8}$$

and r_k is always between the maximum and minimum points. The characteristic angular momentum L_k is defined as

$$L_k^2 = m\left(r^3\frac{dU}{dr}\right)_{r=r_k}. \tag{9}$$

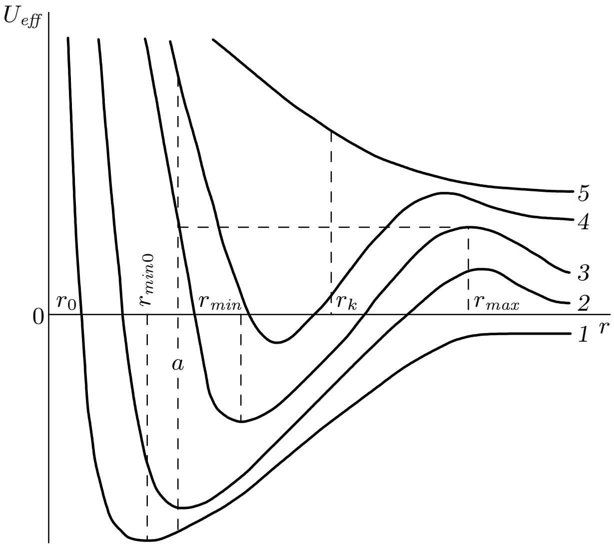

Fig. 2. A qualitative plot of $U_{eff}(r;L)$ vs. r for different values of the angular momentum: *1* — $L \le L_0$, *2* — $L = L_a$, *3* — $L = L_p$, *4* — $L_p < L < L_k$, *5* — $L = L_k$

At $L = L_k$, the extremum points coincide and the function $U_{eff}(r;L)$ has an inflection at this point. If $L > L_k$, the function $U_{eff}(r;L)$ decreases monotonically with increasing r.

4) At $L > L_0$, Eq. (2) has two roots. For a more detailed description of ion motion, we must also determine the angular momentum L_a at which the small root (corresponding to the minimum of $U_{eff}(r;L)$) coincides with the radius a of the grain,

$$\frac{L_a^2}{m} = \left(r^3 \frac{dU(r)}{dr}\right)_{r=a}. \qquad (10)$$

Hence, if the angular momentum is in the range

$$L_0 < L < L_k, \qquad (11)$$

then $U_{eff}(r;L)$ has both maximum and minimum points. The qualitative dependence of $U_{eff}(r;L)$ on r for various values of the angular momentum is shown in Fig. 2. We can now determine the surface that separates the regions of infinite and finite motion of ions in the velocity space (v_r, v_θ), where v_r and v_θ are the velocity components along and across the radial direction. The standard definition of the angular momentum is

$$L = m v_\theta r. \qquad (12)$$

The ions with the angular momentum $L \le L_0$ (or with the velocity component $v_\theta \le (a/r)\sqrt{2|U_\infty|/m}$) and with a negative total energy

$$E(v_r, v_\theta, r) \le 0, \qquad (13)$$

$$E(v_r, v_\theta, r) = \frac{m v_r^2}{2} + U_{eff}(r;L) \qquad (14)$$

can be trapped in the potential well. The ions with the angular momentum in the range $L_0 < L < L_k$ can be trapped if their total energy satisfies the condition

$$E(v_r, v_\theta, r) \le U_{eff}(r_{max}; L). \qquad (15)$$

The ions with the angular momentum larger than L_k $(L > L_k)$ are not trapped. The dependence of $|U_\infty|$ on the grain surface potential $|U(a)|$ for equal electron and ion temperatures $(T_e \approx T_i)$ is depicted on p. 317 in Ref. [31], and shows that one always has

$$\frac{|U_\infty|}{|U(a)|} < \frac{1}{3}.$$

With increasing $|U(a)|$, this ratio decreases. A more precise relation between $|U_\infty|$ and $|U(a)|$ can be established from the quasineutrality condition, in the case where the ion and electron densities are roughly equal. This occurs far from the grain $(r \gg \lambda_D)$. Under this condition, a calculation quite similar to that given in Ref. [34] shows that for a nonisothermal plasma, $T_e \gg T$, the inequality

$$|U_\infty| < \frac{1}{2}|U(a)| \qquad (16)$$

is satisfied for the absorbing grains if

$$\frac{|U(a)|}{T_e} > \frac{1}{2}.$$

The latter relation is usually fulfilled with a great reserve both in laboratory and space plasmas [28]. From (13) and (16), it follows that the zero point r_0 of the effective potential energy $U_{eff}(r;L)$ at $L \le L_0$ is always close to the center $(r = 0)$ in comparison with the grain surface, $r_0 < a$. Indeed, we find from Eq. (1) that $U_{eff}(r;L_0)$ at $r = a$ is negative (see curve *1* in Fig. 2),

$$U_{eff}(a;L_0) \le |U_\infty| - |U(a)| < 0, \quad L \le L_0. \qquad (17)$$

Consequently, when L is smaller than a certain critical value L_0, the dust grain surface is within the well and the ions falling into the potential well are therefore immediately lost due to absorption onto the dust grain surface. Hence, the formation of trapped ion clouds that can shield the grain electric field is possible only for $L > L_a$.

According to the general theory [25, 26], for the stationary well, the distribution function of the trapped particles f_{itr} is constant and the value of f_{itr} is defined by the value of the distribution function of untrapped particles at the limiting energy — in our case, by the energy level $U_{eff}(r_{max};L)$. Considering the distribution function of untrapped ions as a Maxwell–Boltzmann one, we obtain

$$f_{itr} = n_0\,(m/2\pi T_i)^{3/2} \times \times \exp\left(-U_{eff}(r_{max}(L);L)/T_i\right). \quad (18)$$

We emphasize that r_{max} here depends on the angular momentum L (see Eq. (2)). Because we are interested in distances not very large compared to λ_D, $r \lesssim r_{max}$ (see also Eq. (21)) we can choose the Debye–Hückel form

$$U(r) = -|U(a)|\frac{a}{r}\exp\left(-\frac{r-a}{\lambda_D}\right) \quad (19)$$

for the potential energy of ion interactions with the dust grain. It should be stressed that the Debye–Hückel law holds even in the nonlinear regime [23]. It is somewhat modified by the ion flow [16]. The latter also produces a wake field, which is not the focus of this paper. The dependence of $U(r)$ on r definitely corresponds to the dependences that are necessary for the classification of ion motion according to the angular momentum (see Sec. 1). We note that dependence (3) is valid only for very large distances r, $r \gg \lambda_D$. From (2), (6), and (19), we find the critical value of the angular momentum L_a and the corresponding maximum point r_{max} of the effective potential energy

$$L_a^2 = ma^2|U(a)|\left(1+\frac{a}{\lambda_D}\right), \quad (20)$$

as

$$r_{max}(L_a) = \lambda_D\left\{\ln\left(\frac{\lambda_D}{a}\right) + \ln\left[\frac{r_{max}(L_a)}{\lambda_D}\left(1+\frac{r_{max}(L_a)}{\lambda_D}\right)\right]\right\} \approx \approx \lambda_D \ln\left\{\frac{\lambda_D}{a}\left[1+\ln\left(\frac{\lambda_D}{a}\right)\right]\ln\left(\frac{\lambda_D}{a}\right)\right\}. \quad (21)$$

Comparing (4) and (20) by means of (16), we find that $L_a > L_0$, and consequently the effective potential energy has both a minimum and a maximum for $L \geq L_a$. At $L = L_a$, the minimum point of the effective potential energy coincides with the dust grain radius. For $L < L_a$, the distance of the minimum point of $U_{eff}(r;L)$ from the center ($r = 0$) is smaller than the grain radius and all trapped ions are absorbed by the dust grain surface. From (2) and (4), it follows that at $L = L_0$, the point of the minimum of the effective potential energy is given by [9]

$$r_{min0} \approx 2a\frac{|U_\infty|}{|U(a)|}. \quad (22)$$

For $|U_\infty| < (1/2)\,|U(a)|$, we have $r_{min0} < a$ (see Fig. 2). This result is physically expected, because the ions with small angular momenta impact the dust grain surface and are absorbed. We can therefore restrict ourselves by considering the ion angular momenta $L \geq L_a$ and distances $r \leq r_{max}(L_a)$, defined by (21). Obviously, the Debye–Hückel potential (19) is applicable for such distances, and we use it for estimations in what follows. For instance, using the Debye–Hückel shielded potential for the critical distance r_k defined by (8), we obtain $r_k \approx 1.61\lambda_D$.

Different kinds of potential wells that give a contribution to ion trapping can be gathered in two groups. For the angular momentum in the range

$$L_a \leq L < L_p, \quad (23)$$

or

$$ma^2\,|U(a)|\left(1+\frac{a}{\lambda_D}\right) \leq L^2 < 2ma^2\,|U(a)|\,, \quad (24)$$

the distribution function is defined by (18) (with the corresponding $r_{max}(L)$) and only the ions with the energy

$$E \leq U_{eff}(a;L) \quad (25)$$

can take part in forming the cloud shielding the grain field. The ions with larger energy $E \geq U_{eff}(a;L)$ disappear due to absorption on the dust surface. Conditions (25), (12), and (14) allow us to define the limiting value for the velocity component along the radial direction,

$$v_r^2 \leq \left[\frac{L^2}{m^2}\left(\frac{1}{a^2}-\frac{1}{r^2}\right) - \frac{2}{m}\left(|U(a)|-|U(r)|\right)\right] \times \times\left[\Theta(r-a)-\Theta(r-\bar{r}(L)\right], \quad (26)$$

where $\Theta(x)$ is the step function, ($\Theta(x) = 1$ if $x \geqslant 0$ and $\Theta(x) = 0$ if $x < 0$). The turning point $\bar{r}(L)$ is the solution of the equation

$$\frac{L^2}{m^2}\left(\frac{1}{a^2}-\frac{1}{\bar{r}^2}\right) - \frac{2}{m}\left(|U(a)|-|U(\bar{r})|\right) = 0. \quad (27)$$

It turns out that for a given L, the distance r can change in the range

$$a \leq r \leq \bar{r}(L) \quad (28)$$

defined by the energy level $U_{eff}(a)$. According to the general definition, the number density of trapped ions in the angular momentum range (23) is

$$n_1(r) = 2\pi\int v_\theta dv_\theta dv_r f_{itr}, \quad (29)$$

where f_{itr} is defined by (18) and the limits of integration over v_θ and v_r must be chosen according to (12), (23), and (26). Introducing the variable

$$s = L^2/2ma^2\,|U(a)|\,,$$

we find

$$n_1(r) = \frac{2}{\sqrt{\pi}} n_0 \frac{a^2}{r^2}\left(\frac{|U(a)|}{T_i}\right)^{3/2} \times \\ \times \int\limits_{\frac{1}{2}\left(1+\frac{a}{\lambda_D}\right)}^{1} ds \exp\left[\frac{|U(a)|}{T_i}\frac{a^2}{r^2_{max}(s)} s + \frac{|U(r_{max}(s)|}{T_i}\right] \times \\ \times \left[s\left(1 - \frac{a^2}{\lambda_D^2}\right) - \left(1 - \frac{|U(r)|}{|U(a)|}\right)\right]^{1/2} \times \\ \times \left[\Theta(r-a) - \Theta(r - \overline{r}(s)\right], \quad (30)$$

where $r_{max}(s)$ and $\bar{r}(s)$ are to be found from (2), (19), and (27). In what follows, we assume the grain size to be so small that the inequality

$$\frac{|U(a)|}{T_i}\frac{a^2}{\lambda_D^2} \ll 1 \quad (31)$$

is fulfilled. In accordance with (2) and (19), the exponential function in the integrand of (30) can then be replaced by 1 and we obtain after integration that

$$n_1(r) = \frac{4}{3\sqrt{\pi}} n_0 \frac{a^2}{r^2}\sqrt{1 - \frac{a^2}{r^2}}\left(\frac{|U(a)|}{T_i}\right)^{3/2} \times \\ \times \left\{1 - \left[1 - \frac{a}{r}\exp\left(-\frac{r-a}{\lambda_D}\right)\right]\left[1 - \frac{a^2}{r^2}\right]^{-1}\right\} \times \\ \times \left[\Theta(r-a) - \Theta(r - \overline{r}(1)\right], \quad (32)$$

where

$$\overline{r}(1) = \lambda_D \ln\left[\frac{\lambda_D}{a}\ln\left(\frac{\lambda_D}{a}\ln\frac{\lambda_D}{a}\right)\right]. \quad (33)$$

Similar calculations can be performed for ions with the angular momentum in the range

$$L_p \le L < L_k, \quad (34)$$

or

$$2ma^2|U(a)| \le L^2 \le 0.419 \cdot 2ma\lambda_D|U(a)|. \quad (35)$$

In this range, the surface of the grain is outside the well. The energy border of the well is defined by $U_{eff}(r_{max}; L)$ and for the limiting value of the velocity component along the radial direction, we have

$$v_r^2 \le \left[\frac{2}{m}\left(|U(r)| - |U(r_{max}(L))|\right) - \right. \\ \left. - \frac{L^2}{m^2}\left(\frac{1}{r^2} - \frac{1}{r^2_{max}(L)}\right)\right] \times \\ \times \left[\Theta(r - \hat{r}(L)) - \Theta(r - r_{max}(L))\right], \quad (36)$$

where $\hat{r}(L)(\neq r_{max}(L))$ is the solution of the equation

$$\frac{2}{m}\left(|U(\hat{r})| - |U(r_{max}(L))|\right) - \\ - \frac{L^2}{m^2}\left(\frac{1}{\hat{r}^2} - \frac{1}{r^2_{max}(L)}\right) = 0, \quad (37)$$

and $r_{max}(L)$, which is again the maximum point, also satisfies this equation. The procedure, quite analogous to that used above, gives the following expression for the number density $n_2(r)$ of the trapped ions with the angular momentum in the range (34):

$$n_2(r) = \frac{2}{\sqrt{\pi}} n_0 \frac{a^2}{r^2}\left(\frac{|U(a)|}{T_i}\right)^{3/2} \times \\ \times \int\limits_{1}^{0.419\frac{\lambda_D}{a}} ds \exp\left\{-\frac{a^2}{r^2_{max}(s)}\frac{|U(a)|}{T_i}s + \right. \\ \left. + \frac{|U(r_{max}(s))|}{T_i}\right\}\left[\left(|U(r)| - |U(r_{max}(s))|\right)/|U(a)| - \right. \\ \left. - s\left(\frac{a^2}{r^2} - \frac{a^2}{r^2_{max}(s)}\right)\right]^{1/2} \times \\ \times \left[\Theta(r - \hat{r}(s)) - \Theta(r - r_{max}(s))\right]. \quad (38)$$

Here, $r_{max}(s)$ is again defined by (2), (19) and $\hat{r}(s)$ is the root of the expression under the radical (cf. (19) and (37)). From (30) and (38) (and also from the dependence of the wells on the angular momentum, described in Secs. 2 and 3), it follows that a cloud of the trapped ions is localized in a spherical layer restricted by the spheres with the radii a and $\overline{r}(1)$ (the latter is defined by (33)). Integrating the sum of $n_1(r)$ and $n_2(r)$ over the space, we find the total number N of the trapped ions, which we assume to be equal to the charge number Z of the grain,

$$N \approx \frac{4}{3}\sqrt{\frac{2}{3}}\left(\frac{4\pi}{3}n_0\lambda_D^3\right)\left\{\frac{a}{\lambda_D}\frac{|U(a)|}{T_i}\right\}^{3/2} = Z. \quad (39)$$

In estimating (39), we have used the condition of the smallness of the dust grain, $a \ll \lambda_D$. The possibility of such a compensation of charges was recently predicted in [23, 24]. In our model, therefore, the electric charge of a dust grain is screened by the trapped ion cloud and the interaction of the compound particles («dust grain + ion cloud») at large distances cannot be realized as an interaction of charges. In the external (or the induced) electric field, the centers of positive and negative charges within such a compound particle can be shifted, and the particles acquire a dipole moment that can lead to dipole–dipole interactions of the compound particles. Below, we find the electric field that is necessary for shifting the centers of charges over a distance r, with

$$a \ll r \ll \lambda_D, \tag{40}$$

and determine the corresponding induced dipole moment. At shifting distances $r \ll \lambda_D$ (much less than the size of the trapped ion cloud, cf. (33)), we can assume that the form of the dust cloud remains unchanged under shifting. From (32) and (38), the total ion number density at $r \ll \lambda_D$ is given by

$$n(r) = n_1(r) + n_2(r) = \frac{4}{3\sqrt{\pi}} n_0 \left[\frac{|U(a)|}{T_i}\right]^{3/2} \times \\ \times \frac{a}{r}\sqrt{\frac{r}{a}-1}\left(\frac{r}{a}+1\right)^{-1}\Theta(r-a). \tag{41}$$

For the electric field strength, which is defined as

$$\mathbf{E} = \frac{\mathbf{r}}{r^3} 4\pi \int_a^r dr' r'^2 e n(r'), \tag{42}$$

we then obtain

$$\mathbf{E} = \frac{8}{3\sqrt{\pi}}\left(\frac{4\pi}{3} n_0\right)\left[\frac{a}{r}\frac{|U(a)|}{T_i}\right]^{3/2} e\mathbf{r}. \tag{43}$$

Equation (43) represents the electric field within the cloud of trapped ions generated by these ions. Placing a charged grain at the distance r from the center, in order to keep it in equilibrium, one needs to apply an external electric field whose value can be found from Eq. (43). The direction of the external field must be opposite to the displacement of the centers of the positive and negative charges [35, 36, 37].

Inequalities (40) and relation (39) give the following restriction on the electric field:

$$\frac{6}{\pi}\frac{e^2 Z^2}{\lambda_D^4} \ll E^2 \ll \frac{6}{\pi}\frac{e^2 Z^2}{\lambda_D^4}\frac{\lambda_D}{a}. \tag{44}$$

According to (39) and (43), the induced dipole moment and the polarizability of the compound particle («dust grain + ion cloud») are given by

$$\mathbf{P} = Ze\mathbf{r} = \alpha(E)\mathbf{E}, \tag{45}$$

where

$$\alpha(E) = \frac{6}{\pi}\left(\frac{Ze}{\lambda_D^2 E}\right)^3 \lambda_D^3. \tag{46}$$

Due to the specific dependence of the electric field within the compound particle at a distance r from the centre, Eq. (43), the polarizability reveals a nonlinear behavior. At large distances, the interaction energy between the compound particle in the external electric field can be interpreted as the dipole–dipole interaction

$$V = \frac{1}{R^3}\left[\mathbf{P}_1\cdot\mathbf{P}_2 - 3(\mathbf{n}\cdot\mathbf{P}_1)\cdot(\mathbf{n}\cdot\mathbf{P}_2)\right], \tag{47}$$

where $R(\gg \lambda_D)$ is the distance between the dust particles, $\mathbf{n} = \mathbf{R}/R$, and $\mathbf{P}_1$ and $\mathbf{P}_2$ are the dipole moments of the dust particles. Depending on the orientation of the dipole moments, the potential energy can acquire an attractive character. For identical dust particles, the attractive force becomes maximum when the dipole moments are parallel to each other and to $\mathbf{n}$. According to Eqs. (47) and (46), this attractive force is given by

$$\mathbf{F} = \frac{1}{\pi^2}\left(\sqrt{6}\frac{Ze}{\lambda_D E}\right)^6 \frac{E^2}{R^4}\frac{\mathbf{R}}{R}. \tag{48}$$

In Refs. [19, 20], the effective attractive force between two isolated dust grains due to their reciprocal shadowing in the plasma has been investigated. According to Ref. [29], the value of the shadowing force is

$$F_{sh} = \frac{3}{8}\frac{a^2}{\lambda_D^2}\left(\frac{Ze}{R}\right)^2. \tag{49}$$

Comparison of Eqs. (48) and (49) reveals that the shadowing force is smaller than the force due to dipole–dipole interactions for electric field strengths that are typical in laboratory experiments [38–40]. Indeed, relation $|\mathbf{F}| \gg F_{sh}$ is identical to the inequality

$$E^4 \ll 10 E_0^4, \tag{50}$$

where

$$E_0 = \left(7.6\frac{\lambda_D}{\pi a}\frac{\lambda_D}{R}\right)^{1/2}\frac{Ze}{\lambda_D^2}. \tag{51}$$

For $E = E_0$, conditions (44) and (50) can be satisfied if $\lambda_D^2/aR \gg 1$ ($\lambda_D/R \ll 1$). Taking $a \approx 10^{-4}$ cm, $Z \approx 10^4$, $\lambda_D \approx 1.4\cdot 10^{-2}$ cm, and $R \approx 10^{-1}$ cm, we have $E = E_0 \approx 50$ V/cm. According to Eq. (46), the potential energy and the interacting force decrease as the electric field strength increases. Apparently, the dipole–dipole interaction potential energy, as discussed here, may be responsible for the formation of many-layer structures that have been observed in laboratory experiments [8–11] where the behavior of a dust particle cloud in the plasma discharge has been investigated.

3. SUMMARY AND CONCLUSIONS

We have considered complete shielding of the dust grain charge by the trapped ions in plasmas. In the plasma electric field, a neutral compound particle («dust grain + ion cloud») acquires dipole moments, which can lead to their interaction by the potential energy of the dipole–dipole type. We note that our calculations for dipole–dipole interactions are valid under the following assumptions.

1) The surface of the dust grain is absolutely absorbing.

2) For the distances in which we are interested, the spatial dependence of the ion potential energy in the field of a dust grain follows the Debye–Hückel law (see (19)). The latter holds even in the nonlinear regime, as demonstrated in Ref. [23]. The ion flow slightly affects the Debye–Hückel potential [16], and in addition generates a wake field, which is not the topic of the present paper.

3) It is assumed that in a collisionless plasma, ion trapping is the result of adiabatic change of the potential well shape in time [25, 26]. Therefore, the steady state is reached before dust-neutral interactions take place.

4) The number density of trapped ions is small compared to the total ion number density. The trapped ions do not take part in the formation of the potential well.

In conclusion, we mention that some aspects of the interaction observed experimentally [8–11], for example, formation of regular equidistant layers of dust grains, can be explained by the theory developed here. Finally, the present dipole–dipole attractive force can be incorporated in molecular dynamics simulation studies of charged dust particle behavior in dusty plasmas.

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