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# (QUASI)ELASTIC ELECTRON–MUON LARGE-ANGLE SCATTERING WITHIN THE TWO-LOOP APPROXIMATION: VERTEX CONTRIBUTIONS

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We consider quasielastic large-angle electron–muon scattering at high energies with radiative corrections up to the two-loop level. The lowest-order radiative corrections arising from the one-loop virtual photon emission and a real soft emission are presented within a power accuracy. Two-loop corrections are supposed to be of three gauge-invariant classes. One of them, the so-called vertex contribution, is given in the logarithmic approximation. Relation to the renormalization group approach is discussed.

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## 1. INTRODUCTION

Interest in the physics at electron–muon colliders is now increasing. The main attention is paid to the investigation of rare processes, for instance those violating the lepton number conservation law. Another motivation is a test of the models alternative to the Standard Model [1]. The problems of calibration and precise determination of luminosity will be important. For this, the process of quasi-elastic electron–muon scattering can be used.

The processes of quasi-elastic and inelastic large-angle electron–muon scattering (EMS) play an important role in the luminosity calibration at electron–positron colliders. Indeed, they have a clear signature: the scattered leptons move almost back-to-back (in the center-of-mass reference frame) and the cross-section is sufficiently large,

$$\frac{d\sigma_0(\theta)}{d\Omega_e} \approx \frac{200\ \text{nb}}{s\ [\text{GeV}^2]}, \quad \cos\theta \approx \frac{1}{2}, \tag{1}$$

where $s$ is the total energy squared in center-of-mass reference frame, $d\Omega_e$ is the element of angular phase, and $\theta$ is the scattering angle.

The modern experimental requirements to the theoretical accuracy are at the level of per mille or even less and therefore necessitate a detailed knowledge of nonleading terms in the two-loop approximation. Some of these terms were recently calculated in a series of papers [2] devoted to the study of large-angle Bhabha scattering. The contribution of the elastic genuine two-loop virtual correction to the Bhabha amplitude was recently evaluated [3] using the prescription developed in [4] to handle singular terms in QCD at the two-loop level.

In this paper, we consider the EMS process in the two-loop approximation. At this level, we are interested in the contribution to the cross-section given by the interference of the Born amplitude and the two-loop virtual corrections. An attempt to solve this problem was made in a series of papers [5], where a direct calculation was performed; unfortunately, their result is incorrect even in the part containing the infrared divergence. Other papers (see, e.g., [6]) were devoted to the calculation of two-loop Feynman amplitudes within the dimensional regularization scheme. Once again, their results cannot be straightforwardly applied to the real amplitudes of large-angle EMS. One of the reasons is the requirement of distinct masses of the interacting particles.

Here, we consider only virtual and real soft photon contributions to the cross-section of the EMS. In the third order of the perturbation theory, there exist three sets of contributions, each of which is free of

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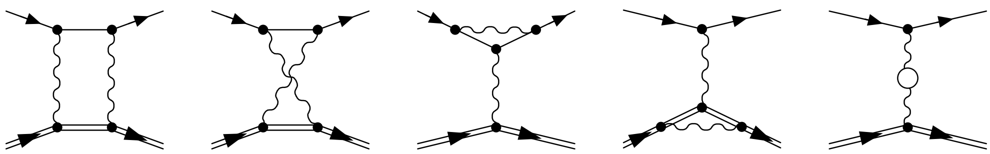

**Fig. 1.** First-order contributions

infrared singularities. They include the contribution coming from the one-loop virtual photon emission corrections (see Fig. 1) and the one given by a soft photon emission (see Fig. 3*a*).

In the fourth order, there are four sets free of infrared singularities. One of them, the dubbed vertex, contains virtual corrections to the lepton vertex function up to the second order of the perturbation theory and relevant inelastic processes with the emission and absorption of real soft photons and lepton pairs by the initial and scattered electrons (and similarly muons). We here use the known expression for the lepton vertex function up to the fourth order of the perturbation theory [7]. Together with the contribution coming from the emission of two real soft photons and a soft charged lepton pair (see Figs. 3*d, e, f*), it is our primary concern in the present paper. We also consider the contribution to the vacuum polarization caused by hadrons and the soft real pion pair production.

Three additional gauge invariant contributions are described by the one-photon exchange containing lepton vertex functions accounting for the vacuum polarization and box-type Feynman diagrams with the self-energy insertion into one of the exchange photon Green's functions. They are left for a separate consideration.

Quasielastic refers to a process with the final particles emitted almost back-to-back in the center-of-mass reference frame. The final particle energies coincide with those of the initial particles up to a small value $\Delta\varepsilon \ll \varepsilon$. This disbalance is due to a possible emission of soft photons and pairs.

We start by giving the results for the Born differential cross-section and first-order corrections. The latter contain radiative corrections due to the emission of virtual photons at the one-loop level and the emission of an additional soft photon. These contributions involve infrared divergences that cancel when the two contributions are added.

The result of the calculations agrees with the renormalization group (RG) prediction in the leading logarithmic approximation,

$$\frac{\alpha}{\pi}\rho_t \sim 1, \quad \frac{\alpha}{\pi} \ll 1, \quad \rho_t = \ln\frac{-t}{m_e m_\mu}, \qquad d\sigma = \frac{d\sigma_0}{|1-\Pi(t)|^2}\mathcal{D}_\Delta^4, \tag{2}$$

where $\Pi(t)$ is defined below (see Eq. (7)), $\mathcal{D}_\Delta$ is the $\Delta$-part of the nonsinglet lepton structure function [9],

$$\begin{aligned} \mathcal{D}_\Delta &= 1 + \sum_{n=1}^{\infty}\left(\frac{\alpha}{2\pi}\rho_t\right)^n P_{n\Delta}, \\ P_{1\Delta} &= 2\ln\Delta + \frac{3}{2}, \\ P_{2\Delta} &= \left(2\ln\Delta + \frac{3}{2}\right)^2 - 4\zeta_2, \\ \Delta &= \frac{\Delta\varepsilon}{\varepsilon} \ll 1, \quad \zeta_2 = \frac{\pi^2}{6}. \end{aligned} \tag{3}$$

Here, $\rho_t$ is the so-called large logarithm, $t$ is the kinematical invariant, and $m_e$ and $m_\mu$ are masses of the leptons.

In addition, we give the explicit form of the nonleading terms and present the result of calculating the lowest-order radiative corrections to a power accuracy,

$$1 + O\left(\frac{\alpha}{\pi}\frac{m^2}{s}\rho_t\right). \tag{4}$$

Our calculation of the second-order contribution is performed in the logarithmic approximation. We keep all the logarithmically enhanced terms including those containing logarithms of the mass ratio and omit the terms of the order $O(1)$.

In calculating radiative corrections in the fourth order of the perturbution theory, we consider three separate gauge-invariant contributions. We call them the vertex contributions, the decorated boxes, and contributions of the eikonal type. The last two involve amplitudes with the electron–muon exchange enhanced by one or two additional virtual (or real soft) photons and by a virtual (real soft) pair. Their contributions are not considered here.

The first set of Feynman diagrams is of the vertex type with second-order radiative corrections (Fig. 2).

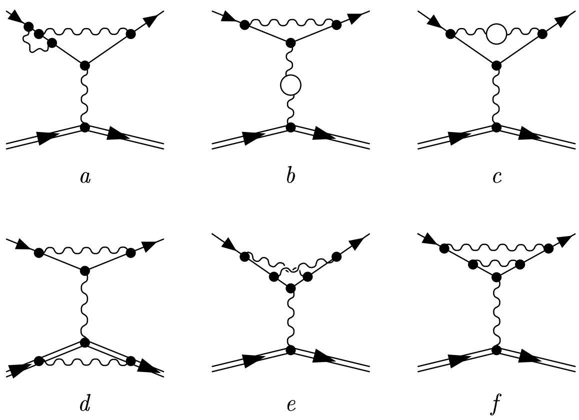


**Fig. 2.** Some of the second-order $V$-type contributions: $a,\ d,\ e,\ f$ — double virtual photon contributions to vertex function, $b,\ c$ — vacuum polarization insertions

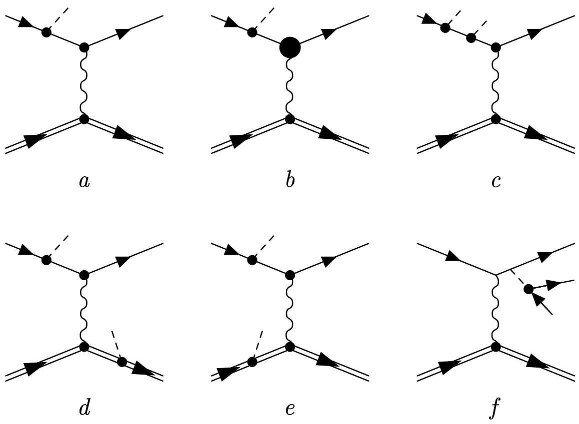


**Fig. 3.** Some of the soft photon contributions. Diagram $a$ corresponds to the first-order radiative corrections; in $b$, the filled circle denotes the vertex one-loop radiative corrections; $c,\ d,\ e$ represent the emission of two soft photons; $f$ represents a soft pair production

The corresponding contribution involves the fourth power of large logarithms and the infrared divergent terms. Combining this with additional contributions coming from the emission and absorption of one and two soft photons by either of the lepton lines results in the cancellation of the fourth and third powers of large logarithms and of all the infrared-divergent terms. The result is found to be in agreement with the RG predictions.

Our paper is organized as follows. After some introductory remarks, we discuss the first-order contribution to the cross-section of the process in Sec. 2. In Sec. 3.1, the radiative corrections coming from the vertex diagrams are considered to the $\alpha^4$ order of the perturbation theory. Section 3.2 is devoted to the study of the vacuum polarization effects including the hadronic contribution to the vertex Feynman diagrams. In Sec. 3.3, we give the contribution due to the emission of one and two soft photons and a soft pair for the cases of emitted (absorbed) leptons with equal and different masses. In conclusion, we summarize the results obtained.

## 2. THE BORN CROSS-SECTION AND LOWEST-ORDER RADIATIVE CORRECTIONS

We recall that we consider the large-angle high-energy electron–muon scattering

$$e^-(p_1) + \mu^-(p_2) \to e^-(p_1') + \mu^-(p_2'), \qquad p_1^2 = p_1'^2 = m_e^2, \quad p_2^2 = p_2'^2 = m_\mu^2, \tag{5}$$

with the kinematic invariants $s$, $t$, and $u$ much larger than the lepton mass squared,

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_1')^2 = -\frac{s}{2}(1-c),$$

$$u = (p_1 - p_2')^2 = -\frac{s}{2}(1+c),$$

where $c = \cos(\widehat{\mathbf{p}_1, \mathbf{p}_1'})$ is the cosine of the scatter angle in the center-of-mass reference frame (this reference frame is implied in what follows). The differential cross-section in the Born approximation is given by

$$\begin{aligned} d\sigma_0 &= \frac{1}{8s} B d\Gamma, \\ B &= \sum |\mathcal{M}_0|^2 = 8(4\pi\alpha)^2 \frac{s^2+u^2}{t^2}, \\ d\Gamma &= \frac{1}{(2\pi)^2} \frac{d^3p_1'}{2\varepsilon_1} \frac{d^3p_2'}{2\varepsilon_2} \times \\ &\times \delta^4(p_1 + p_2 - p_1' - p_2') = \frac{d\Omega_e}{8(2\pi)^2}. \end{aligned} \tag{6}$$

We can then write

$$\frac{d\sigma_0}{d\Omega_e} = \frac{\alpha^2}{2s} \frac{s^2+u^2}{t^2} \left\{ 1 + O\left(\frac{m_\mu^2}{s}\right) \right\}.$$

The lowest-order radiative corrections come from the emission of virtual (one-loop corrections) and real photons. The one-loop radiative corrections are classified into three distinct sets. One of them is related to the vacuum polarization insertion into the propagator of a photon exchanged between leptons. It can be taken into account as

$$\left(\frac{d\sigma}{d\Omega_e}\right)^{vp}=\frac{d\sigma_0}{d\Omega_e}\frac{1}{|1-\Pi(t)|^2},$$
$$\Pi(t)=\frac{\alpha}{3\pi}\left(l_t-\frac{5}{3}\right)+\frac{\alpha}{3\pi}\left(L_t-\frac{5}{3}\right)+$$
$$+\delta_{had}(t)+\frac{\alpha^2}{4\pi^2}(l_t+L_t)+\ldots,$$
$$\delta_{had}(t)=\frac{\alpha}{3\pi}\int\limits_{4m_\pi^2}^{\infty}\frac{dM^2}{M^2}\mathcal{R}(M^2)\frac{t}{t-M^2}, \tag{7}$$
$$l_t=\ln\frac{-t}{m_e^2}=\rho_t+L,$$
$$L_t=\ln\frac{-t}{m_\mu^2}=\rho_t-L,\quad L=\ln\frac{m_\mu}{m_e},$$

where $M^2$ denotes the square of the hadron invariant mass in the process $e\bar{e}\to h$ and

$$\mathcal{R}(M^2)=\frac{\sigma_{e\bar{e}\to h}(M^2)}{\sigma_{e\bar{e}\to\mu\bar{\mu}}} \tag{8}$$

is the known ratio of the single-photon annihilation cross-sections with hadron and muon pairs produced.

Another set of one-loop radiative corrections contains the vertex function (we recall that only the Dirac formfactor of the vertex function applies within power accuracy implied in Eq. (4)),

$$\left(\frac{d\sigma}{d\Omega_e}\right)^{v}=\frac{d\sigma_0}{d\Omega_e}[V_e(l_t)V_\mu(L_t)]^2, \tag{9}$$

with the lowest-order Dirac formfactors of leptons given by (see [7])

$$V_e(l_t)=1+\frac{\alpha}{\pi}f_1^{(2)}(l_t)+\frac{\alpha^2}{\pi^2}f_1^{(4)}(l_t)+\ldots,$$
$$V_\mu(L_t)=V_e(l_t\to L_t),$$
$$f_1^{(2)}(l_t)=l_\lambda(1-l_t)-1+\frac{3}{4}l_t-\frac{1}{4}l_t^2+\frac{1}{2}\zeta_2, \tag{10}$$
$$l_\lambda=\ln\frac{m_e}{\lambda}.$$

Here, $\lambda$ is a fictitious photon mass. It is convenient to represent $f_1^{(4)}(l_t)$ as the sum of two ingredients,

$$f_1^{(4)}=f^{\gamma\gamma}+f^{vp}, \tag{11}$$

where $f^{vp}$ contains QED vacuum polarization effects (to be specified in Sec. 3.2) and

$$f^{\gamma\gamma}=\frac{1}{32}l_t^4-\frac{3}{16}l_t^3+\left(\frac{17}{32}-\frac{1}{8}\zeta_2\right)l_t^2+$$
$$+\left(-\frac{21}{32}-\frac{3}{8}\zeta_2+\frac{3}{2}\zeta_3\right)l_t+$$
$$+\frac{1}{2}l_\lambda^2(l_t-1)^2-l_\lambda(l_t-1)\times$$
$$\times\left(-\frac{1}{4}l_t^2+\frac{3}{4}l_t-1+\frac{1}{2}\zeta_2\right)+O(1),$$
$$\zeta_3\approx 1.2020569. \tag{12}$$

The contribution of the Pauli formfactor is neglected because it is proportional to the lepton mass squared. The remaining one-loop radiative correction is associated with the interference of the Born amplitude with those containing two virtual photons exchanged between lepton lines.

Depending on the photon energy, the soft region ($\omega<\Delta\varepsilon\ll\varepsilon$) and the hard region ($\omega>\Delta\varepsilon$) of the real photon emission can be distinguished. In the quasireal case, only the soft region is relevant,

$$\frac{d\sigma^s}{d\sigma_0}=\frac{-4\pi\alpha}{(2\pi)^3}\int\frac{d^3k}{2\omega}R^2(k),$$
$$\omega=\sqrt{k^2+\lambda^2}<\Delta\varepsilon, \tag{13}$$
$$R(k)=Q_k^{p_1p_1'}+Q_k^{p_2p_2'},\quad Q_k^{pp'}=\frac{p'}{p'k}-\frac{p}{pk}.$$

We now give some useful formulas for the description of a soft photon emission. The center-of-mass reference frame is understood for the initial particles, which implies that the values of 3-momenta of all particles are equal (we consider the elastic EMS).

We first give the expression for a single soft photon emission,

$$\delta_{11'}^s=-\frac{\alpha}{4\pi^2}\int\frac{d^3k}{\omega}\left(\frac{p_1'}{p_1'k}-\frac{p_1}{p_1k}\right)^2\Bigg|_{\omega<\Delta\varepsilon}=$$
$$=\frac{2\alpha}{\pi}\left[(l_t-1)(\ln\Delta+l_\lambda)+\frac{1}{4}l_t^2-\zeta_2+\right.$$
$$\left.+\frac{1}{2}\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right], \tag{14}$$

with the dilogarithm function

$$\mathrm{Li}_2(z)=-\int\limits_0^z\frac{dx}{x}\ln(1-x).$$

By properly squaring this formula, it is easy to derive the quantity $\delta_{11'}^{SS}$ (see Eqs. (21) and (22)).

The contributions to the lowest-order radiative corrections that are free of infrared singularities and originate in two sets containing Dirac formfactors of the leptons and the relevant contribution coming from a soft photon emission can be cast into the form

$$\frac{d\sigma^{(1)}}{d\sigma_0} = 1+\frac{\alpha}{\pi}[\delta^s_{11'}+2f_1^{(2)}(l_t)+\delta^s_{22'}+2f_1^{(2)}(L_t)] = = 1+\frac{\alpha}{\pi}\left[2(\rho_t-1)\left(2\ln\Delta+\frac{3}{2}\right)-2\zeta_2-1+ + 2\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right], \quad (15)$$

which agrees with the structure function approach.

After accounting for the soft photon contributions $\delta^s_{12'}$ and $\delta^s_{21'}$ and the interference of the Born and box-type Feynman diagrams, we obtain

$$d\sigma = d\sigma_0\left\{1+\frac{\alpha}{\pi}\frac{1}{|1-\Pi(t)|^2}\times \times\left[\rho_t(4\ln\Delta+3)-4\ln\Delta-4-2\zeta_2+2\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right]+K\right\},$$

$$K = \frac{\alpha}{\pi}\left\{L_{us}(4\ln\Delta+L_{st}+L_{ut})+2\mathrm{Li}_2\left(\frac{1-c}{2}\right)+ + \frac{t^2}{s^2+u^2}\times \times\left[\frac{u}{t}L_{st}-\frac{s}{t}L_{ut}+\frac{s-u}{2t}(6\zeta_2+L_{st}^2+L_{ut}^2)\right]\right\}, \quad (16)$$

$$L_{st} = \ln\frac{s}{-t}, \quad L_{ut} = \ln\frac{u}{t}, \quad L_{us} = \ln\frac{-u}{s}$$

(details of the lowest-order box Feynman diagram contribution can be found in [2]). The factor $K$ represents the sum of the elastic Born and box-type amplitudes and the corresponding inelastic contributions. The expression for the cross-section given above is in agreement with predictions expected from the RG considerations.

The expression for the EMS cross-section in the leading logarithmic approximation can be brought to the form of a Drell–Yan-like process [9] written in terms of structure functions,

$$d\sigma = \frac{d\sigma_0}{|1-\Pi(t)|^2}\left[\mathcal{D}_\Delta\left(\frac{\alpha(t)}{2\pi}l_t\right)\right]^2\times \times\left[\mathcal{D}_\Delta\left(\frac{\alpha(t)}{2\pi}L_t\right)\right]^2, \quad (17)$$

with the nonsinglet structure function

$$\mathcal{D}_\Delta(z) = 1+zP_{1\Delta}+\frac{1}{2}z^2P_{2\Delta}+\ldots+ + \frac{1}{n!}z^nP_{n\Delta}+\ldots \quad (18)$$

Here, $P_{n\Delta}$ is the $n$-th iteration of the $\Delta$-part of the kernel of evolution equations,

$$P_n(y) = \lim_{\Delta\to 0}[P_{n\Delta}\delta(1-y)+\Theta(1-y-\Delta)P_{n\theta}] = = \int\limits_y^1\frac{dx}{x}P_1(x)P_{n-1}\left(\frac{y}{x}\right), \quad n\ge 2,$$
$$P_{1\theta} = \frac{1+y^2}{1-y}, \quad P_{2\theta} = \frac{1+y^2}{1-y}\left[\ln\frac{(1-y)^2}{y}+\frac{3}{2}\right]+ + \frac{1}{2}(1+y)\ln y-(1-y). \quad (19)$$

Explicit expressions for $P_{1\Delta}$ and $P_{2\Delta}$ are given in (3). The parameter $\Delta$ ($\Delta \ll 1$) can be interpreted as the energy fraction carried by soft real photons and pairs escaping detectors. The running QED coupling constant is

$$\alpha(t) = \frac{\alpha}{1-(\alpha/3\pi)t}.$$

## 3. SECOND-ORDER RADIATIVE CORRECTIONS

Second-order radiative corrections can be represented as the sum of several sets, each of which depends on the choice of the gauge with respect to virtual and real photons. We consider Feynman diagrams describing elastic scattering with the vacuum polarization effects included and with the soft pair production taken into account. They are related to the one-photon exchange Feynman diagrams for both the elastic and quasi-elastic processes and can be specified by the emission of two more (either virtual or real) photons from the same lepton lines.

A keystone to this classification is the soft photon radiator cross-section. In the case of only one soft photon emitted, it takes the form

$$d\sigma^s = \frac{1}{8s}\frac{d\Omega_e}{8(2\pi)^5}\left(\delta\sum|\mathcal{M}|^2\right)_s\frac{d^3k}{2\omega}\bigg|_{2\omega/\sqrt{s}<\Delta},$$
$$\left(\delta\sum|\mathcal{M}|^2\right)_s = = 2\,\mathrm{Re}\sum\mathcal{M}_0^*\mathcal{M}^{(1)}(-4\pi\alpha)R^2(k). \quad (20)$$

For the emission of two soft photons (e.g., by the electron block), we have

$$d\sigma^{ss} = d\sigma_0 \frac{1}{2!}(-4\pi\alpha)^2 \frac{d^3k_1}{2\omega_1(2\pi)^3}\frac{d^3k_2}{2\omega_2(2\pi)^3} \times \times (Q_{k_1}^{p_1p_1'})^2 (Q_{k_2}^{p_2p_2'})^2 \Big|_{2\omega_1/\sqrt{s}+2\omega_2/\sqrt{s}<\Delta}. \quad (21)$$

For the emission of two soft photons such that their total energy does not exceed $\Delta\varepsilon \ll \varepsilon$, we have

$$\left[\int \frac{d^3k_1}{\omega_1}\frac{p_ip_j}{p_ik_1 \cdot p_jk_1} \times \times \int \frac{d^3k_2}{\omega_2}\frac{p_lp_m}{p_lk_2 \cdot p_mk_2}\right]\Bigg|_{\omega_1+\omega_2<\Delta\varepsilon} = = (a_1 \ln\Delta + b_1)(a_2\ln\Delta + b_2) - a_1a_2\zeta_2, \quad (22)$$

where

$$\left[\int \frac{d^3k_1}{\omega_1}\frac{p_ip_j}{p_ik_1\, p_jk_1}\right]\Bigg|_{\omega_1<\Delta\varepsilon} = a_1\ln\Delta + b_1,$$

$$\left[\int \frac{d^3k_2}{\omega_2}\frac{p_lp_m}{p_lk_2\, p_mk_2}\right]\Bigg|_{\omega_2<\Delta\varepsilon} = a_2\ln\Delta + b_2.$$

The general structure of all the above contributions to the differential cross-section reveals the presence of large logarithms up to the fourth power. But the sum involves only their second powers. Such a cancellation is characteristic of each gauge-invariant set of corrections.

### 3.1. Vertex graphs

Three gauge-invariant groups of Feynman diagrams containing one photon exchange contribute

$$\frac{d\sigma^v}{d\sigma_0} = \frac{\alpha^2}{\pi^2}[a_1 + \tilde{a}_1 + a_2]. \quad (23)$$

The quantity $\tilde{a}_1$ is related to the emission of two (virtual and real) photons out of a muon line,

$$\tilde{a}_1 = a_1(l_t \to L_t).$$

Using the results given in Eq. (12) for the electron Dirac formfactor up to the fourth order of the perturbation theory[1)], we can construct the contributions to the squared matrix element of one-photon exchange amplitudes, that are free of infrared singularities

$$a_1 = (f_1^{(2)})^2 + 2f^{\gamma\gamma} + 2f_1^{(2)}\delta^s_{11'} + \delta^{ss}_{11'}, \quad a_2 = 4f_1^{(2)}\tilde{f}_1^{(2)} + 2[f_1^{(2)}\delta^s_{22'} + \tilde{f}_1^{(2)}\delta^s_{11'}] + \delta^s_{11'}\delta^s_{22'}, \quad (24)$$

where $\tilde{f}_1^{(2)}$ corresponds to the muon formfactor, which is identical to the electron one with the electron mass replaced by that of the muon. The quantities $\delta^s_{ij}$ and $\delta^{ss}_{ij}$ correspond to the emission of one and two soft real photons (their energies are restricted by the condition $\Delta\omega_1 + \Delta\omega_2 < \epsilon$) from the fermion lines $i, j$. The corresponding expression is given in Eq. (14). We note the factor $1/2!$ in front of the latter quantities, which is due to the identity of the soft photons emitted.

The relevant contribution to the differential cross-section in the logarithmic approximation is then given by

$$a_1 + \tilde{a}_1 = \rho_t^2 P_{2\Delta} + + \rho_t\left[-\frac{45}{8} + Y + 2\zeta_2 + 6\zeta_3\right] + O(1),$$
$$a_2 = \rho_t^2P_{2\Delta} + \rho_t[-6 + Y + 5\zeta_2] + O(1), \quad (25)$$
$$Y = 2P_{1\Delta}\mathrm{Li}_2\left(\frac{1+c}{2}\right) - - (4\zeta_2 + 14)\ln\Delta - 8\ln^2\Delta.$$

This result is in agreement with the RG form of the large-angle cross-section.

### 3.2. Hadronic vacuum polarization

We study the vacuum polarization effects occurring in considering vertex Feynman diagrams (see Fig. 2*b*, *c*). For this, we use the known expression for the hadronic vacuum contribution to the photon Green's function by making the substitution

$$\frac{1}{k^2} \to \frac{\alpha}{3\pi}\int\limits_{4m_\pi^2}^{\infty}\frac{dM^2}{M^2}\frac{\mathcal{R}(M^2)}{k^2 - M^2}, \quad (26)$$

where $k$ is the 4-momentum of the virtual photon, $M^2$ is the hadron invariant mass squared, and the ratio $\mathcal{R}(M^2)$ is given in Eq. (8).

In the next order of the perturbation theory, we must consider the three gauge-invariant classes of Feynman diagrams for elastic and quasi-elastic processes with a soft photon and a soft pion pair production.

1) Here, we omit the contribution of the vacuum polarization; it is taken into account in what follows.

We first examine the vertex class. The cross-sections can be written as

$$\frac{d\sigma}{d\sigma_0} = 1 + (\delta^s + \delta^v)_{had},$$
$$\delta^v_{had} = \frac{\alpha^2}{6\pi^2}[F(m_e^2, t) + F(m_\mu^2, t)], \tag{27}$$
$$F(m^2, t) = \int\limits_{4m_\pi^2}^{\infty} \frac{dM^2}{M^2}\mathcal{R}(M^2)\, F_1(t, m^2, M^2),$$

where $\delta^s_{had}$ corresponds to the soft hadron emission of soft pion pairs and $F_1$ (with the hadronic vacuum polarization of the virtual photon) is the vertex contribution to the Dirac formfactor of a lepton with the mass $m$. The contribution of the Pauli formfactor $F_2$ is suppressed by the factor $|m^2/t|$.

The standard calculation with the regularization at $t = 0$ leads to

$$F_1(t, m^2, M^2) = 2\int\limits_0^1 dx \int\limits_0^1 y\, dy \left[\ln\frac{d_0}{d} + \frac{a}{d} - \frac{a_0}{d_0}\right] \tag{28}$$

with

$$a = a_0 + t[1 - y + x(1-x)y^2],$$
$$d = d_0 - y^2 x(1-x)t, \tag{29}$$
$$a_0 = -m^2(2-y^2), \quad d_0 = y^2m^2 + (1-y)M^2$$

(for details, see Appendix A). It can be seen that the condition $F_1|_{t=0} = 0$ is satisfied. We now consider two limiting cases for $F_1$. In the case of a large hadron invariant mass squared compared to $-t$, we find

$$F_1(t, m^2, M^2) = \frac{t}{M^2}\left[\frac{2}{3}\ln\frac{M^2}{-t} + \frac{11}{9}\right],$$
$$M^2 \gg -t, \tag{30}$$

and in the case of a small invariant mass squared,

$$F_1(t, m^2, M^2) = -\ln^2\frac{-t}{m^2} - 2\ln\frac{M^2}{m^2}\ln\frac{-t}{m^2} -$$
$$- 5\ln\frac{-t}{m^2} + \frac{\pi^2}{3} - \frac{1}{2}, \quad -t \gg M^2 \gg m_\mu^2. \tag{31}$$

Taking the emission of soft pairs into account (see Appendix B), we obtain the hadronic contribution to the radiative correction

$$(\delta^v + \delta^s)_{had} = \frac{\alpha^2}{6\pi^2}\int\limits_{4m_\pi^2}^{-t}\frac{dM^2}{M^2}R(M^2) \times$$
$$\times \left[-\ln\frac{-t}{M^2}\left[8\ln\frac{M^2}{m_e m_\mu} - 2\ln\Delta + 10\right] -\right.$$
$$\left.-6\ln^2\frac{M^2}{m_e m_\mu} - 10\ln\frac{M^2}{m_e m_\mu} - 6\ln^2\frac{m_\mu}{m_e} + \frac{2}{3}\pi^2 - 1\right]. \tag{32}$$

## 31 Leptonic vacuum polarization and soft lepton pairs

We next study the contribution to the lepton vertex function of the vacuum polarization type. Obviously, there are two possibilities for a vacuum polarization blob to be inserted into the lepton vertex function. The contribution to the elastic cross-section can then be written as

$$\left(\frac{d\sigma^{vp}}{d\sigma_0}\right)_e = 2\frac{\alpha^2}{\pi^2}\Big[Z_1(m_e, m_e) + Z_2(m_e, m_\mu)\Big], \tag{33}$$

where

$$Z_1(m_e, m_e) = -\frac{1}{36}\rho_t^3 + \frac{1}{12}\left(\frac{19}{6} - L\right)\rho_t^2 -$$
$$- \frac{1}{36}\left(6\zeta_2 + \frac{265}{6} + 3L^2 - 19L\right)\rho_t \equiv f^{vp}$$

is the contribution of the electron blob inserted into the electron vertex function (see (11) for the definition of $f^{vp}$) and

$$Z_2(m_e, m_\mu) = -\frac{1}{36}\rho_t^3 + \frac{1}{12}\left(\frac{19}{6} + L\right)\rho_t^2 -$$
$$- \frac{1}{36}\left(6\zeta_2 + \frac{265}{6} + 3L^2 + 63L\right)\rho_t$$

is a muon blob contribution to the electron vertex.

A similar expression is valid for the muon vertex function (the electron blob contribution to the muon vertex),

$$Z_3(m_\mu, m_e) = -\frac{1}{36}\rho_t^3 + \frac{1}{12}\left(\frac{19}{6} - L\right)\rho_t^2 -$$
$$- \frac{1}{36}\left(6\zeta_2 + \frac{265}{6} + 3L^2 - 25L\right)\rho_t.$$

We now turn to the inelastic process of a soft lepton–antilepton pair production (of the mass $\mu$ obeying $2\mu \ll \Delta\varepsilon \ll \varepsilon$). For the differential cross-section, we obtain

$$\frac{d\sigma^{sp}}{d\sigma_0} = \frac{\alpha^2}{6\pi^2}\left[\frac{1}{3}\mathbf{L}^3 + \mathbf{L}^2\left(2\ln\Delta - \frac{5}{3}\right) + \right.$$
$$\left. + \mathbf{L}\left(4\ln^2\Delta - \frac{20}{3}\ln\Delta + \frac{56}{9} - 4\zeta_2 + 2\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right)\right] \tag{34}$$

with

$$\mathbf{L} = \ln(-t/\mu^2).$$

We assume a muon or an electron to be a scattered lepton, and consequently, the quantity $\mu$ stands for the corresponding mass.

The sum of contributions (33) and (34) does not contain cubic powers of large logarithms; for the «electron line corrections», it is found to be (see [11])

$$\left(\frac{d\sigma^{sv,vp}}{d\sigma_0}\right)_e = \left(\frac{\alpha}{\pi}\right)^2 \left\{\left(\frac{2}{3}\ln\Delta + \frac{1}{2}\right)\rho_t^2 + \right.$$
$$+ 2\rho_t\left[-\frac{17}{12} - \frac{11}{9}L + \frac{2}{3}\ln^2\Delta - \frac{10}{9}\ln\Delta - \zeta_2 + \right.$$
$$\left.\left. + \frac{1}{3}\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right]\right\}. \quad (35)$$

For a muon, it is given by

$$\left(\frac{d\sigma^{sv,vp}}{d\sigma_0}\right)_\mu = \left(\frac{\alpha}{\pi}\right)^2 \left\{\left(\frac{2}{3}\ln\Delta + \frac{1}{2}\right)\rho_t^2 + \right.$$
$$+ 2\rho_t\left[-\frac{17}{12} + \frac{11}{6}L + \frac{2}{3}\ln^2\Delta - \frac{10}{9}\ln\Delta - \zeta_2 + \right.$$
$$\left.\left. + \frac{1}{3}\mathrm{Li}_2\left(\frac{1+c}{2}\right)\right]\right\}. \quad (36)$$

It can be seen that the leading terms are in agreement with the RG predictions.

## 4. SUMMARY

We have evaluated the Born cross-section and the first-order radiative correction to it of the EMS process in the quasi-elastic kinematical situation. The relevant formulas are given in (2) and (3) in the leading logarithmic approximation and in (16) with power accuracy.

Among second-order contributions, we have considered gauge-invariant contributions from Feynman diagrams with radiative corrections to the vertex function of either lepton. We have also included soft photon and pair emission with the energies less than $\Delta\varepsilon$.

In the leading logarithmic approximation, the results are in agreement with the RG.

The explicit results for virtual and soft real photon emission are given in Eqs. (23) and (25). For the emission of virtual and soft real lepton pairs, the relevant formulas are given in Eqs. (34) and (35).

In Sec. 3.2, we determined the contributions coming from the hadronic vacuum polarization, where the radiative correction is expressed in terms of an explicit integral of the experimentally measured quantity $R(M^2)$. We also consider a soft pion pair production (see Appendix B). We calculate the hadronic vacuum polarization contribution to the vertex functions of the electron or muon explicitly. The relevant formulas for radiative corrections are given in (32).

The evaluation of contributions of the other gauge-invariant types, the eikonal and the decorated box Feynman diagrams requires additional investigation.

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## APPENDIX A

### Details of the hadronic vacuum polarization

We here consider the details of the vertex hadron function calculation. For the vertex function, we can write

$$V_\mu = \Gamma_1\gamma_\mu + \Gamma_2(\hat{q}\gamma_\mu - \gamma_\mu\hat{q}), \quad (37)$$

where $q = p_2 - p_2'$ and $\Gamma_1$ and $\Gamma_2$ are the Dirac and Pauli formfactors respectively. We write the vertex function as

$$V_\mu = \gamma_\mu[\Gamma_1 + 4m\Gamma_2] - 2(p_2 + p_2')_\mu\Gamma_2 =$$
$$= \gamma_\mu A + (p_2 + p_2')_\mu B, \quad (38)$$

where

$$A = \int y\,dy \int dx\left[\frac{2y2p_2p_2'}{-d} + \frac{4y^2}{d}(m^2 + p_2p_2') - \right.$$
$$\left. -2y\ln\frac{d}{m^2} + \frac{-2m^2y^3x^2}{d} + \frac{2y^3x(1-x)}{d}(-2p_2p_2')\right],$$
$$B = \int y\,dy \int dx\left[\frac{y^2}{d}(-2m) + \right.$$
$$\left. + \frac{2y^3x^2}{d}2m + \frac{2y^3x(1-x)}{d}2m\right]. \quad (39)$$

The quantities $d$ and $d_0$ are defined in Eq. (29). With the regularization at $t = 0$, we have

$$F_1(t, m^2, M^2) = \Gamma_1 - \Gamma_1|_{t=0} =$$
$$= 2\int_0^1 dx \int_0^1 y\,dy\left[\ln\frac{d_0}{d} + \frac{a}{d} - \frac{a_0}{d_0}\right]. \quad (40)$$

The contribution of the Pauli formfactor $\Gamma_2$ is proportional to $B$ and is therefore suppressed by the factor $|m^2/t|$.

## APPENDIX B

### Soft pion pair production

The general expression for the soft pion pair production is

$$\left|\frac{M}{M_0}\right|^2 d\Gamma_\pm = \left(\frac{4\pi\alpha}{q^2}\right)^2 d^4q \int \frac{d^3q_+}{2\varepsilon_+} \times \\ \times \int \frac{d^3q_-}{2\varepsilon_-}(2\pi)^{-6}\delta^4(q_+ + q_- - q) \times \\ \times (q_+ - q_-)_\mu (q_+ - q_-)_\nu J_\mu J_\nu, \quad J_\mu = (Q_q^{p_1 p_1'})_\mu,$$

where

$$m_\mu \ll \sqrt{q^2} \ll \Delta\varepsilon \ll \varepsilon, \quad q_0^2 \gg q^2.$$

Here, $q_\pm$ is the 4-momentum and $\epsilon_\pm$ is the energy of $\pi^\pm$; $q$ is the 4-momentum and $q_0$ is the energy of the soft pair.

Rewriting

$$\int d^4q = \frac{4\pi}{2}\int dq^2 \frac{d\Omega_q}{4\pi} \int\limits_{\sqrt{q^2}}^{\Delta\varepsilon} dq_0 \sqrt{q_0^2 - q^2},$$

$$\int \frac{d^3q_+}{2\varepsilon_+}\int \frac{d^3q_-}{2\varepsilon_-}(2\pi)^{-6}\delta^4(q_+ + q_- - q)(q_+ - q_-)_\mu \times \\ \times (q_+ - q_-)_\nu = \frac{1}{3}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \times \\ \times 2^{-7}\pi^{-5}(4m^2 - q^2)\sqrt{1 - \frac{4m^2}{q^2}},$$

we obtain

$$\left|\frac{M}{M_0}\right|^2 d\Gamma_\pm = -\frac{\alpha^2}{4\pi^3}\frac{(q^2 - 4m^2)^{3/2}}{(q^2)^3} \times \\ \times \int \frac{d\Omega_q}{4\pi}\int dq_0 \sqrt{q_0^2 - q^2}\, J^2,$$

where

$$J^2 = J_\mu J_\nu \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right).$$

Separate contributions are given by

$$\int \frac{d\Omega_q}{4\pi}\frac{m^2}{(p_1 q)^2} = O\left(\Delta^2 \frac{m^2}{q^2}\right),$$

$$\int \frac{d\Omega_q}{4\pi}\frac{p_1 p_2}{p_1 q p_2 q} = \int \frac{d\Omega_q}{4\pi}\frac{p_1' p_2'}{p_1' q p_2' q} = \\ = \frac{1}{2}\ln^2\left(\frac{2\Delta\varepsilon}{\sqrt{q^2}}\right) - \ln 2$$

and the master integral is

$$\int\limits_{\sqrt{q^2}}^{\Delta\varepsilon} dq_0 \sqrt{q_0^2 - q^2}\int \frac{d\Omega_q}{4\pi}\frac{p_1 p_1'}{p_1 q p_1' q} = \\ = \ln^2\left(\frac{2\Delta\varepsilon}{\sqrt{q^2}}\right) + \ln\left(\frac{2\Delta\varepsilon}{\sqrt{q^2}}\right)\ln\left(\frac{1-c}{2}\right) - \zeta_2.$$

The soft pion pair production contribution to the invariant mass distribution (from the emission of both the electron and muon blocks) is given by

$$\frac{M^2}{\sigma_0}\frac{d\sigma}{dM^2} = \frac{\alpha^2}{3\pi^2}\left[\ln^2\frac{-t}{M^2} + \ln\frac{-t}{M^2}\ln\frac{\Delta\varepsilon}{\varepsilon} + O(1)\right].$$

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