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GLOBAL MONOPOLE IN GENERAL RELATIVITY

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We consider the gravitational properties of a global monopole on the basis of the simplest Higgs scalar triplet model in general relativity. We begin with establishing some common features of hedgehog-type solutions with a regular center, independent of the choice of the symmetry-breaking potential. There are six types of qualitative behaviors of the solutions; we show, in particular, that the metric can contain at most one simple horizon. For the standard Mexican hat potential, the previously known properties of the solutions are confirmed and some new results are obtained. Thus, we show analytically that solutions with the monotonically growing Higgs field and finite energy in the static region exist only in the interval $1 < \gamma < 3$, where γ is the squared energy of spontaneous symmetry breaking in Planck units. The cosmological properties of these globally regular solutions apparently favor the idea that the standard Big Bang might be replaced with a nonsingular static core and a horizon appearing as a result of some symmetry-breaking phase transition at the Planck energy scale. In addition to the monotonic solutions, we present and analyze a sequence of families of new solutions with the oscillating Higgs field. These families are parameterized by n, the number of knots of the Higgs field, and exist for $\gamma < \gamma_n = 6/[(2n+1)(n+2)]$; all such solutions possess a horizon and a singularity beyond it.

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1. INTRODUCTION

In accordance with the Standard cosmological model [1], the Universe has been expanding and cooling from a split second after the Big Bang to the present moment and remained uniform and isotropic in doing so. In the process of its evolution, the Universe has experienced a chain of phase transitions with spontaneous symmetry breaking, including the Grand Unification and electroweak phase transitions, formation of neutrons and protons from quarks, recombination, and so forth. Regions with spontaneously broken symmetry that are more than the correlation length apart are statistically independent. At interfaces between these regions, the so-called topological defects necessarily arise. A systematic exposition of the potential role of topological defects in our Universe was given by Vilenkin and Shellard [2]. The particular types of defects — domain walls, strings, monopoles, or textures — are determined by topological properties of the vacuum [3]. If the vacuum manifold is not shrinkable to a point after the breakdown, then the Polyakov–t'Hooft monopole-type solutions [4, 5] appear in quantum field theory.

Spontaneous symmetry breaking plays a fundamental role in modern attempts to construct particle theories. In this context, a symmetry is commonly associated with internal rather than space-time transformations, e.g., the isotopic, electroweak, Grand Unification symmetries, and supersymmetry, whose transformations mix bosons and fermions. Topological defects that are caused by spontaneous breaking of internal

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symmetries and are independent of space-time coordinates are said to be global.

A fundamental property of a global symmetry violation is the Goldstone degree of freedom. For the monopole, the term related to the Goldstone boson in the energy–momentum tensor decreases rather slowly away from the center. As a result, the total energy of a global monopole grows linearly with the distance, or in other words, diverges. Without gravity, this divergence is a general property of spontaneously broken global symmetries. In his pioneering paper [4], Polyakov mentioned two possibilities of avoiding this difficulty. The first one was to combine the monopole with the Yang–Mills field. This idea was independently considered by t'Hooft [5]. This, among other reasons, gave rise to numerous papers on gauge (magnetic) monopoles. The second possibility was to consider a bound monopole–antimonopole system, whose total energy would be large (proportional to the distance between the components) but finite.

One more possibility is to take the self-gravity of global monopoles into account; this can in principle remove the above self-energy problem and is also necessary for potential astrophysical applications. Such a study was first performed by Barriola and Vilenkin [6], who found that the gravitational field outside a monopole is characterized by a solid angle deficit proportional to the energy scale of the spontaneous symmetry breaking. Harari and Lousto [7] showed that the gravitational mass of a global monopole, calculated using the Tolman integral, is negative. Solutions with a horizon for supermassive global monopoles were found by Liebling [8], who also confirmed the estimate in Ref. [9] for the upper value of the symmetry breaking energy compatible with a static configuration. The existence of de Sitter cores inside global monopoles and other topological defects have led to the idea of «topological inflation» [10–12].

For global strings in flat space, the energy per unit length (without gravitation) also diverges with growing distance from the axis, but only logarithmically. But in general relativity, integration over the cross-section yields a finite result [13, 14]. The gravitational interaction thus leads to self-localization of a global string. Does a similar effect occur for a global monopole? An attempt to answer this question, which does not appear to be answered in the existing papers, was one of the motivations for reconsidering the gravitational properties of a global monopole.

The previous studies have used the boundary condition according to which the symmetry-breaking potential must vanish at spatial infinity. Our approach is different: we do not even assume the existence of a spatial asymptotics, but require regularity at the center and try to observe the properties of the entire set of global monopole solutions. In doing so, among other quantities, we discuss the behavior of the total scalar field energy, which turns out to be finite in static regions of supermassive global monopoles.

In Sec. 2, we present the complete sets of equations for a static spherically symmetric gravitating global monopole in two most convenient coordinate systems, those with quasiglobal and harmonic radial coordinates. The general properties of static global monopoles are summarized in Sec. 3. In Sec. 4, we analytically and numerically analyze the specific features of a global monopole in the particular case of the Mexican hat potential. Section 5 contains a general discussion of our results, including their possible cosmological interpretation.

2. EQUATIONS AND BOUNDARY CONDITIONS

2.1. General setting of the problem

We begin with the most general form of a static spherically symmetric metric, without specifying the radial coordinate $x^1 = u$,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = e^{2F_0}dt^2 - e^{2F_1}du^2 - e^{2F_\Omega}d\Omega^2. \quad (1)$$

Here, $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is the linear element on a unit sphere and F_0, F_1, and F_Ω are functions of u. The nonzero components of the Ricci tensor are (the prime denotes d/du)

$$\begin{aligned} R_0^0 &= e^{-2F_1}[F_0'' + F_0'(-F_1' + 2F_\Omega' + F_0')], \\ R_1^1 &= e^{-2F_1}[F_0'' + 2F_\Omega'' + 2F_\Omega'^2 + F_0'^2 - \\ &- F_1'(2F_\Omega' + F_0')], \\ R_2^2 &= R_3^3 = -e^{-2F_\Omega} + \\ &+ e^{-2F_1}[F_\Omega'' + F_\Omega'(-F_1' + 2F_\Omega' + F_0')]. \end{aligned} \quad (2)$$

We consider the Lagrangian describing a triplet of real scalar fields ϕ^a $(a = 1, 2, 3)$ in general relativity,

$$L = \frac{R}{16\pi G} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^a - V(\phi), \quad (3)$$

where R is the scalar curvature, $V(\phi)$ is a potential depending on $\phi = \pm\sqrt{\phi^a\phi^a}$, and G is the gravitational constant. We use the natural units such that

$$\hbar = c = 1, \quad (4)$$

and therefore, $G = m_{Pl}^{-2}$, where $m_{Pl} = 1.22 \cdot 10^{19}$ GeV is the Planck mass.

To obtain a global monopole with unit topological charge [2], we assume that the metric has form (1) and ϕ^a comprise the following «hedgehog» configuration:

$$\phi^a = \phi(u) n^a, \; n^a = \{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}. \quad (5)$$

The Einstein equations can be written as

$$R_\mu^\nu = -8\pi G \tilde{T}_\mu^\nu = -8\pi G \left(T_\mu^\nu - \frac{1}{2}\delta_\mu^\nu T_\alpha^\alpha \right), \quad (6)$$

where T_μ^ν is the energy-momentum tensor and the nonzero components of $\tilde{T}_\mu^\nu$ are

$$\tilde{T}_0^0 = -V, \quad \tilde{T}_1^1 = -V - e^{-2F_1}\phi'^2, \quad \tilde{T}_2^2 = \tilde{T}_3^3 = -V - e^{-2F_\Omega}\phi^2. \quad (7)$$

The conditions for metric (1) to be regular at the center are that

$$e^{F_\Omega} \to 0, \quad F_0 = F_{0\,c} + O(e^{2F_\Omega}), \quad e^{-F_1+F_\Omega}|F_\Omega'| \to 1 \quad (8)$$

at the corresponding value u_c of the coordinate $x^1 = u$. The last condition is necessary for local flatness and ensures the correct ratio of the circumference to the radius for coordinate circles at small $r = e^{F_\Omega}$.

The scalar field energy, defined as the partial time derivative of the scalar field action, $E = -\partial S/\partial t$, is a conserved quantity for our static system,

$$E = \int \sqrt{-g}\, T_0^0 d^3x = 4\pi \int e^{F_0+F_1+2F_\Omega} \times \left(\frac{1}{2} e^{-2F_1}\phi'^2 + e^{-2F_\Omega}\phi^2 + V \right) du, \quad (9)$$

where g is the determinant of the metric tensor.

In what follows, we make some general inferences without specifying the potential $V(\phi)$ and then perform a more detailed study for the simplest and most frequently used symmetry-breaking potential

$$V(\phi) = \frac{1}{4}\lambda(\phi^a\phi^a - \eta^2)^2 = \frac{1}{4}\eta^4\lambda(f^2-1)^2, \quad (10)$$

where $\eta > 0$ characterizes the energy of symmetry breaking, λ is a dimensionless constant, and $f(u) = \phi(u)/\eta$ is the normalized field magnitude playing the role of an order parameter. The model has a global $SO(3)$ symmetry, which can be spontaneously broken to $SO(2)$ by potential wells ($V = 0$) at $f = \pm 1$.

We now explicitly write the Einstein equations and the boundary conditions in the two coordinate frames to be used.

2.2. The quasiglobal coordinate ρ

The first choice is the coordinate $u = \rho$ specified by the condition $F_0 + F_1 = 0$. Setting $e^{2F_0} = e^{-2F_1} = A(\rho)$ and $e^{F_\Omega} = r(\rho)$, we obtain the metric

$$ds^2 = A(\rho)dt^2 - \frac{d\rho^2}{A(\rho)} - r^2(\rho)d\Omega^2. \quad (11)$$

The scalar field equation

$$\Box\phi^a + \partial V/\partial\phi^a = 0, \quad (12)$$

where $\Box = \nabla^\alpha\nabla_\alpha$ is the d'Alembert operator, and certain combinations of the Einstein equations are given by

$$(Ar^2\phi')' - 2\phi = r^2 dV/d\phi, \quad (13)$$

$$(A'r^2)' = -16\pi G r^2 V, \quad (14)$$

$$2r''/r = -8\pi G\phi'^2, \quad (15)$$

$$A(r^2)'' - r^2 A'' = 2(1 - 8\pi G\phi^2), \quad (16)$$

$$A'rr' + Ar'^2 - 1 = 8\pi G\left(\frac{1}{2}Ar^2\phi'^2 - \phi^2 - r^2 V \right), \quad (17)$$

where the prime denotes $d/d\rho$. Only three of these five equations are independent: scalar field equation (13) follows from the Einstein equations and Eq. (17) is a first integral of the others. Given a potential $V(\varphi)$, this is a determined set of equations for the unknowns r, A, and ϕ.

This choice of the coordinates is preferable for considering Killing horizons, which correspond to zeros of the function $A(\rho)$, because such zeros are regular points of Eqs. (13)–(17); moreover, in a close neighborhood of a horizon, the coordinate ρ defined in this manner varies (up to a positive constant factor) as the manifestly well-behaved Kruskal-like coordinates used for analytic continuation of the metric [15, 16]. Therefore, the regions at both sides of a horizon can be simultaneously considered in terms of ρ and the entire range of ρ can contain several horizons in general. For this reason, the coordinate ρ can be called *quasiglobal*.

The regularaity conditions at the center, Eq. (8), are satisfied if

$$A(\rho) = A_c + O((\rho-\rho_c)^2), \quad r(\rho) \approx (\rho-\rho_c)/\sqrt{A_c} \quad (18)$$

near some value ρ_c of the coordinate ρ.

In regions where $A < 0$ (sometimes called T-regions [1]), whenever they exist, the coordinate ρ is timelike and t is spacelike. Changing the notation as $t \to x \in \mathbb{R}$ and introducing the proper time of a comoving observer in the T-region,

$$\tau = \int d\rho/\sqrt{|A(\rho)|}, \qquad (19)$$

we can rewrite the metric as

$$ds^2 = d\tau^2 - |A(\tau)|dx^2 - r^2(\tau)d\Omega^2. \qquad (20)$$

The space-time geometry then corresponds to a homogeneous anisotropic cosmological model of the Kantowski–Sachs type [17, 18], where spatial sections have the topology of $\mathbb{R} \times \mathbb{S}^2$.

2.3. The harmonic coordinate u

Another convenient variable that allows considerably simplifying the form of the equations is the harmonic coordinate u specified by the condition [19][1)]

$$F_1 = 2F_\Omega + F_0, \qquad (21)$$

such that $\Box u = 0$. The field equations can then be written as

$$\phi'' - 2e^{F_0+F_1}\phi = e^{2F_1}dV/d\phi, \qquad (22)$$

$$F_0'' = -8\pi G e^{2F_1}V, \qquad (23)$$

$$F_1'' - 2F_\Omega'(F_\Omega' + 2F_0') = -8\pi G(\phi'^2 + e^{2F_1}V), \qquad (24)$$

$$F_\Omega'' - e^{2(F_0+F_\Omega)} = -8\pi G(\phi^2 e^{2(F_0+F_\Omega)} + e^{2F_1}V), \qquad (25)$$

$$-e^{-2F_\Omega} + e^{-2F_1}(F_\Omega'^2 + 2F_\Omega' F_0') = = 8\pi G\left(\frac{1}{2}e^{-2F_1}\phi'^2 - e^{-2F_\Omega}\phi^2 - V\right), \qquad (26)$$

where the prime denotes d/du.

It is straightforward to obtain that the regularity condition at the center can only correspond to $u \to \pm\infty$; we choose $u \to -\infty$, where we must have

$$e^{F_\Omega} \sim 1/|u|, \quad e^{F_0} = \sqrt{A_c}\,(1 + O(u^{-2})), \quad e^{F_1} \sim 1/u^2, \qquad (27)$$

and A_c is the same as in (18).

1) A cylindrical version of the harmonic radial coordinate has been used previously in the analysis of gravitational properties of current-conducting filaments [20] and cosmic strings [21, 22].

3. GENERAL PROPERTIES OF GLOBAL MONOPOLES

3.1. Monopoles in Minkowski space-time

The Minkowski metric written in the usual spherical coordinates,

$$ds^2 = dt^2 - dr^2 - r^2 d\Omega^2, \qquad (28)$$

is a special case of (11) with $r \equiv \rho$ and $A \equiv 1$. In flat space-time, the only unknown is $\phi(r)$ and the only field equation is (13), which becomes

$$(r^2\phi')' - 2\phi = r^2 dV/d\phi. \qquad (29)$$

For the particular potential in Eq. (10), we have $dV/d\phi = \lambda\phi(\phi^2 - \eta^2)$ and the scalar field equation can then be written in terms of $f = \phi/\eta$ as

$$r^{-2}(r^2 f')' - 2fr^{-2} + \lambda\eta^2 f(1 - f^2) = 0. \qquad (30)$$

The energy integral in Eq. (9) takes the form

$$E = 4\pi \int r^2 \left(\frac{1}{2}\phi'^2 + \frac{\phi^2}{r^2} + V\right) dr. \qquad (31)$$

In the case where $V(\phi) \geq 0$, its convergence implies that all the three terms must vanish as $r \to \infty$ sufficiently rapidly:

$$\phi = o(r^{-1/2}), \quad \phi' = o(r^{-3/2}), \quad V = o(r^{-3}). \qquad (32)$$

This actually implies that a finite-energy configuration is only possible with $V(0) = 0$, contrary to the symmetry breaking assumption according to which V has minima in nonsymmetric states, $\phi \neq 0$. In particular, potential (10) does not give rise to global monopoles with a finite energy. A consideration of self-gravity of the field triad ϕ^a is one of the ways to overcome this difficulty.

In flat space-time, the harmonic coordinate u is related to r as $u - u_0 = \pm 1/r$, where u_0 is an arbitrary constant; choosing the minus sign, we find that u ranges from $-\infty$, which corresponds to the center $r = 0$, to u_0 corresponding to spatial infinity.

3.2. Solutions with constant ϕ

Under the assumption that $\phi = \phi_0 = \text{const}$, the corresponding value of the potential $V(\phi_0) = V_0$ (times $8\pi G$) plays the role of a cosmological constant, and the Einstein equations can be integrated explicitly.

Indeed, in the region where $\phi = \text{const}$, Eq. (15) reduces to $r'' = 0$, whence $r = \alpha\rho + r_0$, where

$\alpha, r_0 =$ const. It remains to find $A(r)$, and this is immediately done by integrating Eq. (16),

$$A(r) = \frac{1-\Delta}{\alpha^2} - \frac{2GM}{r} + Cr^2, \quad \Delta = 8\pi G\phi_0^2, \quad (33)$$

where M and C are integration constants. Substituting (33) in (14), we find

$$C = -8\pi G V_0/3. \quad (34)$$

Thus, the solution is essentially determined by the values of ϕ_0, V_0, and M. One more constant, α, reflects the freedom in choosing the unit of time. We note that this is not a monopole solution. Even if we set $M = 0$, which is evidently necessary for regularity at $r = 0$, this solution with constant $\phi \neq 0$ is singular at the center: for $A(r)$ given by (33) with $M = 0$, the Kretschmann scalar is $\mathcal{K} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \approx 4\Delta^2/r^4$ at small r.

Regarding the global monopole, two cases of the solution in Eq. (33) are of interest. The case where $\phi_0 \equiv 0$ describes the symmetric state and the case where $V_0 = 0$ gives a possible asymptotic behavior at spatial or temporal infinity.

In the case where $\phi_0 = 0$ (the symmetric state), setting $M = 0$ (which is necessary for a regular center), we arrive at the de Sitter metric

$$ds^2 = \left(1 - \frac{r^2}{r_h^2}\right) dt^2 - \left(1 - \frac{r^2}{r_h^2}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad r_h^2 = \frac{8\pi G V_0}{3}. \quad (35)$$

This metric has a horizon at $r = r_h$. At $r > r_h$, outside the horizon, r becomes a timelike coordinate and t is a spacelike one. Changing the notation as in (19) and (20), we obtain the metric

$$ds^2 = d\tau^2 - \mathrm{sh}^2(\tau/r_h)dx^2 - r_h^2 \,\mathrm{ch}^2(\tau/r_h)d\Omega^2. \quad (36)$$

This is the Kantowski–Sachs cosmology with the isotropic inflationary expansion at late times ($\tau \to \infty$).

In the other case, $\phi_0 \neq 0$ but $V_0 = 0$ (the case of broken symmetry, such as $\phi = \eta$ in potential (10)), the metric becomes [6]

$$ds^2 = \left(\frac{1-\Delta}{\alpha^2} - \frac{2GM}{r}\right) dt^2 - \left(\frac{1-\Delta}{\alpha^2} - \frac{2GM}{r}\right)^{-1} dr^2 - r^2 d\Omega^2, \quad (37)$$

where the constant M has the meaning of mass in the sense that test particles at rest experience the acceleration $-GM/r^2$ in gravitational field (37) at large r. Furthermore, a nonzero value of ϕ_0 leads to a solid angle deficit Δ defined in (33) in the asymptotic region as $r \to \infty$ (see [2] for more detail) and to a linear divergence of integral (9) at large r.

The general case of Eq. (33) describes the large-r asymptotic behavior of any solution to Eqs. (13)–(17), provided that such an asymptotic form exists and ϕ tends to a constant value sufficiently rapidly.

For the monopoles to be studied, metric (37) gives a large-r asymptotic behavior in the case where $\Delta < 1$. We also consider solutions with $\Delta > 1$, for which a static asymptotic regime is absent. Metric (37) then describes cosmological evolution at late times.

3.3. General properties of solutions with varying ϕ

We now consider the general form of Eqs. (13)–(17) with varying ϕ, without specifying the potential $V(\phi)$.

We first note that because of (15), we have $r'' \leq 0$, which forbids any nonsingular configurations without a center such as wormholes and horns (see Theorem 1 in Ref. [16] for further details).

Second, Eq. (16) can be rewritten as

$$(r^4 B')' = -2(1 - 8\pi G\phi^2), \quad B \stackrel{\text{def}}{=} A/r^2, \quad (38)$$

and at a point where $B' = 0$, we have $r^4 B'' = -2(1 - 8\pi G\phi^2)$. Hence, it follows that as long as $\phi^2 < 1/(8\pi G)$ (i.e., the ϕ field does not reach trans-Planckian values), $B'' < 0$ at possible extrema of the function B. In other words, B cannot have a regular minimum.

Our interest is in systems with a regular center satisfying conditions (18) with $A(\rho) > 0$ and $B(\rho) > 0$ near $\rho = \rho_c$. At a possible horizon $\rho = h$, both A and B vanish, and because this cannot be a minimum of B, $B < 0$ at $\rho > h$ near the horizon. At greater ρ, the function $B(\rho)$, having no minima, can only decrease and never returns to zero; therefore, $A = Br^2 < 0$ at $\rho > h$. We conclude that there can be no more than one horizon, and if it exists, it is simple (corresponds to a simple zero of $A(\rho)$). Because the global causal structure of space-time is determined (up to possible identifications of isometric hypersurfaces) by the number and disposition of Killing horizons [23–25], we have the following result.

Statement 1. *Under the assumption that $\phi^2 < 1/(8\pi G)$ in the entire space, our system with a regular center can have either no horizon or one simple horizon; in the latter case, its global structure is the same as that of de Sitter space-time.*

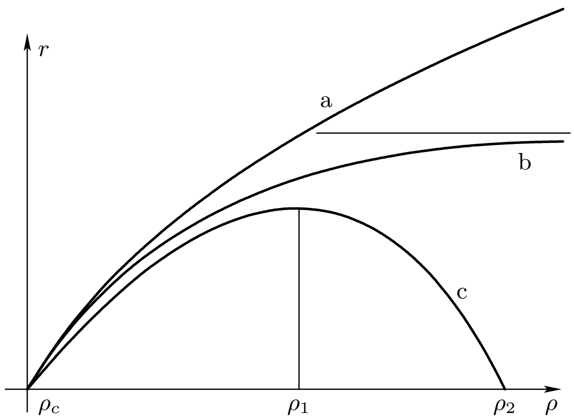

Fig. 1. Possible behavior of $r(\rho)$ in global monopole solutions

The above reasoning is essentially the same as in the proof of Theorem 2 in Ref. [16] on the disposition of horizons in scalar-vacuum space-times. It uses only Eq. (16), which does not involve the potential V. The conclusion is therefore valid for systems with any potentials, positive or negative.

We now return to Eq. (15), according to which $r'' \le 0$. Because $r' > 0$ at a regular center, this leaves three possibilities for the function $r(\rho)$ (Fig. 1):

(a) monotonic growth with a decreasing slope, but $r \to \infty$ as $\rho \to \infty$,

(b) monotonic growth with $r \to r_{\max} < \infty$ as $\rho \to \infty$, and

(c) growth up to $r_{\max}$ at some $\rho_1 < \infty$ and further decrease, reaching $r = 0$ at some finite $\rho_2 > \rho_1$.

In each case, according to Statement 1, a horizon can occur at some $\rho = h$ within the range of ρ, and we therefore have a T-region with the geometry of the Kantowski–Sachs cosmological model at $\rho > h$.

We conclude that there are six classes of qualitative behaviors of the solutions, i.e., (a), (b), and (c), each with or without a horizon, which we indicate with the respective symbols 1 or 0. Thus, all solutions with a spatial asymptotic behavior belong to class (a0). Class (b0) includes space-times ending with a «tube» consisting of two-dimensional spheres of equal radii. Solutions in class (c0) contain a second center at $\rho = \rho_2$, and this center can *a priori* be regular or singular. We thus obtain a static analogue of closed cosmologies. Classes (a1), (b1), and (c1) describe different late-time cosmological behaviors in the two directions corresponding to $\mathbb{S}^2$, whereas the fate of the third spatial direction ($\mathbb{R}$) is determined by the function $A(\rho)$. In particular, the possible de Sitter asymptotic metric in Eq. (36) belongs to class (a1) solutions, and the expansion is isotropic at late times in this case. On the other hand, class-(c1) contains models that at late times behave as the Schwarzschild space-time inside the horizon, contracting to $r = 0$.

This classification is obtained without any assumptions about $V(\phi)$. Solutions with given $V(\phi)$ contain some of these classes, not necessarily all of them.

In the case $V \ge 0$, where Eq. (14) leads to one more important observation: because $A'r^2 = 0$ at a regular center, we can write (14) in the integral form

$$A'r^2 = -16\pi G \int_0^{\rho} V(\bar\rho)\, r^2(\bar\rho)\, d\bar\rho, \qquad (39)$$

and therefore, $A(\rho)$ is a decreasing function unless $V \equiv 0$. Equation (39) leads to the following conclusions.

Statement 1a. *If $V(\phi) \ge 0$, our system with a regular center can have either no horizon or one simple horizon; in the latter case, its global structure is the same as that of de Sitter space-time.*

Statement 2. *If $V(\phi) \ge 0$, the second center in class-(c0) solutions is singular.*

Statement 3. *If $V(\phi) \ge 0$ and the solution is asymptotically flat, the mass M of the global monopole is negative.*

Statement 1a shows that for nonnegative potentials, the assumption $\phi^2 < 1/(8\pi G)$ in Statement 1 is unnecessary, and the causal structure types are known for any magnitudes of ϕ.

Statement 2 follows from $A'(\rho_2) < 0$, whereas at a regular center, it should be $A' = 0$, see (18). The equality $A'(\rho_2) = 0$ could only be possible with $V \equiv 0$, but in this case, the only solution with a regular center is trivial (flat space, $\phi = 0$).

In Statement 3, the asymptotic flatness is understood up to the solid angle deficit, i.e., $r = \rho$ and A is given by (33) with $C = 0$ at large ρ. As $\rho \to \infty$, we then obtain $2GM$ in the left-hand side of Eq. (39) and a negative quantity in the right-hand side.

To our knowledge, this simple conclusion, valid for all nonnegative potentials, has so far been obtained only numerically for the particular potential (10) [9]. We note that Statement 3 is an extension to global monopoles of the so-called generalized Rosen theorem [16, 26], previously known for scalar-vacuum configurations.

Therefore, even before studying particular solutions with potential (10), we have a more or less complete knowledge of what can be expected of such global monopole systems.

4. THE MEXICAN HAT POTENTIAL

4.1. Equations and boundary conditions

In what follows, we analyze the particular Mexican hat potential in Eq. (10). For numerical integration, we prefer to use the harmonic coordinate u and to work with Eqs. (22)–(25). This variable enters the equations only via derivatives and is therefore invariant under translations $u \to u + \text{const}$.

Introducing the dimensionless quantities

$$\tilde{u} = u/(\sqrt{\lambda}\eta), \quad e^{\tilde{F}_\Omega} = \sqrt{\lambda}\eta e^{F_\Omega}, \quad e^{\tilde{F}_1} = \lambda\eta^2 e^{F_1}, \tag{40}$$

we eliminate the parameter λ from the equations. Indeed, omitting the tildes, we obtain

$$f'' = e^{2(F_0+F_\Omega)}[2 - e^{2F_\Omega}(1-f^2)]f, \tag{41}$$

$$F_0'' = -\frac{\gamma}{4}e^{2(F_0+2F_\Omega)}(f^2-1)^2, \tag{42}$$

$$F_\Omega'' = e^{2(F_0+F_\Omega)}\left[1 - \gamma f^2 - \frac{\gamma}{4}e^{2F_\Omega}(1-f^2)^2\right]. \tag{43}$$

Condition (21) is preserved for the newly defined quantities, but the metric becomes

$$ds^2 = e^{2F_0}dt^2 - \frac{e^{2F_1}du^2 + e^{2F_\Omega}d\Omega^2}{\lambda\eta^2}. \tag{44}$$

The boundary conditions as $u \to -\infty$ are given by

$$f = 0, \quad F_0 = 0, \quad F_0' = 0, \quad F_\Omega = -\ln(-u) + o(1/|u|). \tag{45}$$

They follow from the regularity requirement at the center and a particular choice of the time unit ($F_0 = 0$) and of the origin of the u coordinate (the fourth condition).

There remains only one dimensionless parameter in Eqs. (41)–(43),

$$\gamma = 8\pi G\eta^2, \tag{46}$$

which is the squared energy of symmetry breaking in Planck units.

It is easy to obtain that $\gamma = 1$ is a critical value of this parameter. Indeed, if we assume the existence of a large-r asymptotic behavior such that $f \to 1$, i.e., the field tends to the minimum of potential (10), then the asymptotic form of the metric at large r is given by (37) with $\Delta = \gamma$. Consequently, the asymptotics can be static only if $\gamma \le 1$, whereas for $\gamma > 1$, the large-r asymptotics can be only cosmological (the Kantowski–Sachs type), and there is a horizon separating such an outer region from the static monopole core.

On the other hand, if a configuration with $\gamma < 1$ possesses a horizon, there is again the Kantowski–Sachs cosmology outside it, but there cannot be a large-r asymptotic form, and in accordance with Sec. 3, the solutions belong to classes (b1) or (c1).

Now, leaving aside the sufficiently well studied case of solutions with a static asymptotics [2, 6, 7] belonging to class-(a0) in accordance with Sec. 3, we suppose that there is a horizon and return to Eqs. (41)–(43). The horizon corresponds to $u \to +\infty$. In such cases, in addition to (45), we impose the boundary condition

$$f(u) \to f_h, \quad |f_h| < \infty \quad \text{as} \quad u \to +\infty. \tag{47}$$

This condition is necessary for the regularity of a solution on the horizon and is applicable to classes (a1), (b1), and (c1).

For class-(a0) solutions, having a spatial asymptotic behavior and no horizon, condition (47) is meaningless. Moreover, the coordinate u then ranges from $-\infty$ to some $u_0 < \infty$ such that $r(u_0) = \infty$.

For configurations of classes (a0) and (a1), the commonly used boundary condition is

$$f \to 1 \quad \text{as} \quad r \to \infty. \tag{48}$$

It is of interest that in case (a1), to which both conditions are applicable, condition (47), being less restrictive, still leads to solutions satisfying (48) because of the properties of the physical system itself.

The set of equations (41)–(43) with boundary conditions (45) and (47) comprise a well-posed nonlinear eigenvalue problem. Its trivial solution, with $f = 0$ and de Sitter metric (35), describes the symmetric state (with unbroken symmetry). Nontrivial solutions describing hedgehog configurations with spontaneously broken symmetry can be found numerically and yield a sequence of eigenvalues γ_n, $n = 0, 1, \ldots$, and the corresponding values of the horizon radius $r_{h,n}$ for each given value of f_h. Conversely, for a given (admissible) value of γ, we obtain a sequence of values of f_h and r_h.

4.2. The linear eigenvalue problem

Liebling [8] has empirically found the upper critical value $\gamma_0 \approx 3$ for the existence of static solutions[2)]. In this section, we find a theoretical ground for this limit.

[2)] In the notation of Ref. [10], $\eta^* \approx \sqrt{3/(8\pi)}$.

Actually, we analytically find a sequence of critical values γ_n, $n = 0, 1, \ldots$, such that for $\gamma < \gamma_n$, there exist static configurations with the field magnitude $f(u)$ changing its sign n times.

Only the analysis for $f(u) > 0$ can be found in the literature. Our numerical integration of Eqs. (41)–(43) shows that in addition to solutions with monotonically growing $f(u)$ (which exist for $\gamma < \gamma_0 = 3$, Fig. 2*a*), there exist regular solutions for $\gamma < \gamma_1 = 2/3$ with $f(u)$ changing its sign once (Fig. 2*b*). Solutions with two zeros of $f(u)$ exist for $\gamma < \gamma_2 = 0.3$ (Fig. 2*c*), etc. All these solutions have a horizon, and the absolute value of f on the horizon, $|f_{h,n}| = |f_n(\infty)|$, is a decreasing function of γ, vanishing as $\gamma \to \gamma_n - 0$ (Fig. 3).

As $\gamma \to \gamma_n$, the function $f(u)$ vanishes in the entire range of u, and it is this circumstance that allows us to find the critical values γ_n analytically. In a close neighborhood of γ_n, the field $f(u)$ is small within the horizon, $f^2 \ll 1$, and Eq. (41) therefore reduces to a linear equation with given background functions F_0 and F_Ω corresponding to de Sitter metric (35). In terms of the dimensionless spherical radius r, Eq. (41) becomes

$$\frac{d}{dr}\left[r^2\left(1 - \frac{r^2}{r_h^2}\right)\frac{df}{dr}\right] - (2 - r^2)f = 0, \qquad (49)$$

where $r_h = \sqrt{12/\gamma}$ is the value of r on the horizon. The boundary conditions are

$$f\Big|_{r=0} = 0, \quad |f(r_h)| < \infty. \qquad (50)$$

Nontrivial solutions of (49) with these boundary conditions exist for a sequence of eigenvalues $\gamma = \gamma_n$, $n = 0, 1, 2, \ldots$, and the corresponding eigenfunctions $f_n(r)$, which are regular in the interval $0 \le r \le r_h$, are simple polynomials,

$$f_n(r) = \sum_{k=0}^{n} a_k \left(\frac{r}{r_h}\right)^{2k+1}. \qquad (51)$$

Substituting (51) in (49), we find the eigenvalues

$$r_{h,n}^2 = 2(2n+1)(n+2), \quad \gamma_n = \frac{3}{(n+1/2)(n+2)} \qquad (52)$$

and the recurrent relation

$$a_k = a_{k-1}\frac{(2k-1)(2k+2) - r_{h,n}^2}{(2k+1)(2k+2) - 2}, \quad k = 1, 2, \ldots, \qquad (53)$$

allowing us to express all a_k, $k = 1, 2, \ldots, n$, in terms of a_0. Because Eq. (49) is linear and homogeneous, a_0

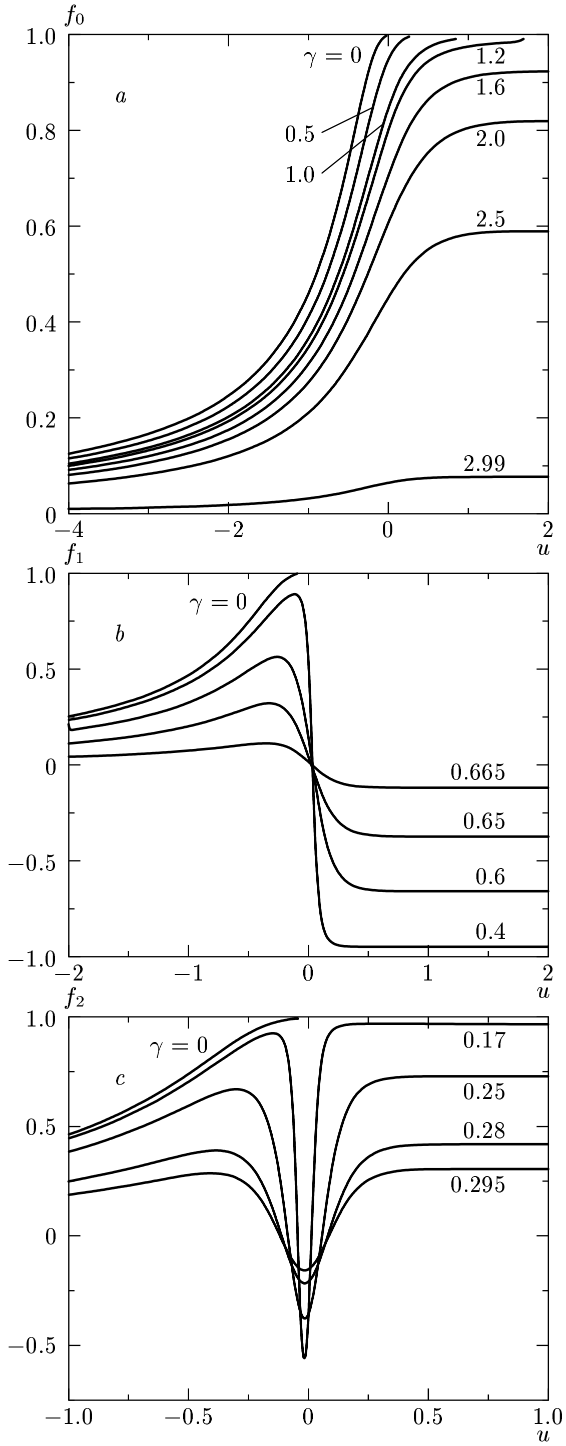

Fig. 2. The field magnitude f as a function of the harmonic coordinate u for different values of γ. Solutions with monotonically growing $f = f_0(u)$ (a) exist for $0 < \gamma < \gamma_0 = 3$. In the region $\gamma < \gamma_1 = 2/3$, there are solutions with $f = f_1(u)$ changing their sign once (b); in the region $\gamma < \gamma_2 = 0.3$, there are solutions with $f = f_2(u)$ changing their sign twice (c). As $\gamma \to \gamma_n - 0$, the function $f_n(u)$ vanishes in the entire range $-\infty < u < \infty$ from the center to the horizon

is an arbitrary constant[3]. For fixed n, the coefficients a_k in (51) are

$$a_k = a_0 \prod_{i=1}^{k} \frac{(2i-1)(2i+2) - r_{h,n}^2}{(2i+1)(2i+2) - 2}, \quad n > 0, \quad 1 \le k \le n. \tag{54}$$

The case where

$$n = 0, \quad r_{h,0} = 2, \quad f_0(r) = a_0 r / r_{h,0}$$

gives a monotonically growing function $f(u)$ in a close vicinity of $\gamma = \gamma_0 = 3$, see Fig. 2a. Thus, the upper limit $\gamma_0 = 3$ for the existence of static monopole solutions, previously found by Liebling [8] numerically, is now obtained analytically.

The case where

$$n = 1, \quad r_{h,1} = 3\sqrt{2},$$

$$f_1(r) = a_0 \frac{r}{r_{h,1}} \left[1 - \frac{7}{5}\left(\frac{r}{r_{h,1}}\right)^2\right]$$

describes the function $f(u)$ changing its sign once, at γ close to $\gamma_1 = 2/3$, see Fig. 2b. The case where $n = 2$, $\gamma_2 = 3/10$, $r_{h,2} = 2\sqrt{10}$, and

$$f_2(r) = a_0 \frac{r}{r_{h,2}} \left[1 - \frac{18}{5}\left(\frac{r}{r_{h,2}}\right)^2 + \frac{99}{35}\left(\frac{r}{r_{h,2}}\right)^4\right]$$

gives the field function $f(u)$ changing its sign twice (Fig. 2c).

For $n \gg 1$, the function $f_n(r)$ rapidly oscillates,

$$f_n(r) = a_0 \frac{\cos\left[r_{h,n} \arcsin\sqrt{\dfrac{r^2-2}{r_{h,n}^2-2}} - \sqrt{2}\arcsin\left(\dfrac{\sqrt{2}}{r}\sqrt{\dfrac{1-(r/r_{h,n})^2}{1-2/r_{h,n}^2}}\right)\right]}{\sqrt[4]{r^2(r^2-2)\left[1-(r/r_{h,n})^2\right]}}.$$

But this semiclassical formula is not valid near the left turning point[4] $r = \sqrt{2}$, see the dashed curve in Fig. 4. Its applicability range is $1 \ll r < r_{h,n} \approx 2n$, $n \gg 1$.

We have not previously met regular monopole configurations with the field function $f(u)$ changing its sign. It seems that this is their first presentation.

4.3. Solutions with monotonically growing $f(u)$

As is clear from the aforesaid, the interval $0 < \gamma < 3$ of the existence of nontrivial solutions with monotonically growing $f(u)$ splits into two qualitatively different regions separated by $\gamma = 1$.

In the interval $0 < \gamma < 1$, the solutions have spatial asymptotics (37); according to our general classification, they belong to class (a0). The spherical radius $r(u) = e^{F_\Omega(u)}$ varies from zero to infinity, $f(u)$ grows from zero to unity, and $A(u)$ decreases from unity to its limiting positive value (cf. Eq. (37))

$$A\Big|_{r\to\infty} = \frac{1-\gamma}{\alpha^2}, \quad \alpha = \frac{dr}{d\rho}\Big|_{\rho\to\infty} = 1 - \frac{\gamma}{2}\int_0^\infty f'^2(\rho) r(\rho) d\rho, \tag{55}$$

and the energy integral (9) diverges.

In the interval $1 < \gamma < 3$, solutions with monotonically growing $f(u)$ belong to class (a1). Instead of a spatial asymptotics, there is a horizon and the Kantowski–Sachs cosmology outside it. The functions A and r inside and outside the horizon are presented in Fig. 5 for $\gamma = 2$. In the presence of a global monopole, the cosmological expansion is slower than the de Sitter one, Eq. (36). As $\tau \to \infty$, the radius $r(\tau)$ grows linearly, while A tends to the negative constant value $-(\gamma - 1)/\alpha^2$.

Within the horizon, $f(u)$ monotonically grows from zero at $u = -\infty$ to a value $f_h = f_{h,0}(\gamma)$ on the horizon, $u \to \infty$, see Fig. 2a. As a function of γ, the value $f_{h,0}$ of f on the horizon decreases from unity at $\gamma = 1$ to zero at $\gamma = \gamma_0 = 3$, see Fig. 3. Integral (9) taken over the static region converges, and we can conclude that at $1 < \gamma < 3$, the gravitational field is sufficiently strong to suppress the Goldstone divergence and to localize the monopole. At $\gamma > 3$, gravity is probably so strong that it restores the high symmetry of the system.

Outside the horizon, the field f as a function of the proper time τ grows from $f_{h,0}$ on the horizon to unity

[3] As $\gamma \to \gamma_n - 0$, the general equation (41) has the same solution as (49), with $a_0 \ll 1$. To find the dependence $a_0(\gamma)$, we must take the next terms that are nonlinear in f into account.

[4] We recall that in view of the substitution (40), the distances are measured in the units $(\sqrt{\lambda}\eta)^{-1}$.

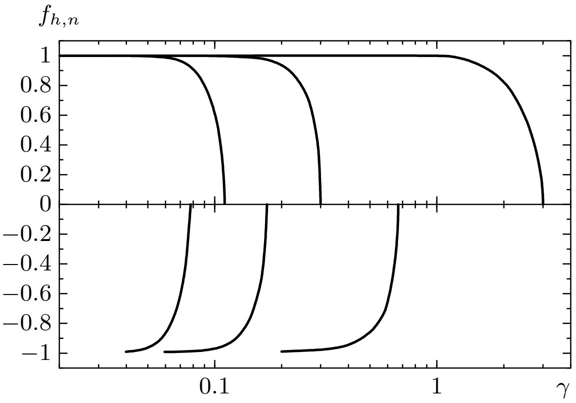

Fig. 3. The γ dependence of the values of $f(u)$ on the horizon, $f_{h,n} = f_n(u)|_{u\to\infty}$

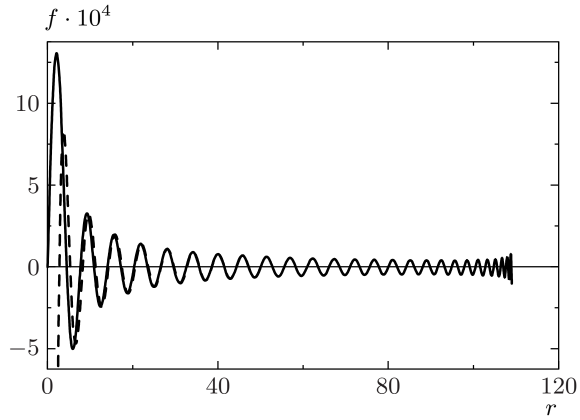

Fig. 4. The field magnitude f as a function of the spherical radius r for $\gamma = 0.001$

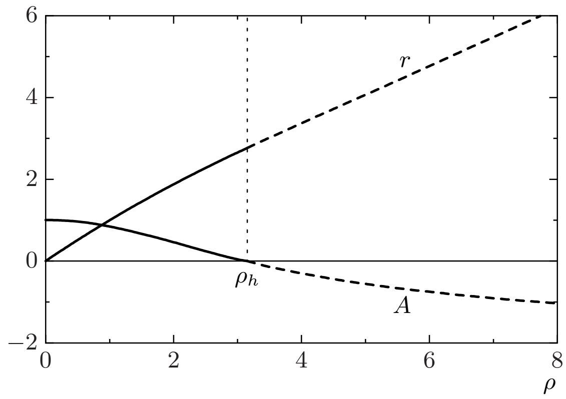

Fig. 5. The functions $A(\rho)$ and $r(\rho)$ form unified smooth curves in the regions inside (solid curves) and outside (dashed) the horizon; $\gamma = 2$

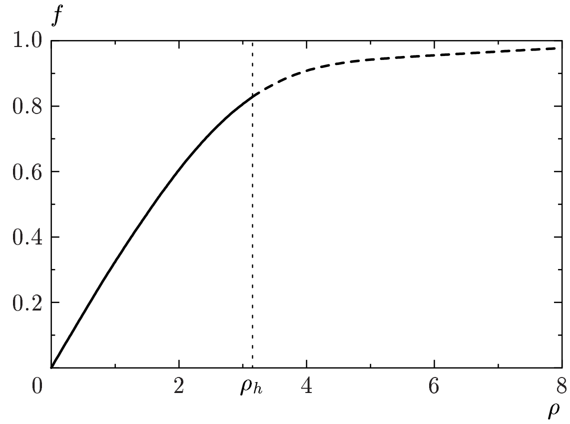

Fig. 6. The function $f(\rho)$ inside (solid curve) and outside (dashed curve) the horizon; $\gamma = 2$

as $\tau \to \infty$. Introducing the proper radial length l inside the horizon by the relation $dl = d\rho/\sqrt{A}$, we can ascertain that the functions $f(l(\rho))$ at $\rho < h$ and $f(\tau(\rho))$ at $\rho > h$ are two parts of a single smooth curve (Fig. 6).

When the parameter γ is close to its critical value $\gamma = 1$ separating the (a0) and (a1) branches of the solution, i.e., when

$$0 < \gamma - 1 \ll 1, \qquad (56)$$

the horizon radius $r_{h,0}$ and the scalar field value on the horizon $f_{h,0}$ can be found analytically under certain additional assumptions on the system behavior that follow from the results of numerical analysis. In particular, there is an «intermediate» region of the u range, $1 \ll u \ll u_0 = \text{const}$, where the first term f'' in the scalar field equation (41) is very small, whereas the function $e^{2(F_0+F_\Omega)}$ is quite large (despite the fact that this function eventually vanishes as $u \to \infty$). In this region, the expression in square brackets in (41) must therefore be small, i.e.,

$$e^{2F_\Omega}(1 - f^2) \approx 2.$$

This relation can be used for further estimates. The results are

$$\ln r_{h,0} \approx \ln[1/(\gamma - 1)] \gg 1, \qquad f_{h,0} \approx 1 - C(\gamma - 1)^2, \qquad (57)$$

where the constant C can be found by comparison with the numerical results; our estimate is $C \approx 0.2$.

The behavior of the solution in the critical regime, $\gamma = 1$, can be characterized as a globally static model with a «horizon at infinity» [8], because $A \to 0$ as $r \to \infty$.

The fact that monotonic solutions with horizons are absent for $\gamma < 1$ becomes evident from the analysis of

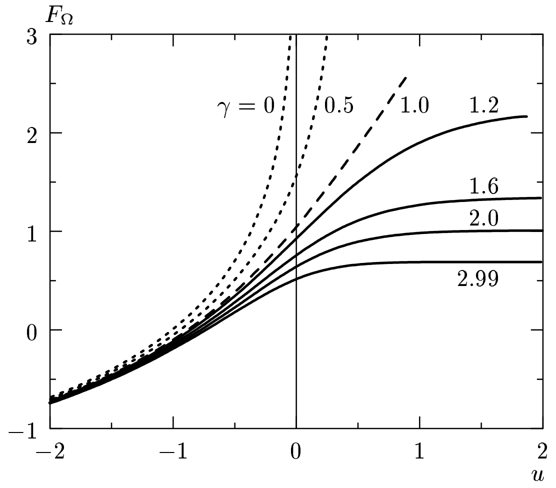

Fig. 7. The function $F_\Omega(u)$ for different values of γ. At $\gamma < 1$, there is a limiting value u_{max} of u such that $F_\Omega \to \infty$ as $u \to u_{max}(\gamma)$ (dotted curves). As $\gamma \to 1$, the value $u_{max}(\gamma) \to \infty$ (dashed curve); for $\gamma > 1$, the function $F_\Omega(u)$ tends to a finite constant value as $u \to \infty$ (solid curves)

the inflection point $u = u_{inf}$ of the function $F_\Omega(u)$. A horizon, whenever it exists, corresponds to $u \to \infty$, where F_Ω remains finite. Because it behaves logarithmically as $u \to -\infty$, there is (at least one) inflection point where the second-order derivative is zero, and it follows from Eq. (43) that

$$1 - \gamma f^2 - \frac{\gamma}{4} e^{2F_\Omega}(f^2 - 1)^2 = 0, \quad u = u_{inf}.$$

This is a quadratic equation for $1 - f^2$, and hence,

$$1 - f^2 = 2e^{-2F_\Omega}\left(1 \pm \sqrt{1 - \frac{\gamma - 1}{\gamma} e^{2F_\Omega}}\right). \qquad (58)$$

A monotonically growing function $f(u)$ corresponds to greater values of f, i.e., to the «minus» branch of (58) (as is confirmed by numerical results). But the right-hand side of (58) is then negative for $\gamma < 1$, leading to $f^2 > 1$, which cannot occur because $f^2 = 1$ is the maximum attainable value for the solutions under study. Therefore, for $\gamma < 1$, all solutions with monotonically growing $f(u)$ belong to class (a0) and possess a spatial asymptotics with a solid angle deficit and a divergent field energy.

Numerical integration confirms these conclusions. The different behavior of $F_\Omega(u)$ for $\gamma < 1$ and $\gamma > 1$ is shown in Fig. 7.

4.4. Solutions with $f(u)$ changing its sign

For $\gamma < \gamma_1 = 2/3$, there are solutions with the function $f(\phi)$ changing its sign once, see Fig. 2*b*. For $\gamma < \gamma_2 = 0.3$, there are solutions with $f(\phi)$ changing its sign twice, see Fig. 2*c*, etc. Unlike the monotonic solutions discussed in Sec. 4.3, all of them possess a horizon and in agreement with the general inferences in Sec. 4.1, belong to class (c1). This implies that beginning with a regular center, the spherical radius $r(\rho)$ first grows, then passes its maximum r_{max} at some ρ_1, and then decreases to zero at finite $\rho = \rho_2$, which is a singularity. The horizon occurs at some $\rho = h < \rho_2$, which can be greater or smaller than ρ_1, but in any case, the singularity occurs in a T-region and is of the cosmological nature. The dependence $r(\rho)$ before and after the horizon is a single smooth curve (Fig. 8*a*).

Beyond the horizon, $|A(\tau)|$ grows from zero at $\tau = 0$ (the horizon) to infinity as $\tau \to \tau_s = \tau(\rho_2)$ (the singularity) as a function of the proper time τ of a comoving observer, see Fig. 8*b*. Beyond the horizon, the scalar field magnitude $|f|$ first grows and then slightly varies around unity. Approaching the singularity, $f(\rho(\tau))$ changes its sign and finally $|f(\rho(\tau))| \to \infty$ as $\tau \to \tau_s$, see Fig. 8*c*.

5. CONCLUSION AND DISCUSSION

We have performed a general study of the properties of static global monopoles in general relativity. We have shown that independently of the shape of the symmetry breaking potential, the metric can contain either no horizon or one simple horizon, and in the latter case, the global structure of space-time is the same as that of the de Sitter space-time. Outside the horizon, the geometry corresponds to homogeneous anisotropic cosmological models of the Kantowski–Sachs type, where spatial sections have the topology $\mathbb{R} \times \mathbb{S}^2$. In general, all possible solutions can be divided into six classes with different qualitative behaviors. This classification is obtained without any assumptions about $V(\phi)$. Solutions with given $V(\phi)$ contain some of these classes, not necessarily all of them. This qualitative analysis gives a complete picture of what can be expected of global monopole systems with particular symmetry breaking potentials.

Our analytical and numerical analysis for the Mexican hat potential confirms the previous results of other authors concerning the configurations with the monotonically growing Higgs field magnitude f. Among other things, we have analytically obtained the upper

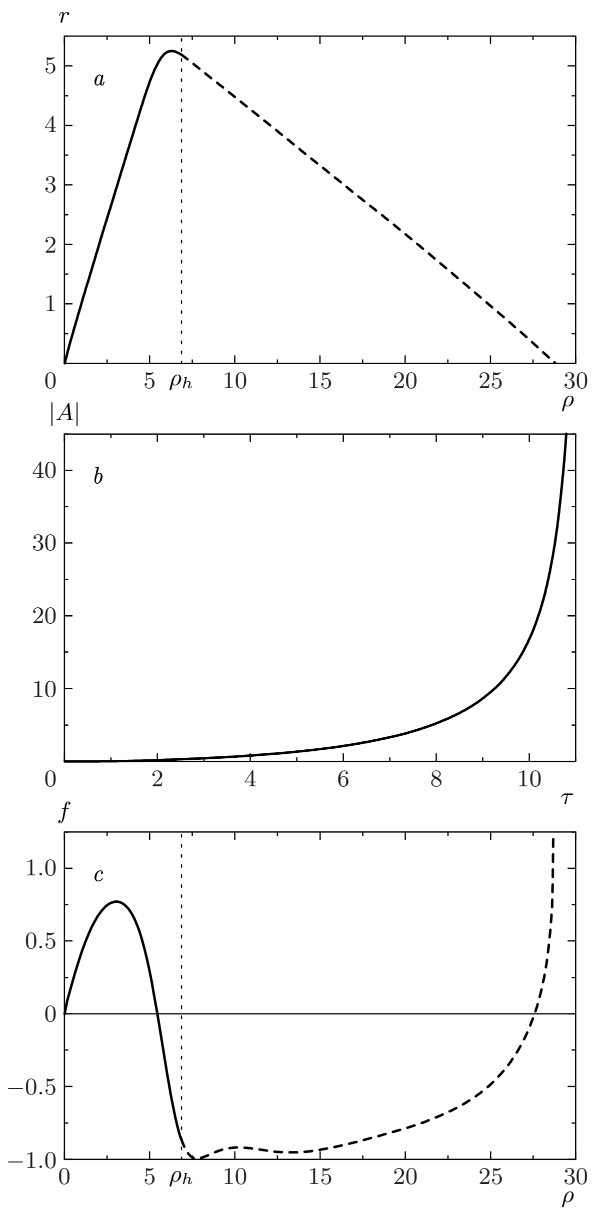

Fig. 8. Solutions with f of alternating sign: the functions (a) $r(\rho)$, (b) $|A(\tau)|$, and (c) $f(\rho)$ for $\gamma = 0.5$ before (solid lines) and after (dashed lines) the horizon

limit $\gamma_0 = 3$ for the existence of static monopole solutions, previously found numerically by Liebling [8]. We have also found and analyzed a new family of solutions with the field function f changing its sign, which we have not met in the existing literature.

Of particular interest can be the class-(a1) solutions with a static nonsingular monopole core and the Kantowski–Sachs cosmological model outside the horizon. Its anisotropic evolution is determined by the functions of the proper time $|A(\tau)|$ (the squared scale factor in the $\mathbb{R}$ direction), $r(\tau)$ (the scale factor in the two $\mathbb{S}^2$ directions), and the field magnitude $f(\tau)$. For a comoving observer in the T-region, the expansion starts with a rapid growth of $|A(\tau)|$ from zero to finite values, resembling inflation, and ends with $A \to$ const as $\tau \to \infty$. The expansion in the $\mathbb{S}^2$ directions described by $r(\tau)$ is comparatively uniform and linear at late times, i.e., much slower than $\mathrm{ch}^2(\tau/r_h)$ corresponding to de Sitter's space-time, see (35). We stress that all such models with the de Sitter-like causal structure (i.e., with a static core and expansion beyond the horizon) drastically differ from the standard Big Bang models in that the expansion starts from a nonsingular surface and cosmological comoving observers can receive information in the form of particles and light quanta from the static region situated in the absolute past with respect to them. Moreover, the static core is nonsingular in our case, and it therefore provides an example of an entirely nonsingular cosmology in the spirit of papers by Gliner and Dymnikova [27–30].

The nonzero symmetry-breaking potential plays the role of a time-dependent cosmological constant, a kind of hidden vacuum matter. Because the field function f tends to unity as $\tau \to \infty$, the potential vanishes and the «hidden vacuum matter» disappears.

The lack of isotropization at late times does not seem to be a fatal shortcoming of the model for two reasons. First, if the model is used to describe the near-Planck epoch of the Universe evolution, then at the next stage, the anisotropy can probably be damped by diverse particle creation, and the further stages with lower energy densities may conform to the standard picture (with possible further phase transitions). Second, if we add a comparatively small positive quantity Λ to potential (10) («slightly raise the Mexican hat»), this must change nothing but the late-time asymptotics, which then becomes the de Sitter one corresponding to the cosmological constant Λ. In our view, these ideas deserve a further study.

Evidently, the present simple model cannot be directly applied to our Universe. It would be too naive to expect that a macroscopic description based on a simple toy model of a global monopole with only one dimensionless parameter γ can explain all the variety of early-Universe phenomena. Nevertheless, it may be considered as an argument in favor of the idea that the standard Big Bang might be replaced with a nonsingular static core and a horizon appearing as a result of some symmetry-breaking phase transition at the Planck energy scale.

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