© 2002

IS THE CRITICAL REYNOLDS NUMBER UNIVERSAL?

S. Novopashin[*], *A. Muriel*

Institute of Thermophysics, Russian Academy of Sciences
630090, Novosibirsk, Russia

This paper is devoted to checking whether the critical Reynolds number is universal in identical conditions for the flow of different fluids. The laminar–turbulent transition in a circular pipe flow has been tested experimentally. The flows of inert gases (He, Ne, Ar, Kr, Xe), molecular gases (N_2, CO, CO_2, SF_6), and two similar liquids (H_2O, D_2O) have been tested. A considerable, up to 40 %, difference in critical Reynolds numbers was observed. The possible reasons of nonuniversality of critical Reynolds number are discussed.

PACS: 47.27.Ak

1. INTRODUCTION

The mystery of turbulent flows has been intriguing researchers in mechanics, synergetics, hydrodynamics, plasma physics, geophysics, chemistry, and biology. In spite of more than two centuries of history, this problem is still unsolved. Numerous experiments since Reynolds's paper [1] show that the stationary flow of fluids is possible only if the Reynolds number is less than some critical value. It is confidently known that the Navier–Stokes equations govern laminar flows. The breakdown of the stationary flow is associated with the loss of stability with increasing the Reynolds number. The analysis of stability of solutions sometimes allows predicting the critical Reynolds number. Most part of the research in the stability of laminar flows was devoted to incompressible flows. In this case, the analysis is considerably simplified because the only dimensionless parameter — the Reynolds number — determines the regime of the flow. Its value depends on the nature of the flow, but must be universal for different liquids in the same flow.

The Hagen–Poiseuille flow [2–4] — the flow in a long circular pipe — is stable with respect to infinitesimal disturbances [5, 6]. The transition to turbulence occurs as a result of finite perturbations or insufficiently smooth boundary conditions at the pipe entrance. Depending on the boundary conditions and external noise, the critical Reynolds number R_c can vary in a wide range of magnitudes: from $2 \cdot 10^3$ to more than 10^5. In the transition to the turbulent regime, the drag coefficient increases sharply, which makes it possible to monitor the critical Reynolds number reliably. This paper reports experimental results on the transition to turbulence in different gases and some liquids in the same pipe. The experiments are directed to check whether the critical Reynolds number is universal for the flows of different fluids.

2. EXPERIMENTAL

The experimental set-up is shown in Fig. 1. The vessel *1* (with the volume 0.1 m^3) can be pumped up to the pressure 0.1 Torr and then filled by any gas up to the pressure 1500 Torr. To study the transition in liquids, the basin *2* is installed inside. The air in the chamber can be compressed up to 750 Torr above the atmospheric pressure. Both gases and liquids can outflow into the atmosphere through the glass pipe *3* with the internal diameter 1.3 mm and the length 300 mm. The gas pressure inside the chamber varies the pressure drop on the pipe. It is measured by a membrane-type pressure gauge *4*. By varying the quality (roughness) of the pipe inlet, it was possible to change the critical Reynolds number in a wide range. The quality of the pipe inlet was chosen such that the critical Reynolds number was about 3500 for Nitrogen. All noble gases (He, Ne, Ar, Kr, Xe), some molecular gases (N_2, CO, CO_2, SF_6), double distillate water, and 99.9 heavy water were used in experiments. The temperature of the liquids was controlled with the accuracy 0.5 K. The flow rate was measured as a function of the pressure drop. For liquids, it was measured by collecting the liquid

_____

[*]E-mail: sanov@itp.nsc.ru

Fig. 1.

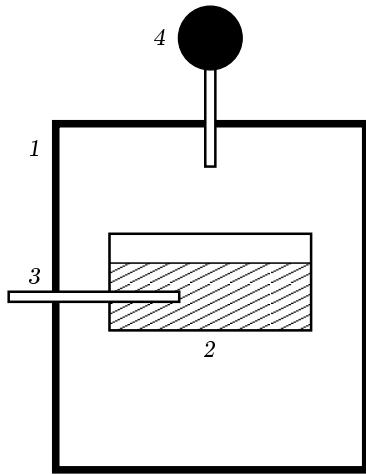$Re_c(H_2O) = 3020$

$Re_c(D_2O) = 3480$
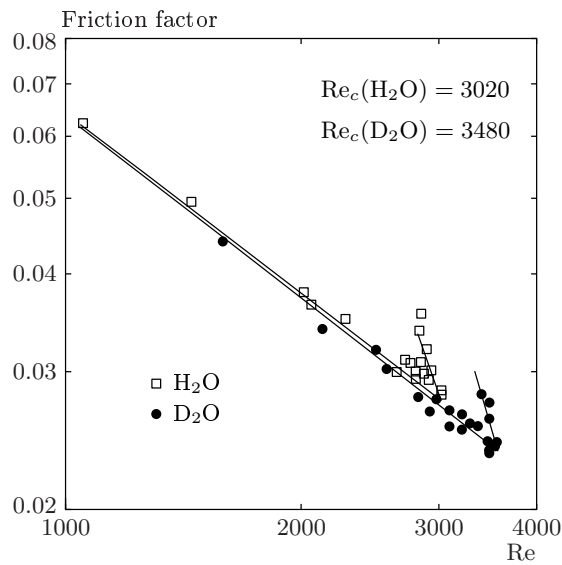
□ H₂O
● D₂O

Fig. 2.

Fig. 3.

for a definite time period (normally, 1 minute). The gas flow rate was measured by controlling the rate of pressure decrease. As an example, the data reduction for H_2O and D_2O is shown in Fig. 2. It is clearly seen that the dependence of the friction factor on Re is close to the theoretical one in the laminar flow, $64/\text{Re}$. The transition to turbulence results in a sharp increase of the friction factor, which allows determining the critical Reynolds number with high accuracy.

3. RESULTS AND DISCUSSION

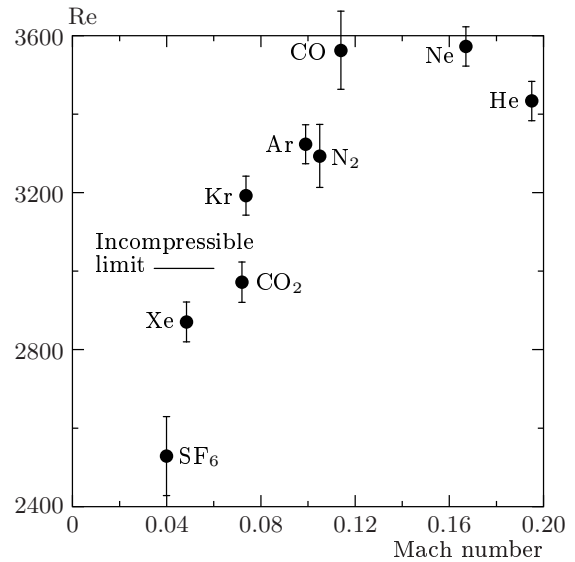The data of all measurements are collected in Tables 1–3. The tables show that the critical Reynolds number varies in the range 2500–3570 (SF_6–Ne). The experiments were carried out at absolutely identical background conditions. The data obtained therefore demonstrate a nonuniversality of the critical Reynolds number, contrary to the conventional tenet.

For the Navier–Stockes equations, the Reynolds number is not the only parameter that can influence the flow stability. For compressible flows, the Mach number is the second important parameter. Figure 3 shows the dependence of the critical Reynolds number on the Mach number for gas flows at the transition point. We can see some correlation between the value of the critical Reynolds number and the Mach number. With the decrease of the Mach number, R_c should reach the limit determined by the incompressible flow. The data for water plotted by the horizontal solid line demonstrate that R_c for water is far from the limit. This means that the Mach number cannot be the parameter that governs the difference of critical Reynolds numbers for gas flows.

The Navier–Stocks equations include three dissipative terms: normal viscosity, bulk viscosity, and heat conductivity. We first consider the role of bulk viscosity. Bulk viscosity is related to the relaxation of the molecular internal degrees of freedom; in particular, it is strictly equal to zero for inert gases. In [7, 8], the difference in the critical Reynolds numbers for N_2 and CO was explained by the difference in rotational relaxation. But additional experiments have shown [8] that the critical Reynolds numbers differ in a range that is sufficiently wide even for noble gases (see Table 1).

Table 1.   Noble gases

Property/Gas	He	Ne	Ar	Kr	Xe
Molecular mass	4.003	20.18	39.95	83.80	131.3
Density, kg/m^3 (101325 Pa, 293 K)	0.1785	0.900	1.784	3.73	5.897
Dyn. visc., 10^6 Pa·s (10^5 Pa, 300 K)	19.9	31.75	22.75	25.54	23.3
Speed of sound, m/s (300 K)	1012	454	334	222	177.4
Critical Reynolds number	3430	3570	3320	3190	2870
Mach number	0.20	0.17	0.10	0.074	0.048
Second virial coefficient, cm^3/mol	11.15	11.02	−16.85	−53	−134.6

Table 2.   Molecular gases

Property/Gas	N$_2$	CO*	CO$_2$	SF$_6$
Molecular mass	28	28	44	146
Density, kg/m^3 (101325 Pa, 293 K)	1.25	1.25	1.977	6.5
Dyn. visc., 10^6 Pa·s (10^5 Pa, 300 K)	17.9	17.9	15.0	15.9
Speed of sound, m/s (300 K)	334	334	274	134.9
Critical Reynolds number	3290	3560	2970	2530
Mach number	0.105	0.114	0.072	0.04
Second virial coefficient, cm^3/mol	−5.47	−10.0	−97.9	−292

Table 3.   Liquids

Property/Liquids	H$_2$O	D$_2$O
Molecular mass	18	20
Density, kg/m^3 (101325 Pa, 293 K)	1000	1104
Dyn. visc., Pa·s (295 K)	0.00096	0.0012
Critical Reynolds number	3020	3480

This fact allows one to conclude that the relaxation of molecular internal degrees of freedom cannot be the only additional parameter that determines R_c.

The difference of thermal conductivities could be important for gas flows because of its expansion and cooling during the flow. The plot of the critical Reynolds number against thermal conductivity is shown in Fig. 4. In spite of some correlation, we must admit that the dissipation due to thermal conductivity
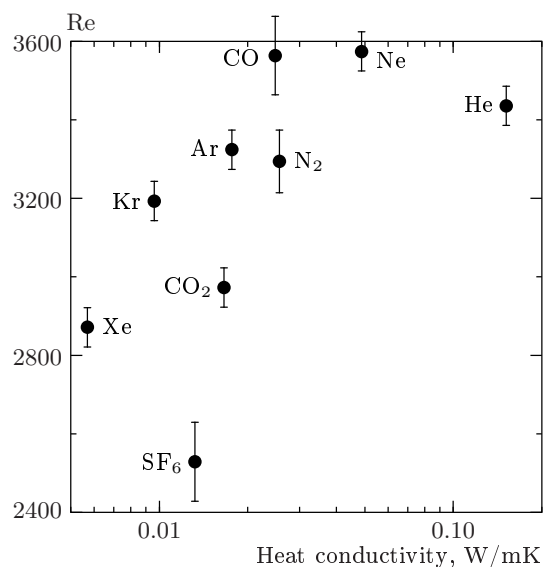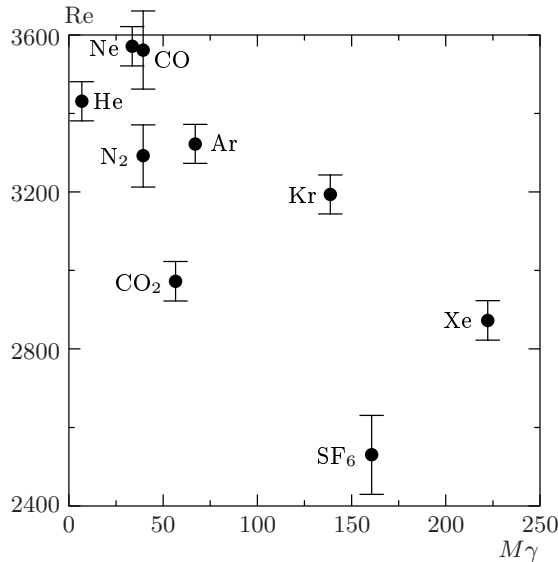
Fig. 4.

Fig. 5.

cannot be an important parameter.

The next reason that could play a role is the influence of the external noise. Although the experiments were carried out under the same external conditions, the role of the noise could be different for different substances. To characterize the noise, we suppose that the spectral components of the noise pressure P_ω are the same. The characteristic dimension of the pressure is ρC^2, where ρ is the density and C is the speed of sound. The characteristic frequency ω is C/D, where D is the characteristic size of the flow (e.g., the diameter of the pipe). Finally, to obtain a dimensionless parameter P_ω, we must normalize this value to $(\rho C^2)/(C/D)$. D is the same for all experiments and C is proportional to $\sqrt{\gamma T/\rho}$, where γ is the adiabatic exponent and T is the temperature (it is the same for all gases). The reduction of the above formulas results in the parameter to normalize as $\gamma\rho$, or γM, where M is the molecular mass. This plot is shown in Fig. 5. It is clearly seen that the experimental points scatter out of any regular dependence. We therefore conclude that the difference in susceptibilities cannot be the reason for the observed nonuniversality of the critical Reynolds number.

We also note the analysis in [9] based on weak nonideality of gases at normal conditions that allowed generalizing the experimental data for all tested gases as a function of the second virial coefficient. The flows of incompressible liquids are simpler in theory because the Reynolds number is the only parameter that should define the regime of the flow. Contrary to the conventional tenet, even in this case (see Table 3), the crit-

ical Reynolds numbers differ for water and heavy water. For liquids, the statistical approach similar to that for gases [9] is considerably more complex. The difficulty is in the exact calculation of the partition function and the individual phase volume even for simple liquids [10, 11].

4. CONCLUSION

The data obtained show that the critical Reynolds number is not universal and that the process of the laminar–turbulent transition is influenced by the individual molecule properties for both gas and liquid flows. Taking references [12–14] and the present research into account, we conclude that a rigorous theory of turbulence should be based on a synthesis of hydrodynamic, statistical, and possibly, quantum theories.

REFERENCES

1. O. Reynolds, Philos. Trans. **174**, 935 (1883).

2. G. Hagen, Pogg. Ann. **46**, 423 (1839).

3. J. Poiseuille, Comptes Rendus **11**, 961 (1840); **12**, 112 (1841).

4. L. Prandtl and O. Tietjens, *Hydro- und Aeromechanik*, Verlag von Julius Springer, Berlin (1931).

5. J. A. Fox, M. Lessen, and W. V. Bhat, Physics of Fluids **11**, 1 (1968).

6. H. Salwen, F. W. Cotton, and C. E. Grosch, J. Fluid Mech. **92**, 273 (1980).

7. O. A. Nerushev and S. A. Novopashin, Phys. Lett. A **232**, 243 (1997).

8. O. A. Nerushev and S. A. Novopashin, Pis'ma Zh. Eksp. Teor. Fiz. **64**, 47 (1996).

9. S. A. Novopashin and A. Muriel, Pis'ma Zh. Eksp. Teor. Fiz. **68**, 582 (1998).

10. S. A. Novopashin and A. Muriel, Techn. Phys. Lett. **26**, 231 (2000).

11. L. D. Landau and E. M. Livshits, *Statistical Physics*, Nauka, Moscow (1976), Part 1.

12. H. N. V. Temperlley, J. S. Rowlinson, and G. S. Rushbrooke, *Physics of Simple Liquids*, North-Holland Publishing Company, Amsterdam (1968).

13. A. Muriel, Physica D **124**, 225 (1998).

14. A. Groisman and V. Steinberg, Nature **405**, 53 (2000).

15. X. de Hemptinne, *Non-Equilibrium Statistical Thermodynamics*, World Scientific, Singapore–New Jersey–London–Hong Kong (1992).