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TWO-DIMENSIONAL ELECTRON LIQUID WITH DISORDER IN A WEAK MAGNETIC FIELD

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We present the effective theory for the low-energy dynamics of two-dimensional interacting electrons in the presence of a weak short-range disorder and a weak perpendicular magnetic field, with the filling factor $\nu \gg 1$. We investigate the exchange enhancement of the g factor, the effective mass, and the decay rate of the simplest spin wave excitations at $\nu = 2N + 1$. We obtain the enhancement of the field-induced gap in the tunneling density of states and the dependence of the tunneling conductivity on the applied bias.

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1. INTRODUCTION

A two-dimensional electron gas in the perpendicular magnetic field has attracted much attention from both theoretical and experimental standpoints. The effects in a strong magnetic field when only the lowest Landau level is occupied have been investigated since the discovery of the quantum Hall effect [1]. Several efforts [2] are made in order to involve larger filling factors $\nu > 1$ into the problem discussed. However, the existence of a small parameter, the ratio of the Coulomb energy at the magnetic field length to the cyclotron energy, has been assumed. In a weak magnetic field, the Coulomb energy at the magnetic field length actually exceeds the cyclotron energy and some attempts [3] have been undertaken to investigate the case of the large filling factor $\nu > 1$.

Experimental investigations of the tunneling density of states for the system under consideration were performed at small ($\nu < 1$) [4] and large ($\nu > 1$) [5] filling factors. In the case of a weak magnetic field ($\nu \gg 1$), the gap in the tunneling density of states has been obtained in the framework of the hydrodynamical approach [6]. The progress was made by Aleiner and Glazman [7] who developed the effective theory for low-energy excitations on a partially filled Landau level at large filling factors $\nu \gg 1$.

Recently, after the prediction that the unidirectional charge-density wave state occurs at half-filled high Landau levels within the framework of the Hartree–Fock theory [8] and the experimental discovery of compressible states with the anisotropic magnetotransport properties in high-mobility systems near the half-filling of the high Landau levels [9], the two-dimensional electron liquid in a weak magnetic field was intensely studied [10].

In this paper, we develop the low-energy effective theory for electrons at the partially filled Landau level with a large filling factor in the presence of disorder (Sec. 2). As an example, the effect of disorder on the exchange enhancement of the effective g factor and the simplest spin-wave excitations are discussed in Sec. 4. Electron tunneling into the electron liquid is considered in Sec. 5. Conclusions are given in Sec. 6.

2. DERIVATION OF THE EFFECTIVE ACTION

2.1. Introduction

We consider the system of two-dimensional electrons with the Coulomb interaction in the presence of disorder in a perpendicular magnetic field H. The system possesses a partially filled high Landau level with the level index $N \gg 1$ equal to the integer part of half the filling factor ν, $N = [\nu/2]$. The presence of a random potential, which is considered to be short-range, results in a broadening of the Landau levels.

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We assume that the elastic collision time satisfies the condition

$$\tau_0 \gg \omega_c^{-1},$$

where $\omega_c = eH/m$ is the cyclotron frequency with the electron charge e and the electron mass m. In this case, the broadening of Landau levels, which is of the order $\sqrt{\omega_c \tau_0}/\tau_0$, is much less than the distance between them.

The conventional parameter characterizing the coupling strength of the Coulomb interaction is

$$r_s = \sqrt{2}e^2/v_F,$$

where v_F is the Fermi velocity. We assume that electrons are weakly interacting, i.e., $r_s < 1$. In this case, we can treat the problem in the random phase approximation. We also assume that the number N is sufficiently large, and the condition $Nr_s \gg 1$ is therefore satisfied. This means that the cyclotron radius $R_c = \sqrt{\nu/m\omega_c}$ is supposed to be much larger than the Bohr radius $a_B = 1/me^2$,

$$R_c \gg a_B.$$

2.2. The formalism

The system is described by the grand canonical partition function in the path-integral representation,

$$Z = \int \mathcal{D}[\overline{\psi}, \psi] \int \mathcal{D}[V_{dis}]\, \mathcal{P}[V_{dis}(\mathbf{r})] \times \times \exp\left\{S[\overline{\psi}, \psi, V_{dis}]\right\}, \quad (1)$$

$$S = \sum_{\alpha=1}^{N_r} \int_0^{1/T} d\tau \times \times \int d\mathbf{r} \left[\overline{\psi}^{\alpha,\sigma}(\mathbf{r},\tau)\left(-\partial_\tau + \mu - \mathcal{H} - V_{dis}(\mathbf{r})\right)\psi^{\alpha,\sigma}(\mathbf{r},\tau) - \right. - \frac{1}{2}\int d\mathbf{r}_1\, \overline{\psi}^{\alpha,\sigma}(\mathbf{r},\tau)\psi^{\alpha,\sigma}(\mathbf{r},\tau)U_0(\mathbf{r},\mathbf{r}_1) \times \left. \times\, \overline{\psi}^{\alpha,\sigma_1}(\mathbf{r}_1,\tau)\psi^{\alpha,\sigma_1}(\mathbf{r}_1,\tau)\right], \quad (2)$$

where the Grassmann variables $\psi^{\alpha,\sigma}$ and $\overline{\psi}^{\alpha,\sigma}$ are defined on the imaginary time interval $\tau \in [0, 1/T]$ with the antiperiodic condition $\psi(\mathbf{r}, 1/T) = -\psi(\mathbf{r}, 0)$. The symbol T stands for the temperature, μ is the chemical potential of the system, and $\sigma, \sigma_1 = \pm 1$ are spin indices. The Hamiltonian

$$\mathcal{H} = \frac{(-i\nabla - e\mathbf{A})^2}{2m}$$

describes the electron with mass m propagating in the two-dimensional space in the perpendicular magnetic field

$$H = \epsilon_{ab}\partial_a A_b.$$

The random potential $V_{dis}(\mathbf{r})$ is chosen to have the Gaussian distribution function

$$\mathcal{P}[V_{dis}(\mathbf{r})] = \sqrt{\rho\tau_0}\, \exp\left(-\pi\rho\tau_0 \int d\mathbf{r}\, V_{dis}^2(\mathbf{r})\right), \quad (3)$$

where ρ denotes the thermodynamical density of states.

To average $\ln Z$ over disorder, N_r replicated copies of the system are introduced; we let $\alpha = 1, \ldots, N_r$ be the replica indices.

The Matsubara representation seems to be more convenient for the above problem. We therefore use the Fourier transform from the imaginary time τ to the Matsubara frequencies. Because the fermionic fields are antiperiodic within the interval $[0, 1/T]$, the frequencies permitted for $\overline{\psi}$ and ψ are

$$\omega_n = \pi T(2n+1),$$

where n is an integer. The Fourier-transformed fields are defined as

$$\overline{\psi}^{\alpha}(\tau) = \sqrt{T}\sum_{n=-\infty}^{\infty} \overline{\psi}_n^{\alpha} e^{i\omega_n\tau}, \qquad \psi^{\alpha}(\tau) = \sqrt{T}\sum_{n=-\infty}^{\infty} \psi_n^{\alpha} e^{-i\omega_n\tau}. \quad (4)$$

In what follows, we omit the limits in the frequency and replica series for brevity.

In the Matsubara representation, action (2) becomes

$$S = \int d\mathbf{r} \times \times \sum_{\alpha,n}\left[\overline{\psi}_n^{\alpha,\sigma}(\mathbf{r})\left(i\omega_n + \mu - \mathcal{H} - V_{dis}(\mathbf{r})\right)\psi_n^{\alpha,\sigma}(\mathbf{r}) - \right. - \frac{T}{2}\sum_{l,m}\int d\mathbf{r}_1\, \overline{\psi}_m^{\alpha,\sigma}(\mathbf{r})\psi_{m-n}^{\alpha,\sigma}(\mathbf{r})U_0(\mathbf{r},\mathbf{r}_1) \times \left. \times\, \overline{\psi}_l^{\alpha,\sigma_1}(\mathbf{r}_1)\psi_{l+n}^{\alpha,\sigma_1}(\mathbf{r}_1)\right]. \quad (5)$$

The Zeeman term in action (2) is neglected because the g factor is small. In fact, the condition $g \ll 1$ is usually satisfied. The Zeeman term can nevertheless be included in the effective action after performing the integration over fast degrees of freedom. To simplify the notation, the spin indices are associated with the replica ones whenever convenient.

2.3. The plasmon field and the average over disorder

The Coulomb term entering action (5) is quartic in the fermionic fields. This quartic term can be

eliminated by the Hubbard–Stratonovich transformation, introducing an extra path integration over bosonic fields $\lambda_n^\alpha(\mathbf{r})$. With the help of the so-called plasmon field, the Coulomb term can be presented as

$$\int \mathcal{D}[\lambda] \exp\left[-\frac{T}{2}\iint d\mathbf{r}\, d\mathbf{r}_1\, \lambda^\dagger(\mathbf{r}) U_0^{-1}(\mathbf{r},\mathbf{r}_1)\lambda(\mathbf{r}_1) + \right. \\ \left. + iT\int d\mathbf{r}\, \psi^\dagger(\mathbf{r})\hat{\lambda}(\mathbf{r})\psi(\mathbf{r})\right], \quad (6)$$

where U_0^{-1} stands for the inverse operator to U_0. The matrix notation is used for the combined replica and frequency indices,

$$\psi^\dagger(\dots)\psi = \sum_{n,m}^{\alpha,\beta} \overline{\psi}_n^\alpha (\dots)_{nm}^{\alpha\beta}\psi_m^\beta, \\ \lambda^\dagger\lambda = \sum_n^\alpha \lambda_{-n}^\alpha \lambda_n^\alpha. \quad (7)$$

The quantities with the hat are defined as

$$\hat{z} = \sum_{\alpha,n} z_n^\alpha I_n^\alpha$$

with the matrix

$$(I_n^\alpha)_{kl}^{\beta\gamma} = \delta^{\alpha\beta}\delta^{\alpha\gamma}\delta_{k-l,n}.$$

The matrices I_n^α represent the diagonals shifted in the frequency space; they are the generators of the $U(1)$ gauge transformations in general. The measure of the path integral over the plasmon field λ is introduced such that integral (6) equals unity for the vanishing fermionic fields $\psi^\dagger$ and ψ.

In order to perform the averaging over disorder in partition function (1), we must integrate over the random potential $V_{dis}(\mathbf{r})$. This leads to the quartic term

$$\frac{1}{4\pi\rho\tau}\int d\mathbf{r} \sum_{n,m}^{\alpha,\beta} \overline{\psi}_n^\alpha(\mathbf{r})\psi_n^\alpha(\mathbf{r})\overline{\psi}_m^\beta(\mathbf{r})\psi_m^\beta(\mathbf{r}) \quad (8)$$

in the action. This term can be decoupled by the Hubbard–Stratonovich transformation. An extra path integration over the Hermitian matrix field variables $Q_{nm}^{\alpha\beta}(\mathbf{r})$ can be introduced [11, 12],

$$\int \mathcal{D}[Q] \times \\ \times \exp\int d\mathbf{r}\left[-\pi\rho\tau_0 \operatorname{tr} Q^2(\mathbf{r}) + i\psi^\dagger(\mathbf{r})Q(\mathbf{r})\psi(\mathbf{r})\right], \quad (9)$$

where the symbol «tr» denotes the matrix trace over the Matsubara, replica, and spin spaces. The measure of the path integral over the matrix field Q is defined in the same way as for the plasmon field, i.e., integral (9) equals unity for vanishing fermionic fields $\psi^\dagger$ and ψ.

After the above calculations, the partition function becomes

$$Z = \int \mathcal{D}[\overline{\psi},\psi,\lambda,Q]\exp\left\{S[\overline{\psi},\psi,\lambda,Q]\right\}, \quad (10)$$

$$S = -\pi\rho\tau_0\int d\mathbf{r}\, \operatorname{tr} Q^2 - \\ - \frac{T}{2}\iint d\mathbf{r}\, d\mathbf{r}_1\, \lambda^\dagger(\mathbf{r})U_0^{-1}(\mathbf{r},\mathbf{r}_1)\lambda(\mathbf{r}_1) + \\ + \int d\mathbf{r}\, \psi^\dagger(\mathbf{r})\left(i\omega + \mu - \hat{\mathcal{H}} + iT\hat{\lambda} + iQ\right)\psi(\mathbf{r}), \quad (11)$$

where ω is the unit matrix in the replica space, while in the Matsubara space, it is a matrix containing the frequencies ω_n on the diagonal,

$$(\omega)_{nm}^{\alpha\beta} = \omega_n\delta_{nm}\delta^{\alpha\beta}.$$

2.4. Elimination of the N-th Landau level

The fermionic fields $\psi^\dagger$ and ψ refer to all Landau levels. In order to integrate over all fermionic degrees of freedom not belonging to the partially filled N-th Landau level, we separate the fermionic fields into two kinds. The first field refers to the N-th Landau level,

$$\Psi(\mathbf{r}) = \sum_k \psi_{Nk}\varphi_{Nk}(\mathbf{r}), \\ \Psi^\dagger(\mathbf{r}) = \sum_k \psi_{Nk}^\dagger\varphi_{Nk}(\mathbf{r}). \quad (12)$$

The second one involves the other levels,

$$\Phi(\mathbf{r}) = \sum_{p\neq N,k} \psi_{pk}\varphi_{pk}(\mathbf{r}), \\ \Phi^\dagger(\mathbf{r}) = \sum_{p\neq N,k} \psi_{pk}^\dagger\varphi_{pk}(\mathbf{r}), \quad (13)$$

where $\varphi_{pk}(\mathbf{r})$ are the eigenfunctions of the Hamiltonian $\mathcal{H}$ and $p = 0, 1, \dots, N, \dots$ labels Landau levels with the energies $\epsilon_p = \omega_c(p+1/2)$. In addition, we introduce two types of the Green's functions. One is for the N-th Landau level,

$$G(\mathbf{r},\mathbf{r}_1;Q,\lambda) = \\ = \sum_{k,k'} \varphi_{Nk}^*(\mathbf{r})G_{Nk,Nk'}(Q,\lambda)\varphi_{Nk'}(\mathbf{r}_1), \quad (14)$$

and the other is for the other levels,

$$\tilde{G}(\mathbf{r},\mathbf{r}_1;Q,\lambda) = \\ = \sum_{p,p'\neq N}\sum_{k,k'} \varphi_{pk}^*(\mathbf{r})G_{pk,p'k'}(Q,\lambda)\varphi_{p'k'}(\mathbf{r}_1), \quad (15)$$

where the inverse of the Green's function for the ψ_{pk} and $\psi^\dagger_{p'k'}$ operators is given by

$$(G^{-1})_{pk,p'k'} = (i\omega + \mu - \epsilon_p)\delta_{pp'}\delta_{kk'} + \\ + iT\hat{\lambda}_{pk,p'k'} + iQ_{pk,p'k'} \quad (16)$$

with the matrix elements

$$f_{pk,p'k'} = \int d\mathbf{r}\, \varphi^*_{p'k'}(\mathbf{r}) f(\mathbf{r}) \varphi_{pk}(\mathbf{r}). \quad (17)$$

The action (11) is bilinear in the fermionic fields $\psi^\dagger$ and ψ, and obviously, also in the fermionic fields $\Phi^\dagger$ and Φ. We can therefore integrate over the fermionic fields $\Phi^\dagger$ and Φ; this gives

$$S = -\int \operatorname{tr}\ln\tilde{G} - \pi\rho\tau_0 \int \operatorname{tr} Q^2 + \\ + \int \Psi^\dagger \left[i\omega + \mu - \hat{\mathcal{H}} + iT\hat{\lambda} + iQ \right] \Psi - \\ - \frac{T}{2} \iint \lambda^\dagger U_0^{-1} \lambda + \\ + \iint \left[\Psi^\dagger Q \tilde{G} Q \Psi + 2T\Psi^\dagger \hat{\lambda} \tilde{G} Q \Psi + T^2 \Psi^\dagger \hat{\lambda} \tilde{G} \hat{\lambda} \Psi \right]. \quad (18)$$

Hereafter, the space indices are omitted. It should be noted that the last term arises in action (18) due to the interaction between electrons belonging to the partially filled N-th Landau level and the other electrons.

2.5. Integration over the Q field

The Q matrix field must be divided into the transverse V and the longitudinal P components as $Q = V^{-1}PV$. Here, the longitudinal component P has a block-diagonal structure in the Matsubara space, i.e., $P^{\alpha\beta}_{nm} \propto \Theta(nm)$, where $\Theta(x)$ is the Heaviside step function. The transverse component V corresponds to a unitary rotation, see [13, 14] for a review.

The change of variables from Q to P and V is motivated by the saddle-point structure of action (18) in the absence of the plasmon field λ and at zero temperature, i.e., as $\omega_n \to 0$. This saddle-point solution can be written as

$$Q_{sp} = V^{-1} P_{sp} V,$$

where the matrix P_{sp} obeys the equation

$$2\pi\rho\tau_0 P_{sp} = i\left[G_0(\mathbf{r},\mathbf{r}) + \tilde{G}_0(\mathbf{r},\mathbf{r}) \right] \quad (19)$$

that coincides with the self-consistent Born approximation equation [15]. Here, the Green's function G_0 is a special case of G, namely,

$$G_0(\mathbf{r},\mathbf{r}_1) = G(\mathbf{r},\mathbf{r}_1; P_{sp}, 0),$$

and similarly for $\tilde{G}_0$.

In the case of small disorder, $\omega_c\tau_0 \gg 1$, the solution of Eq. (19) is given by

$$(P_{sp})^{\alpha\beta}_{nm} = \frac{\operatorname{sgn} n}{2\tau}\delta_{nm}\delta^{\alpha\beta}, \quad \tau = \pi\sqrt{\frac{\rho}{m}}\frac{\tau_0}{\sqrt{\omega_c\tau_0}}. \quad (20)$$

The presence of the plasmon field λ results in a shift of the saddle-point value (20) of the P field; this shift can be found by expanding action (18) to the second order in both λ and $\delta P = P - P_{sp}$. We thus obtain

$$S = S_0 + S_1[\delta P, \lambda] + S_2[\overline{\Psi}, \Psi, \delta P, \lambda], \quad (21)$$

$$S_0 = \int \Big(-\operatorname{tr}\ln\tilde{G}_0 - \pi\rho\tau_0 \operatorname{tr} Q_{sp}^2 + \\ + \Psi^\dagger \left[i\omega + \mu - \hat{\mathcal{H}} + iT\hat{\lambda} + iQ_{sp} \right] \Psi \Big), \quad (22)$$

$$S_1 = iT \int \operatorname{tr}\tilde{G}_0\hat{\lambda} - \pi\rho\tau_0 \int \operatorname{tr}(\delta P)^2 - \frac{T}{2}\iint \lambda^\dagger U_0^{-1}\lambda + \\ + \frac{1}{2}\iint \operatorname{tr}\left[T\hat{\lambda} + \delta P \right] \pi_0 \left[T\hat{\lambda} + \delta P \right], \quad (23)$$

$$S_2 = \iint \Big(\Psi^\dagger \left[T\hat{\lambda} + \delta P \right] \tilde{G}_0 \left[T\hat{\lambda} + \delta P \right] \Psi - \\ - 2\operatorname{tr}\left[T\hat{\lambda} + \delta P \right] \tilde{G}_0 \left[T\hat{\lambda} + \delta P \right] G_0 \Big), \quad (24)$$

where the bare polarization operator π_0 is understood to be a matrix in accordance with the rule

$$\operatorname{tr} A\pi_0 B = \\ = \sum_{n,m}^{\alpha,\beta} A^{\alpha\beta}_{m+n,m}(\mathbf{r}) \pi_0^m(n;\mathbf{r},\mathbf{r}_1) B^{\beta\alpha}_{m,m+n}(\mathbf{r}_1) \quad (25)$$

and is defined by

$$\pi_0^m(n;\mathbf{r},\mathbf{r}_1) = -2\Big(\tilde{G}_0^{m+n}(\mathbf{r},\mathbf{r}_1)\tilde{G}_0^m(\mathbf{r}_1,\mathbf{r}) + \\ + \tilde{G}_0^{m+n}(\mathbf{r},\mathbf{r}_1) G_0^m(\mathbf{r}_1,\mathbf{r}) + \\ + G_0^{m+n}(\mathbf{r},\mathbf{r}_1)\tilde{G}_0^m(\mathbf{r}_1,\mathbf{r}) \Big). \quad (26)$$

After decomposing the matrix field Q into the block-diagonal Hermitian matrix field P and the unitary matrix field V, the measure of the functional integral in (21) becomes

$$\mathcal{D}[Q] = \mathcal{D}[V]\mathcal{D}[\delta P] I[\delta P],$$

where [13]

$$\ln I[\delta P] = -\frac{1}{(\pi\rho)^2} \times \times \int \sum_{n,m}^{\alpha,\beta} [1-\Theta(nm)]\, \delta P^{\alpha\alpha}_{nn}\, \delta P^{\beta\beta}_{mm}. \quad (27)$$

The terms that are quadratic in δP in the part S_1 of action (21) together with the contribution of measure (27) determine the propagator of the δP fields,

$$\langle \delta P^{\alpha\beta}_{m_1 m_2}(\mathbf{q}) \delta P^{\gamma\delta}_{m_3 m_4}(-\mathbf{q})\rangle = = \frac{\delta_{m_1 m_4}\delta_{m_2 m_3}\delta^{\alpha\delta}\delta^{\beta\gamma}\dfrac{\Theta(m_1 m_3)}{2\pi\rho\tau_0}}{1+\dfrac{\pi_0^{m_1}(m_3-m_1;\mathbf{q})}{2\pi\rho\tau_0}} - - \frac{2\,[1-\Theta(m_1 m_3)]}{(2\pi^2\rho^2\tau)^2} \frac{\delta_{m_1 m_2}\delta^{\alpha\beta}}{1+\dfrac{\pi_0^{m_1}(0;\mathbf{q})}{2\pi\rho\tau_0}} \times \times \frac{\delta_{m_3 m_4}\delta^{\delta\gamma}}{1+\dfrac{\pi_0^{m_3}(0;\mathbf{q})}{2\pi\rho\tau_0}}. \quad (28)$$

We note that the propagator of the longitudinal fluctuations (28) proves to be analogous to that previously obtained in the problem of the behavior of a free electron gas in the perpendicular magnetic field [13].

Using expression (28) for the propagator of the δP fields, we can integrate action (21) over the longitudinal fluctuations in the quadratic approximation. This gives

$$S = S_0 + S_\lambda + S_\mu, \quad (29)$$

where S_0 given by Eq. (22) describes the electrons at the partially filled N-th Landau level coupled to the plasmon and Q_{sp} fields. The term S_λ corresponds to the screening of the Coulomb interaction due to the influence of electrons from the other Landau levels and is given by

$$S_\lambda = iT \int d\mathbf{r}\ \mathrm{tr}\, \tilde{G}_0(\mathbf{r},\mathbf{r})\hat{\lambda}(\mathbf{r}) - - \frac{T}{2}\int \frac{d\mathbf{q}}{(2\pi)^2} \sum_n^\alpha \lambda^\alpha_{-n}(\mathbf{q}) U_0^{-1}(q)\varepsilon(n,q)\lambda^\alpha_n(-\mathbf{q}), \quad (30)$$

where the dielectric function is given by

$$\varepsilon(n,q) = 1 + U_0(q)\Pi(n,q)$$

with the polarization operator[1)]

$$\Pi(n,q) = = T\sum_m \pi_0^m(n,q)\left[1 - \frac{\Theta(n(n+m))\dfrac{\pi_0^m(n,q)}{2\pi\rho\tau_0}}{1+\dfrac{\pi_0^m(n,q)}{2\pi\rho\tau_0}}\right] + + T\frac{\delta_{n,0}}{(\pi^2\rho^2\tau)^2}\sum_{k,m}\frac{[1-\Theta(km)]\pi_0^m(0,q)}{1+\dfrac{\pi_0^m(0,q)}{2\pi\rho\tau_0}} \times \times \frac{\pi_0^k(0,q)}{1+\dfrac{\pi_0^k(0,q)}{2\pi\rho\tau_0}}. \quad (31)$$

The third term S_μ in action (29) contains the terms that affect the chemical and thermodynamic potentials of the system (See Appendix A).

It is worthwhile to mention that the saddle-point approximation in which the integration over the Q field is performed is valid because the condition

$$\mu\tau = N\omega_c\tau \gg 1$$

is satisfied.

2.6. Integration over the plasmon field

As a final step of the procedure, action (29) must be integrated over the plasmon field λ. The integration can be performed in the quadratic approximation in the λ fields. The corresponding propagator is determined by the second term in Eq. (30). After that, we obtain the effective action for electrons on the partially filled Landau level,

$$S_{eff} = -\frac{\Omega}{T} + \int d\mathbf{r}\,\Psi^\dagger(\mathbf{r})\left[i\omega + \tilde{\mu} - \hat{\mathcal{H}} + iQ_{sp}\right] \times \times \Psi(\mathbf{r}) - \pi\rho\tau_0\int d\mathbf{r}\ \mathrm{tr}\, Q^2_{sp}(\mathbf{r}) - - \frac{T}{2}\iint d\mathbf{r}\, d\mathbf{r}_1 \sum_{n,m,k}^\alpha \overline{\Psi}^{\alpha,\sigma}_m(\mathbf{r})\Psi^{\alpha,\sigma}_{m+n}(\mathbf{r}) U_{eff}(\mathbf{r}-\mathbf{r}_1) \times \times \overline{\Psi}^{\alpha,\sigma_1}_k(\mathbf{r}_1)\Psi^{\alpha,\sigma_1}_{k-n}(\mathbf{r}_1) + + \frac{g\omega_c}{2}\int d\mathbf{r}\sum_n^\alpha \sigma\overline{\Psi}^{\alpha,\sigma}_n(\mathbf{r})\Psi^{\alpha,\sigma}_n(\mathbf{r}), \quad (32)$$

which is the main result of the paper.

[1)] A similar form of the polarization operator but with a different bare polarization operator $\pi_0^m(n,q)$ was first derived by Baranov and Pruisken [16].

We have incorporated the Zeeman term into the effective action. The Fourier transform of the effective interaction potential

$$U_{eff}(q) = \frac{U_0(q)}{\varepsilon(q)}$$

is determined by the static dielectric function $\varepsilon(q) \equiv \equiv \varepsilon(0,q)$. In general, the low-energy properties of the system under consideration can be described with the help of the retarded interaction alone (see action (29)). However, the description within the framework of the effective action with the instantaneous interaction seems to be a rather good approximation in this problem [7]. This is because transitions between the Landau levels have a characteristic time scale about ω_c^{-1}, while the typical energy scale in the effective theory is of the order of the exchange energy $\Delta_{ex} \ll \omega_c$ (see Sec. 4).

The existence of the other Landau levels except the partially filled N-th level affects both the thermodynamic and the chemical potentials. The thermodynamic potential Ω in action (32) can be represented as

$$\Omega = \Omega_0 + \Delta\Omega, \tag{33}$$

where

$$\Omega_0 = T \int d\mathbf{r}\ \mathrm{tr} \ln \tilde{G}_0(\mathbf{r},\mathbf{r}) \tag{34}$$

is the thermodynamic potential of the system of noninteracting electrons for the completely filled Landau levels in the presence of disorder and the quantity $\Delta\Omega$ is analogous to the first-order exchange and the correlation correction equivalent to the sum of ring diagrams contributing to the ground state energy of a clean electron liquid [17],

$$\Delta\Omega = \frac{T}{2} \int d\mathbf{r} \sum_n \int \frac{d\mathbf{q}}{(2\pi)^2} \ln \varepsilon(n,\mathbf{q}). \tag{35}$$

The chemical potential $\tilde{\mu}$ in action (32) can be written as

$$\tilde{\mu} = \mu + \delta\mu, \tag{36}$$

where the shift of the chemical potential

$$\delta\mu = 2\pi l^2 T \sum_n \int d\mathbf{r}\, \tilde{G}_0^n(0,\mathbf{r}) P_N(0,\mathbf{r}) U_{eff}(n,\mathbf{r}) \tag{37}$$

involves corrections similar to the exchange and correlation ones in a clean electron liquid. Here, $l = 1/\sqrt{m\omega_c}$ is the magnetic field length. The quantity $U_{eff}(n,\mathbf{r})$ is the Fourier transform of $U_0(\mathbf{q})/\varepsilon(n,\mathbf{q})$ and

$$P_N(\mathbf{r}_1,\mathbf{r}_2) = \sum_k \varphi_{Nk}^*(\mathbf{r}_2)\varphi_{Nk}(\mathbf{r}_1) \tag{38}$$

is the projection operator onto the partially filled N-th Landau level.

We note that corrections to the thermodynamic and chemical potentials contain additional terms except those presented above. They are neglected in the limit of a weak disorder $\omega_c\tau \gg 1$ (see Appendix A).

The integration over the plasmon field is performed in the Gaussian approximation. This can be justified if the fluctuations of the plasmon field are small. The long- and short-range fluctuations are different physically. In the case of a large length scale $r \gg R_c$, only the dipole transitions between the adjacent Landau levels are induced. The long-range fluctuations are small if the condition $Nr_s \gg 1$ is satisfied [7]. Physically, this condition means that the characteristic length scale R_c^2/a_B of the long-range fluctuations must be much greater than the cyclotron radius R_c. The short-range fluctuations correspond to the case of a small length scale $r \ll R_c$. Transitions between distant Landau levels are possible in this case. The condition $r_s \ll 1$ of the smallness of short-range fluctuations is just the criterion of the perturbation theory applicability to the Coulomb interaction.

3. EFFECTIVE INTERACTION, THE THERMODYNAMIC AND CHEMICAL POTENTIALS

The results of the previous section allow us to find effective action (32) for the electrons on the partially filled N-th Landau level. The main physical quantity that affects the dynamics of the electrons is the effective electron–electron interaction. It is completely determined by the static dielectric function $\varepsilon(q)$. The other two interesting quantities in effective action (32) are the thermodynamic and chemical potentials.

3.1. The effective interaction

The most pronounced effect of electrons on the completely filled Landau levels is the screening of the electron–electron interaction on partially filled Landau level. This screening is determined by the static dielectric function $\varepsilon(q)$.

In accordance with Eq. (31) for the polarization operator $\Pi(n,q)$, the dielectric function can be obtained for arbitrary values of the disorder parameter

$\omega_c\tau$. However, the situation of a small Landau level broadening due to disorder is most interesting from the physical standpoint. In this case, the expression for the static dielectric function can be simplified drastically,

$$\varepsilon(q) = 1 + \frac{2\pi e^2}{q} T \sum_n \pi_0^n(0,q), \quad \omega_c\tau \gg 1. \tag{39}$$

The evaluation of the static dielectric function is presented in Appendix B. The result can be written as

$$\varepsilon(q) = 1 + \frac{2}{qa_B}\left(1 - \mathcal{J}_0^2(qR_c) - \frac{\pi}{3\omega_c\tau}\eta(qR_c)\right), \tag{40}$$

where the function $\eta(x)$ can be found analytically only in the asymptotic regions of small and large values of x (see Appendix B),

$$\eta(x) = \begin{cases} \dfrac{x^2}{4}, & x \ll 1, \\ \dfrac{\ln^2(3.57x)}{\pi x}, & x \gg 1. \end{cases} \tag{41}$$

Here, $\mathcal{J}_0(x)$ is the Bessel function of the first kind. Expression (40) for the static dielectric function is the main result of the paper.

It is worthwhile to note that the asymptotic expressions (in the $qR_c \ll 1$ and $qR_c \gg 1$ domains) for the static dielectric function $\varepsilon(q)$ in a clean system ($\tau^{-1} = 0$) were obtained earlier by Kukushkin, Meshkov, and Timofeev [18]. The general expression for the static dielectric function in a clean system was derived by Aleiner and Glazman [7].

We mention that the asymptotic expressions for the static dielectric function in a clean system can be obtained from a clear physical picture [18, 7]. The behavior of the static dielectric function in the region $qR_c \ll 1$ can be explained by dipole transitions between the adjacent Landau levels. The result for the static dielectric function in the region $qR_c \gg 1$ is explained by the standard Thomas–Fermi screening. But there is no clear physical picture in the case of a weakly dirty system. We have no other opportunity to obtain the dielectric function except the derivation of the effective action for electrons on a partially filled Landau level.

It follows from (40) that in the domain $qR_c \ll 1$, the static dielectric function is given by

$$\varepsilon(q) = 1 + \left(1 - \frac{\pi}{6\omega_c\tau}\right)\frac{R_c^2 q}{a_B}. \tag{42}$$

This shows that the disorder suppresses the effect of the screening. We can expect that the screening decreases as disorder increases. We can estimate the disorder threshold τ^*, i.e., the point of vanishing screening, as $\omega_c\tau^* \sim 1/2\pi$.

From Eq. (40), we can obtain the expression for the static dielectric function in the domain $qR_c \gg 1$,

$$\varepsilon(q) = 1 + \frac{2}{qa_B}\left(1 - \frac{\ln^2(3.57qR_c)}{3\omega_c\tau qR_c}\right). \tag{43}$$

The disorder also suppresses the screening in the region of large wave vectors $qR_c \gg 1$.

Equations (40)–(41) allow us to obtain the asymptotic behavior of the effective interaction $U_{eff}(r)$ in the coordinate space. The polarization is insignificant for the very large length scale $r \gg R_c^2/a_B$ and the effective interaction coincides with the bare Coulomb interaction

$$U_{eff}(r) = \frac{e^2}{r}\left(1 - \frac{R_c^4}{a_B^2 r^2}\left[1 - \frac{\pi}{6\omega_c\tau}\right]\right). \tag{44}$$

At the intermediate scale $R_c^2/a_B \gg r \gg R_c$, the polarization becomes important and the effective interaction is given by

$$U_{eff}(r) = \frac{\omega_c}{2N\left(1 - \dfrac{1}{2\pi\omega_c\tau}\right)} \times \times \ln\left(1 + \frac{R_c^2\left(1 - \dfrac{\pi}{6\omega_c\tau}\right)}{a_B r}\right). \tag{45}$$

We note that while disorder increases, the effective interaction tends to the bare Coulomb interaction. For the small scale $R_c \gg r \gg a_B$, the Thomas–Fermi screening occurs and the effective interaction is given by

$$U_{eff}(r) = \frac{e^2 a_B^2}{r^3} + \frac{\pi}{3\omega_c\tau}\frac{e^2 a_B}{rR_c}\left[\ln^2\frac{1.31R_c}{r} + 1\right]. \tag{46}$$

We emphasize that disorder in the system most strongly affects the electron–electron interaction within the intermediate length scale $R_c^2/a_B \gg r \gg R_c$. Physically, this is the case where the dipole transitions between the adjacent Landau levels are possible.

3.2. The thermodynamic and chemical potentials

The thermodynamic and chemical potentials in Eqs. (34)–(37) can be evaluated in the leading orders in $1/N$. The detailed calculations are presented in Appendix C.

The thermodynamic potential for the system of noninteracting electrons in the presence of disorder for the completely filled Landau levels is given by

$$\Omega_0 = \frac{L_x L_y}{\pi l^2}\left[\frac{N(N-1)}{2}\omega_c - \mu - \frac{\ln(2\omega_c\tau)-1}{\pi\tau}\right], \quad (47)$$

where L_x and L_y are the sizes of the system. The first-order exchange correction to the thermodynamic potential is given by

$$\Delta\Omega = -\frac{L_x L_y}{\pi l^2}\frac{e^2}{\pi l}(2N)^{3/2}\left[\frac{2}{3} + \frac{2\ln 2}{\pi\omega_c\tau}\frac{1}{2N}\right]. \quad (48)$$

The presence of disorder changes the dependence of $\Delta\Omega$ on the magnetic field, i.e., on N. For the dirty system, the second term in brackets in Eq. (48) is proportional to $1/N$. This is in contrast to the clean system, where the correction is much smaller and is proportional to $1/N^2$ [7].

The shift of the chemical potential due to the exchange correction can be written as

$$\delta\mu = \frac{2e^2}{\pi l}(2N)^{1/2}\left[1 - \frac{\ln N}{8N} + \frac{1}{\pi\omega_c\tau}\frac{1}{2N}\right]. \quad (49)$$

We note that $\delta\mu$ contains only the exchange correction and does not involve the correlation correction due to normal ordering of the $\Psi^\dagger$ and Ψ fields (see Ref. [7]).

4. SPIN EXCITATIONS

In the previous section, we analyzed the renormalization of the electron–electron interaction on the partially filled N-th Landau level due to the existence of the other levels. In this section, we investigate the enhancement of the g factor and the simplest spin excitations at the filling factor $\nu = 2N+1$.

The electrons on the partially filled N-th Landau level at the filling factor $\nu = 2N+1$ possess the maximum spin in the ground state, because the ground state does not contain skyrmions at large ν [19]. This ground state is obviously fully spin-polarized and is described by the wave function

$$|N_{el} = N_\Phi, S_z = N_\Phi/2\rangle,$$

where N_{el} is the number of electrons on the partially filled N-th Landau level and

$$N_\Phi = \frac{L_x L_y}{2\pi l^2},$$

is the number of states on the Landau level. The simplest excitations are described by the state with the energy $E_\uparrow$ with an extra hole and the state with the energy $E_\downarrow$ with an extra electron. The width of the spin gap Δ_s is related to the energies of the excited states and to the energy E_0 of the ground state [20, 21, 7] as

$$\Delta_s = E_\uparrow + E_\downarrow - 2E_0.$$

We can obtain that the width of the spin gap equals

$$\Delta_s = \Delta_{ex} + g\omega_c,$$

where the shift of the chemical potential Δ_{ex} due to the exchange interaction [21, 22] is determined by

$$\Delta_{ex} = 2\pi l^2 \int d\mathbf{r}\, U_{eff}(\mathbf{r}) P_N(0,\mathbf{r}) P_N(\mathbf{r},0). \quad (50)$$

Using expression (38) for the projection operator P_N, we can evaluate the effective g factor. It is defined as $g_{eff} = \Delta_s/\omega_c$ and is given by

$$g_{eff} = g + \frac{r_s}{\pi\sqrt{2}}\ln\frac{2\sqrt{2}}{r_s} + \frac{E_h}{\omega_c} + \frac{r_s}{\pi\omega_c\tau}\frac{\ln^2(14.28N)}{4\sqrt{2}\pi^2 N}, \quad (51)$$

where the «hydrodynamic» term is

$$E_h = \frac{\omega_c}{2N\left(1 - \dfrac{\pi}{6\omega_c\tau}\right)} \times \ln\left[1 + \sqrt{2}r_s N\left(1 - \frac{\pi}{6\omega_c\tau}\right)\right]. \quad (52)$$

The disorder in the system results in the appearance of a strong dependence of the effective g factor on the magnitude of the magnetic field as $\ln^2 N/N$.

We now discuss the neutral excitations, spin waves [21, 23] at the filling factor $\nu = 2N+1$. They are described by the wave function

$$\sum_q \exp(ik_x q l^2)\overline{\Psi}_{N,q,\downarrow}\Psi_{N,q-k_y,\uparrow}\left|N_\Phi, \frac{N_\Phi}{2}\right\rangle. \quad (53)$$

Following Ref. [21], we must take three contributions into account. They are the difference of the exchange self-energy of an electron in the excited Landau level and the self-energy in the level from which the electron was removed, the direct Coulomb interaction, and the exchange energy. We then obtain the equation that determines the spectrum of the spin wave excitations,

$$\omega = g\omega_c + \int \frac{d\mathbf{q}}{(2\pi)^2}\frac{U_0(q)}{\varepsilon(q,\omega)}\left[L_N\left(\frac{q^2l^2}{2}\right)\right]^2 \times \exp(-q^2l^2/2)\left(1 - \exp\left[i(\mathbf{k}\cdot\mathbf{q})l^2\right]\right), \quad (54)$$

where $L_N(x)$ is the Laguerre polynomial. The dielectric function $\varepsilon(q,\omega)$ contains the imaginary part (see Eq. (B.4)), which is of order $1/\omega_c\tau$. It results in the decay rate of the spin wave excitations. Physically, the spin wave excitations decay because of the scattering on impurities. We mention that the decay rate also apperears in the magnetoplasmon spectrum.

The energy of the spin-wave excitations is much less than ω_c: $\omega(k) \ll \omega_c$. We can therefore calculate the real part $E_{SW}(k)$ and the imaginary part $\Gamma_{SW}(k)$ of the spin-wave energy separately. We set $\omega = 0$ in the right-hand side of Eq. (54). The evaluation of Eq. (54) then leads to a quadratic dispersion relation for the small wave vectors $kR_c \ll 1$,

$$E_{SW}(k) = g\omega_c + \frac{r_s\omega_c}{\pi\sqrt{2}} \times \times \left[1+\frac{r_s}{\sqrt{2}}\left(1-\frac{1}{3\omega_c\tau}\,\frac{\ln^2(14.28N)}{4N}\right)\right]^{-1} (kR_c)^2. \quad (55)$$

An additional dependence of the effective mass of the spin-wave excitations on the magnetic field appears because of the presence of disorder in the system. The disorder suppresses the effective mass of the spin-wave excitations. For sufficiently large wave vectors

$$1 \ll kR_c \ll R_c^2/l^2,$$

the energy of the spin wave is given by

$$E_{SW}(k) = \Delta_{ex} - E_h - \frac{r_s\omega_c}{\pi\sqrt{2}} \times \times \left[\ln\left(1+\frac{(\sqrt{2}r_skR_c)^{-1}}{1-\dfrac{1}{3\omega_c\tau}\,\dfrac{\ln^2(7.14N/kR_c)}{2N/kR_c}}\right) + \right. + \left. \frac{\sin 2kR_c}{2kR_c}\left(1+\frac{r_s}{\sqrt{2}}\left(1-\frac{1}{3\omega_c\tau}\,\frac{\ln^2(14.28N)}{4N}\right)\right)\right]. \quad (56)$$

To obtain the decay rate of spin-wave excitations, we take into account that the imaginary part ε'' of the dielectric function is small. We then obtain

$$\Gamma_{SW}(k) = -\int \frac{d\mathbf{q}}{(2\pi)^2}\,\frac{U_0(q)\varepsilon''(q,E_{SW})}{\varepsilon_0^2(q,E_{SW})} \times \times \left[L_N\left(\frac{q^2l^2}{2}\right)\right]^2 \times \times \exp(-q^2l^2/2)\left(1-\exp[i(\mathbf{k}\cdot\mathbf{q})l^2]\right). \quad (57)$$

The evaluation of Eq. (57) for small wave vectors $kR_c \ll 1$ yields

$$\Gamma_{SW} = -\frac{\operatorname{arctg}(2\omega_c\tau g)}{6\omega_c\tau}\,\frac{e^2}{a_B}(kR_c)^2 \times \times \frac{1}{(1+l^2/a_BR_c)^2}\,\frac{2-\sin(4N)}{(4N)^2} \quad (58)$$

and for the large wave vectors $kR_c \gg 1$,

$$\Gamma_{SW} = -\frac{\operatorname{arctg}(2\omega_c\tau g_{eff})}{\pi\omega_c\tau}\,\frac{e^2}{a_B} \times \times \left[\left(\frac{a_B}{R_c}\right)^2 \ln\frac{R_c}{a_B} + \frac{\operatorname{arcch}(2kR_c)}{2(4N)^2}\right]. \quad (59)$$

We note that the decay rate Γ_{SW} is of the same order as the correcrions to the real part of the spin-wave energy E_{SW} due to the presence of disorder.

5. ZERO-BIAS ANOMALY

In this section, we consider the electron tunneling into a two-dimensional electron liquid with disorder in a weak magnetic field. We investigate suppression of the tunnelling conductivity near zero bias, the so-called zero-bias anomaly. The properties of the electron tunnelling into an electron system are usually described by a dependence of the tunnelling conductivity $G(V)$ on the bias V. Recently, the effective action approach to the zero-bias problem was developed by Levitov and Shytov [24]. The effective action describes spreading of the tunnelling electron within the electron system in imaginary time ζ.

Following Ref. [24], the action of a spreading charge for zero bias $V = 0$ is determined by

$$S_0(\zeta) = 4\int_0^{+\infty}\frac{d\omega}{2\pi} \times \times \int_0^{+\infty}\frac{q\,dq}{2\pi}\,\frac{\sin^2(\omega\zeta)}{\omega+Dq^2}\,\frac{U_{eff}(q)}{\omega+Dq^2+\sigma q^2U_{eff}(q)}, \quad (60)$$

where σ and D are the conductivity and the diffusive constant of the electron system, respectively. They are related by the Einstein formula $\sigma = e^2\rho D$.

Using asymptotic expression (42) for the static dielectric function $\varepsilon(q)$, we evaluate action (60) in the large time limit $\zeta \gg 1$ as

$$S_0(\zeta) = \frac{e^2}{8\pi^2\sigma\eta}\ln\frac{2\zeta}{\tau_0}\ln\left(\frac{2\zeta}{\tau_0}\beta^{4\eta}\right), \quad (61)$$

where we introduce two dimensionless parameters

$$\beta = \frac{a_B}{\sqrt{2}\, l_{el}}, \quad \eta = \left(1 - \frac{\pi}{6\omega_c\tau}\right)\left(\frac{R_c}{\sqrt{2}\, l_{el}}\right)^2 \qquad (62)$$

with the bare elastic mean free path $l_{el} = R_c\omega_c\tau_0$. In accordance with the inequality $a_B \ll R_c \ll l_{el}$, the parameters β and η are small, $\beta \ll 1$ and $\eta \ll 1$.

Taking the work done by the voltage source into account, we obtain the total action of the spreading charge

$$S(\zeta) = S_0(\zeta) - 2eV\zeta.$$

We must then find the optimum time ζ_* determined by the minimum of the action $S(\zeta)$. The optimum time ζ_* plays the role of the charge accommodation time in the problem. It can be written as

$$\begin{aligned} \zeta_* &= \tau_0 \frac{V_0}{2V} \ln \frac{V_0}{\beta^{2\eta} V}, \\ eV_0 &= \left(1 - \frac{\pi}{6\omega_c\tau}\right)^{-1} \frac{1}{\pi m R_c^2}. \end{aligned} \qquad (63)$$

The theory must be self-consistent in the hydrodynamical limit, i.e., for $\zeta_* \geq \tau_0$. Therefore, the theory is applicable for the bias $V \leq V_0$.

Assuming the contribution from the barrier to be a constant at a small bias, we can write the tunnelling conductivity as

$$G(V) = G_0 \exp[-S_0(\zeta_*) + 2eV\zeta_*]. \qquad (64)$$

After the evaluation, we obtain the dependence of the tunneling conductivity for a small bias,

$$\begin{aligned} G(V) &= G_0 \left(\frac{V}{V_0}\right)^{\alpha(V)}, \\ \alpha(V) &= \frac{e^2}{8\pi^2\sigma\eta} \ln \frac{V_0}{V\beta^{4\eta}}. \end{aligned} \qquad (65)$$

Equation (65) shows that the screening of the electron–electron interaction results in increasing the suppression of the tunneling conductivity. We note that the above result is valid for the bias in the range $V \leq V_0$.

Expression (65) for the tunneling conductivity contains the energy scale eV_0 that coincides with the «hydrodynamic» term E_h in Eq. (52) except for the logarithm. A hydrodynamical model for the low-energy excitations of a clean ($\tau^{-1} = 0$) electron liquid in a weak magnetic field was considered by Aleiner, Baranger, and Glazman [6]. They showed that the tunneling density of states exhibits the gap $2E_h$ related to the Fermi energy. Equation (52) describes the same gap for a weak disorder $\omega_c\tau \gg 1$. Apparently, the disorder is responsible for the fact that the gap is about $0.05\omega_c$ in a wide range of the applied magnetic field [5].

As the magnetic field increases, the factor α increases and becomes of the order of unity. The zero-bias anomaly in the tunneling conductivity crossovers from weak to strong. Expression (65) shows that the factor α depends on the bias V and the magnetic field. This results in the shift of the crossover point V_c along the bias V as the applied magnetic field changes,

$$V_c = V_0 \exp\left(-\frac{4\pi\mu}{\omega_c^2\tau_0}\right), \qquad (66)$$

where μ is the chemical potential. The crossover was observed by Ashoori et al. [5] in the tunneling current from a normal metal into two-dimensional electrons in the presence of a magnetic field. In the experiment, the ohmic conductance was measured as a function of the temperature T. For low temperatures, the conductance corresponds to the zero temperature conductance taken at $V = T/e$. The two-dimensional electrons were relatively clean, with the elastic collision time $\tau_0 \approx 4 \cdot 10^{-12}$ s. The chemical potential calculated from the elctron density was $\mu = 10$ mV. Using Eq. (66), the dependence of the crossover temperature on the magnetic field can be written as

$$T_c = 2.9 \exp\left(-\left[\frac{3.2}{H}\right]^2\right), \qquad (67)$$

where the temperature is measured in Kelvin and the magnetic field in Tesla. Equation (67) demonstrates a good agreement with the results reported in Ref. [5].

6. CONCLUSIONS

We have considered the system of a two-dimensional electron gas in the presence of disorder and the Coulomb interaction in a weak perpendicular magnetic field. The effective low-energy theory describing electrons at the partially filled N-th Landau level was derived in the case of a weak magnetic field ($Nr_s \gg 1$) and a weak interaction ($r_s \ll 1$). The modified electron–electron interaction for electrons on a partially filled Landau level involves the screening from the other electrons on the occupied Landau levels. We also presented the exchange corrections to the thermodynamic and chemical potentials in the presence of disorder.

The theory proposed here allows us to account for the effects of disorder in the problems connected with the behavior of a two-dimensional electron gas in a

weak magnetic field. It can be investigated how disorder affects the formation of stripes, bubble phase, tunneling density of states, spin excitations, tunneling conductivity, etc.

We discussed the effect of disorder on the exchange enhancement of the g factor and the simplest spin excitations on the partially filled Landau level. We obtained an additional dependence of the effective g factor as a function of the magnetic field, the suppression of the effective mass and the decay rate of the spin-wave excitations.

We also investigated the electron tunneling into a two-dimensional electron liquid with a weak disorder in a weak magnetic field. We obtained the enhancement of the gap in the tunneling density of states and a nonlinear dependence of the tunneling conductivity on the applied bias.

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APPENDIX A

In this appendix, we calculate the term S_μ in action (29). This term appears after performing the integration over the longitudinal fluctuations and is given by

$$S_\mu = \frac{1}{2}\langle \left(S_2[\overline{\Psi}, \Psi, \delta P, \lambda]\right)^2 \rangle_{\delta P}, \qquad (A.1)$$

where $\langle \dots \rangle_{\delta P}$ denotes the average with the propagator of the δP field in Eq. (28). We then obtain

$$S_\mu = \delta S_1 + \delta S_2 + \delta S_3, \qquad (A.2)$$

where

$$\delta S_1 = \\ = T^2 \iint d\mathbf{r}_1\, d\mathbf{r}_2 \left(\Psi^\dagger \hat{\lambda} \tilde{G}_0 \hat{\lambda} \Psi - 2 \operatorname{tr} \hat{\lambda} \tilde{G}_0 \hat{\lambda} G_0 \right), \qquad (A.3)$$

$$\delta S_2 = -T^2 \int d\mathbf{r}_1 \dots \int d\mathbf{r}_4 \times \\ \times \sum_{k,n,m}^{\alpha,\beta} \lambda^\beta_m(\mathbf{r}_1) D^{\alpha\alpha}_{nm}(k; \mathbf{r}_3, \mathbf{r}_4) \times \\ \times \pi^n_0(m; \mathbf{r}_1, \mathbf{r}_2) L^{\alpha\beta,\alpha\alpha}_{n,n+m}(\mathbf{r}_2, \mathbf{r}_4), \qquad (A.4)$$

$$\delta S_3 = \frac{T^2}{2} \int d\mathbf{r}_1 \dots \int d\mathbf{r}_4 \sum_{k,l,n,m}^{\alpha,\beta,\gamma,\delta} D^{\alpha\beta}_{nm}(k; \mathbf{r}_1, \mathbf{r}_2) \times \\ \times D^{\gamma\delta}_{nm}(l; \mathbf{r}_3, \mathbf{r}_4) L^{\alpha\beta,\gamma\delta}_{n,n+m}(\mathbf{r}_2, \mathbf{r}_4). \qquad (A.5)$$

Here,

$$D^{\alpha\beta}_{nm}(k; \mathbf{r}_1, \mathbf{r}_2) = \\ = \left(\overline{\Psi}^\alpha_k(\mathbf{r}_1) \Psi^\beta_{n+m}(\mathbf{r}_2) - 2\delta_{k,n+m} \delta^{\alpha,\beta} G^{n+m}_0(\mathbf{r}_2, \mathbf{r}_1) \right) \times \\ \times \lambda^\alpha_{n-k}(\mathbf{r}_1) \tilde{G}^n_0(\mathbf{r}_1, \mathbf{r}_2) + \\ + \left(\overline{\Psi}^\beta_n(\mathbf{r}_1) \Psi^\alpha_k(\mathbf{r}_2) - 2\delta_{k,n} \delta^{\alpha,\beta} G^n_0(\mathbf{r}_2, \mathbf{r}_1) \right) \times \\ \times \lambda^\beta_{k-n-m}(\mathbf{r}_1) \tilde{G}^{n+m}_0(\mathbf{r}_1, \mathbf{r}_2), \qquad (A.6)$$

and $L^{\alpha\beta,\gamma\delta}_{m_1,m_2}$ is the propagator of longitudinal fluctuations in Eq. (28).

Integrating over the plasmon field, we obtain from the S_μ term

$$S_\mu \to \frac{\Delta\Omega_1 + \Delta\Omega_2 + \Delta\Omega_3}{T} + (\delta\mu_1 + \delta\mu_2 + \delta\mu_3) \times \\ \times \int d\mathbf{r}\, \Psi^\dagger(\mathbf{r}) \Psi(\mathbf{r}), \qquad (A.7)$$

where second-order corrections to the thermodynamic potential are given by

$$\frac{\Delta\Omega_1}{L_x L_y} = -T^2 \times \\ \times \sum_{n,m} \int d\mathbf{r} G^n_0(0, \mathbf{r}) \tilde{G}^m_0(\mathbf{r}, 0) U_{eff}(m-n, \mathbf{r}), \\ \frac{\Delta\Omega_2}{L_x L_y} = 2T^2 \sum_{n,m} \int d\mathbf{r}\, d\mathbf{r}_1\, d\mathbf{r}_2 \times \\ \times \left(G^n_0(0, \mathbf{r}) \tilde{G}^{n+m}_0(\mathbf{r}, 0) + \right. \\ \left. + \tilde{G}^n_0(\mathbf{r}, 0) G^{n+m}_0(0, \mathbf{r}) \right) \times \\ \times U_{eff}(m, \mathbf{r}_1) \pi^n_0(m, \mathbf{r}_1 - \mathbf{r}_2) L^{\alpha\alpha,\alpha\alpha}_{n,n+m}(\mathbf{r}_2 - \mathbf{r}), \qquad (A.8) \\ \frac{\Delta\Omega_3}{L_x L_y} = 2T^2 \sum_{n,m} \int d\mathbf{r}\, d\mathbf{r}_1\, d\mathbf{r}_2 \times \\ \times \left(G^n_0(0, \mathbf{r}) \tilde{G}^{n+m}_0(\mathbf{r}, 0) + \right. \\ \left. + G^{n+m}_0(0, \mathbf{r}) \tilde{G}^n_0(\mathbf{r}, 0) \right) \times \\ \times \left(G^n_0(0, \mathbf{r}_1) \tilde{G}^{n+m}_0(\mathbf{r}_1, 0) + \right. \\ \left. + G^{n+m}_0(0, \mathbf{r}_1) \tilde{G}^n_0(\mathbf{r}_1, 0) \right) \times \\ \times U_{eff}(m, \mathbf{r}_2) L^{\alpha\alpha,\alpha\alpha}_{n,n+m}(\mathbf{r} - \mathbf{r}_1 + \mathbf{r}_2).$$

The above corrections are negligible in the parameter N^{-1} compared to the correction determined by

Eq. (35). The corrections to the chemical potential are given by

$$\frac{\delta\mu_1}{2\pi l^2} = T\sum_m \int d\mathbf{r}\, P_N(0,\mathbf{r})\tilde{G}_0^m(\mathbf{r},0)U_{eff}(m,\mathbf{r}),$$

$$\frac{\delta\mu_2}{2\pi l^2} =$$

$$= -4T\sum_m \int d\mathbf{r}\, d\mathbf{r}_1\, d\mathbf{r}_2\, P_N(0,\mathbf{r})\tilde{G}_0^m(\mathbf{r},0)\times$$

$$\times U_{eff}(m,\mathbf{r}_1)\pi_0^0(m,\mathbf{r}_1-\mathbf{r}_2)L_{0,m}^{\alpha\alpha,\alpha\alpha}(\mathbf{r}_2-\mathbf{r}), \qquad (A.9)$$

$$\frac{\delta\mu_3}{2\pi l^2} =$$

$$= -8T^2\sum_m \int d\mathbf{r}\, d\mathbf{r}_1\, d\mathbf{r}_2\, P_N(0,\mathbf{r})\tilde{G}_0^m(\mathbf{r},0)\times$$

$$\times U_{eff}(m,\mathbf{r}_1)L_{0,m}^{\alpha\alpha,\alpha\alpha}(\mathbf{r}-\mathbf{r}_1+\mathbf{r}_2)\times$$

$$\times\left(G_0^0(0,\mathbf{r}_1)\tilde{G}_0^m(\mathbf{r}_1,0)+G_0^m(0,\mathbf{r}_1)\tilde{G}_0^0(\mathbf{r}_1,0)\right).$$

The second and third corrections are negligible in the parameter N^{-1} compared to the first term. The shift of the chemical potential $\delta\mu$ is therefore mainly determined by the first correction $\delta\mu_1$.

APPENDIX B

In this appendix, we evaluate the polarization operator $\Pi(\omega_n,q)$. The condition $\omega_c\tau \gg 1$ is assumed to hold. Then

$$\Pi(\omega_n,q) = T\sum_m \pi^m(\omega_n,q). \qquad (B.1)$$

The calculation of the polarization operator $\Pi(\omega_n,q)$ is analogous to that given in Ref. [7]. The wave vectors $q \ll R_c/l^2$ are considered.

Using Eq. (31), we immediately obtain

$$\Pi(\zeta_n,Q) =$$

$$= \frac{m}{\pi}\left[1-\sum_{j=-\infty}^{+\infty}\frac{\mathcal{J}_j^2(Q)}{j^2+\zeta_n^2}\left(\zeta_n^2+\frac{\pi}{3\tau\omega_c}\mathcal{L}_j(\zeta_n)\right)\right], \qquad (B.2)$$

where

$$\mathcal{L}_j(\zeta_n) = \Theta(|j|-2)\times$$

$$\times\sum_{n=1}^{|j|-1}\left[\frac{|j|}{n}-\frac{1}{2}\frac{j^2-\zeta_n^2}{j^2+\zeta_n^2}\ln\frac{\zeta_n^2+n^2}{n^2}+\frac{2|j|\zeta_n}{(j^2+\zeta_n^2)^2}\operatorname{arctg}\frac{\zeta_n}{n}\right]+$$

$$+\Theta(|j|-1)\left[\frac{1}{2}+\frac{\zeta_n|j|\operatorname{arctg}\dfrac{\zeta_n}{|j|}}{j^2+\zeta_n^2}+\frac{1}{4}\frac{j^2-\zeta_n^2}{j^2+\zeta_n^2}\times\right.$$

$$\left.\times\ln\left(\frac{\zeta_n^2+j^2}{j^2}(1+2\tau\omega_c\zeta_n)^2\right)\right]. \qquad (B.3)$$

Two parameters $\zeta_n = \omega/\omega_c$ (with $\omega = 2\pi Tn$) and $Q = qR_c$ are introduced here. The transformation of series (B.2) into the integral form yields the asymptotic form of the polarization operator in the different regimes. In the static limit $\zeta_n \ll 1$,

$$\Pi(\zeta_n,Q) = \frac{m}{\pi}\left(1-\mathcal{J}_0^2(Q)-\frac{\pi}{3\omega_c\tau}\eta(Q)+\right.$$

$$\left.+\frac{\ln(1+2\omega_c\tau\zeta_n)}{2\pi\omega_c\tau}\xi(Q)+O(\zeta^2)\right), \qquad (B.4)$$

where the function $\eta(x)$ is defined as

$$\eta(x) = \frac{1}{2\pi}\int_0^\pi\frac{dy}{\pi}\mathcal{J}_0\left(2x\sin\frac{y}{2}\right)\times$$

$$\times\left[\ln^2\left(2\sin\frac{y}{2}\right)-\frac{\pi^2}{12}\right], \qquad (B.5)$$

and its asymptotic form is given in Eq. (41). The function $\xi(x)$ is defined as

$$\xi(x) = \int_0^\pi\frac{dy}{\pi}\mathcal{J}_0\left(2x\sin\frac{y}{2}\right)\left[(y-\pi)^2-\frac{\pi^2}{3}\right], \qquad (B.6)$$

and its asymptotic form is given by

$$\xi(x) = \begin{cases} x^2, & x \ll 1, \\ \dfrac{\pi}{3x}(2-\sin 2x), & x \gg 1. \end{cases} \qquad (B.7)$$

In the hydrodynamic limit $qR_c \ll 1$, we obtain

$$\Pi(\zeta_n,Q) = \frac{m}{2\pi}\frac{Q^2}{1+\zeta_n^2}\times$$

$$\times\left[1-\frac{\pi}{6\tau\omega_c}+\frac{1}{2\pi\tau\omega_c}\frac{1-\zeta_n^2}{1+\zeta_n^2}\times\right.$$

$$\times\ln\left((1+\zeta_n^2)(1+2\tau\omega_c\zeta_n)^2\right)-$$

$$\left.-\frac{\pi}{3\tau\omega_c}\frac{\zeta_n\operatorname{arctg}\zeta_n}{1+\zeta_n^2}\right]. \qquad (B.8)$$

APPENDIX C

In this appendix, we evaluate the corrections to the thermodynamic and chemical potentials.

C.1. Correction to the thermodynamic potential

Using Eq. (35), we can split the correction to the thermodynamic potential into the exchange and correlation ones as

$$\Delta\Omega = \Delta\Omega_{ex} + \Delta\Omega_c, \tag{C.1}$$

$$\frac{\Delta\Omega_{ex}}{L_x L_y} = \frac{T}{2}\sum_n \int \frac{d\mathbf{q}}{(2\pi)^2} U_0(q)\Pi(n,q), \tag{C.2}$$

$$\frac{\Delta\Omega_c}{L_x L_y} = -\frac{T}{2}\sum_n \int \frac{d\mathbf{q}}{(2\pi)^2} \times \times \int_0^1 d\alpha \frac{\alpha U_0^2(q)\Pi^2(n,q)}{1+\alpha U_0(q)\Pi(n,q)}. \tag{C.3}$$

The exchange correction gives the leading contribution [7] and can be written as

$$\frac{\Delta\Omega_{ex}}{L_x L_y} = -\frac{e^2}{2\pi l^3}\sum_{m\neq N}\int_0^\infty dx\, e^{-x^2/2} L_N^1\left(\frac{x^2}{2}\right) \times \times L_m\left(\frac{x^2}{2}\right)\left(\Theta(N-m) + \frac{1}{\pi\omega_c\tau}\frac{1}{m-N}\right), \tag{C.4}$$

where L_n^m stands for the Laguerre polynomials. The above equation goes into Eq. (48) in the case where $N \gg 1$.

C.2. Correction to the chemical potential

Using Eq. (37), we can split the correction to the chemical potential into the exchange and correlation ones,

$$\delta\mu = \delta\mu_{ex} + \delta\mu_c, \tag{C.5}$$

$$\delta\mu_{ex} = 2\pi l^2 T \sum_n \int d\mathbf{r}\, U_0(r) P_N(0,\mathbf{r}) \tilde{G}_0^n(\mathbf{r},0), \tag{C.6}$$

$$\delta\mu_c = -2\pi l^2 T \sum_n \int \frac{d^2\mathbf{q}}{(2\pi)^2} P_N(q)\tilde{G}_0^n(q) \times \times \frac{U_0^2(q)\Pi(n,q)}{1+U_0(q)\Pi(n,q)}. \tag{C.7}$$

The exchange correction gives the leading contribution [7] and can be written as

$$\delta\mu_{ex} = -\frac{e^2}{l}\sum_{m\neq N}\int_0^\infty dx\, e^{-x^2/2} L_N\left(\frac{x^2}{2}\right) L_m\left(\frac{x^2}{2}\right) \times \times \left(\Theta(N-m) + \frac{1}{2\pi\omega_c\tau}\frac{1}{m-N}\right). \tag{C.8}$$

For $N \gg 1$, the above equation leads to

$$\delta\mu_{ex} = \frac{2e^2}{\pi l^2}(2N)^{1/2}\left[1 - \frac{\ln N}{8N} + \frac{1}{4\pi\omega_c\tau}\frac{1}{2N} \times \times \left(\int_1^\infty \frac{dt}{t}\ln(1-e^{-t}) + \int_0^1 \frac{dt}{t}\ln\frac{1-e^{-t}}{t} - \frac{\pi^2}{3}\right)\right]. \tag{C.9}$$

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