

VIOLATION OF THE FACTORIZATION THEOREM IN LARGE ANGLE RADIATIVE BHABHA SCATTERING

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The lowest order QED radiative corrections to the radiative large angle Bhabha scattering process in the region where all kinematic invariants are large compared to the electron mass are considered. We show that the leading logarithmic corrections do not factorize before the Born cross section, contrary to the picture assumed in the renormalization group approach. The leading and non leading contributions for typical kinematics of the hard process at the energy of the Φ factory are estimated.

1. INTRODUCTION

The large angle Bhabha scattering process (LABS) plays an important role in e^+e^- colliding beam physics [1]. First, it is traditionally used for calibration, because it has a large cross section and can be recognized easily. Second, it might provide essential background information in a study of quarkonia physics. The result obtained below can also be used to construct Monte Carlo event generators for Bhabha scattering processes.

In our previous papers we considered the following contributions to the large angle Bhabha cross section: pair production (virtual, soft [2], and hard [3]) and two hard photons [4]. This paper is devoted to the calculation of radiative corrections to a single hard-photon emission process. We consider the kinematics essentially of type $2 \rightarrow 3$, in which all possible scalar products of 4-momenta of external particles are large compared to the electron mass squared.

Considering virtual corrections, we identify gauge invariant sets of Feynman diagrams. Loop corrections associated with emission and absorption of virtual photons by the same fermionic line are called as Glass-type (G) corrections. The case in which a loop involves exchange of two virtual photons between different fermionic lines is called Box-type (B) Feynman diagrams. The third class includes the vertex function and vacuum polarization contributions (Π -type). We see explicitly that all terms that contain the square of large logarithms $\ln(s/m^2)$, as well as those that contain the infrared singularity parameter (fictitious photon mass λ), cancel out in the total sum, where the emission of an additional soft photon is also considered.

We note here that the part of the general result associated with scattering-type diagrams (see Fig. 1 (1, 5)) was used to describe radiative deep inelastic scattering with radiative corrections taken into account in Ref. [5] (we labeled it the Compton tensor with heavy photon). A similar set of Feynman diagrams can be used to describe the annihilation channel [3].

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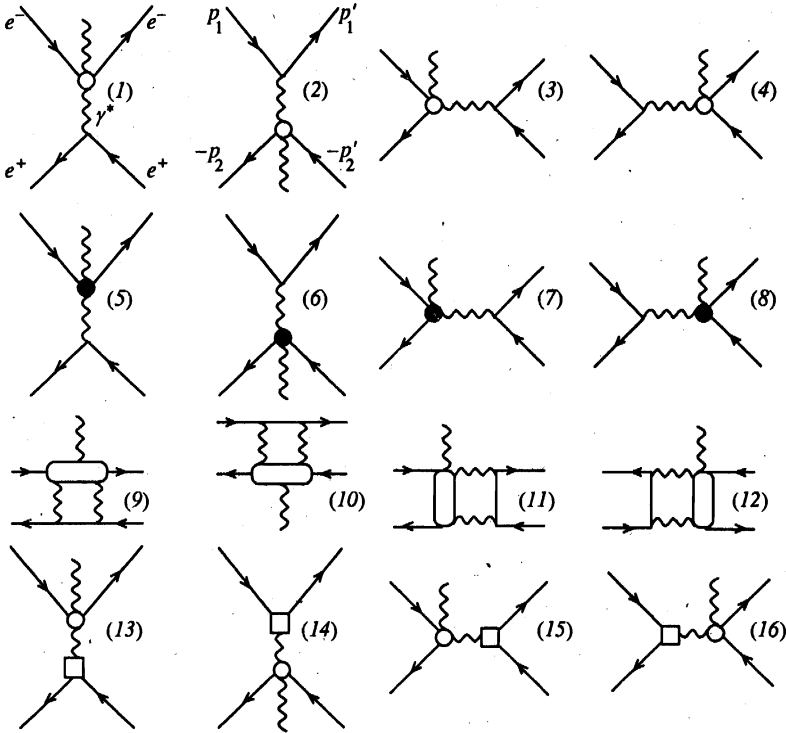


Fig. 1. G- and B-type Feynman diagrams for radiative Bhabha scattering

The problem of virtual radiative corrections calculations at the one-loop level is cumbersome for the process

$$e^+(p_2) + e^-(p_1) \rightarrow e^+(p'_2) + e^-(p'_1) + \gamma(k_1). \quad (1)$$

Specifically, if at the Born level we need to consider eight Feynman diagrams, then at the one-loop level we have as many as 72. Furthermore, performing loop momentum integration, we introduce scalar, vector, and tensor integrals up to the third rank with 2, 3, 4, and 5 denominators (a set of relevant integrals is given in our preprint [6]). A high degree of symmetry of Feynman diagrams for a cross section can be exploited to calculate the matrix element squared. Using it, we can restrict ourselves to the consideration of interferences of the Born-level amplitudes (Fig. 1 (1-4)) with those that contain one-loop integrals (Fig. 1 (5-16)). Our calculation is simplified since we omit the electron mass m in evaluating the corresponding traces due to the kinematic region under consideration:

$$\begin{aligned} s \sim s_1 \sim -t_1 \sim -t \sim -u \sim -u_1 \sim \chi_{1,2} \sim \chi'_{1,2} \gg m^2, \\ s = 2p_1 p_2, \quad t = -2p_2 p'_2, \quad u = -2p_1 p'_2, \quad s_1 = 2p'_1 p'_2, \\ t_1 = -2p_1 p'_1, \quad u_1 = -2p_2 p'_1, \quad \chi_{1,2} = 2k_1 p_{1,2}, \quad \chi'_{1,2} = 2k_1 p'_{1,2}, \\ s + s_1 + t + t_1 + u + u_1 = 0, \quad s + t + u = \chi'_1, \\ s_1 + t + u_1 = -\chi_1, \quad t + \chi_1 = t_1 + \chi'_1. \end{aligned} \quad (2)$$

We found that some kind of local factorization took place both for the G- and B-type Feynman diagrams: the leading logarithmic contribution to the matrix element squared, summed over spin states, arising from interference of one of the four Feynman diagrams at the Born level (Fig. 1 (1-4)) with some one-loop-corrected Feynman diagrams (Fig. 1 (5-16)), turns out to be proportional to the interference of the corresponding amplitudes at the Born level. The latter has the form

$$\begin{aligned}
 E_0 &= (4\pi\alpha)^{-3} \sum |M_1|^2 = -\frac{16}{t^2} \frac{1}{4} \text{Tr}(\hat{p}'_1 \tilde{O}_{11'} \hat{p}_1 O_{11'}) \frac{1}{4} \text{Tr}(\hat{p}_2 \gamma_\sigma \hat{p}'_2 \gamma_\rho) = \\
 &= -\frac{16}{t\chi_1\chi'_1} (u^2 + u_1^2 + s^2 + s_1^2), \\
 O_0 &= (4\pi\alpha)^{-3} \sum M_1 M_2^* = \frac{8}{tt_1} \left(\frac{s}{\chi_1\chi_2} + \frac{s_1}{\chi'_1\chi'_2} + \frac{u}{\chi_1\chi'_2} + \frac{u_1}{\chi_2\chi'_1} \right) \times \\
 &\quad \times (u^2 + u_1^2 + s^2 + s_1^2), \\
 I_0 &= (4\pi\alpha)^{-3} \sum M_1 (M_3^* + M_4^*) = -(1 + \hat{Z}) \frac{4}{ts_1} \left\{ -\frac{4u_1\chi'_2}{\chi_1} + \right. \\
 &\quad + \frac{4u(s_1 + t_1)(s + t)}{\chi_2\chi'_1} - \frac{2}{\chi_1\chi_2} [2suu_1 + (u + u_1)(uu_1 + ss_1 - tt_1)] + \\
 &\quad \left. + \frac{2}{\chi_1\chi'_1} [2t_1uu_1 + (u + u_1)(uu_1 + tt_1 - ss_1)] \right\}, \\
 O_{11'} &= \gamma_\rho \frac{\hat{p}'_1 + \hat{k}_1}{\chi'_1} \gamma_\mu - \gamma_\mu \frac{\hat{p}_1 - \hat{k}_1}{\chi_1} \gamma_\rho, \quad \tilde{O}_{11'} = O_{11'}(\rho \leftrightarrow \mu),
 \end{aligned} \tag{3}$$

where the \hat{Z} -operator acts as follows:

$$\hat{Z} = \begin{vmatrix} p_1 \leftrightarrow p'_1 & s \leftrightarrow s_1 \\ p_2 \leftrightarrow p'_2 & u \leftrightarrow u_1 \\ k_1 \rightarrow -k_1 & t, t_1 \rightarrow t, t_1 \end{vmatrix}.$$

It can be shown that the total matrix element squared, summed over spin states, can be obtained using symmetry properties realized by means of the permutation operations:

$$\begin{aligned}
 \sum |M|^2 &= (4\pi\alpha)^3 F, \quad F = (1 + \hat{P} + \hat{Q} + \hat{R})\Phi = \\
 &= 16 \frac{ss_1(s^2 + s_1^2) + tt_1(t^2 + t_1^2) + uu_1(u^2 + u_1^2)}{ss_1tt_1} \times \\
 &\quad \times \left(\frac{s}{\chi_1\chi_2} + \frac{s_1}{\chi'_1\chi'_2} - \frac{t}{\chi_2\chi'_2} - \frac{t_1}{\chi_1\chi'_1} + \frac{u}{\chi_1\chi'_2} + \frac{u_1}{\chi_2\chi'_1} \right), \\
 \Phi &= E_0 + O_0 - I_0.
 \end{aligned} \tag{4}$$

The explicit form of the $\hat{P}, \hat{Q}, \hat{R}$ operators is

$$\begin{aligned}
 \hat{P} &= \begin{vmatrix} p_1 \leftrightarrow -p'_2 & s \leftrightarrow s_1 \\ p_2 \leftrightarrow -p'_1 & t \leftrightarrow t_1 \\ k_1 \rightarrow k_1 & u, u_1 \rightarrow u, u_1 \end{vmatrix}, \\
 \hat{Q} &= \begin{vmatrix} p_2 \leftrightarrow -p'_1 & s \leftrightarrow t_1 \\ p'_2 \rightarrow p'_2 & s_1 \leftrightarrow t \\ p_1, k_1 \rightarrow p_1, k_1 & u, u_1 \rightarrow u, u_1 \end{vmatrix},
 \end{aligned} \tag{5}$$

$$\hat{R} = \left| \begin{array}{cc} p_1 \longleftrightarrow -p'_2 & s \longleftrightarrow t \\ p'_1 \rightarrow p'_1 & s_1 \longleftrightarrow t_1 \\ p_2, k_1 \rightarrow p_2, k_1 & u, u_1 \rightarrow u, u_1 \end{array} \right|.$$

The differential cross section at the Born level in the case of large angle kinematics (2) was found in Ref. [7]:

$$d\sigma_0(p_1, p_2) = \frac{\alpha^3}{32s\pi^2} F \frac{d^3 p'_1 d^3 p'_2 d^3 k_1}{\varepsilon'_1 \varepsilon'_2 \omega_1} \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2 - k_1), \tag{6}$$

where $\varepsilon_1, \varepsilon_2,$ and ω_1 are the energies of the outgoing fermions and photon, respectively. The collinear kinematic regions (real photon emitted in the direction of one of the charged particles) corresponding to the case in which one of the invariants χ_i, χ'_i is of order m^2 yields the main contribution to the total cross section. These require separate investigation, and will be considered elsewhere.

Our paper is organized as follows. In Sec. 2 we consider the contribution due to the set of Feynman diagrams Fig. 1 (5-8) called *glasses* here (G-type diagrams). Using crossing symmetry, we construct the whole G-type contribution from the gauge-invariant set of Feynman diagrams in Fig. 1 (5). Moreover, only the set of Feynman diagrams depicted in Fig. 2 (d) can be considered in practical calculations, due to an additional mirror symmetry in the diagrams of Fig. 2 (d, e). We therefore start by checking the gauge invariance of the Compton tensor described by the Feynman diagrams of Fig. 2 (d, e) for all fermions and one of the photons on the mass shell. In Sec. 3 we consider the contribution of amplitudes containing vertex functions and the virtual photon polarization operator shown in Fig. 1 (13-16) and Fig. 2 (f, g). In Sec. 4 we take into account the contribution of Feynman diagrams with virtual two-photon exchange, shown in Fig. 1 (9-12), called *boxes* here (B-type diagrams). Again, using the crossing symmetry of Feynman diagrams, we show how to use only the Feynman diagrams of Fig. 1 (9) in calculations. We show that the terms containing infrared singularities, as well as these containing large logarithms, can be written in simple form, related to certain contributions to the radiative Bhabha cross section in the Born approximation (3). We also control terms in the matrix element squared that do not contain large logarithms and are infrared-finite. Thus our considerations permit us to calculate the cross section in the kinematic region (2), in principle, to power-law accuracy, i.e., neglecting terms that are

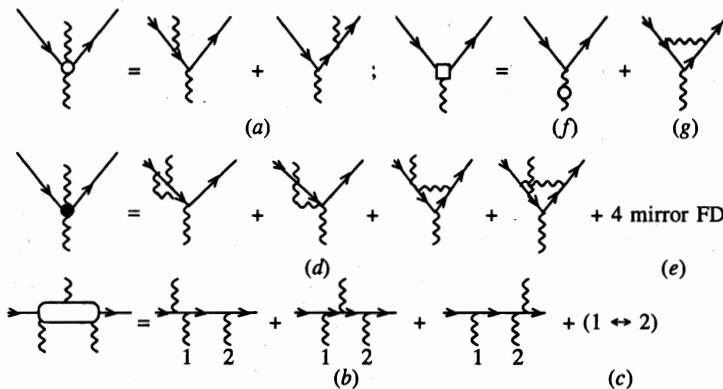


Fig. 2. Content of the notation for Fig. 1

$$\mathcal{O}\left(\frac{\alpha}{\pi} \frac{m^2}{s} L_s^2\right), \tag{7}$$

as compared to $\mathcal{O}(\alpha/\pi)$ terms calculated in this paper. Note that the terms in (7) are less than 10^{-4} for typical moderately high energy colliders (DAΦNE, VEPP-2M, BEPS). Unfortunately, the non leading terms are too complicated to be presented analytically, so we have only estimated them numerically. In Sec. 5 we consider emission of an additional soft photon in our radiated Bhabha process. To conclude, we note that the expression for the total correction, taking into account virtual and real soft photon emission in the leading logarithmic approximation, has a very elegant and handy form, although it differs from what one might expect in the approach based on renormalization group ideas. Besides analytic expressions, we also give numerical values, along with the non leading terms for a few points under typical experimental conditions.

2. CONTRIBUTION OF G-TYPE DIAGRAMS

We begin by explicitly checking the gauge invariance of the tensor

$$\bar{u}(p'_1)R_{1,1'}^{\sigma\mu}u(p_1). \tag{8}$$

This was done indirectly in Ref. [5], where the Compton tensor for a heavy photon was written in terms of explicitly gauge invariant tensor structures. We use the expression

$$R_{1,1'}^{\sigma\mu} = R^{X_1} + R^{X'_1}, \tag{9}$$

$$R^{X_1} = A_2\gamma_\sigma\hat{k}_1\gamma_\mu + \int \frac{d^4k}{i\pi^2} \left\{ \frac{\gamma_\lambda(\hat{p}'_1 - \hat{k})\gamma_\sigma(\hat{p}_1 - \hat{k}_1 - \hat{k})\gamma_\lambda(\hat{p}_1 - \hat{k}_1)\gamma_\mu}{-\chi_1(0)(2)(q)} + \frac{\gamma_\lambda(\hat{p}'_1 - \hat{k})\gamma_\sigma(\hat{p}_1 - \hat{k}_1 - \hat{k})\gamma_\mu(\hat{p}_1 - \hat{k})\gamma_\lambda}{(0)(1)(2)(q)} \right\}, \tag{10}$$

where

$$(0) = k^2 - \lambda^2, \quad (2) = (p'_1 - k)^2 - m^2, \quad (1) = (p_1 - k)^2 - m^2, \tag{11}$$

$$(q) = (p_1 - k_1 - k)^2 - m^2, \quad A_2 = \frac{2}{\chi_1} \left(L_{X_1} - \frac{1}{2} \right), \quad L_{X_1} = \ln \frac{\chi_1}{m^2}.$$

The quantity R^{X_1} corresponds to the Feynman diagrams depicted in Fig. 2d, while $R^{X'_1}$ corresponds to those in Fig. 2 (e). The first term on the right-hand side of Eq. (10) corresponds to the first two of Fig. 2d under conditions (2). The gauge invariance condition $R_{1,1'}^{\sigma\mu}, k_\mu = 0$ is clearly satisfied. The gauge invariance condition regarding the heavy photon Lorentz index provides some check of the loop momentum integrals, which can be found in Ref. [6]:

$$\bar{u}(p'_1)R_{1,1'}^{\sigma\mu}u(p_1)q_\sigma e_\mu(k_1) = Ak_1^\mu e_\mu(k_1), \quad A = -2\frac{L_{X_1} - 2}{\chi_1} - 6\frac{L_{X_1} - 1}{\chi_1}. \tag{12}$$

The gauge invariance is thus satisfied due to the Lorentz condition for the on shell photon, $e(k_1)k_1 = 0$. As stated above, the use of crossing symmetries of amplitudes permits us to consider

only R^{X_1} . For interference of amplitudes at the Born level (see Fig. 1 (1-4) and Fig. 1 (5-8)), we obtain in terms of the replacement operators

$$(\Delta|M|^2)_G = 2^5 \alpha^4 \pi^2 (1 + \hat{P} + \hat{Q} + \hat{R})(1 + \hat{Z}) [E_{15}^{X_1} + O_{25}^{X_1} - I_{35}^{X_1} - I_{45}^{X_1}], \quad (13)$$

with

$$\begin{aligned} E_{15}^{X_1} &= \frac{16}{t^2} \frac{1}{4} \text{Tr}(\hat{p}'_1 R^{X_1} \hat{p}_1 O_{11'}) \frac{1}{4} \text{Tr}(\hat{p}_2 \gamma_\rho \hat{p}'_2 \gamma_\sigma), \\ O_{25}^{X_1} &= \frac{16}{tt_1} \frac{1}{4} \text{Tr}(\hat{p}'_1 R^{X_1} \hat{p}_1 \gamma_\rho) \frac{1}{4} \text{Tr}(\hat{p}_2 \gamma_\sigma \hat{p}'_2 O_{22'}), \\ I_{35}^{X_1} &= \frac{4}{ts_1} \frac{1}{4} \text{Tr}(\hat{p}'_1 R^{X_1} \hat{p}_1 O_{12} \hat{p}_2 \gamma_\sigma \hat{p}'_2 \gamma_\rho), \\ I_{45}^{X_1} &= \frac{4}{ts} \frac{1}{4} \text{Tr}(\hat{p}'_1 R^{X_1} \hat{p}_1 \gamma_\rho \hat{p}_2 \gamma_\sigma \hat{p}'_2 O_{1'2'}), \\ O_{11'} &= \gamma_\rho \frac{\hat{p}'_1 + \hat{k}_1}{\chi'_1} \gamma_\mu - \gamma_\mu \frac{\hat{p}_1 - \hat{k}_1}{\chi_1} \gamma_\rho, \\ O_{22'} &= \gamma_\mu \frac{-\hat{p}'_2 - \hat{k}_1}{\chi'_2} \gamma_\rho - \gamma_\rho \frac{-\hat{p}_2 + \hat{k}_1}{\chi_2} \gamma_\mu, \\ O_{12} &= -\gamma_\mu \frac{\hat{p}_1 - \hat{k}_1}{\chi_1} \gamma_\rho - \gamma_\rho \frac{-\hat{p}_2 + \hat{k}_1}{\chi_2} \gamma_\mu, \\ O_{1'2'} &= \gamma_\rho \frac{\hat{p}'_1 + \hat{k}_1}{\chi'_1} \gamma_\mu + \gamma_\mu \frac{-\hat{p}'_2 - \hat{k}_1}{\chi'_2} \gamma_\rho. \end{aligned} \quad (14)$$

In the logarithmic approximation, the G-type amplitude contribution to the cross section has the form

$$\begin{aligned} d\sigma_G &= \frac{d\sigma_0}{F} \frac{\alpha}{\pi} (1 + \hat{P} + \hat{Q} + \hat{R}) \Phi \left[-\frac{1}{2} L_{t_1}^2 + \frac{3}{2} L_{t_1} + 2L_{t_1} \ln \frac{\lambda}{m} \right], \\ L_{t_1} &= \ln \frac{-t_1}{m^2}. \end{aligned} \quad (15)$$

3. VACUUM POLARIZATION AND VERTEX INSERTION CONTRIBUTIONS

Let us examine a set of $\Gamma\Pi$ -type Feynman diagrams. The contribution of the Dirac form factor of fermions and vacuum polarization (see Fig. 1 (13-16)) can be parametrized as $(1 + \Gamma_t)/(1 - \Pi_t)$, while the contribution of the Pauli form factor is proportional to the fermion mass, and is omitted here. We obtain

$$d\sigma_{\Gamma\Pi} = \frac{d\sigma_0}{F} \frac{\alpha}{\pi} 2(1 + \hat{P} + \hat{Q} + \hat{R})(\Gamma_t + \Pi_t)\Phi, \quad (16)$$

where

$$\begin{aligned} \Gamma_t &= \frac{\alpha}{\pi} \left\{ \left(\ln \frac{m}{\lambda} - 1 \right) (1 - L_t) - \frac{1}{4} L_t - \frac{1}{4} L_t^2 + \frac{1}{2} \zeta_2 \right\}, \\ \Pi_t &= \frac{\alpha}{\pi} \left(\frac{1}{3} L_t - \frac{5}{9} \right), \quad L_t = \ln \frac{-t}{m^2}. \end{aligned} \quad (17)$$

In realistic calculations, the vacuum polarization due to hadrons and muons can be taken into account in a very simple fashion [8], just by adding it to Π_t .

4. CONTRIBUTION OF THE B-TYPE SET OF FEYNMAN DIAGRAMS

A procedure resembling the one used in the previous section, applied to the B-type set of Feynman diagrams (Fig. 1 (9-12a)), enables us to use only certain one-loop diagrams in practical calculations, specifically three of those in the scattering channel with uncrossed exchanged photon legs:

$$(\Delta|M|^2)_B = 2^5 \alpha^4 \pi^2 \operatorname{Re} (1 + \hat{P} + \hat{Q} + \hat{R}) [(1 - \hat{P}_{22'}) I_{19}^{X_1} + (1 + \hat{P}_{22'}) I_{29}^{X_1} - I], \tag{18}$$

where

$$\hat{P}_{22'} = \begin{vmatrix} p_2 \longleftrightarrow -p'_2 & s \longleftrightarrow u \\ p_1 \longleftrightarrow p_1 & s_1 \longleftrightarrow u_1 \\ p'_1, k_1 \rightarrow p'_1, k_1 & t, t_1 \rightarrow t, t_1 \end{vmatrix}, \tag{19}$$

and

$$\begin{aligned} I_{19}^{X_1} &= \int \frac{d^4 k}{i\pi^2} \frac{1}{(0)(q)((p_2 + k)^2 - m^2)} \frac{16}{t} \frac{1}{4} \operatorname{Tr}(\hat{p}'_1 B^{X_1} \hat{p}_1 O_{11'}) \times \\ &\quad \times \frac{1}{4} \operatorname{Tr}(\hat{p}_2 \gamma_\sigma (-\hat{p}_2 - \hat{k}) \gamma_\lambda \hat{p}'_2 \gamma_\rho), \\ I_{29}^{X_1} &= \int \frac{d^4 k}{i\pi^2} \frac{1}{(0)(q)((p_2 + k)^2 - m^2)} \frac{16}{t_1} \frac{1}{4} \operatorname{Tr}(\hat{p}'_1 B^{X_1} \hat{p}_1 \gamma_\rho) \times \\ &\quad \times \frac{1}{4} \operatorname{Tr}(\hat{p}_2 \gamma_\sigma (-\hat{p}_2 - \hat{k}) \gamma_\lambda \hat{p}'_2 O_{22'}), \\ I &= \int \frac{d^4 k}{i\pi^2} \frac{1}{(0)(q)} \left\{ \frac{4}{s_1} \frac{1}{4} \operatorname{Tr}(\hat{p}'_2 \gamma_\rho \hat{p}'_1 B^{X_1} \hat{p}_1 O^{12} \hat{p}_2 (\hat{A} + \hat{B})) + \right. \\ &\quad \left. + \frac{4}{s} \frac{1}{4} \operatorname{Tr}(\hat{p}'_2 O_{1'2'} \hat{p}_1 B^{X_1} \hat{p}_1 \gamma_\rho \hat{p}_2 (\hat{A} + \hat{B})) \right\}, \\ \hat{A} &= \frac{\gamma_\sigma (-\hat{p}_2 - \hat{k}) \gamma_\lambda}{(p_2 + k)^2 - m^2}, \quad \hat{B} = \frac{\gamma_\lambda (-\hat{p}'_2 + \hat{k}) \gamma_\sigma}{(-p'_2 + k)^2 - m^2}. \end{aligned} \tag{20}$$

Here

$$\begin{aligned} B^{X_1} &= \frac{\gamma_\lambda (\hat{p}_1 - \hat{k}_1 - \hat{k}) \gamma_\sigma (\hat{p}_1 - \hat{k}_1) \gamma_\mu}{-\chi_1(d)} + \frac{\gamma_\lambda (\hat{p}_1 - \hat{k}_1 - \hat{k}) \gamma_\mu (\hat{p}_1 - \hat{k}) \gamma_\sigma}{(d)(1)} + \\ &\quad + \frac{\gamma_\mu (\hat{p}'_1 + \hat{k}_1) \gamma_\lambda (\hat{p}_1 - \hat{k}) \gamma_\sigma}{\chi'_1(1)}, \end{aligned} \tag{21}$$

$$(q) = (p_2 - p'_2 + k)^2 - \lambda^2, \quad (d) = (p_1 - \bar{k}_1 - k)^2 - m^2, \quad (1) = (p_1 - k)^2 - m^2, \quad (0) = k^2 - \lambda^2.$$

Analytic evaluations divulge a lack of both double logarithmic ($\propto L_s^2$) and infrared logarithmic ($\propto \ln(\lambda/m)L$) terms in the box contribution. In spite of the explicit proportionality of the

individual contributions to the structures E_0 , O_0 , and I_0 , the overall expression turns out to be somewhat convoluted, despite it has a factorized form in each gauge-invariant subset of diagrams. We parametrize the correction coming from the B-type Feynman diagrams as follows:

$$d\sigma_B = d\sigma_0 \frac{\alpha}{\pi} L_s \Delta_B, \quad \Delta_B = 2 \ln \frac{ss_1}{uu_1} + \frac{2}{F} (\Phi_Q + \Phi_R) \ln \frac{tt_1}{ss_1}. \tag{22}$$

The total virtual correction to the cross section has the form

$$d\sigma^{virt} = d\sigma_G + d\sigma_{\Gamma\Pi} + d\sigma_B = d\sigma_0 \frac{\alpha}{\pi} \left[-L_s^2 + L_s \left(\frac{11}{3} + 4 \ln \frac{\lambda}{m} + \Delta_G + \Delta_{\Gamma\Pi} + \Delta_B \right) + \mathcal{O}(1) \right], \tag{23}$$

$$\Delta_G + \Delta_{\Gamma\Pi} = \frac{1}{F} \left(\Phi \ln \frac{s^2}{tt_1} + \Phi_R \ln \frac{t^2}{ss_1} + \Phi_Q \ln \frac{t_1^2}{ss_1} + \Phi_P \ln \frac{s_1^2}{tt_1} \right),$$

where $\Phi_P = \hat{P}\Phi$, $\Phi_Q = \hat{Q}\Phi$, and $\Phi_R = \hat{R}\Phi$.

5. CONTRIBUTION FROM ADDITIONAL SOFT PHOTON EMISSION

Consider now radiative Bhabha scattering accompanied by emission of an additional soft photon in the center of mass reference frame. By soft we mean that its energy does not exceed some small quantity $\Delta\varepsilon$, compared to the energy ε of the initial beams. The corresponding cross section has the form

$$d\sigma^{soft} = d\sigma_0 \delta^{soft}, \tag{24}$$

$$\delta^{soft} = -\frac{4\pi\alpha}{16\pi^3} \int \frac{d^3k_2}{\omega_2} \left(-\frac{p_1}{p_1 k_2} + \frac{p'_1}{p'_1 k_2} + \frac{p_2}{p_2 k_2} - \frac{p'_2}{p'_2 k_2} \right)^2 \Big|_{\omega_2 < \Delta\varepsilon}$$

The soft photon energy does not exceed $\Delta\varepsilon \ll \varepsilon_1 = \varepsilon_2 \equiv \varepsilon \sim \varepsilon'_1 \sim \varepsilon'_2$. In order to calculate the right-hand side of Eq. (24), we use the master equation [9]:

$$-\frac{4\pi\alpha}{16\pi^3} \int \frac{d^3k}{\omega} \frac{(q_i)^2}{(q_i k)^2} \Big|_{\omega < \Delta\varepsilon} = -\frac{\alpha}{\pi} \ln \left(\frac{m\Delta\varepsilon}{\lambda\varepsilon_i} \right), \quad \omega = \sqrt{k^2 + \lambda^2}, \tag{25}$$

$$\frac{4\pi\alpha}{16\pi^3} \int \frac{d^3k}{\omega} \frac{2q_1 q_2}{(kq_1)(kq_2)} \Big|_{\omega < \Delta\varepsilon} = \frac{\alpha}{\pi} \left[L_q \ln \left(\frac{m^2(\Delta\varepsilon)^2}{\lambda^2\varepsilon_1\varepsilon_2} \right) + \frac{1}{2} L_q^2 - \frac{1}{2} \ln^2 \left(\frac{\varepsilon_1}{\varepsilon_2} \right) - \frac{\pi^2}{3} + \text{Li}_2 \left(\cos^2 \frac{\theta}{2} \right) \right]. \tag{26}$$

Here we used the notation

$$L_q = \ln \frac{-q^2}{m^2}, \quad q_1^2 = q_2^2 = m^2, \quad -q^2 = -(q_1 - q_2)^2 \gg m^2, \tag{27}$$

$$q_{1,2} = (\varepsilon_{1,2}, \mathbf{q}_{1,2}), \quad \theta = \widehat{\mathbf{q}_1 \mathbf{q}_2},$$

where ε_1 , ε_2 , and θ are the energies and angle between the 3-momenta \mathbf{q}_1 , \mathbf{q}_2 , respectively, and λ is the fictitious photon mass (all defined in the center of mass system).

The contributions of each possible term on the right-hand side of Eq. (24) are

$$\frac{\pi}{\alpha} \delta^{soft} = -\Delta_1 - \Delta_2 - \Delta'_1 - \Delta'_2 + \Delta_{12} + \Delta_{1'2'} + \Delta_{11'} + \Delta_{22'} - \Delta_{12'} - \Delta_{1'2}, \quad (28)$$

$$\Delta_1 = \Delta_2 = \ln \frac{m\Delta\varepsilon}{\varepsilon\lambda}, \quad \Delta'_1 = \ln \frac{m\Delta\varepsilon}{\varepsilon'_1\lambda}, \quad \Delta'_2 = \ln \frac{m\Delta\varepsilon}{\varepsilon'_2\lambda},$$

$$\Delta_{12} = 2L_s \ln \frac{m\Delta\varepsilon}{\varepsilon\lambda} + \frac{1}{2}L_s^2 - \frac{\pi^2}{3},$$

$$\Delta_{1'2'} = L_{s_1} \ln \left(\frac{(m\Delta\varepsilon)^2}{\varepsilon'_1\varepsilon'_2\lambda^2} \right) + \frac{1}{2}L_{s_1}^2 - \frac{1}{2} \ln^2 \left(\frac{\varepsilon'_1}{\varepsilon'_2} \right) - \frac{\pi^2}{3} + \text{Li}_2 \left(\cos^2 \frac{\theta_{1'2'}}{2} \right),$$

$$\Delta_{11'} = L_{t_1} \ln \left(\frac{(m\Delta\varepsilon)^2}{\varepsilon'_1\varepsilon\lambda^2} \right) + \frac{1}{2}L_{t_1}^2 - \frac{1}{2} \ln^2 \left(\frac{\varepsilon'_1}{\varepsilon} \right) - \frac{\pi^2}{3} + \text{Li}_2 \left(\cos^2 \frac{\theta_{1'}}{2} \right),$$

$$\Delta_{22'} = L_t \ln \left(\frac{(m\Delta\varepsilon)^2}{\varepsilon\varepsilon'_2\lambda^2} \right) + \frac{1}{2}L_t^2 - \frac{1}{2} \ln^2 \left(\frac{\varepsilon'_2}{\varepsilon} \right) - \frac{\pi^2}{3} + \text{Li}_2 \left(\sin^2 \frac{\theta_{2'}}{2} \right),$$

$$\Delta_{12'} = L_{u_1} \ln \left(\frac{(m\Delta\varepsilon)^2}{\varepsilon\varepsilon'_1\lambda^2} \right) + \frac{1}{2}L_{u_1}^2 - \frac{1}{2} \ln^2 \left(\frac{\varepsilon'_1}{\varepsilon} \right) - \frac{\pi^2}{3} + \text{Li}_2 \left(\sin^2 \frac{\theta_{1'}}{2} \right),$$

$$\Delta_{1'2} = L_u \ln \left(\frac{(m\Delta\varepsilon)^2}{\varepsilon\varepsilon'_2\lambda^2} \right) + \frac{1}{2}L_u^2 - \frac{1}{2} \ln^2 \left(\frac{\varepsilon}{\varepsilon'_2} \right) - \frac{\pi^2}{3} + \text{Li}_2 \left(\cos^2 \frac{\theta_{2'}}{2} \right),$$

$$L_u = \ln \frac{-u}{m^2}, \quad L_{u_1} = \ln \frac{-u_1}{m^2}, \quad \text{Li}_2(z) \equiv - \int_0^z \frac{dx}{x} \ln(1-x),$$

where $\varepsilon'_1, \varepsilon'_2$ are the center of mass energies of the scattered electron and positron, respectively; $\theta_{1'}, \theta_{2'}$ are their scattering angles (measured from the initial electron momentum direction); and $\theta_{1'2'}$ is the angle between the scattered electron and positron momenta.

Separating out large logarithms, we obtain

$$\delta^{soft} = \frac{\alpha}{\pi} \left\{ 4(L_s - 1) \ln \frac{m\Delta\varepsilon}{\lambda\varepsilon} + L_s^2 + L_s \ln \frac{tt_1}{uu_1} + L_s \ln \frac{1 - c_{1'2'}}{2} + \theta(1) \right\}, \quad (29)$$

$$c_{1'2'} = \cos \theta_{1'2'}.$$

This can be written in another form, using experimentally measurable quantities, the relative energies of the scattered leptons and the scattering angles (see Table):

$$y_i = \frac{\varepsilon'_i}{\varepsilon}, \quad c_i = \cos \theta'_i, \quad \frac{1}{2}(1 - c_{1'2'}) = \frac{y_1 + y_2 - 1}{y_1 y_2},$$

$$-\frac{t}{s} = y_2 \frac{1 + c_2}{2}, \quad -\frac{u}{s} = y_2 \frac{1 - c_2}{2}, \quad -\frac{t_1}{s} = y_1 \frac{1 - c_1}{2},$$

$$\frac{s_1}{s} = y_1 + y_2 - 1, \quad -\frac{u_1}{s} = y_1 \frac{1 + c_1}{2}.$$
(30)

Numerical estimates of Δ_L and Δ versus y_1, y_2, c_1, c_2

\mathcal{N}	y_1	y_2	c_1	c_2	Δ_L	Δ
1	0.36	0.89	-0.70	-0.10	10.70	-24.53
2	0.59	0.66	0.29	-0.06	4.86	-11.41
3	0.67	0.67	0.50	0.30	5.82	-35.58
4	0.68	0.65	0.60	-0.50	4.10	-10.45

6. CONCLUSIONS

The double logarithmic terms of type L_s^2 and those proportional to $L_s \ln(\lambda/m)$ cancel in the overall sum with the corresponding terms from the soft photon contribution (29). Omitting vacuum polarization, we obtain in the logarithmic approximation

$$\begin{aligned}
 d\sigma^{soft+virt} &= d\sigma_0 \frac{\alpha}{\pi} \left[L_s \left(4 \ln \frac{\Delta\varepsilon}{\varepsilon} + \Delta_L \right) + \Delta(y_1, y_2, c_1, c_2) \right], \tag{31} \\
 \Delta_L &= 3 + \ln \frac{(1-c_1)(1-c_2)}{(1+c_1)(1+c_2)} + \ln \frac{y_1 + y_2 - 1}{y_1 y_2} + \\
 &+ \frac{1}{F} \left[\Phi \ln \frac{s^2}{tt_1} + \Phi_P \ln \frac{s_1^2}{tt_1} + \Phi_Q \ln \frac{t_1^2}{ss_1} + \Phi_R \ln \frac{t^2}{ss_1} \right] + \\
 &+ 2 \ln \frac{ss_1}{uu_1} + \frac{2}{F} (\Phi_Q + \Phi_R) \ln \frac{tt_1}{ss_1}.
 \end{aligned}$$

The function $\Delta(y_1, y_2, c_1, c_2)$ is quite complicated. To compare it with Δ_L , we give their numerical values (omitting vacuum polarization) for a certain set of points from physical regions (32) and $y_1 + y_2 > 1, D > 0$ (see Table). Considering the kinematics typical of large angle inelastic Bhabha scattering, we show the lowest-order contribution previously obtained [10] and the radiative corrections calculated in this work.

After performing loop integration and shifting logarithms ($L_i = L_s + L_{is}$), one can see that the terms containing infrared singularities and double logarithmic terms $\sim L_s^2$, are associated with a factor equal to the corresponding Born contribution. This is true of all types of contributions.

The phase volume

$$d\Gamma = \frac{d^3 p'_1 d^3 p'_2 d^3 k_1}{\varepsilon'_1 \varepsilon'_2 \omega_1} \delta^{(4)}(p_1 + p_2 - p'_1 - p'_2 - k_1)$$

can be transformed in various ways [10]. We introduce the variables (see Eq. (30))

$$y_i = \frac{\varepsilon'_i}{\varepsilon}, \quad c_i = \cos \theta'_i, \quad \theta'_i = \widehat{\mathbf{p}_1, \mathbf{p}'_i}, \quad 0 < y_i < 1, \quad -1 < c_{1,2} < 1, \tag{32}$$

which parametrize the kinematics of the outgoing particles (these do not include a common degree of freedom, a rotation about the beam axis). The phase volume then takes the form

$$\begin{aligned}
 d\Gamma &= \frac{\pi s dy_1 dy_2 dc_1 dc_2}{2\sqrt{D}(y_1, y_2, c_1, c_2)} \Theta(y_1 + y_2 - 1) \Theta(D(y_1, y_2, c_1, c_2)), \\
 D(y_1, y_2, c_1, c_2) &= \rho^2 - c_1^2 - c_2^2 - 2c_1 c_2, \\
 \rho^2 &= 2(1 - c_1 c_2) \frac{(1 - y_1)(1 - y_2)}{y_1 y_2}.
 \end{aligned}
 \tag{33}$$

The allowed region of integration is a triangle in the y_1, y_2 plane and the interior of the ellipse $D > 0$ in the c_1, c_2 plane.

We now discuss the relation of our result to the renormalization group approach. The dependence on $\Delta\varepsilon/\varepsilon$ in (31) disappears when one takes into account hard two-photon emission. The leading contribution arises from the kinematics when the second hard photon is emitted close to the direction of motion of one of the incoming or outgoing particles:

$$\begin{aligned}
 d\sigma^{hard} &= \frac{\alpha}{2\pi} L_s \left[\frac{1+z^2}{1-z} \left(d\sigma_0(zp_1, p_2, p'_1, p'_2) + d\sigma_0(p_1, zp_2, p'_1, p'_2) \right) dz \right. \\
 &\quad \left. + \frac{1+z_1^2}{1-z_1} d\sigma_0 \left(p_1, p_2, \frac{p'_1}{z_1}, p'_2 \right) dz_1 + \frac{1+z_2^2}{1-z_2} d\sigma_0 \left(p_1, p_2, p'_1, \frac{p'_2}{z_2} \right) dz_2 \right], \\
 z &= 1 - x_2, \quad z_i = \frac{y_i}{y_i + x_2}, \quad x_2 = \frac{\omega_2}{\varepsilon}.
 \end{aligned}
 \tag{34}$$

The fractional energy of the additional photon varies within the limits $\Delta\varepsilon/\varepsilon < x_2 = \omega_2/\varepsilon < 1$. This formula agrees with the Drell—Yan form of radiative Bhabha scattering (with switched-off vacuum polarization)

$$d\sigma(p_1, p_2, p'_1, p'_2) = \int dx_1 dx_2 \mathcal{D}(x_1) \mathcal{D}(x_2) d\sigma_0 \left(x_1 p_1, x_2 p_2, \frac{p'_1}{z_1}, \frac{p'_2}{z_2} \right) \mathcal{D}(z_1) \mathcal{D}(z_2) dz_1 dz_2, \tag{35}$$

where the non-singlet structure functions \mathcal{D} are [11]

$$\begin{aligned}
 \mathcal{D}(z) &= \delta(1-z) + \frac{\alpha}{2\pi} L \mathcal{F}^{(1)}(z) + \left(\frac{\alpha}{2\pi} L \right)^2 \frac{1}{2!} \mathcal{F}^{(2)}(z) + \dots, \\
 \mathcal{F}^{(1)}(z) &= \lim_{\Delta \rightarrow 0} \left[\frac{1+z^2}{1-z} \Theta(1-z-\Delta) + \delta(1-z) \left(2 \ln \Delta + \frac{3}{2} \right) \right].
 \end{aligned}
 \tag{36}$$

In our calculations we see explicitly a factorization of the terms containing double logarithmic contributions and infrared single logarithmic ones, which arise from G- and Γ -type Feynman diagrams. To be precise, the corresponding contributions to the cross section have the structure of the Born cross section (6). But the above claim fails to be true for terms containing single logarithms. Hence, the Drell—Yan form (35) is not valid in this case, and the factorization theorem breaks down, because the mass singularities (large logarithms) do not factorize before the Born structure. That is because of plenty of different type amplitudes and kinematic variables, which describe our process. The reason for the violation of a naive usage of factorization in the Drell—Yan form has presumably the same origin with that found in Ref. [12], where the authors claimed that it is necessary to study independently the renormalization group behavior of leading logarithms before different amplitudes of the same process. Note that in the $e\mu \rightarrow e\mu\gamma$ reaction, which can easily be extracted from our results, factorization does take place. We also see from (31) that factorization will take place if all

the logarithmic terms become equal, i.e., $\ln(s_1/m^2) = \ln(s/m^2) = \dots$. The source for the violation of the factorization theorem, we found, might have a relation to some of those found in other problems [13].

Numerical estimates (see Table) for the Φ factory energy range ($\sqrt{s} \simeq 1$ GeV) shows that the contribution of the non leading terms coming from virtual and soft real photon emission might reach 35%. Additional hard photon emission will also contribute to Δ_L and Δ . To get an explicit form of that correction, one has to take into account a definite experimental setup.

Obviously, an analogous phenomenon of the factorization theorem violation takes place in QCD in processes like $q\bar{q} \rightarrow q\bar{q}g$ and $q\bar{q} \rightarrow q\bar{q}\gamma$. A consistent investigation of the latter processes, taking into account the phenomenon found, can give a certain correction to predictions for large angle jet production and direct hard photon emission at proton—antiproton colliders.

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