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RINDLER SOLUTIONS AND THEIR PHYSICAL INTERPRETATION

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We show that the singular behavior of Rindler solutions near horizon testifies to the currents of particles from a region arbitrarily close to the horizon. Besides, the Rindler solutions in right Rindler sector of Minkowski space can be represented as a superposition of only positive- or only negative-frequency plane waves; these states require infinite energy for their creation and possess infinite charge in a finite space interval, containing the horizon. The positive- or negative-frequency representations of Rindler solutions analytically continued to the whole Minkowski space make up a complete set of states in this space, which have, however, the aforementioned singularities. These positive (negative)-frequency states are characterized by positive (negative) total charge, the charge of the same sign in right (left) Rindler sector and by quantum number κ. But in other Lorentz invariant sectors they do not possess positive (negative)-definite charge density and have negative (positive) charge in left (right) Rindler sector. Therefore these states describe both the particle (antiparticle) and pairs, the mean number of which is given by Planck function of κ. These peculiarities make the Rindler set of solutions nonequivalent to the plane wave set and the inference on the existence of thermal currents for a Rindler observer moving in empty Minkowski space is unfounded.

As is known, it is impossible to create a constant uniform gravitational field with acceleration a in a space length of the order of a^{-1}, or the coordinate system in Minkowski space, imitating such a field [1]. So, in the Rindler coordinate system due to hardness requirement the acceleration is inversely proportional to the space coordinate, $a(z) = z^{-1}$, and becomes infinite at a distance a^{-1} from the plane with acceleration a (event horizon). It is clear that such a system is unrealizable by moving bodies up to horizon. At the same time there are statements in literature that the wave equation solutions with the Rindler system symmetry testify to appearance of currents of particles with thermal spectrum for an observer at rest in the Rindler system, i.e., uniformly accelerated in Lorentz system where no particles are present [2, 3]. Moreover, it is stated that this phenomenon imitates black hole evaporation [2–4]. On the other hand, according to the interesting paper by Belinski [5], the already formed black hole should not create particles. The same is expected if the analogy with QED is warranted [6]: both virtual or real pair components are attracted by black hole and no particles should appear far away.

Rindler solution (the solution of Klein–Gordon equation in Rindler wedge), decreasing for $\zeta \to \infty$, is given by the Bessel function of the second kind (McDonald function),

$$\begin{gathered} K_{i\kappa}(\zeta)\exp(-i\kappa v + i\mathbf{p}_\perp \mathbf{x}_\perp), \\ v = at' = \operatorname{arth}\frac{t}{z}, \quad \zeta = m_\perp z' = m_\perp\sqrt{z^2 - t^2}, \end{gathered} \tag{1}$$

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and may be written down [6] as a superposition of positive ($p^0 > 0$) or negative ($p^0 < 0$) frequency plane waves with different rapidity θ:

$$K_{i\kappa}(\zeta)\exp(-i\kappa v) = \frac{1}{2}\exp\left(\mp\frac{\pi\kappa}{2}\right)\int\limits_{-\infty}^{\infty} d\theta \exp\left[i(p^3 z - p^0 t) \mp i\kappa\theta\right], \quad (2)$$
$$p^0 = \pm m_\perp \operatorname{ch}\theta, \quad p^3 = m_\perp \operatorname{sh}\theta.$$

It is an analytic function of variables $x_\pm = t \pm z$ with branch points at $x_\pm = 0$. This solution can be analytically continued to the whole Minkowski space from semiaxes $x_- < 0, x_+ > 0$ to semiaxes $x_- > 0, x_+ < 0$; for $p^0 > 0$ the continuation is performed through the lower and for $p^0 < 0$ through the upper half-planes of complex $x_\pm$. According to the lower or upper ways of continuation we get the positive- or the negative-frequency solutions $\Phi_\kappa^{(+)}$ or $\Phi_\kappa^{(-)}$ coinciding in Rindler sector R and differing in other sectors:

$$\begin{aligned}
R: &\quad \Phi_\kappa^{(\pm)} = K_{i\kappa}(\zeta)\exp(-i\omega t'), \quad \omega = a\kappa, \\
F: &\quad \Phi_\kappa^{(\pm)} = \exp\left(\mp\frac{\pi\kappa}{2}\right) K_{i\kappa}(\pm i\tau)\exp(-i\omega z'), \quad \tau = m_\perp\sqrt{t^2 - z^2}, \\
P: &\quad \Phi_\kappa^{(\pm)} = \exp\left(\mp\frac{\pi\kappa}{2}\right) K_{i\kappa}(\mp i\tau)\exp(-i\omega z'), \quad az' = \operatorname{arth}\frac{z}{t}, \\
L: &\quad \Phi_\kappa^{(\pm)} = \exp(\mp\pi\kappa)K_{i\kappa}(\zeta)\exp(-i\omega t').
\end{aligned} \quad (3)$$

So, the solutions $\Phi_\kappa^{(\pm)}$ are characterized by frequency sign and by real parameter κ (the latter instead of plane wave momentum).

Solutions $\Phi^{(-)}$ may be obtained from $\Phi^{(+)}$ by complex conjugation and changing the sign of κ:

$$\Phi_\kappa^{(-)} = \Phi_{-\kappa}^{(+)*}. \quad (4)$$

If instead of $\Phi^{(\pm)}$ one considers $\phi^{(\pm)} = \exp(\pm\pi\kappa/2)\Phi^{(\pm)}$ then these solutions $\phi^{(\pm)}$ will be «similarly normalized» in whole Minkowski space by values differing only in sign. The solution $\phi^{(+)}$ corresponds to positive charge located mainly in R or L for κ greater or less than 0 and $\phi^{(-)}$ corresponds to negative charge located mainly in L or R for the same κ.

For $\Phi^{(+)}$ we have the following current density components $j_\pm = j^0 \pm j^3$:

$$\begin{aligned}
R: &\quad j_\pm = \mp\frac{2\kappa}{x_\mp}K_{i\kappa}^2(\zeta), \\
F: &\quad j_\pm = \frac{e^{-\pi\kappa}}{x_\mp}[\pi \mp 2\kappa|K_{i\kappa}(i\tau)|^2], \\
P: &\quad j_\pm = \frac{e^{-\pi\kappa}}{x_\mp}[-\pi \mp 2\kappa|K_{i\kappa}(-i\tau)|^2], \\
L: &\quad j_\pm = \mp e^{-2\pi\kappa}\frac{2\kappa}{x_\mp}K_{i\kappa}^2(\zeta).
\end{aligned} \quad (5)$$

In sectors R and L the current flows along hyperbolae with constant ζ and in sectors F and P both along the hyperbolae with constant τ and along the rays outgoing from (in F) and ingoing to (in P) the origin of coordinates:

$$R: \qquad j_\alpha = j_\alpha^{hyp} = -\frac{2\kappa}{z^2 - t^2} K_{i\kappa}^2(\zeta)\epsilon_{\alpha\beta}x^\beta, \tag{6}$$

$$F: \qquad j_\alpha = j_\alpha^{lin} + j_\alpha^{hyp},$$
$$j_\alpha^{lin} = \frac{\pi e^{-\pi\kappa}}{t^2 - z^2}x_\alpha, \quad j_\alpha^{hyp} = \frac{2\kappa e^{-\pi\kappa}}{t^2 - z^2}|K_{i\kappa}(i\tau)|^2\epsilon_{\alpha\beta}x^\beta, \tag{7}$$

$\epsilon_{\alpha\beta}$ is the antisymmetric tensor, $\epsilon_{03} = -\epsilon_{30} = 1$.

The linear and hyperbolic current densities are orthogonal:

$$j_\alpha^{lin} j^{hyp\alpha} = 0.$$

Singular behavior of current density at $x_\pm = 0$ is evident.

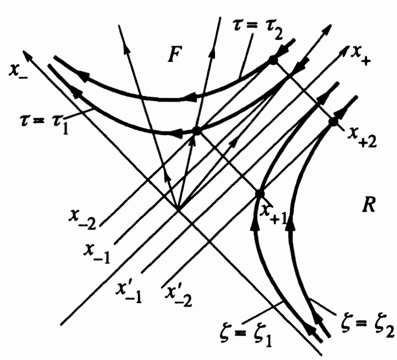
Due to current density conservation the current

$$J = \frac{1}{2}\int\limits_C (j_+ dx_- - j_- dx_+)$$

through a contour C lying in F and having ends on the hyperbola $\tau = \tau_1$ at the points (x_{+1}, x_{-2}) and (x_{+2}, x_{-1}) is equal to the current through a broken line formed by segments of the straight lines $x_+ = x_{+2}$ and $x_- = x_{-2}$ crossing on the hyperbola $\tau = \tau_2 \equiv m_\perp\sqrt{x_{-2}x_{+2}}$ (see Figure):

$$\frac{1}{2}\int\limits_{x_{-1}}^{x_{-2}} \frac{dx_-}{x_-}e^{-\pi\kappa}[\pi - 2\kappa|K_{i\kappa}(i\tau)|^2] +$$
$$+\frac{1}{2}\int\limits_{x_{+1}}^{x_{+2}} \frac{dx_+}{x_+}e^{-\pi\kappa}[\pi + 2\kappa|K_{i\kappa}(i\tau)|^2] = 2\pi e^{-\pi\kappa}\ln\frac{\tau_2}{\tau_1}. \tag{8}$$

It is seen that the hyperbolic parts of the currents cancel each other and the linear parts are equal, and yield the result presented in (8). The former is evident beforehand as the contour



can be deformed to the arc of hyperbola $\tau = \tau_1$ between the chosen points on it, while the latter follows from (7) and the relation

$$\frac{x_{-2}}{x_{-1}} = \frac{x_{+2}}{x_{+1}} = \left(\frac{\tau_2}{\tau_1}\right)^2. \tag{9}$$

If one fixes the coordinates x_{+1}, x_{+2} of the hyperbola arc ends and the parameter τ_1 tends to zero, then the considered arc of hyperbola can be made as close to the segment (x_{+1}, x_{+2}) of x_+-axis as one wishes. Yet the current through this arc of hyperbola remains constant according to (8), (9); the same is true for its components along x_+- and x_--axes. Similarly the current through the contour lying in sector R with ends on hyperbola $\zeta = \zeta_1$ at points (x_{+1}, x'_{-2}) and (x_{+2}, x'_{-1}) is equal to the current through the broken line formed by segments of straight lines $x_- = x'_{-2}$ and $x_+ = x_{+2}$ crossing on the hyperbola $\zeta = \zeta_2 \equiv \sqrt{-x'_{-2}x_{+2}}$:

$$\frac{1}{2}\int\limits_{x_{+1}}^{x_{+2}} \frac{dx_+}{x_+}[-2\kappa K_{i\kappa}^2(\zeta)] + \frac{1}{2}\int\limits_{x'_{-1}}^{x'_{-2}} \frac{dx_-}{x_-}2\kappa K_{i\kappa}^2(\zeta) = 0. \tag{10}$$

It is equal to zero as in R the current flows only along hyperbolae and the choosen contour can be deformed to the arc of hyperbola $\zeta = \zeta_1$. So the arcs of hyperbolae $\tau = \tau_1$ and $\zeta = \zeta_1$, lying in F and R and having the projection (x_{+1}, x_{+2}) on x_+-axis, for $\tau_1, \zeta_1 \to \infty$ press themselves to the projection from both sides but the current flowing through the both arcs remains constant and equal to (8). If the ends of contours in F and R are placed not on hyperbolae, but on straight lines $x_- = x_{-2} > 0$ and $x_- = x'_{-2} < 0$ at the points with coordinates x_{+1}, x_{+2}, then the current through these contours will be given by the second and the first integrals in (8) and (10) as these contours can be deformed to the corresponding segments of straight lines. Choosing $x'_{-2} = -x_{-2}$ (what is equivalent to $\zeta_1 = \tau_1$) it is possible to show that the total current crossing both segments, when they approach each other (i.e., for $\tau_{1,2} \to 0$, but $\tau_2/\tau_1 = \text{const}$), differs from zero and oscillates with increasing frequency:

$$e^{-\pi\kappa}\left\{\pi \ln\frac{\tau_2}{\tau_1} + 2\kappa\int\limits_{\tau_1}^{\tau_2}\frac{d\tau}{\tau}|K_{i\kappa}(i\tau)|^2\right\} - 2\kappa\int\limits_{\tau_1}^{\tau_2}\frac{d\zeta}{\zeta}K_{i\kappa}^2(\zeta) \approx$$
$$\approx \frac{2\pi}{\kappa(e^{\pi\kappa}+1)}\sin\left(\kappa\ln\frac{\tau_2}{\tau_1}\right)\cos\left(\beta + \kappa\ln\frac{\tau_1\tau_2}{4}\right), \tag{11}$$

where $\beta = \arg(\Gamma^2(1-i\kappa))$, $\tau_1\tau_2 = m_\perp^2\sqrt{x_{+1}x_{+2}}x_{-1} \to 0$.

So the current through the closed contour, going through the ends of the segment (x_{+1}, x_{+2}) on the x_+-axis and lying in F and R, due to singular behavior of current density at the x_+-axis depends on the way of calculating the integrals near the singularity. Calculations of principal values of integrals by means of «hyperbolic» or «linear» approaches give respectively finite (8) or indefinite (11) values.

Note, that the charge and the energy of the state (2) in any finite volume containing horizon are infinite due to singularities of charge density j^0 and energy density T^{00} on the horizon.

• Let us consider the scalar or «inner» product of two positive-frequency solutions (3) in Rindler sector R:

$$J^R_{\kappa\kappa'} = \int\limits_{|t|}^{\infty} dz K_{-i\kappa}(\zeta)e^{i\omega t'} i \overleftrightarrow{\partial_t} K_{i\kappa'}(\zeta)e^{-i\omega' t'}. \tag{12}$$

Replacing the integration variable z by the variable ζ, it is not difficult to see that the integrand is total differential of function

$$-J^R_{\kappa\kappa'}(\zeta) = \frac{\zeta}{\kappa-\kappa'} \exp\left[i(\kappa-\kappa')\,\mathrm{arsh}\,\frac{m_\perp t}{\zeta}\right]\left[K_{-i\kappa}(\zeta)K'_{i\kappa'}(\zeta) - K'_{-i\kappa}(\zeta)K_{i\kappa'}(\zeta)\right], \tag{13}$$

which exponentially decays at the upper limit (for $\zeta \to \infty$) and oscillates with infinitely increasing frequency at the lower limit (for $\zeta \to 0$). So, the integral $J^R_{\kappa\kappa'}$ which is equal to the limit of the function $J^R_{\kappa\kappa'}(\zeta)$ for $\zeta \to 0$ does not exist in ordinary sense. Let us consider the function $J^R_{\kappa\kappa'}(\zeta)$ instead of it, i.e., the integral (12) in which the low limit $z = |t|$ is substituted by $z_1(t) = \sqrt{t^2 + (\zeta/m_\perp)^2}$, where ζ is the parameter of hyperbola which intersects the straight line of constant t at the point $z_1(t)$. For this function there is also another representation:

$$J^R_{\kappa\kappa'}(\zeta) = \exp\left[i(\kappa-\kappa')\,\mathrm{arsh}\,\frac{m_\perp t}{\zeta}\right](\kappa+\kappa')\int\limits_{\zeta}^{\infty}\frac{dx}{x}K_{-i\kappa}(x)K_{i\kappa'}(x). \tag{14}$$

For the specisl case $\kappa = \kappa'$ the function $J^R_{\kappa\kappa}(\zeta)$ is real, positive and represents the charge of the state (1), pertaining to the region $z_1(t) \le z < \infty$ and related to unit area of plane z_1. For $\zeta \to \infty$ this charge exponentially decreases,

$$J^R_{\kappa\kappa}(\zeta) \approx \frac{\pi\kappa}{2\zeta^2}e^{-2\zeta}, \quad \zeta \to \infty, \tag{15}$$

and for $\zeta \to 0$ it increases logarithmically:

$$J^R_{\kappa\kappa}(\zeta) = \frac{\pi}{\mathrm{sh}\,\pi\kappa}\left[\ln\frac{2}{\zeta} + \mathrm{Re}\,(\psi(1-i\kappa)) + \frac{1}{2\kappa}\sin\left(\beta + 2\kappa\ln\frac{\zeta}{2}\right) + \cdots\right], \quad \zeta \to 0. \tag{16}$$

Here $\psi(x)$ is logarithmic derivative of Γ-function, and $\beta = 2\arg(\Gamma(1-i\kappa))$.

So, the charge in any finite volume becomes arbitrarily large when this volume approaches the horizon $z = |t|$.

Using equations (13), (14) and asymptotic expressions for the McDonald functions, it is easy to show that for $\kappa \neq \kappa'$ and $\zeta \to 0$

$$\int\limits_{\zeta}^{\infty}\frac{dx}{x}K_{-i\kappa}(x)K_{i\kappa'}(x) \approx \frac{\pi}{2\kappa\kappa'}\sqrt{\frac{\kappa\kappa'}{\mathrm{sh}(\pi\kappa)\,\mathrm{sh}(\pi\kappa')}}\times$$
$$\times\left\{\frac{1}{\kappa-\kappa'}\sin\left[(\kappa-\kappa')\ln\frac{2}{\zeta} - \frac{\beta-\beta'}{2}\right] - \frac{1}{\kappa+\kappa'}\sin\left[(\kappa+\kappa')\ln\frac{2}{\zeta} - \frac{\beta+\beta'}{2}\right]\right\}, \tag{17}$$

where the omitted terms go to zero for $\zeta \to 0$.

Using the representation

$$\lim_{N\to\infty}\frac{\sin(Nx)}{x} = \pi\delta(x), \tag{18}$$

let us rewrite the right-hand side of (17) in the form

$$\frac{\pi^2}{2\kappa\kappa'}\sqrt{\frac{\kappa\kappa'}{\operatorname{sh}(\pi\kappa)\operatorname{sh}(\pi\kappa')}}\left\{\delta(\kappa-\kappa')-\delta(\kappa+\kappa')\right\}. \tag{19}$$

Then

$$\lim_{\zeta\to 0} J^R_{\kappa\kappa'}(\zeta)=\frac{\pi^2}{\operatorname{sh}(\pi\kappa)}\delta(\kappa-\kappa'). \tag{20}$$

The scalar product of the same two solutions (3) in sector F

$$J^F_{\kappa\kappa'}=e^{-\pi(\kappa+\kappa')/2}\int\limits_{-t}^{t} dz K_{-i\kappa}(-i\tau)e^{i\omega z'} i \overleftrightarrow{\partial}_t K_{i\kappa'}(i\tau)e^{-i\omega' z'} \tag{21}$$

is considered as a limit for $\tau\to 0$ of function $J^F_{\kappa\kappa'}(\tau)$, which differs from the integral (21) in that the limits of integration $z=\mp t$ are substituted by limits $z_{1,2}(t)=\mp\sqrt{t^2-(\tau/m_\perp)^2}$, where τ is the parameter of limiting hyperbola which intersects the straight line of constant t at the points $z_{1,2}$. It can be shown that

$$\begin{aligned} J^F_{\kappa\kappa'}(\tau)=&-\frac{2\exp\left[-(\pi/2)(\kappa+\kappa')\right]}{\kappa-\kappa'}\sin\left[(\kappa-\kappa')\operatorname{arch}\frac{m_\perp t}{\tau}\right]\times\\ &\times\tau\left[K_{-i\kappa}(-i\tau)K'_{i\kappa'}(i\tau)+K'_{-i\kappa}(-i\tau)K_{i\kappa'}(i\tau)\right]= \end{aligned}\tag{22}$$

$$\begin{aligned} &=\frac{2\exp\left[-(\pi/2)(\kappa+\kappa')\right]}{\kappa-\kappa'}\sin\left[(\kappa-\kappa')\operatorname{arch}\frac{m_\perp t}{\tau}\right]\times\\ &\times\left(\pi-i(\kappa^2-\kappa'^2)\int\limits_{\tau}^{\infty}\frac{dx}{x}K_{-i\kappa}(-ix)K_{i\kappa'}(ix)\right). \end{aligned}\tag{23}$$

For $\kappa=\kappa'$ the function

$$J^F_{\kappa\kappa}(\tau)=2\pi e^{-\pi\kappa}\operatorname{arch}\frac{m_\perp t}{\tau}. \tag{24}$$

It gives the charge in region F between planes $z=z_1$ and $z=z_2$ per unit area of one of the planes. As seen from (24) this charge is positive, increases logarithmically in time and also as the limits of integration approach the horizons.

For $\kappa\neq\kappa'$ and $\tau\to 0$ it follows from (22) and (18)

$$\lim_{\tau\to 0} J^F_{\kappa\kappa'}(\tau)=2\pi^2 e^{-\pi\kappa}\delta(\kappa-\kappa'). \tag{25}$$

Equations(22)–(25) also hold for sector P, if one subjects them to complex conjugation and notes that $\operatorname{arch} x$ is an even function [7].

Finally, scalar products of two positive-frequency solutions (3) in sectors L and R are related by the expression

$$J^L_{\kappa\kappa'}(\zeta)=-\exp\left(-\pi(\kappa+\kappa')\right)J^{R*}_{\kappa\kappa'}(\zeta). \tag{26}$$

Hence in the region $-\infty < z \leq -z_1(t)$ of sector L the charge is negative, makes up the $\exp(-2\pi\kappa)$ part of the charge in symmetrical region of sector R and increases logarithmically for $\zeta \to 0$, i.e., as the limiting plane approaches the horizon:

$$J^L_{\kappa\kappa}(\zeta) \approx -\frac{\pi \exp(-2\pi\kappa)}{\mathrm{sh}(\pi\kappa)} \left\{ \ln\frac{2}{\zeta} + \mathrm{Re}(\psi(1-i\kappa)) + \frac{1}{2\kappa}\sin\left(\beta + 2\kappa\ln\frac{\zeta}{2}\right)\right\}. \quad (27)$$

At the same time for $\kappa \neq \kappa'$ and $\zeta \to 0$

$$\lim_{\zeta\to 0} J^L_{\kappa\kappa'}(\zeta) = -\frac{\pi^2\exp(-2\pi\kappa)}{\mathrm{sh}\,\pi\kappa}\delta(\kappa-\kappa'). \quad (28)$$

Summing (20), (25), and (28) yields

$$\lim_{\zeta,\tau\to 0}\left[J^R_{\kappa\kappa'}(\zeta) + J^F_{\kappa\kappa'}(\tau) + J^L_{\kappa\kappa'}(\zeta)\right] = 4\pi^2 e^{-\pi\kappa}\delta(\kappa-\kappa'), \quad (29)$$

which may be considered as the orthogonality and normalization condition of two positive-frequency solutions in the whole space.

For negative-frequency solutions $\Phi^{(-)}$ the integrals $J_{\kappa\kappa'}$ may be obtained from (12)–(16), and (20)–(29) by complex conjugation, changing the signs of κ and κ' and changing the overall sign, and for «similarly normalized» solutions $\phi^{(\pm)}$ they differ from those obtained for $\Phi^{(\pm)}$ by multipliers $\exp[\pm(\pi/2)(\kappa+\kappa')]$.

All given formulae hold both for positive and negative κ. Changing sign of κ is equivalent to reflection $z \to -z$:

$$\phi^{(\pm)}_{-\kappa}(z) = \phi^{(\pm)}_{\kappa}(-z). \quad (30)$$

In other words, the sign of κ is not connected with the sign of total charge, but is connected with its spatial distribution.

Note the important relation

$$\phi^{(-)}_{\kappa} = \phi^{(+)*}_{-\kappa}, \quad (31)$$

which is satisfied also by functions $\Phi^{(\pm)}_{\kappa}$.

Two solutions $\phi^{(+)}_{\kappa}$ and $\phi^{(-)}_{\kappa'}$ with different frequency signs are orthogonal in Minkowski space, moreover, they are orthogonal in $R+L$, F and P separately.

The energy density of the state $\Phi^{(+)}$ in sector R is

$$T_{00} = \frac{m_\perp^2(t^2+z^2)}{z^2-t^2}|K'|^2 + \frac{\kappa^2(z^2+t^2)}{(z^2-t^2)^2}|K|^2 + m_\perp^2|K|^2, \quad (32)$$

where $K = K_{i\kappa}(\zeta)$, $\zeta = m_\perp\sqrt{z^2-t^2}$, and in sector F it is

$$T_{00} = e^{-\pi\kappa}\left\{\frac{m_\perp^2(t^2+z^2)}{t^2-z^2}|K'|^2 + \frac{\kappa^2(z^2+t^2)}{(t^2-z^2)^2}|K|^2 + m_\perp^2|K|^2 - \frac{2\pi\kappa z t}{(t^2-z^2)^2}\right\}, \quad (33)$$

where $K = K_{i\kappa}(i\tau)$, $\tau = m_\perp\sqrt{t^2-z^2}$. Comparision with charge density j^0 shows that T_{00} diverges for $z \to \pm t$ stronger then j^0 both in R and in F.

The infinite charge and infinite energy in finite volume containing the horizon testify to nonequivalence of the set of Rindler solutions to usual complete sets of wave equation solutions [8]. That the horizons act as sources of particle and antiparticle pairs is confirmed also by changing of charge density sign at crossing horizon. Finally, the positive-frequency solution $\phi^{(+)}$ possesses in sectors L and R the charges $Q_L^{(+)}$ and $Q_R^{(+)}$ of opposite signs and the total positive charge $Q^{(+)}$. These charges are connected by the relation

$$Q_L^{(+)} = -e^{-2\pi\kappa}Q_R^{(+)} = -\frac{1}{2}(e^{2\pi\kappa}-1)^{-1}Q^{(+)}, \quad Q^{(+)} > 0. \tag{34}$$

Similar connection between the charges in sectors L, R and total charge for negative-frequency state $\phi^{(-)}$ is

$$Q_R^{(-)} = -e^{-2\pi\kappa}Q_L^{(-)} = -\frac{1}{2}(e^{2\pi\kappa}-1)^{-1}Q^{(-)}, \quad Q^{(-)} < 0. \tag{35}$$

Any ratio of the charges of opposite signs may be taken in Minkowski space as a measure of intensity of pair production in this space.

The general feature of the states $\phi^{(\pm)}$ and their arbitrary superposition (36) is that their total charge is equally divided between the sectors $R+L$ and F (or P).

The fact that the state $\phi_\kappa^{(+)}(\phi_\kappa^{(-)})$, being a superposition of positive (negative)-frequency plane waves, not everywhere posseses the positive (negative) charge density, is another manifestation of nonequivalence of $\phi_\kappa^{(\pm)}$-system to plane wave system.

As the functions $\phi^{(\pm)}$ differ in each of the sectors R and L only by constant factor, it is always possible to find such superpositions:

$$\phi_\kappa^R = \alpha\phi_\kappa^{(+)} + \beta\phi_\kappa^{(-)}, \quad \phi_\kappa^L = \beta\phi_\kappa^{(+)} + \alpha\phi_\kappa^{(-)}, \tag{36}$$

which a) are identically zero in L- and R-sectors correspondingly and b) have only positive and only negative charges and their densities in other sectors. For this it is necessary, that

$$a)\ \beta = -\alpha e^{-\pi\kappa}, \quad b)\ \beta^2 = \alpha^2 - 1 = \frac{1}{e^{2\pi\kappa}-1}, \quad \kappa > 0. \tag{37}$$

Therefore it is possible to consider the states ϕ_κ^R and ϕ_κ^L, $\kappa > 0$, as describing the positively and negatively charged particles, i.e., the particle and antiparticle.

Then in representations

$$\phi_\kappa^{(+)} = \alpha\phi_\kappa^R - \beta\phi_\kappa^L, \quad \phi_\kappa^{(-)} = -\beta\phi_\kappa^R + \alpha\phi_\kappa^L, \quad \kappa > 0, \tag{38}$$

inverse to (36), the squares of the coefficients α and β must be interpreted as mean numbers of particles and antiparticles in the state $\phi_\kappa^{(+)}$ with total charge $+1$, and as mean numbers of antiparticles and particles in the state $\phi_\kappa^{(-)}$ with total charge -1. Relation (34) shows that in sector L there is only half (i.e., $\beta^2/2$) of all antiparticles of the state $\phi_\kappa^{(+)}$. The other half is in sector F or P. Similarly, half of all particles of the state $\phi_\kappa^{(-)}$ are in sector R while the other half is in sector F or P.

It is interesting to note that the positivity of charge density j^0 for the state ϕ_κ^R in F and P sectors ($|z| < |t|$),

$$j^0 = \frac{\pi}{t^2 - z^2}\left[|t| - z\frac{\pi\kappa}{\text{sh}(\pi\kappa)}|J_{i\kappa}(\tau)|^2\right], \tag{39}$$

mathematically is the consequence of inequality

$$\frac{\pi\kappa}{\mathrm{sh}(\pi\kappa)}|J_{i\kappa}(\tau)|^2 \le 1, \qquad \kappa,\tau \ge 0, \tag{40}$$

for Bessel function $J_{i\kappa}(\tau)$, which was not found in mathematical literature.

The charge density for the state $\phi_\kappa^{(+)}$ in the same sectors

$$j^0 = \frac{\pi}{t^2 - z^2}\left[|t| - z\frac{2\kappa}{\pi}|K_{i\kappa}(i\tau)|^2\right] \tag{41}$$

has both signs.

According to unconvincing arguments in Ref. [3], the vanishing of ϕ^R in L means that for Rindler observer (uniformly accelerated in sector R of Minkowski space, i.e., at rest in Rindler system) the β^2 turns out to be the measure of pair production intensity, and looks like a thermal Bose spectrum with temperature $a/2\pi$ and frequency $\omega = a\kappa$.

In our opinion, if the exotic state $\phi_\kappa^{(+)}$ is created, then two observers, one at rest in Minkowski system and the other in Rindler's one, can measure and receive information about the charges $Q_{\kappa R}^{(+)}$ and $Q_{\kappa P}^{(+)}$ in sectors R and P. The difference of these charges divided by $Q_{\kappa P}^{(+)}$ for each of the observers is given by the expression

$$\frac{Q_{\kappa R}^{(+)} - Q_{\kappa P}^{(+)}}{Q_{\kappa P}^{(+)}} = \frac{1}{e^{2\pi\kappa} - 1}. \tag{42}$$

The same formula holds also for the charges in R and P sectors of the complex conjugated state $\phi_\kappa^{(+)*} = \phi_{-\kappa}^{(-)}$, as they differ from considered ones only by the sign. Therefore both observers deal with the same field state in Minkowski space, creation of which needs sources of unlimited intensity.

According to Ref. [9], for the quantization of free field in Rindler space it is necessary to satisfy the boundary condition on the Rindler manifold boundary $z' = 0$ with arbitrary t', which corresponds to the point $z = t = 0$ in Minkowski space. So, these authors conclude that the quantization of free fields is quite different in Rindler and Minkowski spaces and their analysis can give no ground for any conclusions about the behavior of uniformly accelerated detector.

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