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THEORY OF THE PSEUDOGAP FORMATION IN 2D ATTRACTING FERMION SYSTEMS

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The two-dimensional fermion system with the indirect Einstein phonon-exchange attraction and additional local four-fermion interaction is considered. It is shown that as a result of the attraction between fermions, the normal phase of such a system is divided into two regions. In one of them, called the pseudogap region, the absolute value of the order parameter exists as essentially nonzero value, but its phase is a random quantity. It is important that in the case of attraction due to the phonons, this abnormal region appears at rather low carrier concentrations, i.e., it decreases appreciably with increasing doping. The relevance of the results obtained for high-temperature superconductors is speculated.

1. INTRODUCTION

The problem of an adequate description of the physical properties of high-temperature superconductors (HTSCs) still remains one of the important problems of the modern solid-state physics. It is connected with some peculiar properties of HTSCs. Among them there are such problems as quasi-$2D$ character of electronic (and magnetic) properties, a relatively low and changeable carrier density n_f, and its influence on the properties of HTSCs (see, for example, the review article [1]).

At present, one of the widely discussed topics on HTSCs is the «pseudogap» (or «spin gap» if magnetic subsystem of HTSCs is taken into account) [2–4], which is experimentally observed, for example, as a loss in the spectral weight of quasiparticle (or spin) excitations in the normal-state samples with lowered carrier density [5–7]. Corresponding samples reveal some specific spectral, magnetic, and thermodynamic pecularities which are not yet sufficiently understood. In addition, the striking difference between the low (underdoped) and high (overdoped) density regions in HTSCs is hotly debated and is considered as one of the very central and key questions in the physics of cuprates [8, 9].

The possibility of experimentally changing the carrier concentration in HTSCs created a general theoretical problem of the description of the crossover from composite boson superfluidity (low n_f) to Cooper pairing (large n_f) when n_f increases (in other words, a description of the continuous transition from the so-called underdoped regime to the overdoped

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one). Such a crossover has already been studied in $3D$ and quasi-$2D$ systems (see the review articles [10, 11]). The $2D$ case has been considered [10, 12] at temperature $T = 0$ in connection with the Hohenberg-Mermin-Wagner theorem, which forbids any homogeneous (i.e., long-range) order in pure $2D$ systems at $T \neq 0$ due to the long-wave fluctuations of the charged order parameter (OP).

The problem of the inhomogeneous condensate (the Berezinskii-Kosterlitz-Thouless (BKT) phase) formation was also considered, despite some difficulties in the 2 + 1 relativistic field models [13], where the fermion concentration effects are irrelevant. At the same time, these effects were studied in the nonrelativistic model in Ref. [14], for example, without allowance for the existence of the neutral order parameter ρ. Its consideration proves to be very important (see Ref. [15]) and, in fact, results in the formation of an equilibrium region with $\rho \neq 0$, which is mainly located in the phase diagram of a system between the ordinary normal phase and the superconducting (here BKT) phase. Because of the fluctuations of the OP phase, this new region of the system, which is a part of tha normal phase, is of course a nonsuperconducting phase.

In this paper an attempt is made to study the crossover and the possibility for the appearance of the above-mentioned new region in the $2D$ fermion system with a more realistic indirect (phonon) and also a direct (local) four-fermion ($4F$) interactions. Thus, this study is to a certain extent a specific and nontrivial generalization of the preliminary short communication [15], where this abnormal region was studied for $4F$ case only, and of the paper [16], in which the Fröhlich model was used to study the crossover at $T = 0$. As will be seen in the boson exchange model (in contrast with the pure $4F$ case), the new region exists when n_f is rather small, which allows one to compare this result qualitatively with the underdoped HTSC compounds. It is actually more interesting to take into account a more realistic situation with an indirect attraction and some kind of local repulsion, which may in principle correspond to the short-range (screened) Coulomb interaction between carriers. In general, however, we assume that $4F$ interaction can be repulsive as well as attractive. Besides, the case of total repulsion allows one to explore the fermion-antifermion (electron-hole) pairing channel, which, despite physical difference, can be formally described in the same manner.

2. MODEL AND BASIC EQUATIONS

Let us choose the simplest Hamiltonian density in the form

$$H(x) = -\psi_\sigma^\dagger(x)\left(\frac{\nabla^2}{2m} + \mu\right)\psi_\sigma(x) H_{ph}(\varphi(x))$$
$$+ g_{ph}\psi_\sigma^\dagger(x)\psi_\sigma(x)\varphi(x) - g_{4F}\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x)\psi_\downarrow(x)\psi_\uparrow(x), \quad (x = \mathbf{r}, t), \tag{1}$$

where $\psi_\sigma(x)$ is the fermionic field with an effective mass m and spin $\sigma = \uparrow, \downarrow$, μ is the chemical potential of the fermions which fixes n_f, $\varphi(x)$ is a phonon field operator, and g_{ph} and g_{4F} are the electron-phonon and the $4F$ interaction coupling constants, respectively. As was indicated above, g_{4F} can be positive (fermion-fermion attraction) or negative (fermion-antifermion attraction); in Eq. (1) we set $\hbar = k_B = 1$.

In Eq. (1) H_{ph} is the Hamiltonian of free phonons, which can be described by the propagator

$$D(i\Omega_n) = -\frac{\omega_0^2}{\Omega_n^2 + \omega_0^2}, \tag{2}$$

where $\Omega_n = 2n\pi T$ (n is an integer) is the Matsubara frequency [17]. As follows from (2), the propagator $D(i\Omega_n)$ was chosen in a very simple form; here ω_0 is the Einstein (dispersionless) phonon frequency. This choice was made for several reasons. First, this propagator makes it possible to integrate the equations which we obtained. Second, the optic phonon and quadrupolar exciton modes with their relatively weak dispersion are widely considered as exchange bosons which can contribute to the hole-hole attraction in HTSCs [1, 18, 19]. Third, the qualitative results concerning retardation effects do not strongly depend on the model studied. On the other hand, the propagator (2) for the model under consideration can hardly be used for quantitative description of the cuprates and their spin-wave branches which, as is wellknown, obey the linear dispersion law.

It is important that the Hamiltonian (1) is invariant under global gauge transformations of two types [20]:

$$\psi_\sigma(x) \to \psi_\sigma(x)e^{i\alpha}, \quad \psi_\sigma^\dagger(x) \to \psi_\sigma^\dagger(x)e^{-i\alpha} \tag{3}$$

and

$$\psi_\uparrow(x) \to \psi_\uparrow(x)e^{i\alpha}, \quad \psi_\downarrow(x) \to \psi_\downarrow(x)e^{-i\alpha},$$

$$\psi_\uparrow^\dagger(x) \to \psi_\uparrow^\dagger(x)e^{-i\alpha}, \quad \psi_\downarrow^\dagger(x) \to \psi_\downarrow^\dagger(x)e^{i\alpha}, \tag{4}$$

which must be taken into account. The phase α in (3) and (4) is real.

To calculate the phase diagram of a system it is necessary to find its thermodynamic potential. It can be calculated by making use of the auxiliary bilocal field method (see, for example, Ref. [21]), which is a generalization of the standard Hubbard-Stratonovich method for the boson-exchange case. The grand partition function Z can then be expressed in terms of the path integral over the fermionic $\psi_\sigma(x)$ and the complex auxiliary fields (for example, $\phi(x, x') \sim \langle\psi_\uparrow^\dagger(x)\psi_\downarrow^\dagger(x')\rangle$).

In the case of model (1) it is convenient, following Ref. [22], to introduce the bispinor

$$\Psi^\dagger(x) = (\psi_\uparrow^\dagger(x), \psi_\downarrow^\dagger(x), \psi_\uparrow(x), \psi_\downarrow(x)) \tag{5}$$

and its Hermitian conjugate, which here are the analogs of the Nambu spinors [23]. After substituting (5) in (1), we can write the Hamiltonian in the form

$$H(x) = -\frac{1}{2}\Psi^\dagger(x)\left(\frac{\nabla^2}{2m} + \mu\right) I \otimes \tau_z \Psi(x) - \frac{1}{2} g_{ph} \Psi^\dagger(x) I \otimes \tau_z \Psi(x)\varphi(x) -$$
$$- \frac{1}{4} g_{4F} \Psi^\dagger(x) I \otimes \tau_z \Psi(x) \Psi^\dagger(x) I \otimes \tau_z \Psi(x) + \varphi(x) D^{-1}(x) \varphi(x), \tag{6}$$

where $I \otimes \tau_z$ is the direct product of the unit I and Pauli τ_z 2×2 matrices, and $D(x)$ is defined by (2). In such a representation of the Hamiltonian (6) and the field variables (5) the Feinman diagram technique is applicable in its ordinary form [22]. Thus, after standard exclusion of the boson field $\varphi(x)$, the Lagrangian of the system can be expressed by the formula

$$L(x_1,y_1,x_2,y_2)=\frac{1}{2}\Psi^\dagger(x)\left[-\partial_\tau+\left(\frac{\nabla^2}{2m}+\mu\right)I\otimes\tau_z\right]\Psi(x)-$$
$$-\frac{1}{4}\Psi(x_1)\Psi^\dagger(y_1)I\otimes\tau_z K(x_1,y_1;x_2,y_2)\Psi(x_2)\Psi^\dagger(y_2)I\otimes\tau_z. \quad (7)$$

The kernel K is the effective, nonlocal, particle-particle interaction function which is explicitly defined in the momentum space below.

In order to explore the pairing possibility in the system we introduce the bilocal auxiliary field or OP,

$$\phi(x_1,y_1)=\tau_z K(x_1,y_1;x_2,y_2)\Psi(x_2)\Psi^\dagger(y_2)I\otimes\tau_z\equiv$$
$$\equiv -i\tau_+\otimes\tau_y\phi^*_{ch}(x_1,y_1)-i\tau_-\otimes\tau_y\phi_{ch}(x_1,y_1)-\tau_z\otimes I\phi_{ins}(x_1,y_1), \quad (8)$$

where $\tau_+=(\tau_x+i\tau_y)/2$, $\tau_-=(\tau_x-i\tau_y)/2$, and the integration over x_2 and y_2 is assumed. Here $\phi_{ch}\sim\langle\psi^\dagger_\downarrow\psi^\dagger_\uparrow\rangle$ and $\phi_{ins}\sim\langle\psi^\dagger_\uparrow\psi_\uparrow\rangle$ are the electron-electron (charged) and the electron-hole (insulating) spin-singlet OP, respectively (we ignore the nonzero spin pairing). The auxiliary fields ϕ_{ch} and ϕ_{ins} are responsible for the dynamic symmetry breaking (in accordance with (3) and (4), respectively).

Adding to (7) a zero term,

$$\frac{1}{4}\left[\phi(x_1,y_1)-K(x_1,y_1;x_1',y_1')\Psi(x_1')\Psi^\dagger(y_1')I\otimes\tau_z\right]K^{-1}(x_1,y_1;x_2,y_2)[\phi(x_2,y_2)-$$
$$-K(x_2,y_2;x_2',y_2')\Psi(x_2')\Psi^\dagger(y_2')I\otimes\tau_z],$$

to cancel the $4F$ interaction, we obtain the Lagrangian in the form

$$L(x_1,y_1;x_2,y_2)=\frac{1}{2}\Psi^\dagger(x_1)\left[-\partial_\tau+\left(\frac{\nabla^2}{2m}+\mu\right)I\otimes\tau_z-\frac{1}{2}I\otimes\tau_z\phi(x_1,y_1)\right]\Psi(y_1)+$$
$$+\frac{1}{4}\phi(x_1,y_1)K^{-1}(x_1,y_1;x_2,y_2)\phi(x_2,y_2). \quad (9)$$

Let us transform the expression for the kernel K; in the momentum space it then is

$$K(x_1,y_1;x_2,y_2)=$$

$$=\int\frac{d^3Pd^3p_1d^3p_2}{(2\pi)^9}K_P(p_1;p_2)\exp\left[-iP(\frac{x_1+y_1}{2}-\frac{x_2+y_2}{2})-ip_1(x_1-y_1)-ip_2(x_2-y_2)\right],$$

where $p_i=(\mathbf{p}_i,\omega_i)$ $(i=1,2)$ and $P=(\mathbf{P},\omega)$ represent the relative and the center-of-mass momenta, respectively. According to the definition, the kernel $K_P(p_1;p_2)$ is in fact independent of P (we can therefore omit the index P below) and acquires a simple form

$$K(p_1;p_2)=g^2_{ph}D(p_1-p_2)-g_{4F}, \quad (10)$$

which is used in (9). The last expression evidently demonstrates that the total character of the effective particle-particle interaction, as it always takes place in such a situation [23, 24], is defined by a possible competition between the first (retarded) and second (nonretarded) terms in (10) or, in other words, by their common action.

The partition function can be written as

$$Z = \int \mathscr{D}\Psi^{\dagger}\mathscr{D}\Psi\mathscr{D}\phi\mathscr{D}\phi^{*} \exp\left[-\beta \int L(\Psi^{\dagger}, \Psi, \phi^{*}, \phi)dxdy\right] \equiv$$
$$\equiv \int \mathscr{D}\phi\mathscr{D}\phi^{*} \exp(-\beta\Omega[\mathscr{G}]), \quad \beta = 1/T,$$

where $\Omega[\mathscr{G}]$ is the thermodynamic potential which in the «leading order» is

$$\beta\Omega[\mathscr{G}] = -\mathrm{Tr}\left[\mathrm{Ln}\,\mathscr{G}^{-1} + \frac{1}{2}(\phi K^{-1}\phi)\right], \tag{11}$$

where Tr includes $2D$ spatial $\mathbf{r}$ and «time» $0 \leq \tau \leq \beta$ integrations, as well as the standard trace operation. The full Green's function of the system is

$$\mathscr{G}^{-1} = -\frac{1}{2}\left[\partial_{\tau} - \left(\frac{\nabla^2}{2m} + \mu\right) I \otimes \tau_z - \phi\right]. \tag{12}$$

From (11) and (12) we obtain to the ϕ-equation (the Schwinger-Dyson equation)

$$\frac{\delta\Omega}{\delta\phi} = \phi - \int \frac{d^2\mathbf{k}d\omega}{(2\pi)^3} K(p; \mathbf{k}, \omega)\mathscr{G}(\mathbf{k}, \omega) = 0. \tag{13}$$

Substituting (13) into (11), we obtain the expression for $\Omega(\mathscr{G})$:

$$\beta\Omega(\mathscr{G}) = -\mathrm{TrLn}\mathscr{G}^{-1} + \frac{1}{2}\mathrm{Tr}\mathscr{G}K\mathscr{G}.$$

This expression is the wellknown Cornwell-Jackiw-Tomboulis formula for the effective action in the one-loop approximation [25]. Using (13), we can rewrite this expression in the form

$$\beta\Omega(\mathscr{G}) = -\mathrm{Tr}\left[\mathrm{Ln}\mathscr{G} + \frac{1}{2}[\mathscr{G}\mathscr{G}_0^{-1} - 1]\right]. \tag{14}$$

As was shown by Thouless et al. [26] (see also Ref. [15]) in the $2D$ case it is more natural to use a new parametrization of the charge OP (Eq. (8)) — its absolute value (modulus) and the phase. In other words[1]:

$$\phi_{ch}(x, y) = \rho_{ch}(x, y) \exp\left[-i\left(\theta(x) + \theta(y)\right)/2\right], \tag{15}$$

where ρ_{ch} is real. As for ρ_{ins}, it corresponds, as can be seen from Eq. (8), to a one-component OP and therefore does not characterize the phase factor.

As will be shown below, with the given kernel (10) only one (ϕ_{ch} or ϕ_{ins}) OP can arise. Therefore, it is necessary to make, simultaneously with (15), the spinor transformation (in accordance with (3) and (4))

$$\Psi^{\dagger}(x) = \chi^{\dagger}(x) \exp(i\theta(x) I \otimes \tau_z/2), \tag{16}$$

[1] It should be noted that Efetov and Larkin, in fact, were the first to use such a parametrization. They studied [27] the effect of interchain hopping and OP phase fluctuations on the superconducting transition temperature in $1D$ superconductors.

$$\Psi^{\dagger}(x) = \chi^{\dagger}(x)\exp(i\theta(x)\tau_z \otimes \tau_z/2) \tag{17}$$

(the spinor $\chi(x)$ is real and formally corresponds to chargeless fermions). Below we shall obtain the θ-corrections for the ϕ_{ch} case only, but the equations for ρ_{ins} are the same up to the substitution $\rho_{ch} \to \rho_{ins}$. The reason is that when $K(p_1, p_2)$ describes the attraction (charge pairing channel), the symmetry of the Lagrangian under operations (3) proves to be crucial for the representation (16); while when $K(p_1, p_2)$ corresponds to the repulsion (chargeless, or electron-hole, pairing channel) the symmetry (4) is already important and the representation (17) must be used as a «working» representation. With this difference, the rest of the calculations for ρ's are almost identical but the «phase effects» persist for the charge channel only. This channel, the most interesting one for metallic (superconducting) systems, we shall therefore consider in detail.

In the variables (16) the Green's function (12) transforms to

$$\mathscr{G}^{-1} = -\frac{1}{2}\left[\partial_\tau - I\otimes\tau_z\left(\frac{\nabla^2}{2m}+\mu\right) + i\tau_x\otimes\tau_y\rho_{ch} - \right.$$
$$\left. - I\otimes\tau_z\left(\partial_\tau\theta + \frac{\nabla\theta^2}{2m}\right) - iI\otimes I\left(\frac{\nabla^2\theta}{2m}+\frac{\nabla\theta\nabla}{m}\right)\right] \equiv G^{-1}(\rho_{ch}) - \Sigma(\partial\theta). \tag{18}$$

Using (18) under assumption that the θ gradients are small (the hydrodynamic approximation) and taking them into account up to the second order, we can divide the effective potential (14) into two parts: $\Omega = \Omega_{kin}(\rho_{ch}, \nabla\theta) + \Omega_{pot}(\rho_{ch})$, where in the $(\nabla\theta)^2$ approximation

$$\beta\Omega_{kin}(\rho_{ch}, \nabla\theta) = \mathrm{Tr}\left[G\Sigma - G_0\Sigma + \frac{1}{2}G\Sigma G\Sigma - \right.$$
$$\left. - \frac{1}{2}G_0\Sigma G_0\Sigma + \tau_x\otimes I\frac{1}{2}i\rho_{ch}G(G\Sigma + G\Sigma G\Sigma)\right]. \tag{19}$$

Assuming by analogy with Ref. [27] (see also Ref. [28]) that $\rho_{ch}(x, y)$ is homogeneous[2)] after rather tedious but otherwise straightforward calculation, we obtain from (19) the expression

$$\Omega_{kin}(\rho_{ch}, \Delta\theta) = \frac{T}{2}\int_0^\beta d\tau \int d^2\mathbf{r} J(\mu, T, \rho_{ch}(\mu, T))(\nabla\theta)^2, \tag{20}$$

where

$$J(\mu, T, \rho_{ch}(\mu, T)) =$$
$$= \frac{1}{8\pi}\left(\sqrt{\mu^2+\rho_{ch}^2} + \mu + 2T\ln\left[1+\exp\left(-\frac{\sqrt{\mu^2+\rho_{ch}^2}}{T}\right)\right]\right. -$$
$$\left. - \frac{T}{4\pi}\left[1 - \frac{\rho_{ch}^2}{4T^2}\frac{\partial}{\partial(\rho_{ch}^2/4T^2)}\right]\int_{-\mu/2T}^{\infty} dx \frac{x+\mu/2T}{\mathrm{ch}^2\sqrt{x^2+\rho_{ch}^2/4T^2}}\right) \tag{21}$$

[2)] Equations for ρ_{ch} and ρ_{ins} are obtained below and, as was shown in Ref. [16], it is an admissible approximation to put in them the value ρ_{ch} (and ρ_{ins}), which is independent of spatial and time variables.

plays the role of the neutral OP stiffness. Note that in comparison with the retardation-free $4F$ model [15], the last expression contains one more term: the term with the derivative.

The equation for the temperature T_{BKT} of the BKT transition can be written after direct comparison of the kinetic term (20) in the effective action with the Hamiltonian of the $2D$ XY model, which formally has the identical form [29]. It is therefore easy to conclude that

$$\frac{\pi}{2} J\,(\mu, T_{BKT}, \rho_{ch}(\mu, T_{BKT})) = T_{BKT}. \tag{22}$$

The basic difference between this equation and the one for the XY model is the inherent dependence of the former on μ (or n_f) and ρ_{ch}.

To complete the set of self-consistent equations, which allow one to trace an explicit dependence of T_{BKT} on n_f, we also give the equations for ρ_{ch} and μ. In particular, a simple equation for $\rho_{ch}(i\omega_n)$ is Eq. (13) with $\nabla\theta = 0$; i.e., the Green's function G of the neutral fermions substitutes $\mathscr{G}$, so that (13) in the frequency-momentum represantation takes the form

$$\begin{pmatrix} \rho_{ch}(i\omega_n) \\ \rho_{ins}(i\omega_n) \end{pmatrix} = T \sum_{m=-\infty}^{\infty} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \begin{pmatrix} -\rho_{ch}(i\omega_m) \\ +\rho_{ins}(i\omega_m) \end{pmatrix} \frac{K(\omega_n, \omega_m)}{\omega_m^2 + \xi^2(\mathbf{k}) + \rho_{ch}^2(i\omega_m) + \rho_{ins}^2(i\omega_m)}, \tag{23}$$

where $\omega_n = (2n+1)\pi T$ is the Matsubara fermionic frequency [23], $\xi(\mathbf{k}) = \mathbf{k}^2/2m - \mu$, and the kernel $K(\omega_m, \omega_n)$ is defined above.

We gave the final equations for both OPs, ρ_{ch}, and ρ_{ins} in order to show that they indeed are the same but alternative if the kernel K changes sign. The analytic solution of these equations, as well as obtaining Eq. (22) and the number equation needed, can be done by assuming that $\rho_{ch}(i\omega_n)$ does not depend on the Matsubara frequencies (see the footnote on page 6).

Making use of this approximation, the equation which follows from the ordinary condition $V^{-1}\partial\Omega[\mathscr{G}]/\partial\mu = -n_f$ (V is the volume of the system) and which is crucial for the crossover description must be added to Eqs. (22) and (23) for self-consistency. We thus obtain

$$\sqrt{\mu^2 + \rho_{ch}^2} + \mu + 2T \ln\left[1 + \exp\left(-\frac{\sqrt{\mu^2 + \rho_{ch}^2}}{T}\right)\right] = 2\epsilon_F, \tag{24}$$

where $\epsilon_F = \pi n_f/m$ is the Fermi energy of free $2D$ fermions with a simple quadratic dispersion law. Thus, in the case under consideration all unknown quantities, ρ_{ch}, μ, and T_{BKT}, are the explicit functions of n_f.

3. ANALYSIS OF THE SOLUTIONS

In contrast with the standard (with the T-independent unit vector) XY model, in the superconducting model two characteristic temperatures can be introduced: T_ρ, where formally the complete OP given by (8) arises but its phase is a random quantity [3)], i.e., $\langle\phi(x,y)\rangle = 0$ and another temperature, $T_{BKT} < T_\rho$, where the phase of the OP becomes ordered, so that $\langle\phi(x,y)\rangle \neq 0$. In other words, we define the temperature T_ρ as the temperature of a relatively sharp change in the neutral OP, which does not break any real symmetry. Therefore, this

[3)] Because ρ_{ch} and ρ_{ins} cannot exist simultaneously (see Eq. (23)), the index ρ is the only OP, which appears at a definite sign of the kernel (10).

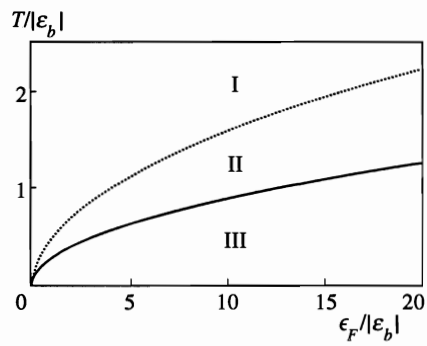

Рис. 1

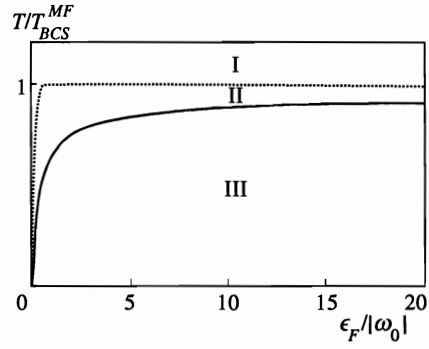

Рис. 2

Fig. 1. The T–n_f phase diagram of the $2D$ metal with $4F$ fermion attraction. The lines correspond to the functions $T_\rho(n_f)$ (the dotted curve) and $T_{BKT(n_f)}$ (the solid curve) at $g_{4F}m/2\pi = 0.5$. Figures I, II, and III show the regions of the normal, pseudogap, and superconducting phases, respectively

Fig. 2. The T–n_f phase diagram of the $2D$ metal with indirect intercarrier attraction for $\lambda = 0.5$. Similarly to Fig. 1, the curves correspond to the functions $T_\rho(n_f)$ and $T_{BKT(n_f)}$ and separate the same regions

temperature (unlike T_{BKT} is not the phase-transition temperature. Nevertheless, it gives (see Refs. [27, 28]) a convenient scale for the description of the neutral OP temperature behavior. Recall that according to the equations obtained above both these temperatures directly depend on the carrier density in the system.

The «critical» temperature T_ρ can be found, for example, from Eqs. (21)–(24) by setting $\rho_{ch} = 0$ (in accordance with the derivation of these equations, it corresponds to the mean-field approximation[4]). As a result, with a decrease in temperature, a $2D$ metal (similarly to a $1D$ metal [27]) passes from the normal phase ($T > T_\rho$) to another phase, where the average homogeneous (charged) OP $\langle\phi(x,y)\rangle = 0$ or, equivalently, the superconductivity is absent, but chargeless OP $\rho_{ch} \neq 0$. It is evident that the pseudogap is formed just in the temperature region $T_{BKT} < T < T_\rho$, because, as follows from the formulas cited above (see, e.g., Eqs. (21)–(24)), $\rho_{ch} = \rho_{ch}(T)$ acquires all the spectral characteristics of a $2D$ metal in the same way as the superconducting gap $\Delta(T)$ enters into corresponding expressions for ordinary superconductors. It justifies why this region can be called «the pseudogap phase». The density of states near ϵ_F in the pseudogap phase is definitely lower than that in the region of the normal phase with $\rho_{ch} = 0$, but does not equal zero as in the superconducting phase. The latter must be checked by direct calculation of the one-particle fermion Green's function, which is most likely a separate problem that is not considered here.

The phase diagram of a system can be found from Eqs. (21)–(24). The quantities $T_\rho(n_f)$ and $T_{BKT}(n_f)$ behave differently for different correlations between interaction constants.

1) $g_{4F} > 0$, $g_{ph} = 0$ (an unretarded interaction).

This case has been partly analyzed in Ref. [15]. It corresponds to fermion-fermion pairing due to the local attraction. Note (see Eq. (23)) that in this case (or in the case of attraction

[4] Despite the fact that the temperature T_ρ is not identical to the BCS critical temperature T_{BCS}^{MF}, they coincide for the large carrier density only (see below).

between carriers) the fermion-antifermion (insulating) pairing channel is absent, i.e., $\rho_{ins} = 0$. The corresponding phase diagram is shown in Fig. 1. We see that the pseudogap phase exists at any carrier concentrations, that the temperature width of this phase region weakly increases with increasing n_f, and that the BKT phase always begins to form when $\rho_{ch}(T_{BKT})$ is finite, which means that fluctuations of the latter near and below T_{BKT} are not essential.

As $\epsilon_F \to 0$, the temperature of the BKT phase formation is defined by the equality $T_{BKT} = \epsilon_F/8$, and T_ρ as a function of n_f can be found from the equation

$$T_\rho \ln(T_\rho/\epsilon_F) = W \exp(-4\pi/g_{4F}m) = -\varepsilon_b^{(4F)}/2,$$

which follows from (23) (W is the conduction band width, and $\varepsilon_b^{(4F)}$ is the two-fermion bound state energy, which is always different from zero in the $2D$ case).

2) $g_{4F} = 0$, $g_{ph} \neq 0$ (a pure indirect interaction).

This is one of the most interesting cases because it corresponds to the widely accepted electron-phonon (or the BCS-Bogolyubov-Eliashberg) model of superconductivity. The numerically calculated phase diagram is shown in Fig. 2. It shows that a comparatively large region with the pseudogap phase exists at rather low carrier concentrations only, and that its temperature area shrinks when $n_f \to \infty$. Such a behavior qualitatively agrees with that which takes place in real HTSCs [5–8] and demonstrates that a pseudogap (and also a spin gap) is mainly observed in underdoped HTSC samples.

It is not difficult to conclude that the asymptotic behavior of $T_\rho(n_f)$ and $T_{BKT}(n_f)$ has the following forms:

i) when the ratio $\epsilon_F/\omega_0 \ll 1$ (very low free fermion density or the local pair case) the first one satisfies the equation

$$T_\rho \ln(T_\rho/\epsilon_F) = \omega_0 \exp(-4\pi/g_{ph}^2 m) = -\varepsilon_b^{(ph)}/2,$$

which immediatedly results in $\partial T_\rho(n_f)/\partial n_f|_{n_f \to 0} \to \infty$ (here similarly to the $4F$ case it is convenient to introduce the bound state energy $\varepsilon_b^{(ph)}$ for the phonon attraction). At the same time, the temperature T_{BKT} in the limit $n_f \to 0$ has identical dependence on the carrier density and, as above, $T_{BKT} = \epsilon_F/8$. This simply means that here again it is proportional to the number $n_f/2$ of composite bosons; in this density region $T_\rho/T_{BKT} \gg 1$ (this inequality is also satisfied for the pure $4F$ interaction).

ii) in the opposite case $\epsilon_F/\omega_0 \gg 1$ (very large fermion density or the Cooper pair case) we easily obtain the standard BCS value:

$$T_\rho = (2\gamma\omega_0/\pi) \exp(-2\pi/g_{ph}^2 m) \equiv T_{BCS}^{MF} = (2\gamma/\pi)\Delta_{BCS}$$

(Δ_{BCS} is the usual one-particle BCS gap at $T = 0$). In other words, in this limit the temperature T_ρ becomes equal, as it should be, to the BCS value[5]. The T_{BKT} asymptotic behavior here is not so evident and requires a more detailed consideration.

First of all, it is natural to assume that for large n_f the temperature $T_{BKT} \to T_\rho$. It is then necessary to check the dependence of ρ on T as $T \to T_\rho$. For this purpose Eq. (23) can be transformed to

[5] Being equal (in the mean-field approximation only), these temperatures (T_ρ and T_{BCS}^{MF}) are in fact different: if T_{BCS}^{MF} immediately decreases to zero as the fluctuations ϕ and ϕ^* are taken into account, T_ρ does not decrease and is renormalized only when ρ fluctuates.

$$\frac{2\pi}{g_{ph}^2 m} = \int\limits_0^\infty dx \left(\frac{\operatorname{th}\sqrt{x^2+\rho_{ch}^2/4T^2}}{\sqrt{x^2+\rho_{ch}^2/4T^2}} - \frac{\operatorname{th}\sqrt{x^2+\rho_{ch}^2/4T^2} - \operatorname{th}(\omega_0/2T)}{2(\sqrt{x^2+\rho_{ch}^2/4T^2} - \omega_0/2T)} - \right.$$

$$\left. - \frac{\operatorname{th}\sqrt{x^2+\rho_{ch}^2/4T^2} + \operatorname{th}(\omega_0/2T)}{2(\sqrt{x^2+\rho_{ch}^2/4T^2} + \omega_0/2T)} \right). \tag{25}$$

(Here it was assumed that in the concentration region under consideration the ratio $\mu/T_\rho \simeq$ $\simeq \epsilon_F/T_\rho \gg 1$ because $\mu \simeq \epsilon_F$ [10–12, 16].)

Since usually $\omega_0/T_\rho \gg 1$, only very small x give the main contribution to the integral (25) (this is seen from the limit $\rho/T_\rho \to 0$, when $\epsilon_F/\omega_0 \to \infty$). Therefore, it takes the approximate form

$$\frac{2\pi}{g_{ph}^2 m} = \int\limits_0^\infty dx \left(\frac{\operatorname{th}\sqrt{x^2+\rho_{ch}^2/4T^2}}{\sqrt{x^2+\rho_{ch}^2/4T^2}} - \frac{1}{x+\omega_0/2T} \right). \tag{26}$$

On the other hand, the accepted condition $\rho_{ch}(T_\rho) = 0$ in (26) directly results in the simple equation

$$\frac{2\pi}{g_{ph}^2 m} = \int\limits_0^\infty dx \left(\frac{\operatorname{th} x}{x} - \frac{1}{x+\omega_0/2T_\rho} \right) \tag{27}$$

for T_ρ. Comparing (26) and (27), we obtain

$$\int\limits_0^\infty dx \left(\frac{\operatorname{th} x}{x} - \frac{\operatorname{th}\sqrt{x^2+\rho_{ch}^2/4T^2}}{\sqrt{x^2+\rho_{ch}^2/4T^2}} \right) = \ln\frac{T_\rho}{T}.$$

Now from the expansions

$$\frac{\operatorname{th}\sqrt{x^2+\rho_{ch}^2/4T^2}}{\sqrt{x^2+\rho_{ch}^2/4T^2}} \simeq \begin{cases} 1 - 3^{-1}\left[x^2+\rho_{ch}^2/4T^2\right], & x \le 1 \\ x^{-1} - \rho_{ch}^2/8T^2x^3, & x > 1 \end{cases},$$

we obtain the expression which we need

$$\rho_{ch}(T) \simeq 2.62 T_\rho \sqrt{T_\rho/T - 1}. \tag{28}$$

Recall that the well-known $3D$ result is $\Delta_{BCS}(T) = 3.06 T_{BCS}^{MF}\sqrt{T_{BCS}^{MF}/T - 1}$ (Ref. [17]) and the small difference can be explained by the above approximation,which is suitable for the qualitative discussion below (see Sec. 4).

The dependence (28) must be substituted in Eq. (22); because $\mu/T_{BKT} \simeq \epsilon_F/T_{BKT} \gg 1$ and $\rho_{ch}(T_{BKT})/T_{BKT} \ll 1$ when $T_{BKT} \to T_\rho$, this equation can be written as

$$\frac{\epsilon_F}{4T_{BKT}} \left[1 - \frac{\rho_{ch}^2}{4T_{BKT}^2} \frac{\partial}{\partial(\rho_{ch}^2/4T_{BKT}^2)} \right] \int\limits_0^\infty dx \left(\frac{1}{\operatorname{ch}^2 x} - \right.$$

$$\left. - \frac{1}{\operatorname{ch}^2\sqrt{x^2+\rho_{ch}^2/4T_{BKT}^2}} \right) = 1. \tag{29}$$

Finally, using the expansion in $\rho_{ch}(T_{BKT})/2T_{BKT}$ in integral (29), the latter can be transformed to

$$\frac{a\epsilon_F}{8T_{BKT}}\left[\frac{\rho_{ch}(T_{BKT})}{2T_{BKT}}\right]^4 = 1, \tag{30}$$

where the numerical constant

$$a = \int_0^\infty dx \frac{\mathrm{th}^2 x - x^{-1}\,\mathrm{th}\, x + 1}{2x^2\,\mathrm{ch}\, x} \simeq 1.98.$$

Combining now (28) and (30), we obtain the final simple relation between T_ρ and T_{BKT} for the large carrier density

$$T_{BKT} \simeq T_\rho\left(1 - 2.34\sqrt{T_\rho/\epsilon_F}\right).$$

In other words, T_{BKT} as a function of n_f (see Fig. 2) actually approaches T_ρ (or T_{BCS}^{MF}).

With regard to the crossover region defined by the equality $\mu \simeq 0$, it is easy to see from Eqs. (21)–(24) and Fig. 2 that the former corresponds to the densities when the temperatures T_ρ and T_{BKT} are essentially different; otherwise, the pseudogap phase really exists here. It is important that because of the relatively, low for the phonon case, the value of the energy of state of the bound pair $\varepsilon_b^{(ph)}$ and the very small region of negative μ (Ref. [16]), the behavior $T_{BKT}(n_f) \sim \epsilon_F$ hardly corresponds to the Bose-Einstein condensation and, in fact, is consistent for carrier densities when $\mu > 0$ (although probably $\mu \neq \epsilon_F$).

3) $g_{4F} \neq 0$, $g_{ph} \neq 0$.

This general case contains the boson exchange and unretarded interactions. The situation closest to the real systems corresponds to the case $g_{4F} < 0$ (or to some sort of short-range repulsion) but total interaction has attractive character; this means that at least $g_{ph}^2 > |g_{4F}|$. There are two qualitatively different cases again: i) low and ii) high carrier densities.

i) $\epsilon_F/\omega_0 \ll 1$. For this inequality we see that T_ρ satisfies the same equation $T_\rho \ln(\epsilon_F/T_\rho) = -\varepsilon_b/2$, where now

$$\varepsilon_b = -2W\left(\frac{\omega_0}{W+\omega_0}\right)^{\lambda/(\lambda-\mu_C^*)} \exp\left(-\frac{2}{\lambda-\mu_C^*}\right)$$

is the two-body bound state energy, and $\lambda = g_{ph}^2 m/2\pi$ and $\mu_C^* = -g_{4F}m/2\pi$ are the ordinary, effective, electron-phonon (attractive) and Coulomb (repulsive) constants; the difference $\lambda - \mu_C^*$ should be positive. We see that ε_b equals to $\varepsilon_b^{(4F)}$ or $\varepsilon_b^{(ph)}$ (if $W \gg \omega_0$) for the previous limiting cases. Here $T_{BKT} = \epsilon_F/8$ and $\partial T_\rho(n_f)/\partial n_f \to \infty$ as $n_f \to 0$.

ii) $\epsilon_F/\omega_0 \gg 1$. In this limit the expression for T_ρ has the form

$$T_\rho = \frac{\gamma}{\pi}\sqrt{\omega_0|\varepsilon_b|\left(\frac{\epsilon_F}{\omega_0}\right)^{\mu_C^*/(\lambda-\mu_C^*)}}. \tag{31}$$

It follows from this expression that the dependence on n_f is still weak, which results in an increase (for $g_{4F} > 0$) or decrease (for $g_{4F} < 0$) of T_ρ when $\epsilon_F \geq \omega_0(\lambda - \mu_C^*)/\mu_C^*$, which is

direct consequence of the model with unretarded $4F$ interaction. The temperature T_{BKT} is described by Eq. (30) with T_ρ, defined by (31).

It must be noted, however, that with more realistic assumptions about Coulomb repulsion μ_C^*, which can be initially represented (see, for example, Ref. [22]) by the matrix elements

$$V(\mathbf{k},\mathbf{k}') = \begin{cases} V_c, & |\xi(\mathbf{k})|, |\xi(\mathbf{k}')| \le |\mu| \\ 0, & |\xi(\mathbf{k})|, |\xi(\mathbf{k}')| > |\mu| \end{cases},$$

it acquires the wellknown Tolmachev logarithmic correction or turns out to be screened at large n_f. At low carrier densities such effect (screening) does not take place, so the local repulsion model can be considered as a good approximation in the physical cases in which the Fermi energy of free fermions is less than or not much greater than the characteristic boson frequency.

4. CONCLUSIONS

The model proposed to describe the possible two-stage superconducting phase transition in $2D$ (and quasi-$2D$) metallic systems was greatly very simplified in order to investigate their most typical and general features. Surprisingly, it gives some essential details which are characteristic of underdoped HTSC copper oxides. In particular, the experimental data show [30, 31] that i) the critical temperature T_c for low n_f indeed is proportional to n_f (which is simply ϵ_F), ii) T_c «becomes saturated» when n_f approaches «optimal doping» (i.e., carrier concentration when T_c as a function of n_f reaches its highest possible valur for the given compound), iii) the ratio T_c/ϵ_F in these and other «exotic» superconductors is as high as $10^{-2}-10^{-1}$ which independently points to rather low Fermi energy, etc. (for details see Ref. [31]). In addition, the standard ratio (see Ref. [23]) $2\Delta(0)/T_c$ can be roughly estimated as $2\rho(0)/T_{BKT}$; this value always exeeds its canonical BCS value and increases approximately $\sim n_f^{-1/2}$ at small values of n_f.

One would think that the pecularities mentioned above receive their natural interpretation on the basis of the model for the metal with indirect fermion-fermion interaction if the temperature T_{BKT} is the critical temperature T_c (this is justifiable for pure $2D$ systems [13]). In a quasi-$2D$ model the third spatial direction and the phase fluctuation stabilization give rise to the true temperature T_c of an ordinary homogeneous ordering arises [31, 32] (see also Ref. [11]), but the region where $T_c \neq T_\rho$ (or T_c^{BCS}) can be conserved [11].

As regards the other temperature (here esimated as T_ρ), it is usually determined empirically as some temperature point T^*, where the observable spectral (or magnetic) properties of HTSCs begin to deviate appreciably from their standard for normal metallic state-behavior [5–9]. As a rule, such a deviation is attributable to the appearance of fluctuating (short-lived) pairs. We showed, however, that a finite number of these pairs does exist or begins to be formed (rapidly) below some definite (in the mean-field approximation) temperature T_ρ, which, as indicated above, does not correspond to a phase transition. Additionally, because of the fluctuations (including quantum fluctuations), $\rho(T_\rho)$ remains nonzero at $T > T_\rho$. In this temperature region the number of pairs is exponentially small, and the fluctuations, which are superconducting (developed in the $2D$ case) can contribute to the temperature behavior of different observables (even at large n_f (Ref. [33])). The only difference from the supposed dependence T^* on the density of doped holes is the decreasing asymptotic behavior at n_f. We have found that this collective temperature also decreases, while (see, for example, Ref. [31]) T^* is usually plotted as one that increases with decrasing n_f. It seems that such a behavior still has no satisfactory

explanation, especially for the $2D$ case, where the bound states do not demand, as in the $3D$ case a strong coupling. Nevertheless, it must be stressed that the above limit, $T_\rho(n_f) \to 0$ when $n_f \to 0$, cannot be considered as sufficiently regular because of the growth of the neutral OP fluctuations; their role was disregarded, and they become very important at small values of n_f, when, for example, any collective behavior cannot exist.

The model under consideration qualitatively correctly describes the explicit narrowing of the pseudogap area as the carrier density increases [such a narrowing results in a rather rapid confluence of the temperatures (T_c and T_ρ and their experimental confluence, rendering them indistinguishable) in the BCS limit]. On the other hand, recent angle-resolved photoemission spectra unexpectedly showed [2, 3] that, unlike T_c, the superconducting gap even in the underdoped samples is essentially independent of doping. Such a difference to some extent also follows from the superconducting transition scenario proposed by us: Indeed, one-particle spectrum gap as a function of n_f is simply defined by the value $\rho(n_f)$ (it was calculated in Ref. [16]), which is proportional to $T_\rho(n_f)$, and the latter (see Fig. 2) very quickly becomes equal to Δ_{BCS}, or a constant, although T_{BKT} (and T_c) not quite yet reach this point. This behavior is a direct consequence of the evident smallness of the negative μ (local pairs and/or strongly developed fluctuations) region for the indirect fermion-fermion interaction model in which the bound states prove to be extremely subtle.

Some important problems still remain unresolved and must be investigated. These problems are: more complete and deep development of the model, which must consider different kinds of dispersion laws for the intermediate bosons; more careful taking into account of the Coulomb repulsion; neutral OP fluctuations, especially for low n_f; generalization of the approach to the case of nonisotropic pairing. On the other hand, high-T_c compounds must be studied in the frame of more realistic models, which include such pecularities of HTSCs as the magnetism of cuprate layers, non-qudratic free carrier dispersion relation with possible van Hove singularities in the hole density of states, and, of course, spatial quasi-two-dimensionality. One of the most interesting problems is to obtain doping and temperature dependent effective action, which is equivalent to the Ginzburg-Landau potential, because in many cases the phenomenology is more preferable.

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