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INFLUENCE OF AN ISOLATED MAGNETIC IMPURITY ON AN UNCONVENTIONAL SUPERCONDUCTING STATE

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The effect of the moment of a magnetic impurity on the order parameter of an unconventional superconductor is examined. The coupling of the magnetic moment to the order parameter induces a locally time-reversal symmetry-breaking state which generates a magnetic field distribution in the vicinity of the impurity. The magnetic field can cause precession of the magnetic moment. The case of a spin polarized muon injected into the superconductor is discussed.

1. INTRODUCTION

Some heavy-fermion superconductors possess complex phase diagrams with various superconducting phases [1]. These phase diagrams provide strong evidence for unconventional superconductivity, because the different phases should be distinguished by symmetry. The two examples of such heavy Fermion superconductors are $U_{1-x}Th_xBe_{13}$ and UPt_3, which both show two consecutive transitions with high- and low-temperature superconducting states. The minimal requirement for such behavior is that the order parameter has more than one component. Considerable effort from theoretical and experimental side has been invested in determining the symmetry of the order parameter in both systems. So far no unambiguous identification of their order parameter symmetry has been achieved. Nevertheless, there is convincing evidence that the low-temperature states in both systems break the time-reversal symmetry $\mathcal{T}$. This fact occurs very naturally in most of the phenomenological theories explaining the phase diagram. $\mathcal{T}$-violating states have particular magnetic properties which can be observed in experiment. The zero-field relaxation rate of injected muons shows an increase when the material enters the low-temperature state [2, 3], though the magnitude of this increase may substantially depend upon the sample quality [4]. This rate is a measure of the internal field distribution and its increase indicates additional magnetization occuring in connection with the lower transition.

The additional magnetic fields are due to spontaneous supercurrents flowing in the vicinity of inhomogeneities of the time reversal symmetry breaking superconducting order parameter, for example, around (non-magnetic) impurities [5–9]. The net magnetization of an isolated impurity vanishes. There are two length scales involved, the London penetration depth λ and the coherence length ξ. While screening currents usually affect the magnetic field over a length λ, the spatial modulation of the currents can lead to an effective canceling of the magnetization on a shorter length comparable with ξ rather than λ. At the same time the possible existence of supercurrent decreasing over a characteristic scale greater than $\xi(T)$ with distance from the impurity, in this approach may be associated with the existence of the continuous degeneracy of the superconducting state, which is lifted due to the interaction with an impurity [5, 8].

In this work we consider the problem of spontaneous currents for the time-reversal symmetric phase above the lower transition in the presence of a static magnetic «impurity». This impurity could be an injected muon whose spin can be considered as static on the relevant time scales of the superconductor. The magnetic moment of the impurity couples to the superconducting order parameter. As we will show, the basic effect is the appearance of a locally $\mathcal{T}$-violating order parameter. By analogy with the case mentioned above, spontaneous supercurrents will be generated. The aim of this paper is to investigate the spatial distribution of these currents and the field pattern. Of particular interest is the magnetic field generated at the impurity site, as it would cause precession of the impurity spin. The essential coupling between impurity and order parameter originates from the combined scattering from the hyperfine and nonmagnetic (and/or spin-orbit and spin-spin) impurity potentials.

2. GINZBURG-LANDAU THEORY

Our discussion is based on a generalized Ginzburg-Landau (GL) functional. To be concrete, we use the example of a two-component order parameter as introduced in theories of the phase diagram of UPt_3. Thus the order parameter $\boldsymbol{\eta} = (\eta_1, \eta_1)$ belongs either to the irreducible representation E_1 or E_2 of either parity (singlet or triplet pairing) [10]. The general free energy functional is identical for both cases and has the following form

$$F = \int dV \left\{ a_1|\eta_1|^2 + a_2|\eta_2|^2 + \beta_1(|\eta_1|^2 + |\eta_2|^2)^2 + \beta_2|\eta_1^2 + \eta_2^2|^2 + \right. \\ + K_{123}(|p_x\eta_1|^2 + |p_y\eta_2|^2) + K_1(|p_x\eta_2|^2 + |p_y\eta_1|^2) + K_2(p_x^*\eta_1^* p_y\eta_2 + p_x\eta_1 p_y^*\eta_2^*) + \\ \left. + K_3(p_x^*\eta_2^* p_y\eta_1 + p_x\eta_2 p_y^*\eta_1^*) + K_4(|p_z\eta_1|^2 + |p_z\eta_2|^2) + \frac{[\nabla \mathbf{A}]^2}{8\pi} \right\}, \tag{1}$$

where $\mathbf{p} = -i\nabla - (2e/c)\mathbf{A}$ ($\mathbf{A}$ is the vector potential), $a_j = \alpha(T - T_{cj})$ and the coefficients are real numbers in the standard notation. We assume $T_{c1} > T_{c2}$ so that in the temperature range $T_{c1} > T > T^*$ only the η_1-component of the order parameter is finite. T^* denotes the low-temperature transition point below which η_2 appears,

$$T^* = \frac{T_{c1} + T_{c2}}{2} - \frac{\beta_1}{2\beta_2}(T_{c1} - T_{c2}). \tag{2}$$

In order to have a $\mathcal{T}$-violating low-temperature phase, it is necessary that $\beta_2 > 0$.

We introduce now the coupling to an impurity located at the origin. The terms in lowest order are

$$F_{imp} = \int dV \left[\kappa(|\eta_1|^2+|\eta_2|^2)+\nu(|\eta_1|^2-|\eta_2|^2)+\gamma(\eta_1\eta_2^*+\eta_1^*\eta_2)+i\mu(\eta_1\eta_2^*-\eta_1^*\eta_2)\right] \delta(\mathbf{r}). \tag{3}$$

From the invariance of this expression under the spatial symmetry group for the system of the crystal and the isolated impurity and from its time-reversal symmetry it follows that ν, γ and μ are not scalar quantities. The coefficients ν and γ differ from zero for a hexagonal crystal only for impurity states breaking the symmetry with respect to rotations around the hexagonal axis through the angle $\pi/3$. The last term describes the linear (or odd order) coupling of the magnetic moment to the order parameter. Note that $\eta_1\eta_2^* - \eta_1^*\eta_2$ is finite only if the order parameter $\boldsymbol{\eta}$ breaks time-reversal symmetry, i.e. the relative phase between the two components is not 0 or π. Hence, the coefficient μ differs from zero only for the time-reversal breaking state of the impurity.

We consider now the effect of the impurity on the order parameter in the high-temperature phase where $\boldsymbol{\eta} = \boldsymbol{\eta}_0 = \eta_0(1,0)$ choosing η_0 real with

$$\eta_0^2 = \frac{-a_1(T)}{2(\beta_1+\beta_2)}. \tag{4}$$

For simplicity we assume that the coupling is weak so that the distortion of the order parameter is small. We consider $\boldsymbol{\eta} = \boldsymbol{\eta}_0 + \boldsymbol{\psi}$, where $\boldsymbol{\psi} = \boldsymbol{\psi}' + i\boldsymbol{\psi}''$ is small compared with η_0. Since for the homogeneous phase the vector potential vanishes we can also assume $\mathbf{A}$ to be small. Therefore we analyze the GL equations linearized in $\boldsymbol{\psi}$ and $\mathbf{A}$. This leads to seven coupled equations, obtained by varying $F + F_{imp}$ with respect to the order parameter,

$$\begin{aligned}
&2a_1\psi_1' + K_{123}\partial^2_{xx}\psi_1' + K_1\partial^2_{yy}\psi_1' + K_4\partial^2_{zz}\psi_1' + K_{23}\partial^2_{xy}\psi_2' = (\kappa+\nu)\eta_0\delta(\mathbf{r}),\\
&\Delta a\psi_2' - K_{123}\partial^2_{yy}\psi_2' - K_1\partial^2_{xx}\psi_2' - K_4\partial^2_{zz}\psi_2' - K_{23}\partial^2_{xy}\psi_1' = -\gamma\eta_0\delta(\mathbf{r});
\end{aligned} \tag{5}$$

$$\begin{aligned}
&K_{123}\partial_x\left(\partial_x\psi_1'' - \frac{2e\eta_0}{c}A_x\right) + K_1\partial_y\left(\partial_y\psi_1'' - \frac{2e\eta_0}{c}A_y\right) +\\
&\quad + K_4\partial_z\left(\partial_z\psi_1'' - \frac{2e\eta_0}{c}A_z\right) + K_{23}\partial^2_{xy}\psi_2'' = 0,\\
&a_*\psi_2'' - K_{123}\partial^2_{yy}\psi_2'' - K_1\partial^2_{xx}\psi_2'' - K_4\partial^2_{zz}\psi_2'' - K_2\partial_y\left(\partial_x\psi_1'' - \frac{2e\eta_0}{c}A_x\right) -\\
&\quad - K_3\partial_x\left(\partial_y\psi_1'' - \frac{2e\eta_0}{c}A_y\right) = -\mu\eta_0\delta(\mathbf{r});
\end{aligned} \tag{6}$$

and with respect to the vector potential

$$\begin{aligned}
\partial_x \operatorname{div}\mathbf{A} - \Delta A_x &= \frac{16\pi e\eta_0}{c}K_{123}\left(\partial_x\psi_1'' - \frac{2e\eta_0}{c}A_x\right) + \frac{16\pi e\eta_0}{c}K_2\partial_y\psi_2'',\\
\partial_y \operatorname{div}\mathbf{A} - \Delta A_y &= \frac{16\pi e\eta_0}{c}K_1\left(\partial_y\psi_1'' - \frac{2e\eta_0}{c}A_y\right) + \frac{16\pi e\eta_0}{c}K_3\partial_x\psi_2'',\\
\partial_z \operatorname{div}\mathbf{A} - \Delta A_z &= \frac{16\pi e\eta_0}{c}K_4\left(\partial_z\psi_1'' - \frac{2e\eta_0}{c}A_z\right).
\end{aligned} \tag{7}$$

Here the following abbreviations were used: $\Delta a = a_2 - a_1$, $a_* = 2\beta\alpha(T-T_*)/(1+\beta)$, $\beta = \beta_2/\beta_1$. Note that the first two equations do not couple to the remaining five. The κ, ν and γ terms in F_{imp} act only on the real part of the order parameter, inducing a finite real η_2-component in the vicinity of the impurity. We will not discuss these two equations further here, since they lead to the distortion of the order parameter without interesting effects concerning the magnetic properties.

Clearly the imaginary part of the order parameter and the vector potential couple in Eqs. (6) and (7). The right-hand sides of the last three equations correspond essentially to the components of the supercurrents $4\pi\mathbf{j}/c$. It is only the last term of F_{imp} which enters into these equations. Obviously, the presence of a magnetic moment drives the imaginary order parameter components. This leads immediately to finite supercurrents and a magnetic field distribution.

A simplification occurs if we take the gauge freedom of the order parameter phase into account. In first-order approximation the quantity ψ_1''/η_0 is in fact a common phase of the order parameter $((\eta_0 + \psi_1' + i\psi_1'', \psi_2' + i\psi_2'') \simeq (\eta_0 + \psi_1', \psi_2' + i\psi_2'')\exp(i\psi_1''/\eta_0))$, whose value is directly associated with a gauge for vector potential $\mathbf{A}$. Therefore we can choose

$$\psi_1'' = 0 \tag{8}$$

as a gauge condition. Furthermore, one can see easily that the first equation in Eqs. (6) is equivalent to the condition $\operatorname{div}\mathbf{j} = 0$, and the same condition obviously follows from the Maxwell equations. Therefore, this equation may be omitted and we reduce the problem with the aid of Eq. (8) to the following four equations for the unknown quantities $\mathbf{A}$, ψ_2'':

$$\begin{gathered}
\partial_{xy}^2 A_y + \partial_{xz}^2 A_z + (\lambda_{123}^{-2} - \partial_{yy}^2 - \partial_{zz}^2)A_x = \frac{16\pi}{c} e\eta_0 K_2 \partial_y \psi_2'', \\
\partial_{xy}^2 A_x + \partial_{yz}^2 A_z + (\lambda_1^{-2} - \partial_{xx}^2 - \partial_{zz}^2)A_y = \frac{16\pi}{c} e\eta_0 K_3 \partial_x \psi_2'', \\
\partial_{xz}^2 A_x + \partial_{yz}^2 A_y + (\lambda_4^{-2} - \partial_{xx}^2 - \partial_{yy}^2)A_z = 0, \\
a_* \psi_2'' - K_1 \partial_{xx}^2 \psi_2'' - K_{123} \partial_{yy}^2 \psi_2'' - K_4 \partial_{zz}^2 \psi_2'' = -\mu\eta_0 \delta(\boldsymbol{r}) - \frac{2e\eta_0}{c}(K_2 \partial_y A_x + K_3 \partial_x A_y).
\end{gathered} \tag{9}$$

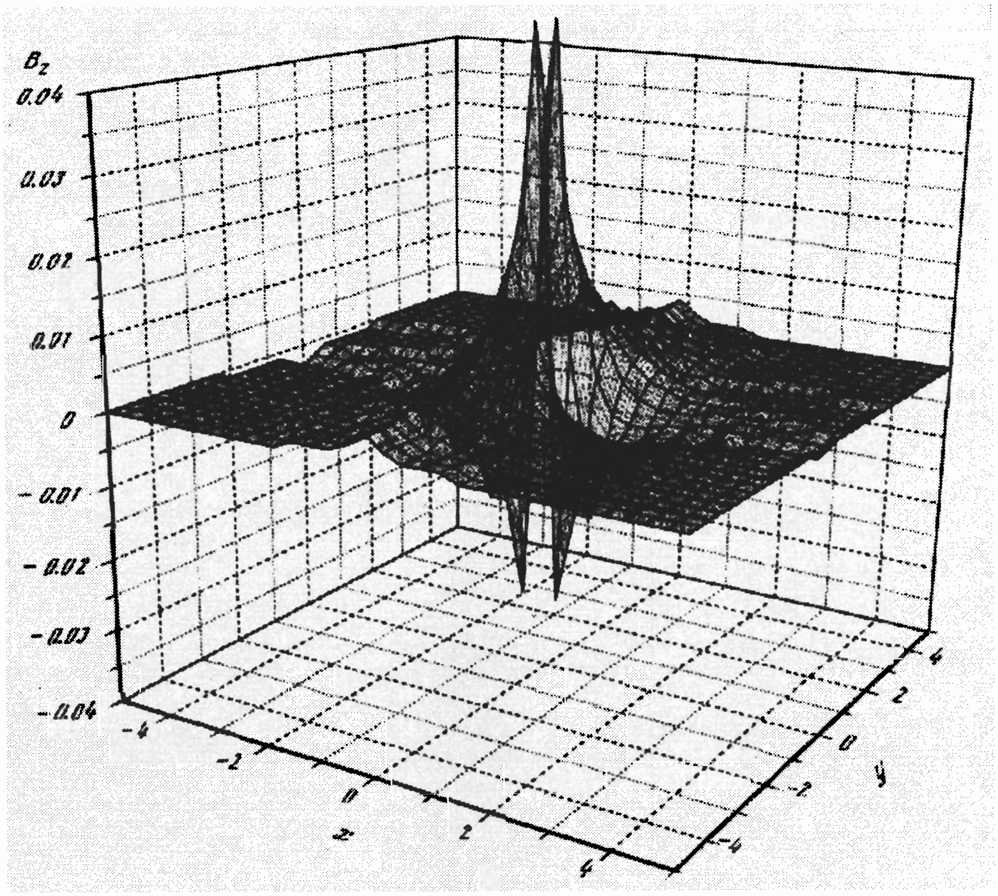
We have introduced here the notation $\lambda_i^{-2} = (32\pi e^2 \eta_0^2/c^2)K_i$. These equations can be easily solved in momentum space. We use the Fourier transformation

$$\begin{aligned}
\tilde{\mathbf{A}}(\mathbf{k}) &= \frac{1}{\sqrt{V}} \int dV \mathbf{A}(\mathbf{r}) e^{i\mathbf{kr}}, \\
\tilde{\psi}_2''(\mathbf{k}) &= \frac{1}{\sqrt{V}} \int dV \psi_2''(\mathbf{r}) e^{i\mathbf{kr}},
\end{aligned} \tag{10}$$

which leads to

$$\begin{gathered}
(\lambda_{123}^{-2} + k_y^2 + k_z^2)\tilde{A}_x - k_x k_y \tilde{A}_y - k_x k_z \tilde{A}_z - \frac{ic}{2e\eta_0}\lambda_2^{-2} k_y \tilde{\psi}_2'' = 0, \\
-k_x k_y \tilde{A}_x + (\lambda_1^{-2} + k_x^2 + k_z^2)\tilde{A}_y - k_y k_z \tilde{A}_z - \frac{ic}{2e\eta_0}\lambda_3^{-2} k_x \tilde{\psi}_2'' = 0, \\
-k_x k_z \tilde{A}_x - k_y k_z \tilde{A}_y + (\lambda_4^{-2} + k_x^2 + k_y^2)\tilde{A}_z = 0, \\
\frac{2ie\eta_0}{ca_*}(K_2 k_y \tilde{A}_x + K_3 k_x \tilde{A}_y) + (1 + \xi_1^2 k_x^2 + \xi_{123}^2 k_y^2 + \xi_4^2 k_z^2)\tilde{\psi}_2'' = -\frac{\mu\eta_0}{a_*}.
\end{gathered} \tag{11}$$

The solution of this Equation is straightforward but gives a rather complicated result (see below Eq. (16) and figure). A good picture of the result can be obtained by solving the last equation for $\tilde{\psi}_2''$ in the absence of the vector potential and then inserting the latter into the other three equations. This approximation may be justified, for example, under the conditions $K_2 \sim K_3 \ll$



The spatial distribution of the magnetic field at $z = 0$ is presented for the parameter values $K_3 = 1.5K_2 \ll K_1 = K_4$, $l = 10$, $l_2 = 0.258$, $l_3 = 0.316$, $l_4 = 1$. The parameters l, l_2, l_3, l_4 are defined as follows: $\lambda_1 = l_2\lambda_2 = l_3\lambda_3 = l_4\lambda_4 = l\xi_1(T)$. All distances are measured in units of $\xi_1(T)$, while the magnetic field is given in units of $(K_2)/(K_1)\ \left(\mu/\pi^3 a^* \xi_1^3(T)\right)\ \left(\phi_0/2\pi\xi_1^2(T)\right)$. The value of $B_z(0)$ is about 77% of its maximum value

$\ll K_1 \sim K_4$. Then the order parameter has the form

$$\tilde{\psi}_2''(\mathbf{k}) = -\frac{\mu\eta_0}{a_*(T)} \frac{1}{1 + \xi_1^2 k_x^2 + \xi_{123}^2 k_y^2 + \xi_4^2 k_z^2}, \tag{12}$$

which corresponds to an anisotropic Yukawa potential-like shape in real space. The induced imaginary component ψ'' of the order parameter leads to a local $\mathscr{T}$-violation. The length scales over which ψ_2'' decays are the anisotropic temperature-dependent coherence lengths $\xi_i^2 = K_i/a_*$ which diverge as T approaches T^*. Obviously, ψ'' is infinite at $r = 0$ in real space, because the use of a delta function in Eq. (3) eliminates the lower cutoff-length scale. Within the Ginzburg–

Landau theory the natural cutoff length is ξ_0. Therefore for the qualitative consideration of quantities at $r = 0$ we need a cutoff which is at least of order the zero-temperature coherence length of η. The order parameter modulation yields supercurrents in the form

$$\begin{aligned} \tilde{j}_x &= 4e\eta_0 i k_y K_2 \tilde{\psi}_2'' , \\ \tilde{j}_y &= 4e\eta_0 i k_x K_3 \tilde{\psi}_2'' , \end{aligned} \tag{13}$$

and $\tilde{j}_z = 0$ if we neglect the screening currents for the moment. We use now these currents as a source and calculate the induced vector potential

$$\begin{aligned} \tilde{A}_x &= \frac{4\pi}{Dc}\left[\tilde{j}_x(R_yR_z - k_y^2k_z^2) + \tilde{j}_y k_x k_y (R_z + k_z^2)\right], \\ \tilde{A}_y &= \frac{4\pi}{Dc}\left[\tilde{j}_y(R_xR_z - k_x^2k_z^2) + \tilde{j}_x k_x k_y (R_z + k_z^2)\right], \\ \tilde{A}_z &= \frac{4\pi}{Dc}\left[\tilde{j}_x k_x k_z (R_y + k_y^2) + \tilde{j}_y k_y k_z (R_x + k_x^2)\right], \end{aligned} \tag{14}$$

where

$$\begin{aligned} D &= R_xR_yR_z - 2k_x^2k_y^2k_z^2 - R_xk_y^2k_z^2 - R_yk_x^2k_z^2 - R_zk_x^2k_y^2 , \\ R_x &= \lambda_{123}^{-2} + k_y^2 + k_z^2 , \quad R_y = \lambda_1^{-2} + k_x^2 + k_z^2 , \quad R_z = \lambda_4^{-2} + k_x^2 + k_y^2 . \end{aligned} \tag{15}$$

We consider now the magnetic field distribution around the impurity site. Using $\mathbf{B} = [\nabla \mathbf{A}]$ we obtain for the Fourier-transformed magnetic field, $\tilde{\mathbf{B}} = i[\mathbf{k}\tilde{\mathbf{A}}]$,

$$\begin{aligned} \tilde{B}_x &= \frac{16\pi e\eta_0\tilde{\psi}_2'' k_x k_z}{Dc}\left[K_2k_y^2(\lambda_4^{-2} - \lambda_1^{-2}) + K_3(\lambda_{123}^{-2}k_x^2 + \lambda_4^{-2}R_x)\right], \\ \tilde{B}_y &= \frac{16\pi e\eta_0\tilde{\psi}_2'' k_y k_z}{Dc}\left[-K_2(\lambda_1^{-2}k_y^2 + \lambda_4^{-2}R_y) + K_3k_x^2(\lambda_{123}^{-2} - \lambda_4^{-2})\right], \\ \tilde{B}_z &= \frac{16\pi e\eta_0\tilde{\psi}_2''}{Dc}\left[K_2k_y^2(\lambda_4^{-2}k_z^2 + R_z\lambda_1^{-2}) - K_3k_x^2(\lambda_4^{-2}k_z^2 + R_z\lambda_{123}^{-2})\right]. \end{aligned} \tag{16}$$

The magnetic field distribution has a rather complicated structure, as we show for the B_z-component in figure. We do not analyze this structure further, but concentrate on the magnetic field at the site of the impurity. For this purpose we have to perform the Fourier transform from momentum space to real space. At $r = 0$ this corresponds simply to the $\mathbf{k}$-integral of $\tilde{\mathbf{B}}(\mathbf{k})$. We see immediately that there are no x- and y-components, because the angular dependence in k-space leads to an exact cancellation. The z-component, however, is finite, if we take the lower cutoff length into account properly.

As a consequence the magnetic field would lead to precession of the magnetic moment around the z-axis. This precession does not change the z-component of the moment so that the coupling term with the superconducting order parameter in Eq.(3) is not changed at all. Therefore the local superconducting state and its field distribution is essentially static despite the precession of the impurity moment. Regarding the muon as an impurity, one could measure the precession in the standard way through the muon decay into positrons. In a very clean material all muons are usually trapped in crystallographically equivalent (very symmetric) points and, consequently, have the same environment. If completely spin-polarized muons are injected, all of them should generate the same local magnetic field distribution and, hence, have the same precession frequency ω. The frequency ω depends, however, on the angle θ between the expectation value of the muon spin and the z-axis of the crystal. Because B_z is proportional to μ the frequency is

$$\omega \propto B_z(r=0) \propto \mu \propto \cos\theta. \tag{17}$$

Of course, the precession of the muon spin can only be seen if $0 < \theta < \pi/2$.

On the other hand, in a dirty sample the trapping positions of the muons may be scattered so that the magnetic field generated at the muons is spread over many values. Then we would not observe a pure precession, but rather a depolarization for the x-y-component of the spin. In both cases the effect should become stronger as we approach the transition at T^*.

3. MICROSCOPIC DERIVATION OF THE IMPURITY TERMS

In the following we discuss briefly the microscopic calculations of the coefficient μ as well as κ, ν and γ, assuming for simplicity hole-particle symmetry for the energy spectrum. In quasiclassical theory the basic equations for the propagators in the presence of the isolated impurity may be written as follows [6]

$$\begin{gathered}[i\varepsilon_n\hat{\tau}_3 - \hat{\sigma}(\mathbf{k}_F,\mathbf{R}), \hat{g}(\mathbf{k}_F,\mathbf{R};\varepsilon_n)] + i\mathbf{v}_F\nabla_R\hat{g}(\mathbf{k}_F,\mathbf{R};\varepsilon_n) = \\ = \left[\hat{t}(\mathbf{k}_F,\mathbf{k}_F;\varepsilon_n), \hat{g}_{int}(\mathbf{k}_F,\mathbf{R}=\mathbf{R}_{imp};\varepsilon_n)\right]\delta(\mathbf{R}-\mathbf{R}_{imp}).\end{gathered} \tag{18}$$

Here $\varepsilon_n = (2n+1)\pi T$ is the Matsubara frequency, $\mathbf{k}_F$ is the momentum direction on the Fermi surface, $\mathbf{v}_F(\mathbf{k}_F)$ is the Fermi velocity, and $\hat{\tau}_3$ is the third Pauli matrix in Nambu space.

The normalization condition for the matrix propagator is

$$\hat{g}^2(\mathbf{k}_F,\mathbf{R};\varepsilon_n) = -\pi^2\hat{1}. \tag{19}$$

Equations (18), (19) must be supplemented by the equation for the quasiparticle scattering t-matrix of the impurity

$$\hat{t}(\mathbf{k}_F,\mathbf{k}'_F;\varepsilon_n) = \hat{v}(\mathbf{k}_F,\mathbf{k}'_F) + N(0)\int\frac{d^2\mathbf{k}''}{4\pi}\hat{v}(\mathbf{k}_F,\mathbf{k}''_F)\hat{g}_{int}(\mathbf{k}''_F,\mathbf{R}=\mathbf{R}_{imp};\varepsilon_n)\hat{t}(\mathbf{k}''_F,\mathbf{k}'_F;\varepsilon_n). \tag{20}$$

Here $\hat{v}(\mathbf{k}_F,\mathbf{k}'_F)$ is the matrix of the impurity potential. The auxiliary quantity $\hat{g}_{int}(\mathbf{k}_F,\mathbf{R};\varepsilon_n)$ obeys the normalization condition and Eq. (18) without the t-matrix impurity term on the right-hand side.

The impurity potential matrix $\hat{v}(\boldsymbol{k}_F,\boldsymbol{k}'_F)$ may be represented in the form

$$\hat{v}(\mathbf{k}_F,\mathbf{k}'_F) = w_{kk'}\hat{1} + iv_{kk'}\hat{\tau}_3 + u_{kk'}\mathbf{M}\hat{\mathbf{S}} + i\mathbf{m}_{kk'}\hat{\mathbf{S}}\hat{\tau}_3. \tag{21}$$

Here terms $w_{kk'}$, $v_{kk'}$, $u_{kk'}$ and $\mathbf{m}_{kk'}$ describe the conventional nonmagnetic potential, the hyperfine interaction, the magnetic spin-spin and spin-orbit coupling respectively. The form of the spin operator $\hat{\mathbf{S}}$ is defined as in [11].

The Ginzburg–Landau equations are obtained by expanding the self-consistency equation

$$\Delta(\mathbf{k}_F, \mathbf{q}) = 2TN(0) \sum_{\varepsilon_n} \int \frac{d^2k'}{4\pi} V(\mathbf{k}_F, \mathbf{k}'_F) f(\mathbf{k}'_F, \mathbf{q}; \varepsilon_n) \tag{22}$$

in powers of the order parameter and its spatial derivatives. For one-dimensional representations the contributions from an isolated nonmagnetic impurity to the free energy functional were considered in [12, 13] for estimation of the vortex pinning potential. Since we are interested in the terms in Eq. (3), we can put $\mathbf{q} = 0$ in Eq. (22) omitting gradient terms. This equation is written in the form valid for singlet pairing, $\Delta(\mathbf{k}_F) = i\sigma_y \psi(\mathbf{k}_F)$, and for the particular kind of triplet pairing ($\Delta(\mathbf{k}_F) = i(\mathbf{d}(\mathbf{k}_F)\boldsymbol{\sigma})\sigma_y$ with $\mathbf{d} \parallel \mathbf{z}$, where $\mathbf{z}$ is the hexagonal crystalline axis), if one makes use of the notations $\Delta(\mathbf{p}) = \psi(\mathbf{p})$ for the former and $\Delta(\mathbf{p}) = d_z(\mathbf{p})$ for the latter cases. Mostly, these types of pairing are discussed for the analysis of experimental data of UPt_3 [10]. We consider the pairing potential of the form

$$V(\mathbf{p}, \mathbf{p}') = -\frac{g}{2} \left[\varphi_1(\mathbf{p})\varphi_1(\mathbf{p}') + \varphi_2(\mathbf{p})\varphi_2(\mathbf{p}')\right]$$

and assume the basis functions to be real and normalized according to $\int d\Omega \varphi^2_{1,2}(\hat{k}) = 4\pi$.

The solution of Eq. (20) in the second Born approximation and its substitution into Eq. (18) are straightforward, since one can use the bulk expression for the quantity $\hat{g}_{int}$ in the case $\sigma/\xi_0^2 \ll 1$, where σ is the quasiparticle cross-section for the impurity potential [5, 6]. From the solution of the Eilenberger equations in this approximation we obtain the impurity contribution to the anomalous propagator

$$f_{imp} = \left(\frac{\pi}{\varepsilon_n}\right)^2 N(0) \int \frac{d\Omega'}{4\pi} \left[-(v^2_{kk'} - m^2_{kk'} + M^2 u^2_{kk'} - w^2_{kk'})\Delta(\mathbf{k}'_F) - \right.$$
$$\left. - (v^2_{kk'} + m^2_{kk'} + M^2 u^2_{kk'} + w^2_{kk'})\Delta(\mathbf{k}_F) + 2i\left(v_{kk'}w_{kk'} - u_{kk'}\mathbf{m}_{kk'}\mathbf{M}\right)\Delta(\mathbf{k}'_F)\right]. \tag{23}$$

Only the last term of this expression for the f-function, substituted into the self-consistency equation (22), yields a finite value of μ,

$$\mu = \frac{\pi^2}{16} M_z \alpha v_F \lambda N^2(0) \int d\Omega \int d\Omega' (\Phi_{kk'} - b u_{kk'}) w_{kk'} (\hat{k}_x \hat{k}'_y - \hat{k}_y \hat{k}'_x) \varphi_1(\mathbf{k}_F)\varphi_2(\mathbf{k}'_F). \tag{24}$$

Here the coupling constant $\lambda = gN(0)$ is expressed in terms of the critical temperature in the conventional way ($T_c \propto \exp(-1/\lambda)$) and the matrix elements for the hyperfine and spin-orbit interactions are represented in the form

$$v_{kk'} = \left([\mathbf{k}_F \mathbf{k}'_F]\mathbf{M}\right) \Phi_{kk'}, \quad \mathbf{m}_{kk'} = b w_{kk'} [\mathbf{k}_F \mathbf{k}'_F].$$

Note, that for the point-like impurity potential, when $w_{kk'}$, $\Phi_{kk'}$ and $u_{kk'}$ do not depend upon momenta $\mathbf{k}$, $\mathbf{k}'$, the coefficient μ vanishes for singlet pairing. It is not the case for triplet superconductors due to the different parity properties of the basis functions $\varphi_{1,2}(\mathbf{k}_F)$ for singlet and triplet superconductors. This result may be justified beyond the Born approximation as well. Since the expression Eq. (24) is proportional to M_z, the coefficient μ changes its sign under the time-reversal operation, which ensures the time-reversal symmetry of the whole expression $i\mu(\eta_1\eta_2^* - \eta_1^*\eta_2)$.

4. CONCLUSION

We have demonstrated phenomenologically that a magnetic impurity can generate a locally $\mathcal{T}$-violating superconducting phase. This leads to a distribution of supercurrents and magnetic fields which acts on the magnetic moment. For the two representations E_1 and E_2 considered here, only the z-component of the magnetic moment couples to the superconducting order parameter and the resulting magnetic field has only a finite z-component at the impurity site. We have shown that this fact yields the precession of the magnetic moment without changing the locally $\mathcal{T}$-violating order parameter configuration. Thus, for injected muons this may lead to precession of the spin. However, it is difficult to estimate whether the generated magnetic field would be sufficiently large to really give an observable precession. Our discussion may also apply to other systems besides the UPt_3 we had in mind here. It is important for the enhancement of effects considered to be in the vicinity of a bulk transition to a superconducting state with broken time reversal symmetry.

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