

**SUPERCONDUCTIVITY FLUCTUATIONS IN A ONE-DIMENSIONAL TWO-BAND ELECTRON-PHONON MODEL WITH STRONG REPULSIVE INTERACTIONS***S. I. Matveenko\**, *A. R. Bishop†*, *A. V. Balatsky†\***\*Landau Institute for Theoretical Physics  
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We study a one-dimensional, two-band model with short-range electron-electron repulsions (onsite  $U$  and nearest-neighbor  $V$  terms) and electron-phonon coupling. We show that there is a region of  $U$ ,  $V$  and band filling in which singlet superconductivity fluctuations are dominant. This region is absent without electron-phonon interactions and includes large values of  $U$  and  $V$ .

**1. INTRODUCTION**

The physics of low-dimensional strongly correlated fermion systems with repulsive interactions is a topic of active interest, largely because the origin of high- $T_c$  superconductivity in cuprate oxides and the role of phonons in these correlated systems are not clearly understood. Using a simple one-dimensional (1D) Cu-O chain model [1], we investigate the effects of both short range electron-electron ( $e-e$ ) repulsive interactions (onsite  $U$  and nearest-neighbor Cu-O repulsion  $V$ ) and electron-phonon ( $e-p$ ) coupling in the ground state of the system. We show that superconducting (SC) correlations are absent in the model if we take into account  $e-e$  interactions only. The inclusion of  $e-p$  interactions leads to the appearance of a  $(U, V, \rho)$  region (where  $\rho$  is the band-filling) in which superconducting fluctuations are dominant. On the other hand, the ground state of the system in the absence of  $e-e$  repulsion is a state with a charge-density wave (CDW) or spin-density wave (SDW) state without a divergent SC response. Thus, the region with dominant SC response results from the combined effect of  $e-e$  and  $e-p$  interactions for this model.

We use a renormalization-group (RG) two-cutoff approach developed in earlier works [2, 3]. With some assumptions on the model parameters, our analysis is valid in the limit of large  $U$  and  $V$ . The possibility of SC fluctuations in quasi-1D systems with strong repulsive  $e-e$  interaction and  $e-p$  coupling was first raised in work of Zimanyi et al. [3], where results are obtained for a massive Thirring model. The two-band model without  $e-p$  interaction was considered in [1], where numerical results are presented, pointing out the possible existence of SC fluctuations in the strong-coupling limit. This statement is based on the numerical results, which point to the presence of phase separation in the strong-coupling limit, and an intuitive assumption that results obtained for the Luttinger liquid in the weak-coupling limit are valid qualitatively in the strong-coupling limit for the quantum lattice model. Then in some vicinity of the phase separation (where the correlation exponent satisfies  $K_\rho \rightarrow \infty$ ) one has a divergent SC response if  $K_\rho > 1$  holds (see Eq. (24) below). Here we investigate RG weak-coupling solutions. Therefore we do not consider phase separation. We show that in a region of sufficiently large

$e$ - $e$  repulsion, where the RG approach is still applicable, a ground state with a dominant SC response can be achieved due to the interplay of  $e$ - $e$  and  $e$ - $p$  interactions. We will show that, due to two-band hybridization, the repulsion amplitudes  $U$  and  $V$  are multiplied by small parameters in the RG equations. This allows us to consider larger  $U$  and  $V$  values, which exceed, for example, the width of the upper band.

The plan of this article is as follows. In Sec. 2 we define our Hamiltonian and calculate the band structure in the absence of  $e$ - $e$  and  $e$ - $p$  terms. In Sec. 3 we take into account  $e$ - $e$  and  $e$ - $p$  terms, and construct the ground-state phase diagram on the basis of our RG analysis. In the Conclusion we discuss our results and their implications.

## 2. THE HAMILTONIAN

We consider a chain consisting of two types of atoms: Cu on odd sites with  $d$ -orbitals and O on even lattice sites with  $p$ -orbitals. The Hamiltonian of the system is

$$H = H_0 + H_{ee} + H_{ep}, \tag{1}$$

$$H_0 = -t \sum_{\langle i,j \rangle} c_{p,i}^+ c_{d,j} + \text{h.c.} + \sum_i \Delta (c_{p,i}^+ c_{p,i} - c_{d,i}^+ c_{d,i}), \tag{2}$$

$$H_{ee} = \sum_{\alpha=d,p} \sum_i U_{\alpha} c_{\alpha,i,\uparrow}^+ c_{\alpha,i,\uparrow} c_{\alpha,i,\downarrow}^+ c_{\alpha,i,\downarrow} + V \sum_{\langle i,j \rangle} c_{d,i}^+ c_{d,i} c_{p,j}^+ c_{p,j}, \tag{3}$$

where  $t$  is the hopping integral,  $\langle i, j \rangle$  are nearest-neighbor sites,  $\Delta = (E_p - E_d)/2$ ,  $E_p$  and  $E_d$  are site energies,  $U_d$  and  $U_p$  are Hubbard onsite repulsive energies, and  $V$  is the repulsion amplitude between nearest-neighbor sites. Direct antiferromagnetic coupling between Cu sites is omitted. Also

$$H_{ep} = H_{ep,1} + H_{ep,2}, \tag{4}$$

and we consider two models of electron-phonon coupling: the molecular crystal (MC) model with the Hamiltonian  $H_{ep,1}$  in which optic phonons couple to the electron site energy, and the Su-Schrieffer-Heeger (SSH) model with the Hamiltonian  $H_{ep,2}$  in which the lattice distortions modulate the electron-hopping matrix element  $t$ . The Hamiltonian  $H_{ep,1}$  consists of two parts:  $H_{ep,1} = H_{ep,d} + H_{ep,p}$ , where each part has the form

$$H_{ep} = \sum_i \frac{P_i^2}{2M} + \frac{1}{2} \kappa q_i^2 + \lambda q_i \rho_i = \sum_k \omega_0 \left( d_k^+ d_k + \frac{1}{2} \right) + \frac{g}{\sqrt{N}} (d_k + d_k^+) \rho_k \tag{5}$$

with  $\omega_0 = \sqrt{\kappa/M}$ ,  $g = \lambda/\sqrt{2M\omega_0}$ ,  $\rho_k = \sum c_{k+q}^+ c_q$ . Here  $M$  is the ion mass,  $\omega_0$  is the optic-phonon frequency,  $\kappa$  is the elasticity constant, and  $\lambda$  is the  $e$ - $p$  coupling constant. All terms in (5) have labels  $d$  or  $p$ , and the sum is over odd or even sites for  $H_{ep,d}$  or  $H_{ep,p}$ , respectively. The Hamiltonian  $H_{ep,2}$  takes into account intermolecular phonon modes:

$$\begin{aligned} H_{ep,2} &= \sum_i \frac{P_i^2}{2M} + \frac{1}{2} \kappa (q_{i+1} - q_i)^2 - \sum_{\langle i,j \rangle} \delta t_{i,j} c_{d,i}^+ c_{p,j} = \\ &= \sum_k \omega_k \left( f_k^+ f_k + \frac{1}{2} \right) + \frac{1}{\sqrt{N}} \sum_{k,q} g(k,q) (f_q + f_{-q}^+) c_{d,k+q}^+ c_{p,k}, \end{aligned} \tag{6}$$

where  $\delta t_{i,j} = \lambda(q_i - q_j)$ ,  $\omega_q = 2\sqrt{\kappa/M} \sin(qa/2)$  is the acoustic-phonon frequency,  $g(k, q) = 4i\lambda \sin(qa/2) \cos(ka + qa/2) / \sqrt{2M\omega_q}$ , and  $a$  is the Cu-O lattice constant.

First we consider the noninteracting Hamiltonian  $H_0$ . Diagonalization gives

$$H_0 = \sqrt{4t^2 \cos^2 ka + \Delta^2} [c_2^+(k)c_2(k) - c_1(k)^+ c_1(k)], \quad (7)$$

where

$$\begin{aligned} c_d(k) &= \cos \theta_k c_1(k) + \sin \theta_k c_2(k), \\ c_p(k) &= -\sin \theta_k c_1(k) + \cos \theta_k c_2(k) \end{aligned} \quad (8)$$

with  $\text{tg}(2\theta_k) = -2t \cos(ka) / \Delta$ ,  $-\pi/2 < 2\theta_k < \pi/2$ . Now we have a two-band electronic structure and consider the case of an entirely filled lower band. The filling factor of the upper band is  $0 < \rho < 2$  (empty for  $\rho = 0$  and filled for  $\rho = 2$ ). With unit cell  $2a$ , the quasi-momenta  $k$  and  $k + \pi/a$  are equivalent, and we may assume that the states in the lower band have quasi-momenta in the interval  $-\pi/2a < k < \pi/2a$  and that in the upper band in the interval  $\pi/2a < |k| < \pi/a$ ; then  $k_F a = \pi/2 + \pi\rho/4$ . The Fermi velocity is

$$v_F = -\frac{2at^2 \sin(2k_F a)}{\sqrt{4t^2 \cos^2(k_F a) + \Delta^2}}. \quad (9)$$

### 3. RG TREATMENT

Since we will use an RG approach we take into account only states in the upper band in the vicinity of  $E_F$  which are described by the operators  $c_2$ . Then  $H_0$  has the form, in the  $x$ -representation,

$$H_0 = v_F \Psi_{2,+}^+ \left( -i \frac{\partial}{\partial x} \right) \Psi_{2,+} + v_F \Psi_{2,-}^+ \left( i \frac{\partial}{\partial x} \right) \Psi_{2,-}, \quad (10)$$

where  $\Psi_{2,\pm}$  include momenta near  $\pm k_F$ , respectively. Below we will omit the subscript 2 and also terms in  $H$  with  $\Psi_1$ . (Taking into account the terms with  $\Psi_1$  can produce a shift of the chemical potential and some renormalization of the Fermi velocity.) Therefore in the Hamiltonian we can make the replacements

$$\Psi_d \rightarrow \sin \theta_F \Psi(x), \quad \Psi_p \rightarrow \cos \theta_F \Psi(x). \quad (11)$$

Note that in the case  $t/\Delta \ll 1$  or  $\rho \ll 1$  we have

$$\sin \theta_F \approx \theta_F \approx \frac{t \sin(\pi\rho/4)}{\Delta}. \quad (12)$$

First we consider  $e$ - $e$  interaction effects. For the Cu-O case it is appropriate to consider  $U_d \gg U_p$ . Let us study the case  $U_p = 0$ . The effect of small  $U_p$  is easily taken into account and will be discussed below. In terms of a «g-ology» model [4], the Hamiltonian  $H_{ee}$  gives the scattering amplitudes

$$g_1 = \frac{Ua}{2} \sin^4 \theta_F + 2Va \sin^2 \theta_F \cos(2k_F a) = g_3, \tag{13}$$

$$g_2 = \frac{Ua}{2} \sin^4 \theta_F + 2Va \sin^2 \theta_F = g_4,$$

where  $g_1$  is the backscattering amplitude, and  $g_2$  and  $g_4$  are forward scattering amplitudes. The «Umklapp» part  $g_3$  exists only for the half-filled case  $\rho = 1$ : for simplicity we will not consider this case. Since we use a RG approach below, we consider  $g_i/\pi v_F \leq 1$ , i.e.,  $Ua, Va \leq \pi v_F$  or  $\sin \theta \ll 1$  for large  $U$  and  $V$ . We have a spin-rotation invariance, i.e.,  $g_{\perp} = g_{\parallel}$ . Therefore, when they are not essential, we will omit spin indices. The effect of the  $g_4$  term is taken into account separately: it simply produces a shift in the velocity of the spin and charge degrees of freedom:  $v_{\sigma} = v_F(1 + g_4)$ ,  $v_{\rho} = v_F(1 - g_4)$ .

The familiar RG equations defining the scaling behavior of the system are [4]

$$g'_1 = \frac{1}{\pi v_{\sigma}} g_1^2, \tag{14}$$

$$g_c \equiv g_1 - 2g_2 = \text{const.} \tag{15}$$

For  $g_1 \geq 0$  the excitation spectrum is gapless,  $g_1 \rightarrow g_1^* = 0$ , while there is a gap if  $g_1 < 0$ . The charge excitation spectrum is gapless for  $g_c \geq 0$  and has a gap  $\Delta_{\rho}$  if  $g_c < 0$ . The ground state has the most divergent singlet (triplet) SC response for  $g_c \geq 0$  and  $g_1 < 0$  (or  $g_c \geq 0$  and  $g_1 \geq 0$ ). In our case

$$g_c = -\frac{Ua}{2} \sin^4 \theta_F + 2Va \sin^2 \theta_F \cos^2 \theta_F [\cos(2k_F a) - 2] < 0. \tag{16}$$

Therefore there is no region in  $(U, V)$  with divergent SC fluctuations, in accordance with results [1] for the weak-coupling case. The possible ground states are a CDW or SDW, depending on the sign of  $g_1$ . (This sign can vary due to the  $\cos k_F a$  term.) We see that in order to obtain SC correlations it is necessary to have large positive  $g_1^*$  or negative  $g_2^*$  terms. As we will see below, this condition can be achieved by taking into account an appropriate  $e$ - $p$  interaction.

Second-order perturbation theory in  $e$ - $p$  interaction produces a retarded  $e$ - $e$  interaction [2] for  $\omega$  less than the Debye frequency,  $\omega < \omega_D \sim \sqrt{\kappa/M}$ . (We consider the case  $\omega_D < E_F$ .) The effective  $e$ - $e$  interaction can be described in «g-ology» terminology:  $g_{1,ph} = -2g^2(k_F, 2k_F)/\omega_{2k_F}$ ,  $g_{2,ph} = -2g^2(k_F, 0)/\omega_0$ ,  $g_{3,ph} = g_{1,ph}$  (half-filled band only). In the case of the MC model (5) we have

$$g_{1,ph} = g_{2,ph} = g_{3,ph} = -\frac{\lambda^2}{4\kappa}, \tag{17}$$

whereas the SSH model (6) gives

$$g_{1,ph} = g_{3,ph} = -4\frac{\lambda^2}{\kappa}(\sin^2 \theta_F \cos^2 \theta_F). \tag{18}$$

The parameters  $\kappa$  and  $\lambda$  in (17) and (18) are, of course, different, as well as the other parameters in the Hamiltonians  $H_{ep,1}$ ,  $H_{ep,2}$ . Note that all terms are negative, and  $g_{2,ph}$  is due solely to onsite  $e$ - $p$  coupling and does not contain renormalization terms  $\sin \theta_F$  and  $\cos \theta_F$ . In the case

$\theta \ll 1$ , the onsite  $e$ - $p$  interaction is dominant. Now we have two types of  $e$ - $e$  interaction with cutoffs  $E_F$  and  $\omega_D$ . Thus we use the RG procedure [2, 3] for a two-cutoff model. The one-loop scaling equations (14) and (15) for  $g_i$  are unaffected by the presence of retarded interaction. The equations for the  $g_{i,ph}$ , taking into account the cross terms  $g_i g_{j,ph}$ , were derived in [3]:

$$g'_{1,ph} = \frac{1}{\pi v_F} \left( \frac{3}{2} g_1 + \frac{1}{2} g_c + g_{1,ph} \right) g_{1,ph}, \quad (19)$$

$$g'_{2,ph} = 0. \quad (20)$$

We shall consider the case  $g_{3,ph} = 0$ . The integration in (19) and (20) is taken from  $E_F$  to  $\omega_0 \sim \omega_D(\omega_0)$ , where  $\omega_D(\omega_0)$  is the renormalized value of  $\omega_D$  [3]. As a result, the combined action of different scattering processes is described by

$$g_i^T = g_i^* + g_{i,ph}^*. \quad (21)$$

The properties of the system at energies small compared to  $\omega_0$  are derived from a model with single interactions  $g_i^T$  and bandwidth  $\omega_0$ .

Now we examine the solutions of Eqs. (14), (15), (19), and (20). The initial conditions for (14) and (15) are defined by (13). The initial conditions for (19) and (20) are defined by (17) and (18). We write  $g_{1,ph}^{(0)} = -\gamma$ ,  $g_{2,ph}^{(0)} = -\tilde{\gamma}$ . If  $g_1^{(0)} \geq 0$  holds (we shall see that this is the situation in the interesting region), from (12) we find that  $g_1$  scales toward small positive values  $g_1^* \ll g_1^{(0)}$ . Note that in the case  $\theta \ll 1$  we have  $g_{1,ph}^{(0)} \approx g_{2,ph}^{(0)}$ , i.e.,  $\tilde{\gamma} \approx \gamma$ . From (13) it follows that  $g_1^* - 2g_2^* = g_1^{(0)} - 2g_2^{(0)}$ . A positive derivative in (19) implies that  $g_{1,ph}$  scales toward large negative values. We consider the opposite case  $g'_{1,ph} < 0$ . Then, at least initially,  $g_{1,ph}$  will scale toward a small negative value. Therefore we demand that

$$\frac{3}{2} g_1^{(0)} + \frac{1}{2} g_c^{(0)} + g_{1,ph}^{(0)} > 0, \quad (22)$$

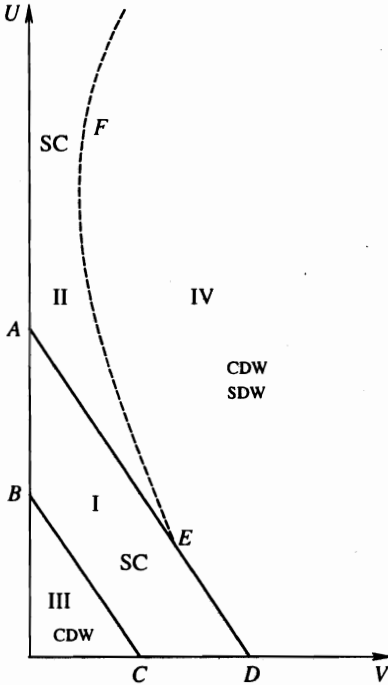
since  $g_{1,ph} < 0$ . The inequality (22) can not be valid throughout the scaling process, since  $g_1$  scales to small values. Therefore the value  $g_{1,ph}^*$  may not be very small. We do not require  $|g_{1,ph}^*| \ll \gamma$ ; for our purposes it is sufficient that  $g_{1,ph}^* > \gamma - 2\tilde{\gamma}$  holds, as we show below. The value  $g_{2,ph}$  is not scaled as follows from (20), i.e.,  $g_{2,ph}^* = -\tilde{\gamma}$ . This value does not contain the renormalization coefficient  $\sin \theta_F$ . As a result of scaling we have the state with  $g_i^T = g_i^* + g_{i,ph}^*$ . The ground state of the system with the new scaling amplitudes has dominant divergent SC susceptibility if

$$g_c^T = g_1^T - 2g_2^T = g_1^{(0)} - 2g_2^{(0)} + g_{1,ph}^* + 2\tilde{\gamma} > 0. \quad (23)$$

Since we assume that  $g_{1,ph}^T \approx g_{1,ph}^* < 0$ , we have a state with spin gap  $\Delta_\sigma$ . Therefore the dominant singularity is the singlet SC response with SC correlation function

$$R(x) \sim x^{-1/K_\rho}, \quad K_\rho = \sqrt{\frac{1 + g_c^T/2\pi v_\rho}{1 - g_c^T/2\pi v_\rho}} > 1. \quad (24)$$

In this case the CDW response can be divergent with a correlation function  $\propto x^{-K_\rho}$ . The inequalities (22) and (23) define the region in which the singlet SC correlations are dominant.



Phase diagram obtained by two-cutoff RG scaling. The divergent SC response regions are I and II

In terms of  $u = (Ua/2) \sin^4 \theta$ ,  $v = Va \sin^2 \theta \cos^2 \theta$  we rewrite (22) and (23) as

$$\gamma + 2v \left( 1 + 2 \cos \frac{\pi \rho}{2} \right) < u < 2\gamma^* - 2v \left( 2 + \cos \frac{\pi \rho}{2} \right), \tag{25}$$

where  $2\gamma^* = 2\tilde{\gamma} + g_{1,ph}^*$ . It is easy to obtain the solution of (25). This is the region  $ABCD$  in Figure bounded by the lines  $u = 0, v = 0, u = \gamma - 2v, u = 2\tilde{\gamma} + g_{1,ph}^* - 2v$ . Recall that in the limit  $\theta \ll 1$ , Eq. (12), the bare repulsive energies satisfy  $U \sim u/\theta^4, V \sim v/\theta^2 \gg \gamma$ . Thus our model includes the case of strong electron repulsion. For any point  $(u, v)$  in the region  $ABCD$  the inequality (25) is valid for

$$\rho > \frac{2}{\pi} \cos^{-1} \left( \max \left\{ \frac{u - \gamma - 2v}{4v}, \frac{2\gamma^* - 4v - u}{2v} \right\} \right). \tag{26}$$

In the limit  $t/\Delta \ll 1$  we can obtain the phase diagram in terms of the bare values  $U$  and  $V$ . Then the coordinates of the points  $A, B, C$ , and  $D$  are  $A = \{0, (4\gamma^*)(\Delta/t)^4\}$ ,  $B = \{0, 2\gamma(\Delta/t)^4\}$ ,  $C = \{(\gamma/2)(\Delta/t)^2, 0\}$ , and  $D = \{(\gamma^*/2)(\Delta/t)^2, 0\}$ . The SC region is deformed to include region II due to the  $\sin(\pi\rho/4)$  term. The equation of the curve  $EF$  is

$$V = \frac{\Delta^2}{t^2} \frac{((2\gamma^* - \gamma)k + 4\gamma + 16\gamma^*)^2}{72(k + 2)(\gamma + 2\gamma^*)}, \quad U = 2kV \frac{\Delta^2}{t^2}. \tag{27}$$

In the limit  $k \rightarrow \infty$  we have  $U \propto V^2$ , but in this region  $\rho \sim 1/U^{1/4} \rightarrow 0$ . The inequality (25) with  $t/\Delta \ll 1$  becomes

$$\begin{aligned} (U_1 + 8V_1)y^2 - 6V_1y - \gamma &> 0, \\ (4V_1 - U)y^2 - 6V_1y + 2\gamma^* &> 0, \end{aligned} \tag{28}$$

where  $V_1 = V(\Delta/t)^2, U_1 = (U/2)(\Delta/t)^4, y = \sin^2(\pi\rho/4)$ . In region I the solution of (28) is  $\rho_0 < \rho < 2$ , where  $\sin^2(\pi\rho_0/4) = y_0$  is the largest root of Eq. (28). In region II we have  $\rho_1(U, V) < \rho < \rho_2(U, V)$ , where  $\rho_1$  and  $\rho_2$  can be easily obtained from (28). If  $V = 0$  holds, the solution is

$$\frac{4}{\pi} \sin^{-1} \left( \frac{\gamma}{U_1} \right)^{1/4} < \rho < \frac{4}{\pi} \sin^{-1} \left( \frac{2\gamma^*}{U_1} \right)^{1/4} \tag{29}$$

for  $U_1 > \gamma$ ; if  $U = 0$  holds, then in the region  $\gamma/2 < V_1 < \gamma^*$  the solution is  $\sin^2(\pi\rho/4) > y_0$ , where  $y_0$  is the largest root of Eq. (28) for  $U = 0$ .

In using the RG approach, we supposed as usual that  $g_i/\pi v_F < 1$ . For small  $t/\Delta$  we have the initial value  $v_F^{(0)} \sim (t^2/\Delta) \sin(\pi\rho/2)$ . Recalling that  $g_i \sim V[t \sin(\pi\rho/4)/\Delta]^2$  or  $U[t \sin(\pi\rho/4)/\Delta]^4, g_{2,ph} = \text{const}$ , we can regard our results as reasonable if we are not too close to band edges, where  $v_F \rightarrow 0$ , i.e.,  $\epsilon_1 < \rho < 2 - \epsilon_2$  and  $\rho \neq 1$  ( $g_3 = 0$ ). It follows from our analysis that in region III we have the large spin and charge gaps, so that there is only a CDW divergent response. In region IV we have  $g_c^T < 0$ , small  $g_1^T < 0$  and thus divergent CDW and SDW (in the limit  $g_1^* \rightarrow 0$ ) responses.

We considered the effect of the lower band only through the renormalization of the bare values  $U$  and  $V$ . Thus, we did not take into account the terms  $(V\Theta^2 + U)\Psi_1^+\Psi_1\Psi_1^+\Psi_1 + (V + U\Theta^2)\Psi_1^+\Psi_1\Psi_2^+\Psi_2$ . Therefore our results are valid in the region  $U, V \leq E_{\text{gap}} \sim \Delta$ . In order to estimate the effect of the cross term, we can rewrite our two-band model in terms of two-chain model and use the results of RG investigation [5]. It is easily to see that electron-hole pair interchain hopping is irrelevant (scales to small values), if  $U > V$ . Thus we can consider the region of large values  $U$  and  $V$  ( $U > V$ ) in comparison with the upper-band width  $\sim t^2/\Delta \ll \Delta$ .

In this treatment we have not taken into account the effects of  $U_p$  repulsion. This is easily achieved by substituting  $u$  into (20) in the form  $u = (Ua \sin^4 \theta_F + U_p a \cos^4 \theta_F)/2$ . For small values of  $U_p$  the RG approach remains valid, and all results continue to hold in terms of the new  $u$  and  $v$ . For  $t/\Delta \ll 1$  we have  $\cos \theta_F \sim 1$ , so that we cannot consider the large- $U_p$  limit in our approach.

#### 4. CONCLUSIONS

In conclusion, using a two-cutoff RG approach we have studied a two-band, 1D tight-binding model with  $e-e$  and  $e-p$  interactions. We included onsite  $U$  and nearest-neighbour  $V$  electron repulsions, as well as intra- and inter-molecular  $e-p$  coupling. We have shown (in accordance with [1]) that there is no  $U, V, \rho, t, \Delta$  parameter region with dominant divergent SC response in the absence of  $e-p$  interaction. In the lowest-order RG approach we found that such a region does occur if we include  $e-p$  coupling with optical intra-molecular modes. Only this form of  $e-p$  interaction produces an effective renormalized  $g_{2,ph}$  term. We have found that the singlet SC region includes large values of the  $U$  and  $V$  repulsive interactions if  $t \sin(\pi\rho/4)/\Delta \ll 1$ . Note that a similar behaviour is possible in a one-band model, for which  $\Delta = 0$ . Then, instead of (25), we have

$$\gamma + 2V [1 - 2 \cos(\pi\rho)] < U < 2\tilde{\gamma} + g_{1,ph}^* - 2V [2 - \cos(\pi\rho)], \tag{30}$$

where  $\gamma = -g_{1,ph}^{(0)}, \tilde{\gamma} = -g_{2,ph}^{(0)}, 0 < \rho < 2$ . The solution of (30) is the same region  $ABCD$  in Figure provided that  $2\tilde{\gamma} + g_{1,ph}^* > \gamma$ . However, the bare values  $U$  and  $V$  must be small, of the

order of phonon scattering strengths. Note also that we have used a RG approach. Therefore we did not consider the strong-coupling limit ( $V, U \gg t, \Delta$ ), where a phase separation instability could take place [1].

The main results of our treatment are the following:

1) Using a 1D two-band model, we have taken into account both  $e$ - $p$  coupling and  $e$ - $e$  repulsion and have shown that there is a region of parameters with dominant divergent SC response. This effect is absent in the model without  $e$ - $p$  coupling and is a result of the interplay of  $e$ - $e$  and  $e$ - $p$  interactions.

2) We have found that weak  $e$ - $p$  interactions and relatively strong  $e$ - $e$  repulsions can result in an effective electron pairing and a divergent SC response. This is possible in the limit  $t/\Delta \ll 1$ , where we can take into account large  $U$  and  $V$  values, since effective  $e$ - $e$  interactions are scaled by a factor  $t/\Delta$ . As a result we find that dominant divergent SC fluctuations are possible in the region  $V(t/\Delta)^2 \sim U(t/\Delta)^4 \sim g_{\text{ph}}$ ,  $V < U < \Delta$ , as shown in Figure.

3) We have found that dominant SC fluctuation states are possible only in a some interval  $\rho_1 < \rho < \rho_2$  of band filling.

4) We have found that only  $e$ - $p$  interaction with optical intramolecule phonon modes can result in SC.

5) Our conclusions are valid also beyond the limit  $t/\Delta \ll 1$  for the two-band model and for the one-band model ( $\Delta = 0$ ). But in these cases the SC fluctuation ground state is possible in the region of relatively small repulsive constants ( $U, V \sim g_{\text{ph}}$ ).

This model without  $e$ - $p$  coupling was studied in [1] where some indications of SC fluctuations in the strong-coupling limit were obtained. We have considered a substantially another region.

We have proposed one possible scenario for the origin of dominant SC fluctuations in quasi-one-dimensional systems as a result of the combined effect of repulsive  $e$ - $e$  and attractive  $e$ - $p$  interactions in a two-band situation. We suggest that features of this picture will survive in analogous two-dimensional models of high- $T_c$  superconductors, in particular in three-band Peierls-Hubbard models [6]. However, the orbital structure of the order parameter in this case ( $s$ -wave vs  $d$ -wave) is unclear without detailed calculations.

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