Electromagnetic radiation accompanying fast phase transitions in polarized bodies

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The processes involved in the transformation of the external static field of ferromagnets and ferroelectrics into radiation are examined. Attention is focused mainly on the case of fast phase transitions, during which the polarization decay (or rise) time is much shorter than the light propagation time over the maximum diameter of the body. It is shown that highly efficient conversion of static polarization energy into directed radiation occurs in this case. The spatiotemporal characteristics of the radiation are found.
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1. INTRODUCTION

A large number of studies of the electrical phenomena accompanying phase transitions, specifically, during crystallization of water and some dielectrics have been performed in the last few years (see, for example, Refs. 1 and 2). The observed effects are weak and difficult to interpret. In the present paper we examine the problem of electromagnetic radiation accompanying phase transitions from a different standpoint, which will make it possible not only to monitor the strength of the observed effects but also to use them for constructing new types of radiators.

It is well known that, as a rule, single-domain ferromagnets at temperatures below the Curie point possess an external magnetic field and at higher temperatures there is no external field. This raises the question of whether the energy stored in the external field goes into radiation or heat. The same question can also be raised for ferroelectrics, though in this case the effect of free surface charges and currents makes the answer less certain. As far as we know, this question was first stated clearly in Ref. 3, where radioemission accompanying phase transitions in ferroelectrics was studied. Unfortunately, the quasistationary approach employed in Ref. 3 does not, in principle, describe the most interesting strong-radiation effects, since during slow phase transitions virtually all of the energy stored in the external field is dissipated in the form of heat and the radiation effects are too weak compared with other effects, for example, the fields of surface charges. Evidently, this is why the experiments performed in Ref. 4 do not agree with the theory of Ref. 3. Nonetheless, it is possible to produce conditions under which a large fraction of the external static electromagnetic energy of a polarized body can be converted into electromagnetic radiation. This was proved in Ref. 5, where it was shown for the case of a rapidly decaying ring current that under certain conditions—for relaxation times shorter than the light propagation time over the maximum diameter of the ring— the system can radiate efficiently. This result is completely applicable also to phase transitions in polarized bodies.

Our objective in the present paper is to investigate the transformation of the energy stored in the external fields of polarized bodies into radiation during rapid phase transitions. In Sec. 2 a qualitative analysis of the problem is presented and it is shown that in quasi-one- or quasi-two-dimensional single-domain samples a fast phase transition and therefore efficient emission of radiation are possible. In Secs. 3 and 4, the analytical results of an investigation of the radiation processes accompanying a fast phase transition in a discoid (quasi-two-dimensional) ferromagnetic sample and a needle-shaped (quasi-one-dimensional) ferroelectric sample, respectively, are presented and it is shown that the energy stored in the external static electric magnetic field can be efficiently converted into radiation. In the concluding section, the results are discussed and the prospects for further investigations are outlined.

2. QUALITATIVE ANALYSIS OF THE PROBLEM

Consider a single-domain sample at a temperature below the Curie point. If it is assumed that somehow the sample can instantaneously go over to a nonconducting paramagnetic state, then a nonzero external field will evolve according to Maxwell's laws for free space and therefore the energy of the sample will be converted completely into electromagnetic radiation. In the opposite case, when the sample is heated adiabatically, of course, there will be no radiation.

In the intermediate case of a phase transition with a characteristic time τ_0 it can be assumed that the energy contained in a region of space of thickness $c\tau_0$ next to the body flows into the body and is dissipated there, and the energy outside this region is radiated, since at times $t \gg \tau_0$ there are no currents in the system and the free fields are converted into radiation (see Fig. 1). For sufficiently short phase-transition times a region of thickness $c\tau_0$ next to the sample contains a very small fraction of the total external energy. This means that in this regime the radiation efficiency is high.

An instantaneous phase transition is impossible, and to obtain a sufficiently rapid transition with minimum energy expenditure quasi-one- or quasi-two-dimensional structures must be used. The main condition for efficient conversion of the energy stored in the static field into radiation can be satisfied by means of the shock action of a laser or other beam on the sample at its thinnest point:[5]

$$c\tau_0 \ll r_{\max}, \tag{1}$$

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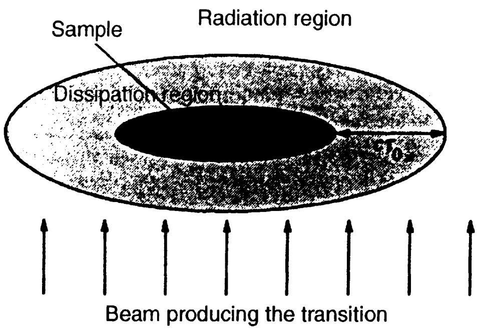

FIG. 1. Geometry of the quasi-two-dimensional problem.

where r_{max} is the maximum characteristic size of the sample and τ_0 is the total phase-transition time. This inequality guarantees that the dissipated energy will be very small compared with the radiated energy. In our case the transition time is determined by the velocity v_s of the (shock) wave of the phase transformation and the scale size r_{min} of the thinnest part of the sample, i.e., efficient energy conversion occurs under the condition

$$\tau_s=\frac{r_{min}}{v_s}\ll\frac{r_{max}}{c}. \qquad (2)$$

As the polarized state decays, part of the energy stored in the external field in the region of space next to the sample will be dissipated in the sample.

Since the ratio of the velocity of the shock wave to the velocity of light is small, a regime of efficient emission of radiation can be obtained quite simply only for thin films or needle-shaped samples. Furthermore, even when the external conditions are produced, the phase transition does not occur instantaneously and along with (2), the relaxation time τ_r must be short:

$$\tau_r\ll\frac{r_{max}}{c}. \qquad (3)$$

3. CHARACTERISTICS OF THE RADIATION ACCOMPANYING FAST RELAXATION OF A SINGLE-DOMAIN DISCOID FERROMAGNET

In this section, to find the radiation fields, we shall examine the case when the magnetization of a sample in the form of an oblate ellipsoid of revolution varies according to a fixed temporal law but in the process remains constant in direction. The variation of the magnetization will be equivalent to a change in an effective surface current. Further, if attention is confined to a thin disc (and only such samples can radiate efficiently), then the surface current will be concentrated mainly far from the symmetry axis of the problem and can be well approximated by a ring current with a corresponding magnitude and radius.

The total effective current I_0 can be easily found from the relation

$$I_0=\int\int_s ds\ c\ \mathrm{curl}(M_0)_\varphi=cM_0 2b, \qquad (4)$$

where M_0 is the initial magnetization.

The effective radius ρ_0 of the approximating ring current can be found from the definition of the magnetic moment of the current

$$\frac{I_0}{c}\pi\rho_0^2=\frac{4}{3}\pi a^2 bM_0, \qquad (5)$$

whence an expression can be found for ρ_0:

$$\rho_0=\sqrt{\frac{2}{3}}a. \qquad (6)$$

Therefore the problem of radiation emission from a discoid ferromagnetic sample is completely equivalent to the problem of a rapidly decaying ring current, which was completely solved in Ref. 5 in the case of a current varying according to the law

$$I(t)=I_0\left(1-\frac{2}{\pi}\arctan\frac{t}{\tau_0}\right). \qquad (7)$$

The electric component of the radiation field is described by the expression

$$E_\varphi=\frac{\sqrt{2}\rho_0 I_0}{c^2 R\tau_0}F(\tau,\theta), \qquad (8)$$

where

$$F(\tau,\theta)\frac{1}{\varepsilon\sqrt{(\tau^2-\varepsilon^2-1)^2+4\tau^2}}$$
$$\times[\sqrt{\sqrt{(\tau^2-\varepsilon^2-1)^2+4\tau^2}-\varepsilon^2-1+\tau^2}$$
$$-\tau\sqrt{\sqrt{(\tau^2-\varepsilon^2-1)^2+4\tau^2}+\varepsilon^2+1-\tau^2}]. \qquad (9)$$

Here the retarded time $\tau=(t-R_0/c)/\tau_0$ is used, and $\varepsilon=\nu\sin\theta$ and $\nu=\rho_0/c\tau_0$. In addition, as usual, R is the distance from the center of the body to the observation point. The electric field in the radiation zone (more accurately, the dimensionless function (9)) as a function of the dimensionless time τ and the observation angle θ, measured from the symmetry axis of the system, is shown in Fig. 2.

The radiation spectrum is described by the expression

$$E_\varphi(\omega)=\int dt\ e^{i\omega t}E_\varphi(t)=\frac{2\pi\rho_0 I_0 i}{c^2 R}J_1\left(\frac{\omega\rho_0\sin\theta}{c}\right)$$
$$\times\exp(-|\omega\tau_0|). \qquad (10)$$

We obtain for the directional pattern of the radiation, by which we mean here the energy passing through a given solid angle

$$W(\theta)=R^2\int dt\ \mathbf{S}(\vartheta,t),\quad \mathbf{S}=\frac{c}{4\pi}\mathbf{E}\times\mathbf{B}, \qquad (11)$$

the expression

$$W(\vartheta)=\frac{\rho_0^2 I_0^2}{\pi c^3\tau_0\varepsilon^2}\left(\frac{2+\varepsilon^2}{\sqrt{1+\varepsilon^2}}K\left(\frac{\varepsilon}{\sqrt{1+\varepsilon^2}}\right)\right.$$
$$\left.-2\sqrt{1+\varepsilon^2}E\left(\frac{\varepsilon}{\sqrt{1+\varepsilon^2}}\right)\right), \qquad (12)$$

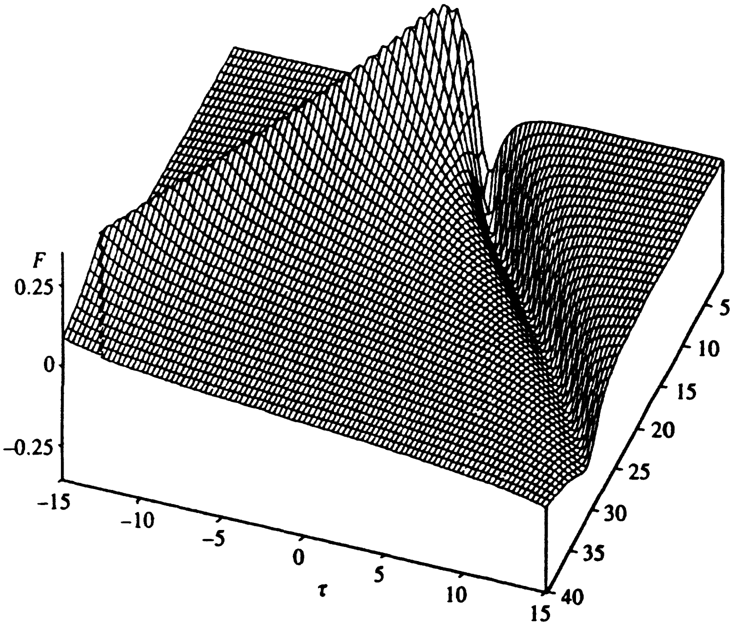

FIG. 2. Dimensionless function $F(\tau,\theta)$ (9) versus the dimensionless time τ and the observation angle θ measured from the symmetry axis of the system for a discoid body.

where K and E are complete elliptic integrals. The concentration factor D

$$D(\theta,\nu)=4\pi\frac{W}{\int d\Omega W} \tag{13}$$

is shown in Fig. 3 as a function of the observation angle and the ratio $\nu=\rho_0/c\tau_0$. It is seen clearly in this figure that for a sufficiently fast phase transition (large values of ν) the radiation is concentrated in a narrow cone near the symmetry axis of the body.

The total radiated energy E_{tot} is obtained by integrating (12) over the solid angle

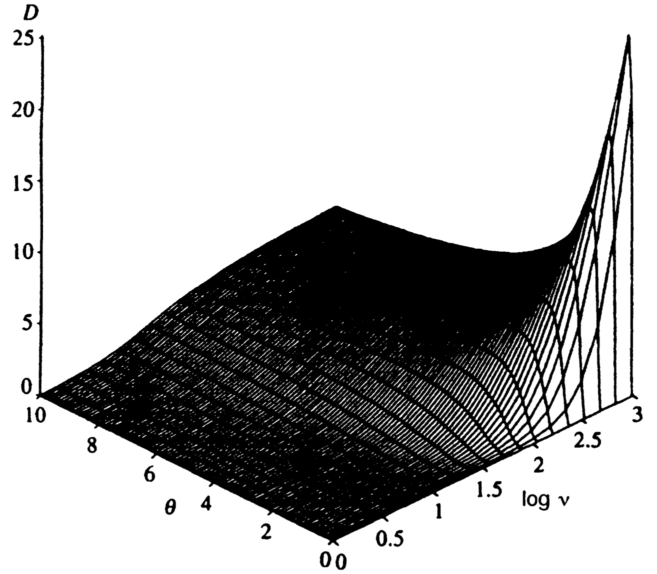

FIG. 3. Radiation concentration factor D versus $\nu=\rho_0/c\tau_0$ and observation angle θ, measured from the symmetry axis of a discoid body.

$$E_{rad}=\frac{2\pi\rho_0 I_0^2}{c^2}\left(\ln(\nu+\sqrt{1+\nu^2})-\frac{2}{\nu}(\sqrt{1+\nu^2}-1)\right). \tag{14}$$

It is easy to see that the energy stored in the static magnetic field outside the ellipsoid is described by the expressions

$$E_{out}=\frac{8\pi^2a^2b}{3}M_0^2\,u_T(\gamma)(1-u_T(\gamma)), \tag{15}$$

$$u_T(\gamma)=\frac{1}{1-\gamma^2}\left[1-\frac{1}{\lambda}\left(\frac{\pi}{2}-\arctan\frac{1}{\lambda}\right)\right],$$

$$\lambda=\sqrt{1-\gamma^2}/\gamma,\quad \gamma=b/a\ll 1, \tag{16}$$

which can be obtained with the aid of well-known expressions.[6]

The ratio $\eta=E_{rad}/E_{out}$, characterizing the radiation efficiency, can be easily found from (14) and (16):

$$\eta=\frac{2\sqrt{6}}{\pi^2}\left(\ln(\nu+\sqrt{1+\nu^2})-\frac{2}{\nu}(\sqrt{1+\nu^2}-1)\right),$$

$$\nu=\sqrt{\frac{2}{3}}\frac{a}{c\tau_0}. \tag{17}$$

In deriving this expression we took into consideration the thickness of the disk, i.e., we set

$$u_T(\gamma)=1-\frac{\pi}{2}\gamma. \tag{18}$$

The function $\eta(\nu)$ (17) is shown in Fig. 4.

In summary, energy is radiated efficiently if the magnetization of the sample changes sufficiently rapidly, and the directional pattern of this radiation is sharp (see Fig. 3). We note that the expressions obtained (including also the expres-

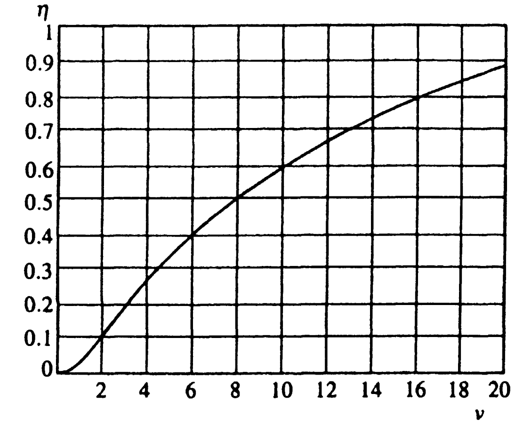

FIG. 4. Radiation efficiency as a function of $\nu=\rho_0/c\tau_0$ for a discoid body.

sion for the efficiency (17)) hold only for phase transitions which are not too fast, since for $c\tau_0\approx b$ our approximation of an infinitely thin disk is not entirely correct.

4. CHARACTERISTICS OF THE RADIATION ACCOMPANYING FAST RELAXATION OF A SINGLE-DOMAIN NEEDLE-SHAPED FERROELECTRIC

We now consider a quasi-one-dimensional polarized structure. Here we shall employ the electric terminology, i.e., we shall study a ferroelectric sample in the form of a strongly prolate ellipsoid of revolution with polarization vector **P** directed along the symmetry axis of the system. The geometry of the problem is shown in Fig. 5.

As the magnitude (but not the direction) of the electric polarization vector varies, a current will flow from one end of the sample to the other. Since the sample is thin, it can be assumed that the current is concentrated only on the axis of the system and is described by the expression

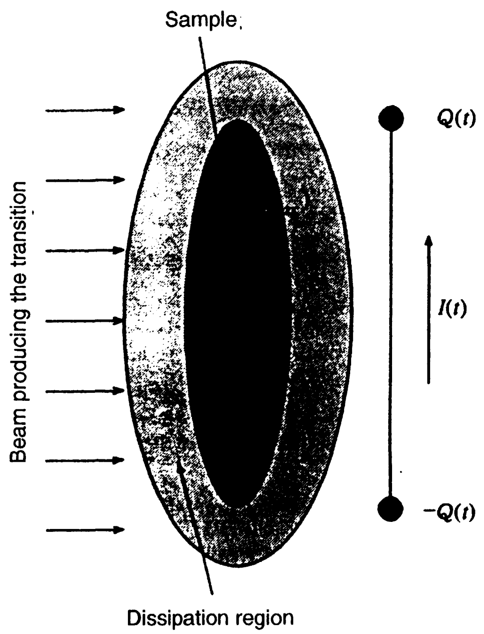

FIG. 5. Geometry of the quasi-one-dimensional problem.

$$j_z=Q(t)\delta(x)\delta(y)\Theta(l_0-z)\Theta(l_0+z) \tag{19}$$

and that the electric charge density will be different from zero only at the ends of the ellipsoid

$$\rho=Q(t)(\delta(z-l_0)-\delta(z+l_0))\delta(x)\delta(y). \tag{20}$$

In Eqs. (20) and (21) δ and Θ are the Dirac and Heaviside functions, respectively. The charge $Q(t)$ can be determined from the standard formula

$$Q(t)=\pi a^2P_z(t), \tag{21}$$

where P_z is the polarization vector, while the effective length l_0 of the dipole can be found, just as in the "magnetic" case, from the definition of the electric dipole moment

$$\frac{4}{3}\pi a^2bP_z(t)=2l_0Q(t). \tag{22}$$

Hence we have for l_0 the expression

$$l_0=\frac{2}{3}b. \tag{23}$$

Therefore the problem of radiation from a prolate ferroelectric ellipsoid of revolution reduces to solving the inhomogeneous Maxwell equations with the prescribed current and charge densities (19) and (20).

The solution of these equations in the wave region can be found by the standard methods. If the polarization of the sample varies according to the law (7) and is directed along the symmetry axis of the problem, only the z component of the vector potential is different from zero. Simple integrations yield the following expression for it:

$$A_z=-\frac{Q_0}{\pi R\cos\vartheta}\left[\arctan\left(\frac{ct'+l_0\cos\vartheta}{c\tau_0}\right)+\arctan\left(\frac{ct'-l_0\cos\vartheta}{c\tau_0}\right)\right], \tag{24}$$

where $t'=t-R/c$ is the retarded time.

Correspondingly, we obtain for the magnetic field, which possesses only an azimuthal component,

$$B_\varphi=-\frac{Q_0}{\pi Rc\tau_0}G(t'/\tau_0,\theta), \tag{25}$$

$$G(t'/\tau_0,\theta)=\tan\theta\left[\frac{1}{1+(t'/\tau_0+\nu\cos\vartheta)^2}-\frac{1}{1+(t'/\tau_0-\nu\cos\vartheta)^2}\right]. \tag{26}$$

Here and below we write $\nu=l_0/c\tau_0=2b/3c\tau_0$. The function $G(\tau,\theta)$ versus the dimensionless time and angle is displayed in Fig. 6.

Integrating the squared magnetic field over time, we find the following expression for the total radiation energy passing into a unit solid angle:

$$W(\theta)=\frac{Q_0^2\nu^2}{4\pi^2c\tau_0}\frac{\sin^2\vartheta}{1+\nu^2\cos^2\vartheta}. \tag{27}$$

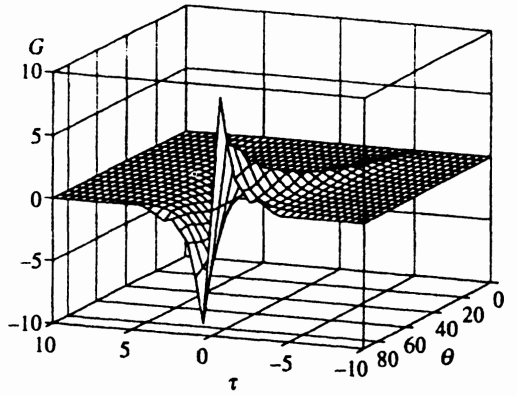

FIG. 6. Dimensionless function $G(\tau,\theta)$ (26) versus the dimensionless time τ and observation angle θ measured from the symmetry axis of a needle-shaped body, $\nu=5$.

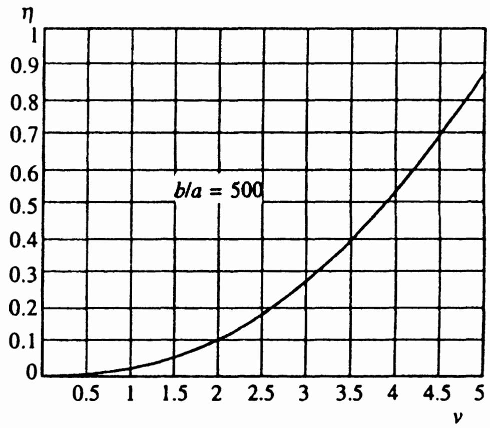

FIG. 8. Radiation efficiency as a function of $\nu=\rho_0/c\tau_0$ for a needle-shaped body with $b/a=500$.

The concentration factor D (13) for the case at hand is shown for different values of ν in Fig. 7. One can see from this figure that as the phase transition accelerates, radiation is concentrated in a plane perpendicular to the symmetry axis of the system.

Integrating over angles, we find the total radiated energy

$$E_{\rm rad}=\frac{Q_0^2}{\pi l_0}[-\nu+(1+\nu^2)\arctan\nu]. \tag{28}$$

An expression for the total initial external energy is easily obtained from the general expression in Ref. 6. In the case of a strongly prolate ellipsoid of revolution we obtain

$$E_{\rm out}=\frac{8\pi^2a^2b}{3}P_0^2\frac{a^2}{b^2}\ln\frac{b}{a}. \tag{29}$$

Knowing the ratio of the radiated energy to the initial energy, we can find the radiation efficiency as

$$\eta=\frac{9}{16\pi\ln(b/a)}[-\nu+(1+\nu^2)\arctan\nu],$$

$$\nu=\frac{2b}{3c\tau_0}. \tag{30}$$

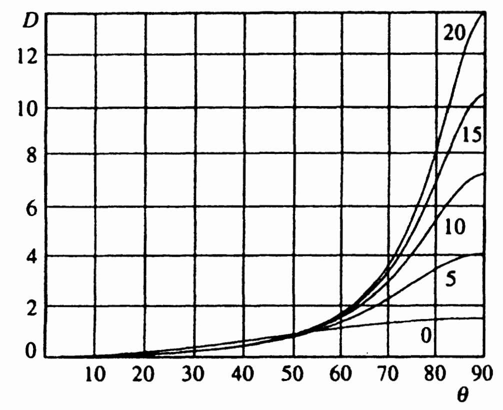

FIG. 7. Radiation concentration factor D versus the observation angle θ measured from the symmetry axis of a needle-shaped body, for different values of $\nu=\rho_0/c\tau_0$.

Just as in the case of the radiation from a disk, the expressions obtained for the radiation efficiency are valid for phase transitions which are not too rapid, since the approximation of an infinitesimally thin disk is incorrect in this case. The correct calculations in this case, as can be demonstrated for the examples of discoid ferromagnet and a ring current, show that the efficiency is close to 1.

5. CONCLUSIONS

The expressions obtained in Secs. 3 and 4 for the space–time and energy characteristics of the radiation accompanying fast phase transitions in oblate and prolate polarized samples solve the problem completely if the dependence of the polarization on the time and the thinness of the samples is given. Of course, the specific law of variation of the polarization does not affect the results obtained, since only the characteristic transition time is important.

We consider now how sufficiently short transition times can be achieved. Here one should keep foremost in mind the fact that if the structural changes occurring in the sample in the process of a phase transition are not too strong, then the shortest achievable transition time is bounded by the relaxation time of the polarization in the sample. Specifically, for ferromagnets the relaxation time is equal to approximately 10 ns, while for ferroelectrics the characteristic relaxation times are of order 0.1–0.01 ns. Hence it follows that the largest dimension of a ferromagnetic sample must be

$$r_{\rm max,magn}\gg c\tau_{r,\rm magn}=3\ \ {\rm m},$$

which is quite large. A more favorable situation obtains in the case of ferroelectrics

$$r_{\rm max,el}\gg c\tau_{r,\rm el}=3\ \ {\rm cm}.$$

In summary, if a thick ferroelectric film ~ 100 cm in diameter and ~ 1 mm is irradiated with a sufficiently powerful energy flux (laser, beam), then a quite powerful and directed pulse of radio radiation can be expected. For example, for a $LiNbO_3$ film with spontaneous polarization ~ 0.7 C/m^2 (Ref. 7) the energy emitted in directed radiation equals ~ 100 kJ over a time ~ 1 ns, i.e., the power is ~ 100 TW!

We note, however, that in this case, as already mentioned in the introduction, the existence of free charge in air will result in the appearance of additional currents, which, evidently, will decrease the radiation effects. A quantitative investigation of this aspect of the problem requires an additional analysis.

Moreover, in the present paper we did not examine the questions concerning the preparation of single-domain, one- and two-dimensional, polarized samples, which in itself presents certain experimental difficulties. These and other questions concerning the problem of radiation accompanying fast phase transitions merit further investigation.

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[1] L. G. Kachurin, S. Kolev, and V. F. Psalomshchikov, Dokl. Akad. Nauk SSSR **267**, 347 (1982) [Sov. Phys. Dokl. **27**, 937 (1982)].
[2] A. A. Vorob'ev, E. K. Zavodovskaya, and V. N. Sal'nikov, Dokl. Akad. Nauk SSSR **220**, 82 (1975).
[3] V. V. Kolesov, Zh. Tekh. Fiz. **60**, 118 (1990) [Sov. Phys. Tech. Phys. **35**, 471 (1990)].
[4] Yu. V. Korobkin, O. A. Pel'tikhin, V. B. Studenev, and A. V. Chernyshov, Pis'ma Zh. Tekh **16**, 19 (1990) [Sov. Tech. Phys. Lett. **16**, 489 (1990)].
[5] Yu. V. Afanas'ev, V. V. Klimov, L. P. Feoktistov, and A. L. Feoktistov, Zh. Éksp. Teor. Fiz. **101**, 1118 (1992) [Sov. Phys. JETP **74**, 596 (1992)].
[6] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill, N. Y., 1941 [Russian translation, Gostekhizdat, Moscow, 1948].
[7] M. Lines and A. Glass, *Principles and Applications of Ferroelectrics and Related Materials*, Clarendon Press, Oxford, 1977 [Russian translation, Mir, Moscow, 1981].

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