The positronium atom in the field of an optical laser: radiation-induced energy shifts and annihilation decay kinetics

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A theory of quantum transitions in an atom of positronium is constructed that takes into account two-photon annihilation decay and optical transitions between two arbitrary states of the positronium atom. The problem is addressed without using perturbation theory by solving sixteen Heisenberg equations for the photon and atomic operators. Solutions to these equations are used to calculate radiative shifts of the energy levels of the positronium atom, phase relaxation times, and the lifetime of the positronium atom itself, taking into account spontaneous transitions and transitions stimulated optically between two quantum states. Radiative and nonradiative interactions of the positronium atom with the photon fields are distinguished. The role of coherent effects in positronium due to feedback from the fields of intrinsic and external photons is discussed. The kinetics of annihilation decay is investigated for various initial conditions. In addition, it is demonstrated that long-lived atoms of parapositronium can form in the field of an optical laser if one of the states of the positronium atom is a Rydberg state. [S1063-7761(96)00710-X]

1. INTRODUCTION

In the physics of positronium atoms, an important role is played by theoretical and experimental investigations directed towards refining the wave functions of the electron and positron, which in turn are reflected in changes in the lifetimes and shifts of energy levels of the positronium atom in its various states.[1–3] In this article we will regard these e^+e^- interactions as nonradiative in nature.

The goal of this article is to neglect nonradiative interactions and investigate theoretically the radiative interactions of a positronium atom with the field of its own and external photons. As we will show below, such a field significantly changes the energies and lifetimes of states of the positronium atom. In contrast to previous works,[4,5] this article will focus on the radiative e^+e^- interaction with light and the absorption of optical photons in the course of annihilation decay. This allows us to clarify the roles of spontaneous and stimulated optical transitions in the radiative interaction of the positronium atom with the field of vacuum annihilation photons.

A number of papers[6–10] have noted the important role an optical laser can play in the process of annihilating a positronium atom. Thus, in Ref. 6, Rivlin discussed the possibility of spontaneously induced 3γ annihilation of an atom of orthopositronium, in which one of the modes of the annihilation photons is optical. In this article it will be shown that the field of an optical laser not only causes significant changes in the energy and lifetime of the positronium atom, but under certain circumstances also makes it possible to obtain a long-lived positronium atom if the radiation of the optical laser excites the positronium atom to a Rydberg state.

2. EQUATIONS OF MOTION FOR PHOTON AND ATOMIC OPERATORS

The effective Hamiltonian of the electron–positron system in the field of annihilation (modes 1, 2, 3, 4) and optical photons takes the form

$$\begin{aligned}H=&\hbar\omega_1^{(+)}\hat{a}_1^+\hat{a}_1+\hbar\omega_1^{(-)}\hat{b}_1^+\hat{b}_1+\hbar\omega_2^{(+)}\hat{a}_2^+\hat{a}_2\\&+\hbar\omega_2^{(-)}\hat{b}_2^+\hat{b}_2+\hbar\omega_1c_1^+c_1+\hbar\omega_2c_2^+c_2+\hbar\omega_3c_3^+c_3\\&+\hbar\omega_4c_4^+c_4+\hbar\omega c^+c+U_1\hat{b}_1^+\hat{a}_1^+\hat{a}_1\hat{b}_1\\&+U_2\hat{b}_2^+\hat{a}_2^+\hat{a}_2\hat{b}_2+S_2(\mathbf{k}_1,\mathbf{k}_2)c_1^+c_2^+\hat{b}_2\hat{a}_2\\&+S_2^*(-\mathbf{k}_1,-\mathbf{k}_2)\hat{a}_2^+\hat{b}_2^+c_1c_2+S_1(\mathbf{k}_3,\mathbf{k}_4)c_3^+c_4^+\hat{b}_1\hat{a}_1\\&+S_1^*(-\mathbf{k}_3,-\mathbf{k}_4)\hat{a}_1^+\hat{b}_1^+c_3c_4+G(\mathbf{k})\hat{a}_2^+\hat{b}_2\hat{a}_1\hat{b}_1c\\&+G^*(-\mathbf{k})\hat{a}_1^+\hat{b}_1^+\hat{a}_2\hat{b}_2c^+,\end{aligned}\tag{2.1}$$

where $\hbar\omega_1^{\mp}>0$, $\hbar\omega_2^{\mp}>0$ are the energies of the electron (positron) in states 1 and 2 of the positronium atom, c, c^+ are annihilation and creation operators for optical photons, and $\hbar\omega_\alpha$ (α=1, 2, 3, 4) are energies of the annihilation photons. The quantities U_1 and U_2 determine the interaction energy of an electron and positron in states 1 and 2 of the positronium atom as second-order quantum electrodynamic effects (Fig. 1a). The quantities S_1, S_2 characterize the probabilities $W_{2\gamma}(1)$, $W_{2\gamma}(2)$ for two-photon decay of a positronium atom in states 1 and 2 respectively; in the absence of higher corrections, these corresponds to a second-order effect (Fig. 1b). The quantities G and G^* define quantum transitions between states 1 and 2 of the positronium atom under the action of the optical laser. Such quantum transitions can be represented as third-order effects in quantum electrodynamics (Fig. 1c). In what follows the quantities S_1, S_2, and G will be determined explicitly. The summation over different photon modes will be carried out in the final formulas.

We use the equal-time commutation relations for photon and electron–positron operators[11] to derive Heisenberg operator equations. After the necessary computations we obtain the following equations for the photon operators:

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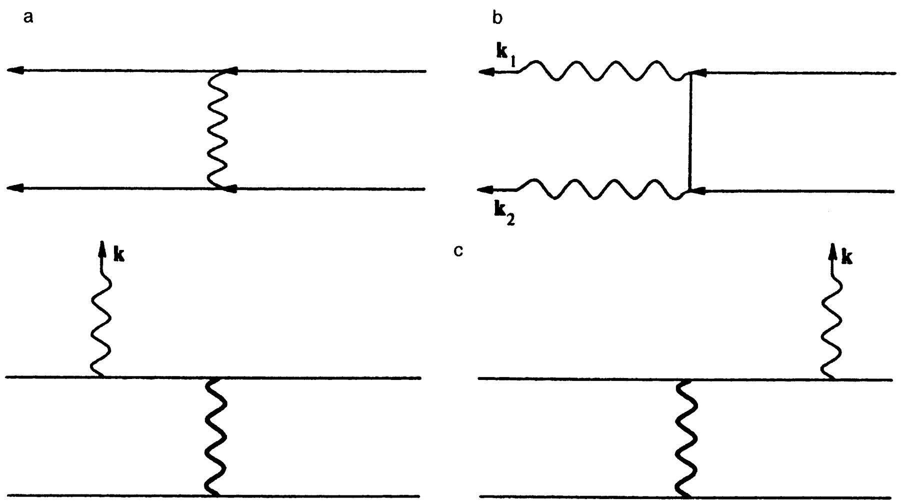

FIG. 1. Feynman diagrams for the e^+e^- interaction in a positronium atom which determine the effective Hamiltonian (2.1).

$$\dot c_1=-i\omega_1c_1+\frac{i}{\hbar}S_2c_2^+P_2,\quad \dot c_2=-i\omega_2c_2+\frac{i}{\hbar}S_2c_1^+P_2,$$

$$\dot c_3=-i\omega_3c_3+\frac{i}{\hbar}S_1c_4^+P_1,\quad \dot c_4=-i\omega_4c_4+\frac{i}{\hbar}S_1c_3^+P_1,$$

$$\dot c=-i\omega c+\frac{i}{\hbar}G^*P_1^+P_2, \tag{2.2}$$

where $P_1=\hat a_1\hat b_1$, $P_2=\hat a_2\hat b_2$ are atomic operators for states 1 and 2 of the positronium atom. These equations must be supplemented by corresponding equations for the atomic operators:

$$\dot P_1=-i\Omega_{01}P_1-\frac{i}{\hbar}U_1P_1-\frac{i}{\hbar}S_1^*n_1c_3c_4 +\frac{i}{\hbar}G^*n_1P_2c^+,$$

$$\dot P_1^+=i\Omega_{01}P_1^++\frac{i}{\hbar}U_1P_1^++\frac{i}{\hbar}S_1n_1c_3^+c_4^+-\frac{i}{\hbar}Gn_1P_2^+c,$$

$$\dot n_1=-\frac{2i}{\hbar}S_1c_3^+c_4^+P_1+\frac{2i}{\hbar}S_1^*c_3c_4P_1^++\frac{2i}{\hbar}GP_2^+cP_1 -\frac{2i}{\hbar}G^*P_2c^+P_1^+, \tag{2.3}$$

where $\Omega_{01}=\omega_1^{(+)}+\omega_1^{(-)}$, $n_1=a_1^+a_1+b_1^+b_1-1$. We obtain the rest of the equations analogously:

$$\dot P_2=-i\Omega_{02}P_2-\frac{i}{\hbar}U_2P_2-\frac{i}{\hbar}S_2^*n_2c_1c_2+\frac{i}{\hbar}GP_1cn_2,$$

$$\dot P_2^+=i\Omega_{02}P_2^++\frac{i}{\hbar}U_2P_2^++\frac{i}{\hbar}S_2n_2c_1^+c_2^+ -\frac{i}{\hbar}G^*P_1^+c^+n_2,$$

$$\dot n_2=-\frac{2i}{\hbar}S_2c_1^+c_2^+P_2+\frac{2i}{\hbar}S_2^*c_1c_2P_2^+-\frac{2i}{\hbar}GP_1cP_2^+ +\frac{2i}{\hbar}G^*P_1^+c^+P_2, \tag{2.4}$$

where $\Omega_{02}=\omega_2^{(+)}+\omega_2^{(-)}$. Equations (2.2)–(2.4) form a closed system, which will be used below to describe the various properties of the positronium atom in the field of intrinsic and external photons.

2.1. Conservation laws

From Eqs. (2.3) and (2.4) we can derive the conservation law:

$$\frac{d}{dt}\left(P_1^+P_1+P_1P_1^++\frac{1}{2}n_1^2\right)=0, \tag{2.5}$$

$$\frac{d}{dt}\left(P_2^+P_2+P_2P_2^++\frac{1}{2}n_2^2\right)=0. \tag{2.6}$$

Therefore, we have the following integrals of the motion:

$$P_1^+P_1+P_1P_1^++\frac{1}{2}n_1^2=n_{01}, \tag{2.7}$$

$$P_2^+P_2+P_2P_2^++\frac{1}{2}n_2^2=n_{02}, \tag{2.8}$$

where n_{01},n_{02} are constants.

The quantities n_{01} and n_{02} can be found from Eqs. (2.7) and (2.8) and the commutation relations

$$[P_\alpha,P_\beta^+]=-\delta_{\alpha\beta}n_\alpha,\quad [P_\alpha,n_\beta]=2P_\alpha\delta_{\alpha\beta},$$
$$[P_\alpha^+,n_\beta]=-2P_\alpha^+\delta_{\alpha\beta},\quad \alpha,\beta=1,2. \tag{2.9}$$

In what follows the initial conditions will be discussed in detail, and a solution to the equations of motion will be obtained for the atomic and photon operators.

2.2. Adiabatic approximation

Let us use the adiabatic approximation, where

$$P_\alpha(t)=P_{0\alpha}(t)e^{-i\Omega_\alpha t},\quad P^+_\alpha(t)=P^+_{0\alpha}(t)e^{+i\Omega_\alpha t},$$

$$c_\mu(t)=c^{(0)}_\mu(t)e^{-i\omega_\mu t},\quad c^+_\mu(t)=c^{(0)+}_\mu(t)e^{i\omega_\mu t},\qquad(2.10)$$

$P_{0\alpha},P^+_{0\alpha},c^{(0)}_\mu,c^{(0)+}_\mu$ are slowly varying amplitudes; Ω_α is the frequency of the positronium atom in states $\alpha=1,2$ $(\Omega_\alpha=\Omega_{0\alpha}+U_\alpha/\hbar)$; and $\mu=1,2,3,4$ labels the modes of the annihilation photons. Then we obtain from the first equation in (2.2)

$$c_1(t)=c_{1v}(t)+d_2(t)c^+_2(t),\qquad(2.11)$$

where $c_{1v}(t)=c_{1v}(0)e^{-i\omega_1 t}$ consists of vacuum-field operators,

$$d_2(t)=d_{02}P_2(t),\quad d_{02}=-\frac{i}{\hbar}|S_2|\zeta^*(-\Omega_2+\omega_1+\omega_2),\qquad(2.12)$$

and $\zeta(x)$ is the Heitler ζ-function:[12]

$$\zeta(x)=\frac{P}{x}-i\pi\delta(x)\equiv\lim_{t\to\infty}\frac{1-e^{-ixt}}{ix}.\qquad(2.13)$$

By analogy we obtain the following operator equations:

$$c_2(t)=c_{2v}(t)+d_2(t)c^+_1(t),$$

$$c^+_1(t)=c^+_{1v}(t)+d^*_2(t)c_2(t),$$

$$c^+_2(t)=c^+_{2v}(t)+d^*_2(t)c_1(t).\qquad(2.14)$$

From Eqs. (2.11), (2.14) we obtain the following operators:

$$c_1(t)=\frac{c_{1v}+c^+_{2v}d_2}{1-d^*_2d_2},\quad c_2(t)=\frac{c_{2v}+c^+_{1v}d_2}{1-d^*_2d_2},$$

$$c^+_1=c^*_1,\quad c^+_2=c^*_2.\qquad(2.15)$$

We transform the operator $(1-d^*_2d_2)^{-1}$, using its expansion in a series

$$\frac{1}{1-d^*_2d_2}=1+d^*_2d_2+(d^*_2d_2)^2+\cdots=1+p_{02}P^+_2P_2,\qquad(2.16)$$

where

$$p_{02}=\frac{|d_{02}|^2}{1-|d_{02}|^2}.\qquad(2.16a)$$

Thus, in place of (2.15) we have

$$c_1=(c_{1v}+c^+_{2v}d_{02}P_2)(1+p_{02}P^+_2P_2),$$

$$c^+_1=(1+p_{02}P^+_2P_2)(c^+_{1v}+d^*_{02}P^+_2c_{2v}),$$

$$c_2=(c_{2v}+c^+_{1v}d_{02}P_2)(1+p_{02}P^+_2P_2),$$

$$c^+_2=(1+p_{02}P^+_2P_2)(c^+_{2v}+d^*_{02}P^+_2c_{1v}).\qquad(2.17)$$

We also introduce the operator

$$d_1(t)=d_{01}P_1(t),\quad d_{01}=-\frac{i}{\hbar}|S_1|\zeta^*(-\Omega_1+\omega_3+\omega_4)\qquad(2.18)$$

and obtain from Eq. (2.2) the following operators:

$$c_3=(c_{3v}+c^+_{4v}d_{01}P_1)(1+p_{01}P^+_1P_1),$$

$$c^+_3=(1+p_{01}P^+_1P_1)(c^+_{3v}+d^*_{01}P^+_1c_{4v}),$$

$$c_4=(c_{4v}+c^+_{3v}d_{01}P_1)(1+p_{01}P^+_1P_1),$$

$$c^+_4=(1+p_{01}P^+_1P_1)(c^+_{4v}+d^*_{01}P^+_1c_{3v}),\qquad(2.19)$$

where

$$p_{01}=\frac{|d_{01}|^2}{1-|d_{01}|^2}.\qquad(2.19a)$$

Factoring out the slowly varying amplitude in the photon operators $c(t)$ and $c^+(t)$, we obtain

$$c(t)=c_v(t)+l_0P^+_1P_2,\quad c^+(t)=c^+_v(t)+l^*_0P^+_2P_1,\qquad(2.20)$$

where

$$l_0=\frac{1}{\hbar}G^*\zeta^*(\Omega_1-\Omega_2+\omega).\qquad(2.20a)$$

In obtaining the operators (2.17), (2.19), and (2.20), we have used the ordinary properties of fermion operators of the electron–positron field. In calculating these operators, arbitrary powers of the interaction constants $|S_1|$ and $|S_2|$ are included, i.e., as is clear from (2.16), restriction on the no perturbation-theoretic series is invoked in our investigation.

By substituting the operators (2.17), (2.19), (2.20) into the equations of motion (2.3) and (2.4) for the atomic operators, we can transform these equations. In this case, it is clear from (2.17) and (2.19) that in our discussion the coupling between photon and atomic operators arises from the optical photon operators (2.20).

3. OCCUPATION NUMBER OF POSITRONIUM-ATOM STATES IN A FIELD OF OPTICAL PHOTONS

From Eqs. (2.3) and (2.4) with normally ordered operators we have

$$\frac{d}{dt}(n_1+n_2)=-\frac{2i}{\hbar}S_1c^+_3c^+_4P_1+\frac{2i}{\hbar}S^*_1P^+_1c_3c_4$$
$$-\frac{2i}{\hbar}S_2c^+_1c^+_2P_2+\frac{2i}{\hbar}S^*_2P^+_2c_1c_2.\qquad(3.1)$$

Let us substitute the photon annihilation operators (2.17) and (2.18) into (3.1). Multiplying out the brackets, we take the quantities d_{01}, d_{02} and p_{01}, p_{02} as parameters for estimating the contribution from different terms. We will also assume that

$$-\frac{i}{\hbar}|S_1|=\frac{1}{\hbar}S_1,\quad -\frac{i}{\hbar}|S_2|=\frac{1}{\hbar}S_2.\qquad(3.2)$$

We assume that there are no bare photons in modes 1,2 and 3,4 of the free (vacuum) field. This implies that the following equations hold:

$$c_{\mu\nu}|\mathrm{vac}\rangle=0,\quad \langle \mathrm{vac}|c^{+}_{\mu\nu}=0,\quad \mu=1,2,3,4,$$

$$c^{+}_{\mu\nu}c_{\mu\nu}|\mathrm{vac}\rangle=0,\quad \langle c^{+}_{\mu\nu}c_{\mu\nu}\rangle=0, \tag{3.3}$$

where $|\mathrm{vac}\rangle$ is the wave function of the vacuum field in the occupation-number representation of the photon states, and $\langle ...\rangle$ denotes an average value computed by using this wave function. After multiplying out the operators in (3.1), we average out the atomic operators in the final expressions with the help of a certain wave function $|\Phi\rangle$ in the occupation number representation of the electron-positron states. In addition to this we will use the following identities:

$$(P^{+}_{\alpha}P_{\alpha})^2=\frac{1}{9}n^2_{0\alpha}+\frac{1}{3}n_{0\alpha}n_{\alpha}+\frac{1}{6}n_{0\alpha},\quad \alpha=1,2,$$

$$n^2_{\alpha}=\frac{2}{3}n_{0\alpha},\quad P^{+}_{\alpha}n^2_{\alpha}P_{\alpha}=\frac{1}{2}n_{\alpha}+\frac{1}{3}n_{0\alpha},$$

$$-n_{\alpha}P_{\alpha}=P_{\alpha},\quad P_{\alpha}P^{+}_{\alpha}=\frac{1}{3}n_{0\alpha}-\frac{1}{2}n_{\alpha}. \tag{3.4}$$

Adding all the terms in the operator equation (3.1), we obtain the following equation for the average value (we omit the symbol $\langle ...\rangle$):

$$\frac{d}{dt}(n_1+n_2)=B_1n_1+B_0+C_1n_2+C_0, \tag{3.5}$$

where

$$B_1=-\left(\frac{2|S_1|}{\hbar}\right)^2\left(\frac{1}{2}+\frac{1}{3}n_{01}p_{01}\right)\pi\delta(-\Omega_1+\omega_3+\omega_4),$$

$$B_0=-\left(\frac{2|S_1|}{\hbar}\right)^2\left[\frac{1}{3}n_{01}+p_{01}\left(\frac{1}{9}n^2_{01}+\frac{1}{6}n_{01}\right)\right]\pi\delta(-\Omega_1+\omega_3+\omega_4),$$

$$C_1=-\left(\frac{2|S_2|}{\hbar}\right)^2\left(\frac{1}{2}+\frac{1}{3}n_{02}p_{02}\right)\pi\delta(-\Omega_2+\omega_1+\omega_2),$$

$$C_0=-\left(\frac{2|S_2|}{\hbar}\right)^2\left[\frac{1}{3}n_{02}+p_{02}\left(\frac{1}{9}n^2_{02}+\frac{1}{6}n_{02}\right)\right]\pi\delta(-\Omega_2+\omega_1+\omega_2).$$

In order to solve Eq. (3.5), it is necessary to determine the explicit time dependence of the occupation numbers n_1 and n_2. For this we once more turn to Eqs. (2.3) and (2.4). Following the computational procedure described above, after averaging over the vacuum from (2.3) we obtain

$$\dot n_1=B_1n_1+B_0+\frac{2i}{\hbar}Gl_0\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right)\left(\frac{1}{3}n_{01}+\frac{1}{2}n_1\right)-\frac{2i}{\hbar}G^*l^*_0\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right)\left(\frac{1}{3}n_{01}-\frac{1}{2}n_1\right). \tag{3.6}$$

Analogously, we obtain from the third term of (2.4)

$$\dot n_2=C_1n_2+C_0-\frac{2i}{\hbar}Gl_0\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right)\left(\frac{1}{3}n_{01}-\frac{1}{2}n_1\right)+\frac{2i}{\hbar}G^*l^*_0\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right)\left(\frac{1}{3}n_{01}+\frac{1}{2}n_1\right). \tag{3.7}$$

We distinguish the nonlinear terms in Eqs. (3.6) and (3.7), i.e.,

$$\dot n_1=a_1n_1+a_2+n_2a_3+n_2n_1a_4, \tag{3.8a}$$

$$\dot n_2=b_1n_2+b_2+n_1b_3+n_2n_1a_4, \tag{3.8b}$$

where the meaning of the coefficients a_μ, b_μ ($\mu=1,2,3,4$) is easily established by a comparison with Eqs. (3.8), (3.7) and (3.6).

Subtracting Eqs. (3.8a) and (3.8b) from one another and using Eq. (3.5), we find the following solution:

$$n_2(t)=-R_3+[n_2(0)+R_3]\exp\left[-\frac{1}{2}(a_3-b_1-C_1)t\right], \tag{3.9}$$

where

$$R_3=\frac{a_2-b_2-B_0-C_0}{a_3-b_1-C_1}. \tag{3.9a}$$

Substituting (3.9) into Eq. (3.8a), we obtain

$$n_1(t)=-\frac{R_1}{B_1}-\frac{2R_2}{a_3-b_1-C_1+B_1}\exp\left[-\frac{1}{2}(a_3-b_1-C_1)t\right]+e^{B_1t}\left[n_1(0)+\frac{R_1}{B_1}+\frac{2R_2}{a_3-b_1-C_1+B_1}\right], \tag{3.10}$$

where

$$R_1=-\frac{1}{2}\frac{(a_3-b_1+C_1)(a_2-b_2-B_0-C_0)}{a_3-b_1-C_1}+\frac{1}{2}(a_2-b_2+B_0+C_0),$$

$$R_2=\frac{1}{2}(a_3-b_1+C_1)\left(n_2(0)+\frac{a_2-b_2-B_0-C_0}{a_3-b_1-C_1}\right). \tag{3.10a}$$

Solving (3.10), (3.9) allows us to determine the time dependence of the occupation numbers of states 1 and 2 of the positronium atom in a field of optical photons in the single-mode approximation. In what follows these solutions will be used to solve Eq. (3.5), taking into account the multimode nature of annihilation decay.

4. COHERENCE OF STATES OF A POSITRONIUM ATOM

Following the terminology of resonance optical spectroscopy,[13] we will estimate the coherent effects in a positronium atom with the help of nonzero average values of the operators P_1, P^{+}_1 and P_2, P^{+}_2. For this we turn to Eq. (2.3) and transform the following atomic-field operator:

$$-\frac{i}{\hbar}S^*_1n_1c_3c_4+\frac{i}{\hbar}G^*c^{+}n_1P_2, \tag{4.1}$$

making use of Eqs. (2.20) and (2.19) and normal ordering of the operators.

After vacuum averaging we obtain the following equation:

$$\dot{P}_1=-i\left(\Omega_{01}+\frac{1}{\hbar}U_1\right)P_1+\frac{i}{\hbar}S_1^*d_{01}\left[1+p_{01}\left(\frac{1}{3}n_{01}+\frac{1}{2}\right)\right]P_1+\frac{i}{\hbar}G^*l_0^*P_1\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right). \quad (4.2)$$

Analogously, we obtain from Eq. (2.4)

$$\dot{P}_2=-i\left(\Omega_{02}+\frac{1}{\hbar}U_2\right)P_2+\frac{i}{\hbar}S_2^*d_{02}\left[1+p_{02}\left(\frac{1}{3}n_{02}+\frac{1}{2}\right)\right]P_2+\frac{i}{\hbar}Gl_0\left(\frac{1}{3}n_{01}-\frac{1}{2}n_1\right)P_2. \quad (4.3)$$

Using the explicit values of (2.20a), (2.18) and (2.12) for the complex quantities d_{01}, d_{02} and l_0, we can identify the real and imaginary parts in Eqs. (4.2) and (4.3), which determine the energy shift and the relaxation time of a positronium atom in states 1 and 2.

4.1. Energy shift of a positronium atom in the field of annihilation and optical photons

According to Eq. (4.2), we determine the energy shift ΔE_1 of the state 1 from the following expression

$$\Delta E_1=-\frac{1}{\hbar^2}|S_1|^2f_1\left[1+\left(\frac{1}{3}n_{01}+\frac{1}{2}\right)\frac{(|S_1|^2/\hbar^2)F_1}{1-(|S_1|^2/\hbar^2)F_1}\right]-\frac{|G|^2}{\hbar^2}\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right)f_0, \quad (4.4)$$

where we introduce the notation

$$f_0=\frac{1-\cos(\Omega_1-\Omega_2+\omega)t}{\Omega_1-\Omega_2+\omega},$$

$$f_1=\frac{1-\cos(-\Omega_1+\omega_3+\omega_4)t}{-\Omega_1+\omega_3+\omega_4},$$

$$F_1=\frac{1-\cos(-\Omega_1+\omega_3+\omega_4)t}{(-\Omega_1+\omega_3+\omega_4)^2}. \quad (4.5)$$

Analogously, we obtain from Eq. (4.3) the energy shift of state 2:

$$\Delta E_2=-\frac{1}{\hbar^2}|S_2|^2f_2\left[1+\left(\frac{1}{3}n_{02}+\frac{1}{2}\right)\frac{(|S_2|^2/\hbar^2)F_2}{1-(|S_2|^2/\hbar^2)F_2}\right]-\frac{|G|^2}{\hbar^2}\left(\frac{1}{3}n_{01}-\frac{1}{2}n_1\right)f_0, \quad (4.6)$$

where

$$f_2=\frac{1-\cos(-\Omega_2+\omega_1+\omega_2)t}{-\Omega_2+\omega_1+\omega_2},$$

$$F_1=\frac{1-\cos(-\Omega_2+\omega_1+\omega_2)t}{(-\Omega_2+\omega_1+\omega_2)^2}. \quad (4.7)$$

Obviously it is necessary to carry out a summation in Eqs. (4.4) and (4.6) over the various frequencies of the annihilation and optical photons, and also over the various directions these photons are emitted in. This procedure does not give rise to any special difficulties, if we take into account that the functions f_1, F_1, f_0, f_2, F_2 have sharp maxima as $t\to\infty$.

We point out several fundamental properties of the shifts ΔE_1 and ΔE_2.

1) In Eqs. (4.4) and (4.6) we have used the representation of the ζ function for finite time intervals. This allows us to trace the time dependence of the shifts ΔE_1 and ΔE_2 for a stepwise change in occupation numbers n_1 and n_2 according to the solutions (3.9) and (3.10).

2) Since we have $(|S_1|/\hbar)^2\sim W_{2\gamma}(1)$ and $(|S_2|/\hbar)^2\sim W_{2\gamma}(2)$, the energy shift of a parapositronium atom amounts to several gigaherz, which is comparable in value to the energy shifts caused by nonradiative interactions in a positronium atom.[1]

3) The energy shifts ΔE_1 and ΔE_2 depend on the choice of initial conditions and excitation conditions of the quantum transition $1\leftrightarrows 2$ involving optical photons.

4.2. Phase relaxation time

Taking into account the imaginary part of the ζ function in Eqs. (4.2) and (4.3), we find the characteristic times $T_{2\gamma}^{-1}(1)$ and $T_{2\gamma}^{-1}(2)$ for phase relaxation of states 1 and 2 in the single-mode approximation:

$$T_{2\gamma}^{-1}(1)=-\left(\frac{|S_1|}{\hbar}\right)^2g_1\left[1+\left(\frac{1}{3}n_{01}+\frac{1}{2}\right)\frac{(|S_1|^2/\hbar^2)F_1}{1-(|S_1|^2/\hbar^2)F_1}\right]+\frac{|G|^2}{\hbar^2}\left(\frac{1}{3}n_{02}+\frac{1}{2}n_2\right)g_0. \quad (4.8)$$

$$T_{2\gamma}^{-1}(2)=-\left(\frac{|S_2|}{\hbar}\right)^2g_2\left[1+\left(\frac{1}{3}n_{02}+\frac{1}{2}\right)\frac{(|S_2|^2/\hbar^2)F_2}{1-(|S_2|^2/\hbar^2)F_2}\right]-\frac{|G|^2}{\hbar^2}\left(\frac{1}{3}n_{01}-\frac{1}{2}n_1\right)g_0, \quad (4.9)$$

where

$$g_1=\frac{\sin(-\Omega_1+\omega_3+\omega_4)t}{-\Omega_1+\omega_3+\omega_4}, \quad g_0=\frac{\sin(\Omega_1-\Omega_2+\omega)t}{\Omega_1-\Omega_2+\omega},$$

$$g_2=\frac{\sin(-\Omega_2+\omega_1+\omega_2)t}{-\Omega_2+\omega_1+\omega_2}. \quad (4.10)$$

The quantities $T_{2\gamma}^{-1}(1)$ and $T_{2\gamma}^{-1}(2)$ are analogs of the phase relaxation times in coherent resonant optics,[13] and determine the time interval during which the average values P_1 and P_2 fall to zero due to annihilation. However, as is clear from (4.8) and (4.9), the phase relaxation time contains contributions with opposite signs, due both to annihilation decay and the influence of optical photons. Competition between these contributions can change the phase relaxation time significantly compared to cases where no optical photons are present.[4]

4.3. Initial conditions

Let us compute the quantities n_{01} and n_{02} using conservation laws (2.7) and (2.8).

The commutation relations (2.9) and the identity (3.4) imply that

$$2P_1P_1^+ + n_1 = \frac{2}{3} n_{01}. \tag{4.11}$$

If we assume that at the initial time $t=0$ the average occupation number satisfies $\langle n_1\rangle_0=1$ and we have $\langle P_1P_1^+\rangle_0=0$, then $n_{01}=3/2$. However, we can show that annihilation of a positronium atom in state 1 is impossible for this initial condition. In fact, let us calculate the operator $d(P_1P_1^+)/dt$ using the operator equation (2.3) and operators (2.19) and (2.20). After multiplying all the operators and averaging the final states we find $d\langle P_1P_1^+\rangle/dt=0$ for $n_{01}=3/2$ and $\langle n_1\rangle_0=1$. Therefore, we will assume that at the initial time $\langle P_1P_1^+\rangle_0=\Delta_{1F}$ holds. Then from Eq. (4.11) we find that $n_{01}=3(\Delta_{1F}+1/2)$. In order to calculate Δ_{1F}, we use the obvious identity $\langle P_1^+P_1\rangle^2=P_1^+P_1$. Then, using (3.4), we obtain

$$\Delta_{1F} = -\langle n_1\rangle_0 - \frac{1}{2}, \tag{4.12}$$

and for $\langle n_1\rangle_0=1$ we have $\Delta_{1F}=-3/2$.

We find n_{02} analogously. From the conservation law (2.8) we have

$$2P_2P_2^+ + n_2 = \frac{2}{3} n_{02}. \tag{4.13}$$

Assuming that at the initial time $\langle P_2P_2^+\rangle_0=\Delta_{2F}$ holds, we obtain

$$\Delta_{2F} = -\langle n_2\rangle_0 - \frac{1}{2}. \tag{4.14}$$

For $\langle n_2\rangle_0=0$ we have $\Delta_{2F}=-1/2$. Substituting the initial conditions (4.12), (4.14) into Eqs. (4.8) and (4.9), and also into (4.4) and (4.6), we can compute the relaxation time and energy shift when the conditions for interaction of the positronium atom with optical photons are specified.

5. LIFETIME OF A POSITRONIUM ATOM IN A FIELD OF OPTICAL PHOTONS

Let us now solve Eq. (3.5) with the initial conditions (4.12) and (4.14). We will take into account the multimode annihilation decay when the quantities B_1, B_0, C_1, and C_0 are δ-functions. For this it is necessary to multiply these quantities by the density of the number of photon states

$$\frac{V_R'}{(2\pi c)^3} d\Omega_A \omega^2 d\omega \tag{5.1}$$

and integrate over the frequency ω, assuming for simplicity $\omega_1=\omega_2$ and $\omega_3=\omega_4$ for the case of a motionless positronium atom in accordance with energy–momentum conservation when states 1 and 2 decay. Here V_R' is the quantization volume of the electromagnetic field, and $d\Omega_A$ is the element of solid angle in whose direction the annihilating photons are emitted. We denote by $\bar{B}_1$, $\bar{B}_0$, $\bar{C}_1$, and $\bar{C}_0$ the corresponding constant quantities on the right side of Eq. (3.5) in which we have used our procedure for taking into account the multimode nature of the process. Similarly, we denote by $\bar{a}_\mu$ and $\bar{b}_\mu$ (μ=1,2,3,4) the quantities a_μ and b_μ in Eq. (3.8), where the multimode nature can also be taken into account. Then we introduce the constant quantities $\bar{a}_\mu$, $\bar{b}_\mu$, $\bar{B}_1$, $\bar{B}_0$, $\bar{C}_1$, and $\bar{C}_0$ into the solutions to Eqs. (3.9) and (3.10).

In keeping with the meaning of the effective Hamiltonian (2.1), the quantities $|S_1|$ and $|S_2|$ correspond to free annihilation decay of a positronium atom in states 1 and 2, without taking into account the feedback of the field of annihilating photons and field of optical photons, with probabilities $\Delta W_{2\gamma}^{(0)}(1)$ and $\Delta W_{2\gamma}^{(0)}(2)$ per unit time for emission of annihilating photons into an element of solid angle $\Delta\Omega$. Then

$$|S_1| = \sqrt{\frac{\Delta W_{2\gamma}^{(0)}(1)\hbar^2(2\pi c)^3}{2\pi V_R'\omega^2\Delta\Omega}},$$

$$|S_2| = \sqrt{\frac{\Delta W_{2\gamma}^{(0)}(2)\hbar^2(2\pi c)^3}{2\pi V_R'\omega^2\Delta\Omega}}. \tag{5.2}$$

For isotropic decay

$$\Delta W_{2\gamma}^{(0)}(1) = \frac{1}{4\pi} W_{2\gamma}^{(0)}(1)\Delta\Omega,$$

$$\Delta W_{2\gamma}^{(0)}(2) = \frac{1}{4\pi} W_{2\gamma}^{(0)}(2)\Delta\Omega, \tag{5.3}$$

and $|S_1|$ and $|S_2|$ do not depend on the element of solid angle. In this case $|S_1|$ and $|S_2|$ consist of the interaction energy of a positronium atom with the field of free (vacuum) annihilation photons.

Let us clarify the meaning of the quantity V_R'. The quantity V_R' has the sense of the effective interaction volume of the positronium atom with the photon field, and is determined by the characteristic wavelength in the spectrum of the positronium atom in state 1 (or 2). For the $1S$ state of a positronium atom we have

$$V_R' = \left(\frac{4\pi c\hbar^3}{me^4}\right)^3, \tag{5.4}$$

where m is the electron mass and e is its charge. The volume V_R' is chosen in such a way as to "resolve" the characteristic frequency of a parapositronium atom in state $1S$. For the state $1S$ we have $W_{2\gamma}^{(0)}(1S)=(\alpha^5mc^2/2\hbar)$, where α is the fine structure constant.[1] Thus, the characteristic frequency for effective interaction of a parapositronium atom in this state with a field of annihilation photons equals

$$|S_1|/\hbar = \alpha^5\sqrt{\alpha c^2}m/8\hbar\pi. \tag{5.5}$$

The constant quantities p_{01} and p_{02}, which are determined by Eqs. (2.16a) and (2.19a), enter into Eq. (3.5). In the integration over frequencies of the annihilating photons we have used the properties of the Heitler ζ-function, and we find that

$$p_{01}=\frac{(|S_1|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_1|/\hbar)^2\tau_{2\gamma}^2},\quad p_{02}=\frac{(|S_2|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_2|/\hbar)^2\tau_{2\gamma}^2},\qquad(5.6)$$

where $\tau_{2\gamma}$ is the characteristic time for observing this process of interaction of the positronium atom with the field of annihilating and optical photons, i.e., the lifetime of a positronium atom taking into account feedback of the photons and optical transitions of the positronium atom between states 1 and 2.

Let us substitute the solutions (3.9) and (3.10) into Eq. (3.5). After some calculations, taking into account the remarks we made above, we find

$$n_1+n_2=Q_1\exp\left[-\frac{1}{2}(\bar{a}_3-\bar{b}_1-\bar{C}_1)t\right]+Q_2\exp(\bar{B}_1t)+Q_3,\qquad(5.7)$$

where

$$\bar{B}_1=-2W_{2\gamma}^{(0)}(1)\left[\frac{1}{2}+\frac{1}{3}n_{01}\frac{(|S_1|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_1|/\hbar)^2\tau_{2\gamma}^2}\right],$$

$$\bar{a}_3-\bar{b}_1-\bar{C}_1=-\frac{2}{3}n_{01}\frac{1}{\tau}+4W_{2\gamma}^{(0)}(2)\left[\frac{1}{2}+\frac{1}{3}n_{02}\frac{(|S_2|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_2|/\hbar)^2\tau_{2\gamma}^2}\right],$$

$$Q_1=\frac{\bar{B}_1-2\bar{C}_1}{\bar{a}_3-\bar{b}_1-C_1}+\bar{B}_1\left[\frac{2\bar{B}_1(\bar{a}_3-\bar{b}_1+\bar{C}_1)}{(\bar{a}_3-\bar{b}_1-\bar{C}_1)(\bar{a}_3-\bar{b}_1-\bar{C}_1+\bar{B}_1)}-\frac{2\bar{C}_1}{\bar{a}_3-\bar{b}_1-\bar{C}_1}\right]\left[n_2(0)+\frac{\bar{a}_2-\bar{b}_2-\bar{B}_0-\bar{C}_0}{\bar{a}_3-\bar{b}_1-\bar{C}_1}\right],$$

$$\bar{a}_3-\bar{b}_1+\bar{C}_1=-\frac{2}{3}n_{01}\frac{1}{\tau},$$

$$Q_2=n_1(0)+\frac{1}{2\bar{B}_1}(\bar{a}_2-\bar{b}_2+\bar{B}_0+\bar{C}_0)-\frac{1}{2\bar{B}_1}\frac{(\bar{a}_3-\bar{b}_1+\bar{C}_1)(\bar{a}_2-\bar{b}_2-\bar{B}_0-\bar{C}_0)}{\bar{a}_3-\bar{b}_1-\bar{C}_1}+\frac{\bar{a}_3-\bar{b}_1+\bar{C}_1}{\bar{a}_3-\bar{b}_1-\bar{C}_1+\bar{B}_1}\left[n_2(0)+\frac{\bar{a}_2-\bar{b}_2-\bar{B}_0-\bar{C}_0}{\bar{a}_3-\bar{b}_1-\bar{C}_1}\right],$$

$$\bar{a}_2-\bar{b}_2-\bar{B}_0-\bar{C}_0=-\frac{4}{9}n_{01}n_{02}\frac{1}{\tau}-2\bar{C}_0,$$

$$\bar{C}_1=-2W_{2\gamma}^{(0)}(2)\left[\frac{1}{2}+\frac{1}{3}n_{02}\frac{(|S_2|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_2|/\hbar)^2\tau_{2\gamma}^2}\right],$$

$$Q_3=-\frac{1}{2\bar{B}_1}\left[\bar{a}_2\bar{b}_2+\bar{B}_0+\bar{C}_0-\frac{(\bar{a}_3-\bar{b}_1+\bar{C}_1)(\bar{a}_2-\bar{b}_2-\bar{B}_0-\bar{C}_0)}{\bar{a}_3-\bar{b}_1-\bar{C}_1}\right]-\frac{\bar{a}_2-\bar{b}_2-\bar{B}_0-\bar{C}_0}{\bar{a}_3-\bar{b}_1-\bar{C}_1},$$

$$\bar{C}_0=-2W_{2\gamma}^{(0)}(2)\left[\frac{1}{3}n_{02}+\left(\frac{1}{9}n_{02}^2+\frac{1}{6}n_{02}\right)\frac{(|S_2|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_2|/\hbar)^2\tau_{2\gamma}^2}\right],$$

$$\bar{a}_2-\bar{b}_2+\bar{B}_0+\bar{C}_0=-\frac{2}{9}n_{01}n_{02}\frac{1}{\tau}+2\bar{B}_0,$$

$$\bar{B}_0=-2W_{2\gamma}^{(0)}(1)\left[\frac{1}{3}n_{01}+\left(\frac{1}{9}n_{01}^2+\frac{1}{6}n_{01}\right)\frac{(|S_1|/\hbar)^2\tau_{2\gamma}^2}{1-(|S_1|/\hbar)^2\tau_{2\gamma}^2}\right],$$

where $1/\tau=4\omega_0^2d^2/3c^3\hbar$ is the Einstein coefficient for spontaneous transitions of a positronium atom from state 2 to state 1 with emission of an optical photon with frequency $\omega_0=\Omega_2-\Omega_1$, and d is the dipole moment of the transition.

Using initial conditions in which we assume that state 2 is not occupied at time $t=0$, we obtain

$$n_{01}=\frac{3}{2},\quad n_1(0)=1-2\Delta_{1F},\quad n_{02}=0,\quad n_2(0)=0,$$

$$Q_1=0,\quad Q_2=n_1(0)+\bar{B}_0/\bar{B}_1.\qquad(5.8)$$

Then we determine the lifetime of the positronium atom from the solution (5.7):

$$\tau_{2\gamma}^{-1}\equiv W_{2\gamma}=\frac{1}{2}W_{2\gamma}^{(0)}(1)\left(1+\sqrt{1+\frac{\alpha}{2\pi}}\right).\qquad(5.9)$$

Let us discuss the physical meaning of this result. Corrections to the probability per unit time of annihilation decay of a positronium atom in the 1^1S_0 state calculated to order $O(\alpha^2\ln\alpha^{-1})$ lead[14] to a decrease of this quantity compared to $W_{2\gamma}^{(0)}(1)$. These corrections will be regarded as nonradiative corrections, since they are caused by the effects of quantum electrodynamics with Feynman diagrams that contain only closed photon lines. In this case the procedure for calculating nonradiative corrections does not include the possibility of creation of a positronium atom due to feedback of the photon field. In our discussion, which is based on the closed system of Heisenberg equations of motion for the photons and atomic operators, this possibility is included automatically. Therefore, in contrast to Ref. 14, and also to other papers in which higher-order nonradiative corrections are discussed, Eq. (5.9) corresponds to different, i.e., radiative corrections. In keeping with the meaning of the effective Hamiltonian (2.1) we include nonradiative corrections of all orders in the interaction constants of the positronium atom with the photon field. Then Eq. (5.9) will be complete enough for comparison with experimental results, in which record accuracy is achieved in measuring the spectroscopic characteristics of a positronium atom.[1] Let us now discuss the solution to (5.7) for other initial conditions where at the initial instant of time state 2 is occupied. In this case

$$n_{01}=\frac{3}{2}, \quad n_1(0)=1-2\Delta_{1F},$$

$$n_{02}=\frac{3}{2}, \quad n_2(0)=1-2\Delta_{2F}, \quad n_2(0)+n_1(0)=1, \tag{5.10}$$

Let us find the lifetime $\tau_{2\gamma}$ of a positronium atom, assuming that at time $t=\tau_{2\gamma}$ the average occupation numbers reduce to zero, i.e., $n_1+n_2=0$. The exact value of $\tau_{2\gamma}$ can be found by numerical methods. However, if we take into account that Q_1 and Q_2 are functions of $\tau_{2\gamma}$ that vary more slowly than an exponential function, we can replace $\tau_{2\gamma}$ in the expressions for Q_1 and Q_2 by, e.g., $[W_{2\gamma}^{(0)}(1)]^{-1}$. Then we obtain the approximate expression:

$$\tau_{2\gamma}=\frac{\ln Q}{[W_{2\gamma}^{(0)}(2)-1/2\tau-W_{2\gamma}^{(0)}(1)]}, \tag{5.11}$$

where $Q=-Q_1/Q_2$, and we assume that $(|S_1|/\hbar)^2\tau_{2\gamma}^2\ll 1$, $(|S_2|/\hbar)^2\tau_{2\gamma}^2\ll 1$. This expression demonstrates quite clearly the relative contributions of annihilation and optical transitions to the lifetime of the positronium atom.

5.1. The role of stimulated optical transitions

The solution to the equations of motion contains a contribution from spontaneous optical transitions 2→1 between states of the positronium atom. The origin of these transitions is connected with inclusion of third-order quantum-electrodynamic effects (Fig. 1c) in the effective Hamiltonian (2.1). The radiative interaction of a positronium atom with the field of annihilating and optical photons is treated in the nonrelativistic theory based on the Heisenberg equations of motion for atomic and photon operators without perturbation theory. Within perturbation theory, an external field can be introduced into quantum electrodynamics in two ways. The first is connected with introducing the field into the Dirac equation, whose solutions are then treated as basis functions for quantum-electrodynamic perturbation theory. This approach was used previously by the author[15] in order to demonstrate the possibility of quantum beating in the annihilation decay of the positronium atom in a mixed state. The mixing of two atomic states of a positronium atom was implemented in Ref. 15 using a powerful resonance optical laser; as was shown in Ref. 15, the probability of annihilation decay oscillates with the frequency of the optical transition. The second way to include an external field is based on corrections to the quantum electromagnetic field of the external field. In the present work, this is the method used to include the effect of an optical laser with frequency ω_0. In light of the previous discussions, the method can be implemented by making the following replacement in solution (5.7):

$$-\frac{2}{\tau}\rightarrow-\frac{2}{\tau}-\frac{\omega_0^3 d^2\Delta\Omega}{2c^3\hbar}N_L, \tag{5.12}$$

where $\Delta\Omega$ is the solid angle within which N_L optical photons of the optical laser act on the positronium atom.

Let us investigate the role of stimulated optical transitions, using the solution (5.7) and the following equations:

$$Q_1+Q_2=n_1(0)+n_2(0),$$

$$Q_1\exp\left[-\frac{1}{2}(\bar{a}_3-\bar{b}_1-\bar{C}_1)\tau_{2\gamma}\right]+Q_2\exp(\bar{B}_1\tau_{2\gamma})=0. \tag{5.7a}$$

We assume that at time $t=0$ states 1 and 2 are unoccupied, i.e., $n_1(0)+n_2(0)=0$. Then $Q_1=-Q_2$ and we must satisfy the equation

$$\bar{a}_3-\bar{b}_1-\bar{C}_1+2\bar{B}_1=0,$$

from which we find the lifetime of the positronium atom after some transformations:

$$\tau_{2\gamma}=\sqrt{\frac{8\pi^2(9a+1)}{\alpha a[W_{2\gamma}^{(0)}(1)]^2}\left[1\pm\sqrt{1-\frac{8a(4a-7)}{(9a+1)^2}}\right]}, \tag{5.7b}$$

where

$$a=-\frac{1}{\tau}\frac{1}{W_{2\gamma}^{(0)}(1)}.$$

For small values of the quantity a ($|a|\ll 1$), as is clear from (5.7b),

$$\tau_{2\gamma}=\frac{8\pi}{\sqrt{2\alpha}}\sqrt{\frac{4a-7}{9a+1}}\frac{1}{W_{2\gamma}^{(0)}(1)},$$

i.e., the lifetime depends weakly on optical transitions between states 1 and 2. The role of the induced transitions in Eq. (5.7b) can be significantly enhanced by the transformation (5.12), which changes the numerical value of the quantity a.

5.2. Long-lived state of a positronium atom in the field of an optical laser

In Ref. 9, Ziock *et al.* investigated optical saturation of the 1^3S-2^3P transition of a positronium atom in a constant magnetic field, and hence in the presence of mixing of the triplet and singlet states. In contrast to Ref. 9, this article treats the selective excitation of singlet states, e.g., 2^1P_1, of a positronium atom using an optical laser. In this case the transition $1^1S_0\rightarrow 2^1P_0$ corresponds to a wavelength $\lambda=2429.6$ Å, and the 2γ-annihilation of a parapositronium atom is treated in the absence of triplet states. Whereas the lifetime of a parapositronium atom in state 1^1S_0 equals 125 ps (Refs. 1 and 9), it is eight times larger[1] in the state 2^1P_0. In this case, a transition of a positronium atom from state 2^1P_0 to state 1^1S_0 due to spontaneous emission of optical photons with a characteristic time $\tau=3.2$ ns takes place.[9] Based on the solution (5.7), let us consider the possibility of creating a long-lived state of a positronium atom under the action of an optical laser. For this we assume $(d/dt)(n_1+n_2)=0$. Then from (5.7) we find

$$\ln Q_4-\frac{1}{2}(\bar{a}_3-\bar{b}_1-\bar{C}_1+2\bar{B}_1)t=0, \tag{5.13}$$

where

$$Q_4=\frac{1}{2}\frac{Q_1}{Q_2}\frac{\bar{a}_3-\bar{b}_1-\bar{C}_1}{\bar{B}_1}.$$

It is possible to satisfy condition (5.13) if the individual terms in (5.13) reduce to zero for appropriate interaction parameters of a positronium atom with the field of annihilating and optical photons, i.e.,

$$\ln Q_4\rightarrow 0,\quad \frac{1}{2}(\bar{a}_3-\bar{b}_1-\bar{C}_1+2\bar{B}_1)\rightarrow 0. \tag{5.14}$$

As follows from (5.7), condition (5.14) can be realized if we assume that state 2 corresponds to a certain highly excited state (large values of the principal quantum number n) of the positronium atom, for which $W_{2\gamma}^{(0)}(2)\rightarrow 0$. We assume that

$$(|S_1|/\hbar)^2\tau_{2\gamma}^2=M,\quad (|S_2|/\hbar)^2\tau_{2\gamma}^2=M/n^3, \tag{5.15}$$

where $M\ll 1$. Then

$$\bar{a}_3-\bar{b}_1-\bar{C}_1+2\bar{B}_1=-\frac{1}{\tau}-\frac{\omega_0^3 d^2\Delta\Omega}{4c^3\hbar}N_L+\frac{1}{M-1}W_{2\gamma}^{(0)}(1)\rightarrow 0, \tag{5.16}$$

where ω_0 is the frequency of the transition $1\rightarrow 2$. Similarly, we can calculate Q_3 when condition (5.15) is satisfied. It is easy to show that this quantity reduces to unity if the quantity (5.16) goes to zero. Thus, by choosing the parameters ω_0, d, $\Delta\Omega$ and N_L we can create a long-lived state of the positronium atom without annihilation into gamma rays if we first place the positronium atom in a certain Rydberg state.

Thus, in this article we have shown that the interaction of a positronium atom with a photon field must be understood as a radiative interaction, and also that radiative interactions appreciably change the kinetics of annihilation decay.

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