Periodic two-phase structures during phase transformations stimulated by Joule heating

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One-dimensional longitudinal and transverse periodic structures in two-dimensional samples are considered. In the longitudinal structures the direction of modulation of the temperature field coincides with the direction of the electric current, and in the transverse structures these directions are mutually orthogonal. The properties of the structures are highly dependent on the relationship between the conductivities of the low-temperature (σ_1) and high-temperature (σ_2) phases. It is shown that there is a stable stationary transverse structure in a fixed-current regime when $\sigma_1<\sigma_2$. A similar longitudinal structure exists in a fixed-voltage regime when $\sigma_1>\sigma_2$. However, this longitudinal structure slowly drifts due to the Peltier effect. The transverse structures in materials with $\sigma_1>\sigma_2$ and the longitudinal structures observed for $\sigma_1<\sigma_2$ are unstable in the general case, because the low-temperature phase in these structures is superheated and the high-temperature phase is supercooled. When the degrees of superheating and supercooling are sufficiently large, the appearance of spontaneous motion of these structures (traveling waves) not associated with the Peltier effect is possible.
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1. INTRODUCTION

The presence of heat sources that are nonlinear with respect to the temperature can result in the propagation of undamped thermal waves and periodically modulated thermal fields.[1,2] One of the possibilities for the appearance of such phenomena is associated with the passage of an electric current along a sample composed of a material in which phase transformations can occur. When an electric current is passed, the temperature of the sample is determined by the balance between the Joule heat and the heat dissipated. The difference between the conductivities of the phases and the associated difference between the amounts of Joule heat evolved can result in the appearance of thermal waves, which are simultaneously phase-transition waves,[3–7] as well as in the formation of stationary and moving two-phase structures.[3,4,8]

In Ref. 9 we considered the morphological instability of planar interfaces when a pure material is heated by an electric current. Both isolated thermal waves and two-phase structures were examined in the large-period limit of these structures, where the thermal interaction of different interfaces can be neglected. In the present paper we wish to examine the possibility of the realization of periodic structures in greater detail for arbitrary relationships between the period of the structure and the characteristic lengths of the thermal interaction of the fronts. We shall confine ourselves here to the geometrically simple situation of two parallel electrodes. We shall distinguish between longitudinal structures, in which the direction of modulation of the thermal field coincides with the direction of the electric field, and transverse structures, in which these two directions are mutually orthogonal. We shall consider both fixed-current and fixed-voltage regimes in each of these geometries.

2. THERMAL FIELDS IN PERIODIC STRUCTURES

We consider simple one-dimensional structures, which can drift with a constant velocity in the general case. The thermal fields in the coordinate system moving together with a structure depend on only one spatial coordinate and are described by the equation

$$T_i''(x)+\frac{v}{D}T_i'(x)+\frac{Q_i}{\kappa}-\frac{1}{h^2}[T_i(x)-T_c]=0. \tag{1}$$

Here the subscript $i=1, 2$ labels the two different phases. To be specific, we understand that phase 1 is the low-temperature phase, i.e., the phase which is thermodynamically stable at low temperatures ($T<T_0$) and metastable at high temperatures ($T>T_0$, where T_0 is the equilibrium temperature of phases *1* and *2*). In Eq. (1) v is the steady-state drift velocity of the structure, κ is the thermal conductivity, $D=\kappa/c$ is the thermal diffusivity, c is the specific heat, and h is a coefficient with the dimensions of length, which characterizes the rate of linear heat exchange with the surrounding medium of temperature T_c. For simplicity the thermal characteristics of both phases are assumed to be identical. The density of the Joule heat evolved Q_i is given by the relation

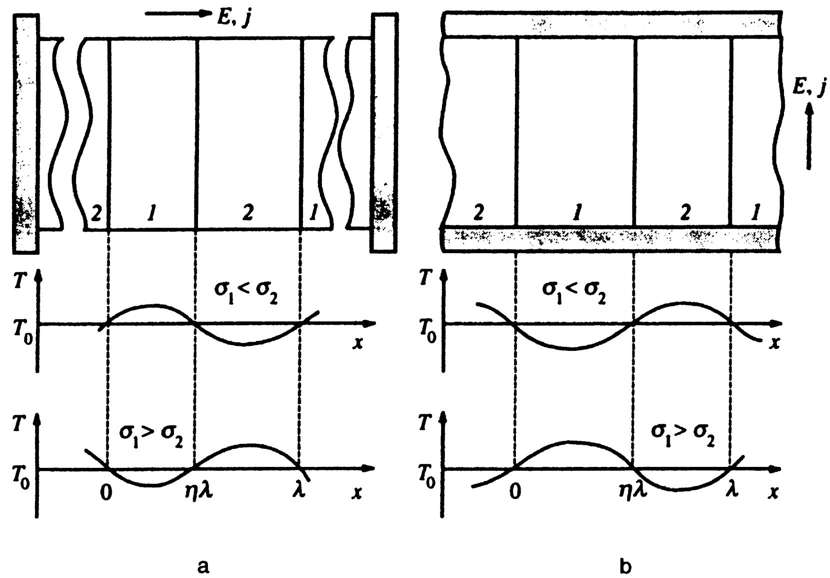

FIG. 1. Schematic representation of periodic two-phase structures and the temperature distribution in them under the longitudinal (a) and transverse (b) geometries.

$$Q_i=\frac{j_i^2}{\sigma_i}=\sigma_i E_i^2, \tag{2}$$

in which j is the current density, E is the electric field, and σ is the conductivity, which is assumed to differ in phases *1* and *2*, but not to depend on the temperature. We stress that we do not restrict the metastability regions of the phases. For example, the low-temperature phase can exist metastably even at $T>T_0$, and its conductivity is σ_1 in that range. Similarly, the conductivity of phase *2* equals σ_2 even at $T<T_0$.

The following equilibrium conditions hold on the phase boundaries, i.e., at $x=0$ and $x=\eta\lambda$ (see Fig. 1):

$$T_1(0)=T_2(\lambda)=T_0, \tag{3}$$

$$T_1(\eta\lambda)=T_2(\eta\lambda)=T_0. \tag{4}$$

Here λ is the period of the structure, and η is the fraction of low-temperature phase *1* in the structure under consideration. A heat balance is maintained on the interfaces, and with consideration of the Peltier effect the conditions defining it have the form

$$D[T_1'(0)-T_2'(\lambda)]=\frac{Lv}{c}+\frac{\Pi j_n}{c}, \tag{5}$$

$$D[T_1'(\eta\lambda)-T_2'(\eta\lambda)]=\frac{Lv}{c}+\frac{\Pi j_n}{c}. \tag{6}$$

In Eqs. (3) and (5) the periodicity of the thermal field is taken into account explicitly, $T_2(0)=T_2(\lambda)$, and $T_2'(0)=T_2'(\lambda)$. In the conditions (5) and (6) L is the latent heat of the transition of phase *2* to phase *1*, Π is the Peltier coefficient for the phase boundary between phases *1* and *2*, and its sign is chosen such that when a current passes from phase 1 to phase 2, i.e., when $j_n>0$, Peltier heat is evolved for $\Pi>0$ and is absorbed for $\Pi<0$. In (5) and (6) j_n is the current component normal to the interface. We note that in a transverse structure $j_n=0$, and the Peltier effect is absent.

The solution of Eq. (1) under the boundary conditions (3)–(6) leads to the following two relations:

$$\Delta_1\frac{(\exp\{s_1\eta\Lambda\}-1)(1-\exp\{-s_2\eta\Lambda\})}{(\exp\{s_1\eta\Lambda\}-\exp\{-s_2\eta\Lambda\})}-\Delta_2\frac{(\exp\{s_1(1-\eta)\Lambda\}-1)(1-\exp\{-s_2(1-\eta)\Lambda\})}{(\exp\{s_1(1-\eta)\Lambda\}-\exp\{-s_2(1-\eta)\Lambda\})}=0, \tag{7}$$

$$\Delta_1\frac{s_2(\exp\{s_1\eta\Lambda\}-1)-s_1(1-\exp\{-s_2\eta\Lambda\})}{(\exp\{s_1\eta\Lambda\}-\exp\{-s_2\eta\Lambda\})}+\Delta_2\frac{s_2\exp\{-s_2(1-\eta)\Lambda\}(\exp\{s_1(1-\eta)\Lambda\}-1)-s_1\exp\{s_1(1-\eta)\Lambda\}(1-\exp\{-s_2(1-\eta)\Lambda\})}{(\exp\{s_1(1-\eta)\Lambda\}-\exp\{-s_2(1-\eta)\Lambda\})}+2V+P=0, \tag{8}$$

where

$$s_1=\sqrt{1+V^2}-V, \qquad s_2=\sqrt{1+V^2}+V. \tag{9}$$

In Eqs. (7)–(9) we have introduced the dimensionless velocity $V=vh/2D$, the dimensionless period $\Lambda=\lambda/h$, the dimensionless Peltier effect $P=\Pi h j_n/LD$, and the dimensionless deviations of the temperatures of the homogeneous phases $T_i=T_c+Q_ih^2/\kappa$ from the equilibrium temperature T_0:

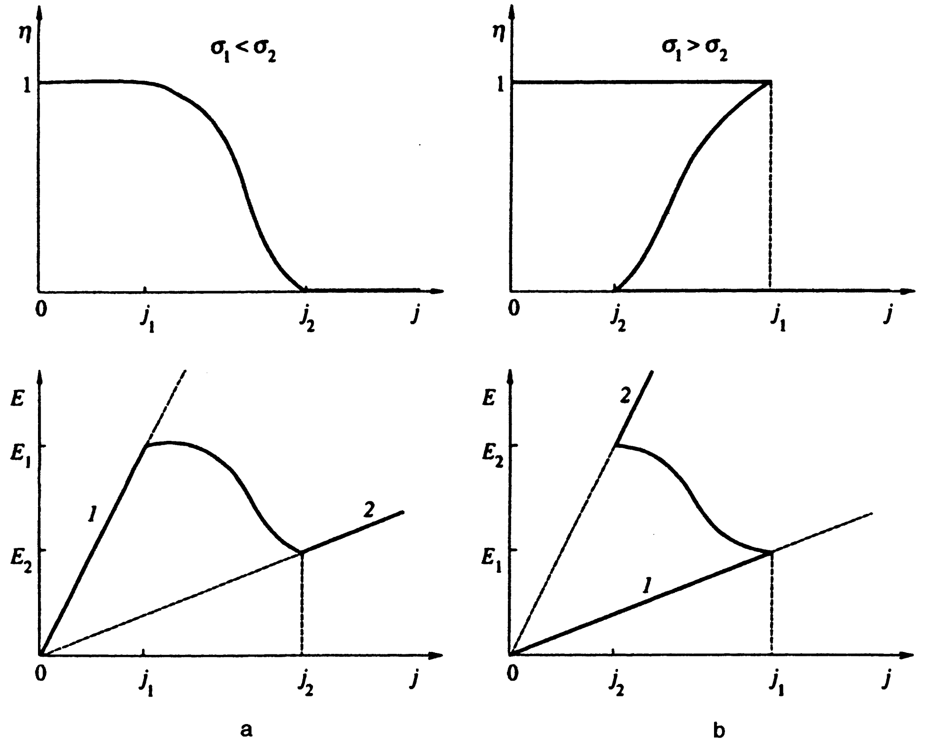

FIG. 2. Qualitative dependence of the fraction η of phase *1* on the current j and corresponding current–voltage characteristic at a fixed period for a longitudinal structure: a) $\sigma_1<\sigma_2$; b) $\sigma_1>\sigma_2$.

$$\Delta_1=\frac{c}{L}\left[T_0-T_c-\frac{Q_1h^2}{\kappa}\right],\qquad \Delta_2=\frac{c}{L}\left[T_c+\frac{Q_2h^2}{\kappa}-T_0\right]. \tag{10}$$

Assuming that the Peltier effect and the corresponding drift velocities are small ($P\ll 1$, $V\ll 1$) and expanding (7) and (8) to terms linear in V, we find

$$\Delta_1\tanh\frac{\eta\Lambda}{2}=\Delta_2\tanh\frac{(1-\eta)\Lambda}{2}, \tag{11}$$

$$V\left[\Delta_1\left(1-\frac{\eta\Lambda}{\sinh(\eta\Lambda)}\right)+\Delta_2\left(1-\frac{(1-\eta)\Lambda}{\sinh(1-\eta)\Lambda}\right)\right]+2V+P=0. \tag{12}$$

Equations (7) and (8) or Eqs. (11) and (12), which correspond to them, allow us to determine η and the drift velocity as functions of the period of the structure Λ under assigned external conditions for passage of the current.

3. LONGITUDINAL PERIODIC STRUCTURES

In this case the periodic structure consists of alternating plates of phase *1* and phase *2* parallel to the electrodes (Fig. 1a). The direction of modulation of the temperature field coincides with the direction of the current. The structure under consideration corresponds to conductors of different conductivity connected in series. The current is constant along the sample, i.e., identical in the two phases. The external conditions can fix either the current density j or the mean electric field strength E. In the latter case the current density in the sample depends on the fractions of the phases:

$$j=\frac{\sigma_1\sigma_2E}{(1-\eta)\sigma_1+\eta\sigma_2}. \tag{13}$$

Fixed-current regime. In Eq. (11), which specifies η, the Δ_i are described according to (10) and (2) by the expressions

$$\Delta_1=\frac{c}{L}\left[T_0-T_c-\frac{j^2h^2}{\kappa\sigma_1}\right],\qquad \Delta_2=\frac{c}{L}\left[T_c+\frac{j^2h^2}{\kappa\sigma_2}-T_0\right] \tag{14}$$

and do not depend on η. It follows from Eq. (11) that a two-phase structure corresponding to $0<\eta<1$ can exist only in a certain range of currents between j_1 and j_2:

$$j_1^2=\frac{\kappa\sigma_1}{h^2}(T_0-T_c),\qquad j_2^2=\frac{\kappa\sigma_2}{h^2}(T_0-T_c), \tag{15}$$

in which Δ_1 and Δ_2 have identical signs (Δ_1 and Δ_2 vanish at j_1 and j_2, respectively). The dependence of η on the current j for a fixed period Λ is shown schematically in Fig. 2.

When $\sigma_1<\sigma_2$ (Fig. 2a), it is seen from (15) that $j_2>j_1$. At currents $j<j_1$ low-temperature phase *1* exists ($\eta=1$), and its temperature $T_1<T_0$; at $j>j_2$ high-temperature phase *2* exists, and its temperature $T_2>T_0$. The existence of a periodic structure with a value of η that depends on the period Λ is possible in the current range $j_1<j<j_2$ (Fig. 3). The temperature is modulated along the direction of the current (Fig. 1a). We stress that the low-temperature phase in this case is superheated and that the high-temperature phase is supercooled with the possible resultant morphological instability of such a structure.[9]

The characteristic value of the current j_0 in Fig. 3 is determined by the behavior of η as the period Λ increases. When $j<j_0$, the fraction η of phase *1* increases and tends to

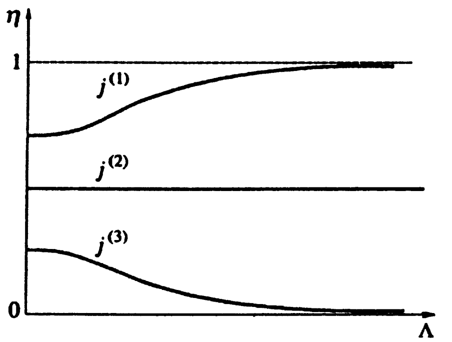

FIG. 3. Qualitative dependence of the fraction η of phase *1* on the period Λ of a longitudinal structure for $\sigma_1<\sigma_2$ and three values of the current j: $j_1<j^{(1)}<j_0$, $j^{(2)}=j_0$, $j_0<j^{(3)}<j_2$.

unity, and when $j>j_0$, it decreases and tends to zero. It is seen from (11) that j_0 is determined from the condition $\Delta_1=\Delta_2$:

$$j_0^2=\frac{2\kappa}{h^2}\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}(T_0-T_c). \tag{16}$$

When $\sigma_1>\sigma_2$ (Fig. 2b), $j_1>j_2$. In this case the regions for the existence of the homogeneous phases determined using the criterion of thermodynamic stability (i.e., $T_1<T_0$ for phase *1* and $T_2>T_0$ for phase 2) overlap. In the current range $j_2<j<j_1$ solutions corresponding to periodic two-phase structures with $0<\eta<1$ formally exist in addition to the homogeneous states. However, a simple analysis reveals the instability of these structures against transitions to the homogeneous states with $\eta=1$ or $\eta=0$. We note that the dependence of η on j in Fig. 2b is typical of kinetic phase transitions of the first kind with overlapping metastability regions $j_2<j<j_1$. The question of the transition point between the homogeneous states of phase *1* and phase *2* can be solved by analyzing the motion of a thermal wave that switches the system from one homogeneous state to the other. When the Peltier effect is small, phase *1* "gobbles up" phase *2* at $j<j_0$, the opposite occurs at $j>j_0$, and the transition current j_0 is specified by (16) (Refs. 5 and 7). As was shown in Ref. 9, the thermal wave front is morphologically stable in the case of $\sigma_1>\sigma_2$ under consideration.

We note that, regardless of the relationship between the conductivities of the phases, the current–voltage characteristics in the region where the two-phase structures exist have segments with a negative differential resistivity, $dE/dj<0$ (Fig. 2).

Fixed-voltage regime. In this case the values of Δ_1 and Δ_2 in Eq. (11) are specified by the relations (14) with j from (13). Now, Δ_1 and Δ_2 depend on the fraction η of phase *1*, and the value of η as a function of the field E and the period Λ is found from the transcendental equation (11). The dependence of η on E at a fixed Λ is qualitatively illustrated in Fig. 4.

As in the case of a fixed current, the metastability regions of the homogeneous states with phase *1* and with phase *2* correspond to $E<E_1\equiv j_1/\sigma_1$ and $E>E_2\equiv j_2/\sigma_2$. Here the currents j_i are assigned by (15) and specify the conditions under which the temperature of each homogeneous phase is equal to the transition temperature T_0. However, in contrast to the fixed-current regime, here the region where the two-phase structures exist is broader than the interval between E_1 and E_2 and depends on the period Λ of the structure. As Λ tends to zero, this region contracts to the interval $[E_1,E_2]$. When $\Delta\ll1$, from Eq. (11) we find

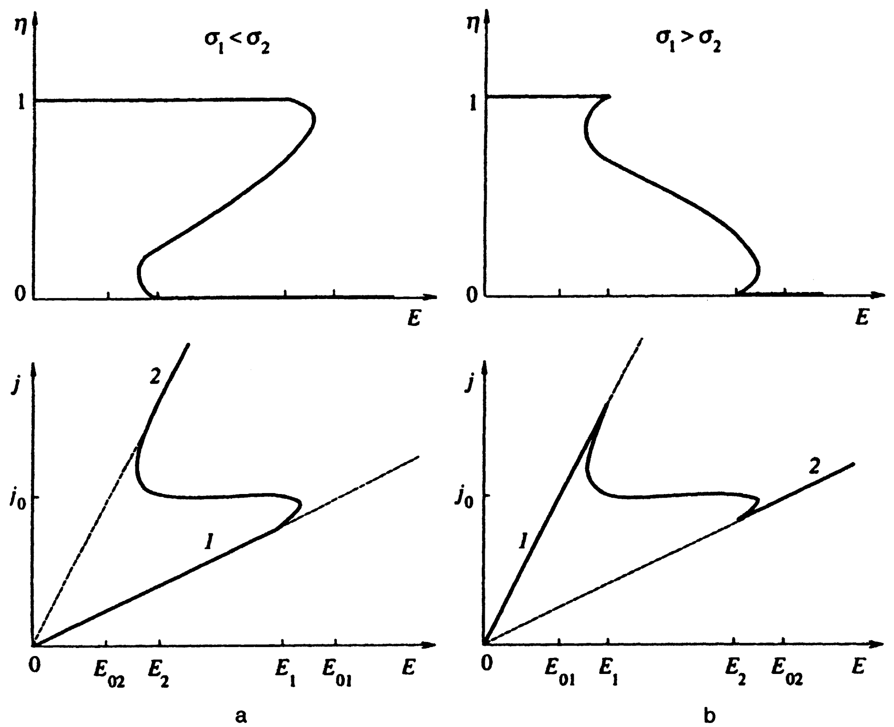

FIG. 4. Qualitative dependence of the fraction η of phase *1* on the field strength E and corresponding current–voltage characteristic for a longitudinal structure with a fixed period: a) $\sigma_1<\sigma_2$, b) $\sigma_1>\sigma_2$.

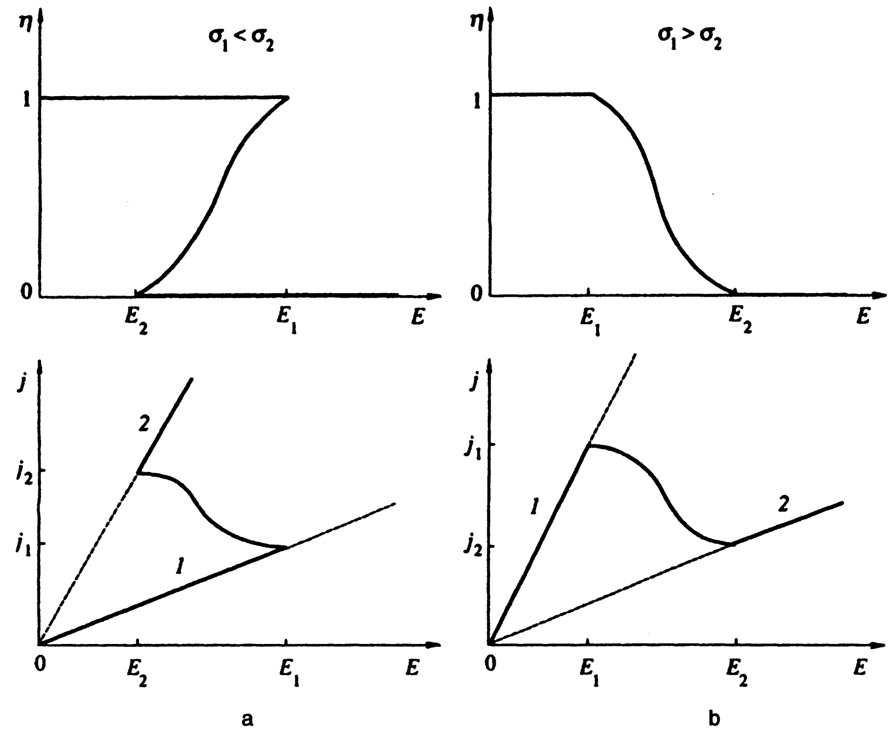

FIG. 5. Same as in Fig. 4, but for a transverse structure.

$$\eta=\left[\frac{\sigma_1\sigma_2h^2E^2}{\kappa(T_0-T_c)}-\sigma_1\right]\bigg/(\sigma_2-\sigma_1). \tag{17}$$

As Λ tends to infinity, this region expands to the interval between $E_{01}\equiv j_0/\sigma_1$ and $E_{02}\equiv j_0/\sigma_2$. The physical meaning of the fact that the fields E_{0i} are determined by the equilibrium current j_0 is as follows. When $\Lambda\to\infty$, the interfaces in the two-phase structure become isolated. Such interfaces can be stationary only when the equilibrium current j_0, at which $\Delta_1=\Delta_2$, passes through the sample. The condition $\Delta_1=\Delta_2$, which follows from Eq. (11) when $\Lambda\to\infty$, specifies the linear dependence of η on the field strength E:

$$\eta=[\sigma_1-\sigma_1\sigma_2E/j_0]/(\sigma_1-\sigma_2), \tag{18}$$

where j_0 is assigned by Eq. (16).

When $\sigma_1<\sigma_2$ (Fig. 4a), the ascending portion of the η versus E curve for the two-phase structures is unstable against transitions to a homogeneous state, in analogy to the situation with $\sigma_1>\sigma_2$ in the fixed-current regime.

When $\sigma_1>\sigma_2$ (Fig. 4b), the two-phase structure is stable. However, unlike the fixed-current regime, under which stability corresponds to $\sigma_1<\sigma_2$, here the transitions from the homogeneous states to the two-phase states correspond to "reverse" bifurcations and the presence of intermediate unstable states. We note that, as was shown in Ref. 9, the interfaces in the two-phase structure in the case of $\sigma_1>\sigma_2$ under consideration are morphologically stable, at least in the limit of large values of Λ.

To conclude this section we note that after determining the fraction η of phase *1* from Eq. (11), we can use (12) to find the drift velocity of the structure, which is proportional to the Peltier effect $P=\Pi hj/LD$. We note that in the fixed-voltage regime the current j, in turn, depends on η according to (13).

4. TRANSVERSE PERIODIC STRUCTURES

In this case the periodic structure consists of alternating plates of phase *1* and phase *2* perpendicular to the electrodes (Fig. 1b). The direction of modulation of the temperature field is perpendicular to the electric field. The structure under consideration corresponds to conductors of different conductivity connected in parallel. Here the electric field strength E is constant along the sample, i.e., identical in the two phases. The external conditions can fix either the field strength E or the mean current density j. In the latter case the value of E in the sample depends on the fractions of the phases:

$$E=j/[\eta\sigma_1+(1-\eta)\sigma_2]. \tag{19}$$

In transverse structures there is no current component perpendicular to the boundary, $j_n=0$, and there is, therefore, no Peltier effect [$P=0$ in Eqs. (8) and (12)]. When stationary periodic structures are considered, it is sufficient to determine η from Eq. (11).

Fixed-voltage regime. In this case the Δ_i appearing in Eq. (11) are described, according to (10) and (2) by the expressions

$$\Delta_1=\frac{c}{L}\left[T_0-T_c-\frac{\sigma_1E^2h^2}{\kappa}\right],$$
$$\Delta_2=\frac{c}{L}\left[T_c+\frac{\sigma_2E^2h^2}{\kappa}-T_0\right] \tag{20}$$

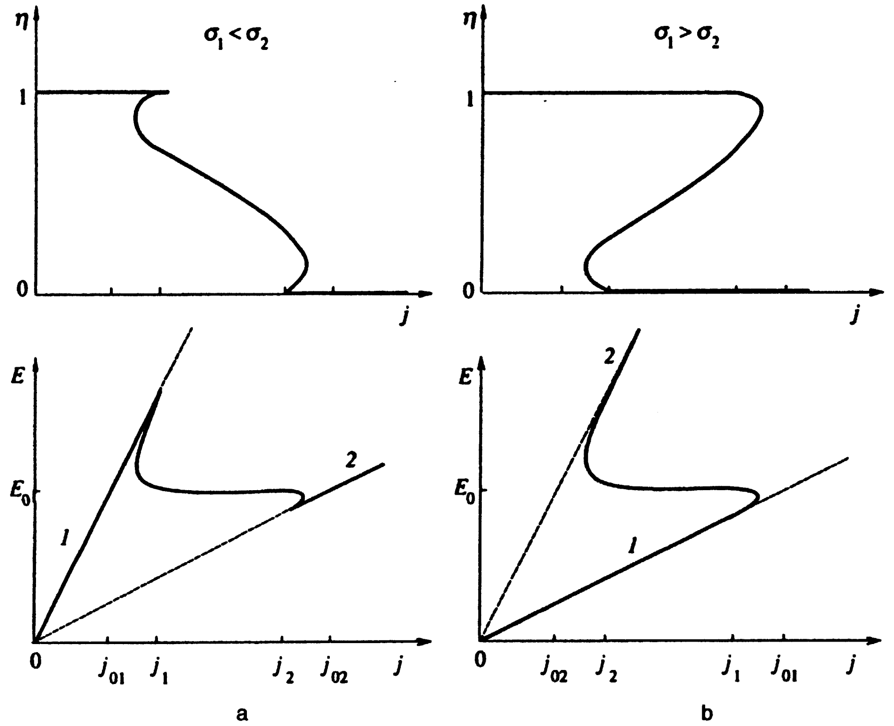

FIG. 6. Same as in Fig. 2, but for a transverse structure.

and do not depend on η.

The dependence of η on E at a fixed value of the period Λ is schematically shown in Fig. 5. The region of two-phase structures is found in the interval between $E_1 \equiv j_1/\sigma_1$ and $E_2 \equiv j_2/\sigma_2$. Here E_1 and E_2, like the currents j_1 and j_2 from (15), correspond to the vanishing of Δ_1 and Δ_2, respectively.

When $\sigma_1 < \sigma_2$ (Fig. 5a), the two-phase structures are unstable against transitions to the homogeneous states. The transition from one homogeneous state to the other can occur upon passage of an isolated thermal wave, which is morphologically stable.[9] In this case phase *2* "gobbles up" phase *1* when $E > E_0$, and phase *1* "gobbles up" phase *2* when $E < E_0$. The critical value of E_0 is determined by the condition $\Delta_1 = \Delta_2$, under which the velocity of the thermal wave equals zero:

$$E_0^2 = \frac{2\kappa(T_0 - T_c)}{h^2(\sigma_1 + \sigma_2)}. \tag{21}$$

When $\sigma_1 > \sigma_2$ (Fig. 5b), the two-phase structures formed in the range of field strengths between E_1 and E_2 is stable against variations of the fractions of the phases in it. However, the temperature distribution in such a structure (Fig. 1b) is characterized by the superheating of phase *1* and the supercooling of phase *2*, which possibly lead to the morphological instability of the interfaces in such a structure.

The current–voltage characteristic for the two-phase structures has a negative differential conductivity regardless of the relationship between the conductivities (Fig. 5), just as in the case of the longitudinal two-phase structures in the fixed-current regime.

Fixed-current regime. The description of this regime is largely similar to the fixed-voltage regime for longitudinal structures. The Δ_i, which are defined by (20) and (19) in terms of the mean current density j, now depend on η. The dependence of η on j at a fixed value of Λ is qualitatively illustrated in Fig. 6, which is similar in many respects to Fig. 4.

The range of currents at which a two-phase structure exists depends on the period Λ of the structure. When $\Lambda \to 0$, this range contracts to the interval $[j_1, j_2]$. In this case, using Eq. (11) and considering (19) and (20), for $\Lambda \ll 1$ we find

$$\eta = \left[\frac{h^2 j^2}{\kappa(T_0 - T_c)} - \sigma_2\right] \bigg/ (\sigma_1 - \sigma_2). \tag{22}$$

The fraction η of phase *1* depends quadratically on the current and is equal to zero and unity at j_2 and j_1, respectively [the currents j_i are defined by (15)]. In the other limiting case, in which $\Lambda \to \infty$, the region where the two-phase structures exist expands to the interval between the points $j_{01} \equiv E_0/\sigma_1$ and $j_{02} \equiv E_0/\sigma_2$, where E_0 is specified by Eq. (21) and corresponds to the condition that each isolated boundary is stationary. Here, setting $E = E_0$, from (19) we find

$$\eta = (j/E_0 - \sigma_2)/(\sigma_1 - \sigma_2). \tag{23}$$

It is not difficult to see that (23) is the solution of Eq. (11) for $\Lambda \gg 1$, in which case $\Delta_1 = \Delta_2$.

When $\sigma_1 < \sigma_2$ (Fig. 6a), as in the case corresponding to Fig. 4b, the two-phase structure is stable, and transitions from the homogeneous states to the two-phase states correspond to reverse bifurcations and the presence of intermediate unstable states. We note that in this case each of the phases in the structure is in its own stability region, i.e., low-temperature phase *1* is at temperatures below T_0, and phase *2* is at temperatures above T_0 (Fig. 1b).

When $\sigma_1 > \sigma_2$ (Fig. 6b), the ascending portion of the $\eta(j)$ curve for the two-phase structures is unstable against transitions to a homogeneous state, as in the cases in Figs. 2b, 4a, and 5a.

5. MOTION OF THE PERIODIC STRUCTURES

As we have already noted, the Peltier effect results in motion of the longitudinal structures, whose direction depends on the direction of the electric current. In addition, motion can also appear spontaneously as a bifurcation from stationary structures. As will become clear, spontaneous motion of the structures is possible only under conditions such that low-temperature phase *1* is superheated and phase *2* is supercooled. This means that such motion is possible for transverse structures when $\sigma_1 > \sigma_2$ and for longitudinal structures when $\sigma_1 < \sigma_2$. In order to stress the spontaneity of this motion, we shall discuss transverse structures, in which the Peltier effect and the motion associated with it are absent. The direction of the spontaneous motion can then be arbitrary; the structure can move either to the right or to the left with velocities of identical magnitude (traveling waves).

Let us consider periodic transverse structures in the fixed-voltage regime. Stationary structures exist in the interval between E_1 and E_2, and when $\sigma_1 > \sigma_2$, they are stable against transitions to the homogeneous states. We shall not consider the fixed-current regime for $\sigma_1 > \sigma_2$, since the corresponding structures are unstable. The dependence of η on E is specified by Eqs. (11) and (20) and is qualitatively illustrated in Fig. 5b. Equation (12) (with $P=0$ in the case under consideration) was written in the linear approximation with respect to V. As usual, the bifurcation to a regime of moving structures is specified by the condition that the coefficient in front of the term that is linear in V vanish:

$$\Delta_1\left[1-\frac{\eta\Lambda}{\sinh(\eta\Lambda)}\right]+\Delta_2\left[1-\frac{(1-\eta)\Lambda}{\sinh(1-\eta)\Lambda}\right]+2=0. \tag{24}$$

The solution of this equation together with (11) specifies values of E within the interval $[E_1, E_2]$ and the fraction η of phase *1* at the bifurcation points for an assigned value of the period Λ. The linear approximation is inadequate for finding the velocity and η in a moving structure, and the general equations (7) and (8) must be considered. An analysis of these equations leads to the following qualitative picture for the velocity V (Fig. 7).

As Λ decreases, the region for the existence of moving structures contracts to the point E_0, at which $\Delta_1=\Delta_2=\Delta_0$ and $\eta \equiv 1/2$:

$$\Delta_0=-\frac{c}{L}(T_0-T_c)\,\frac{1-\sigma_2/\sigma_1}{1+\sigma_2/\sigma_1}. \tag{25}$$

The moving solutions vanish when the two bifurcation points merge and coincide with the point E_0 (Fig. 7). This occurs at the critical values Δ_0 and $\Lambda=\Lambda_0$, which satisfy Eq. (24) at the point E_0:

$$\frac{\sinh(\Lambda/2)}{\Lambda/2-\sinh(\Lambda/2)}=\Delta_0. \tag{26}$$

Here motion is possible in the range of parameters in which

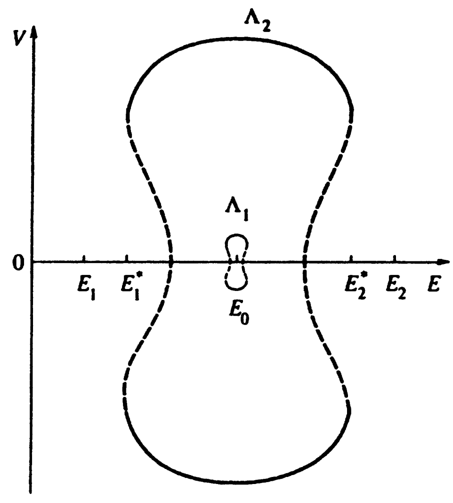

FIG. 7. Qualitative dependence of the rate of spontaneous motion of transverse structures on the field strength E for two different values of the period $\Lambda_2 > \Lambda_1$. The value of Λ_1 is close to the critical value determined from Eq. (26).

$$\frac{\sinh(\Lambda/2)}{\sinh(\Lambda/2)-\Lambda/2}<|\Delta_0|. \tag{27}$$

It is seen from (26) and (27) that for a fixed value of the period Λ motion is possible when $|\Delta_0|$ exceeds a certain critical value that depends on Λ; if Δ_0 is fixed, motion appears at values of Λ greater than a certain critical value that depends on $|\Delta_0|$. Moving solutions are impossible when $|\Delta_0|<1$. We note that since moving solutions exist only for $\Delta_0<0$, spontaneous motion of the transverse structures is possible only in systems with $\sigma_1 > \sigma_2$, and spontaneous motion of the longitudinal structures is possible only in systems with $\sigma_1 < \sigma_2$. In such systems low-temperature phase *1* is superheated, and high-temperature phase *2* is supercooled. Moreover, the degrees of superheating and supercooling must be fairly large: $|\Delta_0|>1$.

At the point E_0, at which $\Delta_1=\Delta_2=\Delta_0$ and $\eta=1/2$, Eq. (7) holds identically, and Eq. (8) becomes significantly simpler and takes the form

$$\frac{\sinh(V\Lambda/2)}{V}=\frac{|\Delta_0|-1}{|\Delta_0|\sqrt{1+V^2}}\sinh\left(\sqrt{1+V^2}\,\frac{\Lambda}{2}\right). \tag{28}$$

Near the boundary where the solutions of (26) vanish the velocity is small ($V \ll 1$) and vanishes as

$$V^2=6\left[\frac{|\Delta_0|-1}{|\Delta_0|}\sinh\left(\frac{\Lambda}{2}\right)-\frac{\Lambda}{2}\right]\Bigg/\left[\frac{3\Lambda}{2}+\left(\frac{\Lambda}{2}\right)^3-3\left(\frac{\Lambda}{2}\right)^2\coth\left(\frac{\Lambda}{2}\right)\right]. \tag{29}$$

We present several evaluations for the case of $\Lambda \gg 1$. In the case in which large values of Λ correspond to conditions close to (26) (this is possible when $|\Delta_0|$ is close to 1), from (29) we find

$$V^2 \simeq 6\left[\frac{|\Delta_0|-1}{2}\exp\left(\frac{\Lambda}{2}\right)-\frac{\Lambda}{2}\right]\left(\frac{\Lambda}{2}\right)^{-3} \ll 1. \tag{30}$$

In the other limiting case the velocity is large. Using Eq. (28) for $\Lambda \gg 1$ and $V \gg 1$, after expanding in $1/V^2$ we obtain the velocity at E_0

$$|V| = \frac{\Lambda}{2} \Big/ \ln \frac{|\Delta_0|}{|\Delta_0|-1}. \qquad (31)$$

In the limit $\Delta \gg 1$ under consideration, the bifurcation point located between E_1 and E_0 is characterized by a value of Δ_1 in the range $1 < |\Delta_1| < 2$. There is a similar range for Δ_2 and the second bifurcation point, which is located between E_0 and E_2. In addition, $|\Delta_i| \to 1$ when $|\Delta_0| \to 1$, and $|\Delta_i| \to 2$ when $|\Delta_0| \gg 1$. The values of the fields at the turning points E_1^* and E_2^* (see Fig. 7) depend on $|\Delta_0|$, but so that the corresponding values $|\Delta_i^*| \simeq 1$.

The qualitative picture shown in Fig. 7 is characteristic of reverse bifurcations. Here the dashed portions of the curves are unstable against transitions either to stationary structures or to structures that move with larger velocities and correspond to the solid portions of the curves in Fig. 7.

As we have already noted, the spontaneous motion of a structure is possible under the conditions of high degrees of superheating and supercooling of metastable phases. Motion of the transverse structures is therefore possible in systems with $\sigma_1 > \sigma_2$. Conversely, spontaneous motion of the longitudinal structures occurs only in systems with $\sigma_1 < \sigma_2$ in the fixed-current regime (in the fixed-voltage regime the structures are unstable against transitions to the homogeneous states). Also, the velocity of these structures is different for motion along and against the current due to the presence of the Peltier effect.

The practical realization of the spontaneous motion of structures requires materials with a broad region of metastability, since the material must withstand high degrees of superheating and supercooling during an experiment.

6. CONCLUSIONS

We have examined both the stationary and moving periodic structures appearing during phase transitions stimulated by Joule heating. These structures depend significantly on the relationship between the conductivities of the low-temperature phase (σ_1) and the high-temperature phase (σ_2). In addition, it is important whether a fixed-current or a fixed-voltage regime is effected and whether the geometry of the structures is longitudinal or transverse.

In materials with $\sigma_1 < \sigma_2$:

1. There is a stable transverse structure in the fixed-current regime (Fig. 6a).

2. In the fixed-current regime there is a longitudinal structure (Fig. 2a) which moves with a small velocity that is proportional to the Peltier coefficient. This structure, however, can be morphologically unstable. In addition, in a certain range of currents a slowly moving structure can become unstable against a transition to more rapid spontaneous motion not associated with the Peltier effect. Such a transition to spontaneous motion is possible under the conditions of strong superheating and supercooling of the phases in the two-phase structure.

3. In fixed-voltage regimes both the longitudinal and transverse structures are unstable against transitions to the homogeneous states in the range of field strengths between E_1 and E_2 (Figs. 4a and 5a). The transition from one homogeneous state to the other can occur as a result of the motion of transverse thermal waves. In this case the low-temperature phase is stable against the transition to the high-temperature phase at $E < E_0$, and the high-temperature phase is stable at $E > E_0$.

In materials with $\sigma_1 > \sigma_2$:

1. There is a stable longitudinal structure in the fixed-voltage regime (Fig. 4b).

2. In the fixed-voltage regime there is a stationary transverse structure (Fig. 5b), in which the low-temperature phase is superheated and the high-temperature phase is supercooled. This structure can be morphologically unstable. At sufficiently high degrees of superheating and supercooling of the phases the stationary structure can become unstable against the appearance of spontaneous motion.

3. In the fixed-current regime the longitudinal and transverse periodic structures are unstable against transitions to the homogeneous states in the range of currents between j_1 and j_2 (Figs. 2b and 6b). Transitions between the homogeneous states can occur as a result of the propagation of longitudinal thermal waves.

We considered very simple periodic structures containing one plane of each of the phases in each period. In the general case these structures exist in a range of periods greater than a certain critical value. In the case of stationary structures or structures that move only because of the Peltier effect, this critical value of the period is equal to zero within the current interval $[j_1, j_2]$ or the field interval $[E_1, E_2]$ (see Figs. 4b and 6a). Transitions of the period-doubling type and the formation of more complicated periodic structures are apparently possible. However, an analysis of these questions is beyond the scope of the present work.

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