Coherent scattering of three-level atoms in the field of a bichromatic standing light wave

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We discuss the coherent scattering of three-level atoms in the field of two standing light waves for two values of the spatial shift. In the case of a zero spatial shift and equal frequency detunings of the standing waves, the problem of scattering of a three-level atoms is reduced to scattering of an effectively two-level atom. For the case of an exact resonance between the waves and transitions we give expressions for the population probability of the states of the three-level atom obtained in the short-interaction-time approximation. Depending on the initial population distribution over the states, different scattering modes are realized. In particular, we show that there can be initial conditions for which the three-level system does not interact with the field of the standing waves, with the result that there is no coherent scattering of atoms. In the case of standing waves shifted by $\pi/2$, there are two types of solution, depending on the values of the frequency detuning. For instance, when the light waves are detuned equally we give the exact solution for arbitrary relationships between the detuning and the standing wave intensities valid for any atom–field interaction times. The case of "mirror" detunings and shifted standing waves is studied only numerically. © *1996 American Institute of Physics.* [S1063-7761(96)00706-8]

1. INTRODUCTION

Coherent scattering of atomic wave packets by a periodic spatial structure formed by standing light waves has always greatly attracted the attention of researchers.[1–4] Primarily the interest lies in various coherent effects, such as the Kapitza–Dirac optical effect and the optical analog of the Stern–Gerlach effect, observable in the scattering process. In the first effect the width of the wave packet of an atom exceeds the wavelength of the light, while in the second it is the wavelength that is greater than the size of the wave packet. Recently several papers have appeared in which the scattering by standing waves is proposed as a method for creating splitters of coherent atomic beams.[5,6]

Note that all the researchers use the traditional approach: they study the coherent scattering of the wave packet of a two-level atom in the field of a single standing light wave. However, even this simple case of scattering of a two-level atom by a standing wave demonstrates the difficulties in describing the scattering processes, difficulties related to the need of directly allowing for the microscopic structure of the light field. This explains the choice of various scattering modes, among which the best-known are the Raman–Nat scattering mode and the scattering mode of the Bragg type.[4] The main difference between the two modes is that in Raman–Nat scattering the solution is sought for interaction times much shorter than the reciprocal recoil frequency (see, e.g., Ref. 7), while in scattering of the Bragg type the atom–field interaction time is much longer than the reciprocal recoil frequency.[2] Generally, i.e., allowing for the finiteness of the atomic recoil energy and arbitrary times of interaction of atoms and the standing-wave field, it is impossible to derive an exact analytical solution of the problem not only for arbitrary values of frequency detuning but also in the case of a zero detuning.[4,8]

There has also been an upsurge of interest in the scattering of three-level atoms in the field of traveling waves.[9–12] For instance, Marte *et al.*[9] and Pfau *et al.*[10] suggested an effective beam splitter for three-level atoms, and the possibility of obtaining effective temperatures of roughly 10^{-12} K by the velocity selection method applied to a beam of three-level atoms in a field of oppositely disected traveling waves was discussed in Refs. 11 and 12.

In this paper we present the results of an analytical and numerical analysis of the scattering of three-level atoms by the field of two standing light waves, $\mathbf{E}_1 \cos(kx)$ and $\mathbf{E}_2 \cos(kx+\phi)$, for two different values of the relative spatial shift ϕ. Here the width of the atomic wave packet in the direction orthogonal to the that of propagation of the atomic beam is assumed to be roughly greater than, or equal to, the wavelength. We show that the problem of scattering of three-level atoms in the case of equal detuning and zero spatial shift between the standing waves is reduced to that of scattering of an effectively two-level atom, since the Hamiltonian H(t) of a three-level system in a two-photon resonance can be expressed in terms of the generators of the subgroups $SU(2)*U(1)$ of the $SU(3)$ group.[13] In other words, for equal detuning of the light waves and a zero spatial shift, the three-level atom can always be represented as an effectively two-level atom interacting with the field and a separate level whose temporal evolution can be obtained fairly simply.

For the case of short interaction times,[8] zero frequency detuning of the standing waves, and normal incidence of the atomic beam we obtain an analytical solution for the population probabilities of the states of a three-level atom. We study the effectiveness of three-level atom scattering as a function of the type of initial conditions and show that under

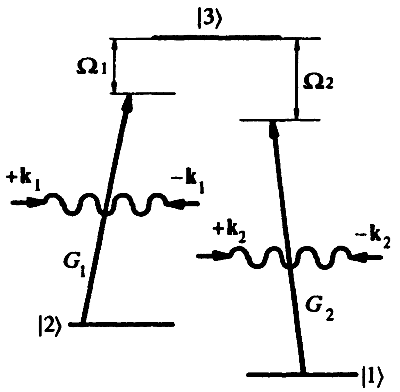

FIG. 1. Atomic level diagram for a three-level Λ-atom interacting with two standing waves: Ω_1 and Ω_2 are the detunings of the light waves with respect to the atomic transition frequencies, G_1 and G_2 are the Rabi frequencies of the light waves, and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors.

certain initial conditions the atomic beam is effectively scattered while under other conditions there is no scattering. In the latter case the dynamics of the three-level system differs essentially from the well-studied case of a two-level atom, where momentum is always transferred from field to atom irrespective of the type of initial conditions.

We study the coherent scattering of three-level atoms in the field of standing light waves with a $\pi/2$ relative spatial shift. Here we examine the case of equal detunings of the light waves (a two-photon resonance) and the case of "mirror" detuning, where the frequency detunings are equal in absolute value but have opposite signs. Note that for the case of shifted standing waves and equal frequency detuning we give the exact solutions with arbitrary interaction times. Here, irrespective of the values of the frequency detuning and the times of interaction of the atom and the field of the standing waves, the effective scattering is only measured by a quantity that is an integral multiple of the recoil momentum $|\Delta p=0,\hbar k,2\hbar k|$, which sets this type of three-level–atom scattering apart from two-level–atom scattering, where the momentum acquired by the atom in the scattering process is proportional to the time of interaction of the atom and the field of the standing wave. At the same time, for the case of mirror detunings and a $\pi/2$ value of the spatial shift, the scattering pattern basically corresponds to that of scattering of a two-level atom by a single standing wave. However, the scattering process happens to be more effective, which suggests that such a scheme of interaction of atoms and light fields can serve as a splitter of neutral-atom beams.[14]

2. SCATTERING OF ATOMS IN THE FIELD OF TWO SPATIALLY IN-PHASE STANDING LIGHT WAVES

Let us take the three-level diagram of atomic levels (Fig. 1) in the field of standing waves with a zero spatial shift:

$$\mathbf{E}=\mathbf{e}_1E_1\cos(k_1z)\exp(i\omega_1t)+\mathbf{e}_2E_2\cos(k_2z)\exp(i\omega_2t), \quad (1)$$

where $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit polarization vectors, E_1 and E_2 are the field amplitudes, k_1 and k_2 are the wave vectors, and ω_1 and ω_2 are frequencies in resonance with the frequencies ω_{13} and ω_{23} of the optical transitions in the three-level atom. We assume that the atomic beam crosses the z axis at right angles.

Note that from the practical viewpoint the notion of a spatial shift between two standing waves is of a local nature. For instance, if the wave vectors differ by Δk and both standing waves are formed by reflection from a common mirror, even at a distance $D=\pi/2\Delta k$ from the mirror a zero spatial shift changes by $\pi/2$. For instance, for a potassium atom excited on the optical transition $3S\rightarrow3P$ the distance D is approximately 5 cm if for the lower states $|1\rangle$ and $|2\rangle$ of the three-level system we take the sublevels of the hyperfine structure of the ground state.

We assume that there can be no spontaneous relaxation in the system. From the standpoint of physics this corresponds to small times of interaction of the atom and the field of the standing waves. In other words, for zero frequency detunings of the standing waves the time of interaction of the atom and the field (1) can be much shorter than the time of spontaneous decay of the upper excited level in the three-level system (see Fig. 1). But if the detunings are much larger than the natural atomic-transition linewidth and the light waves have low intensities, coherent scattering can take place in interaction times of roughly equal to, or longer than, the spontaneous decay time, since in this case we can ignore the population of the excited state as in the scattering of a three-level atom in the field of two counterpropagating traveling waves.[11,12] Restrictions on the interaction time $T_{\rm int}$ in numerical calculations are imposed only by the existence of spontaneous decay in real systems. If we assume that there is no relaxation, initially no restrictions on the interaction time are introduced. The coherent dynamics of scattering of the wave packet of a three-level atom in the field (1) is described by a wave function of the type

$$\Psi(z,\xi,t)=\sum_i \tilde{a}_i(z,t)\psi_i(\xi)\exp\left(-\frac{i}{\hbar}\varepsilon_it\right), \quad (2)$$

where z is the center-of-mass coordinate of the atom (the motion is solely along the z axis), $\tilde{a}_i(z,t)$ are the time-dependent amplitudes describing the translational dynamics of the atom, ξ stands for the set of coordinate characterizing internal motion, and ε_i are the level energies. Note that in studying coherent scattering we are interested in the variations of the momentum distribution in the atomic beam only along the z axis. Here we also assume that all the atoms, irrespective of their transverse momenta, take the same time interval $T_{\rm int}$ to fly through the interaction region, an interval that determines the interaction. The width of the atomic wave packet is assumed to be equal to, or greater than, the wavelength of the light field.

For the Hamiltonian of the system under investigation we can write

$$\hat{H}=\hat{H}_0+\hat{V}+\hat{P}^2/2M, \quad (3)$$

where $\hat{H}_0$ corresponds to the internal states of the atom, $\hat{V}$ is the operator representing the dipole interaction of the atom and the field (1), $\hat{P}^2/2M$ is the operator of the kinetic energy of atomic motion along the z axis, and M is the mass of the atom. Substituting (2) into the time-dependent Schrödinger equation with (3) as the Hamiltonian, in the resonance approximation we arrive at the following system of equations for the time-dependent probability amplitudes $a_i(z,t)$ describing the translational dynamics of three-level atoms in the field (1):

$$i\frac{da_1}{dt}=-\frac{\hbar}{2M}\nabla^2 a_1-G_1a_3\cos(kz)+\Omega_1a_1,$$

$$i\frac{da_2}{dt}=-\frac{\hbar}{2M}\nabla^2 a_2-G_2a_3\cos(kz)+\Omega_2a_2, \tag{4}$$

$$i\frac{da_3}{dt}=-\frac{\hbar}{2M}\nabla^2 a_3-G_1a_1\cos(kz)-G_2a_2\cos(kz),$$

where G_i and Ω_i are, respectively, the Rabi frequencies and the frequency detunings of the standing waves, and we have eliminated the explicit time dependence by introducing $\widetilde{a_i}=a_i\exp(i\Omega_it)$. In deriving the system of equations (4) we ignored the difference in the wave vectors and assumed that $k_1\approx k_2$ and $|\omega_1-\omega_2|\ll\omega$, which imposes restrictions on the characteristic spatial size of the atomic beam, D_{tr}, since only in this case can we speak of a definite relative spatial phase of the two standing waves. Here for beams with widths $D\geqslant\pi/2\Delta k$ there can be effects caused by the difference Δk in the wave vectors of the excitation waves, which directly affects the nature of scattering.

The natural way to solve equations of the form (4) is to go to the momentum representation:

$$a_i(p,t)=\frac{1}{\sqrt{2\pi}}\int a_i(z,t)\exp\left(-\frac{ipz}{\hbar}\right)dz. \tag{5}$$

Introducing new variables r, s, and a by the formulas $r=a_1-a_2$, $s=a_1+a_2$, and $a=\sqrt{2}a_3$ into (4), we get

$$i\frac{dr(p,t)}{dt}=\frac{p^2}{2M\hbar}r,$$

$$i\frac{ds(p,t)}{dt}=\frac{p^2}{2M\hbar}s-\widetilde{G}[a(p+\hbar k)+a(p-\hbar k)]+\Omega s, \tag{6}$$

$$i\frac{da(p,t)}{dt}=\frac{p^2}{2M\hbar}a-\widetilde{G}[s(p+\hbar k)+s(p-\hbar k)],$$

where we have assumed that the detunings of the standing waves are $\Omega_1=\Omega_2=\Omega$, and that $G_1=G_2=G$, with $\widetilde{G}=G/\sqrt{2}$.

Equations (6) clearly show that the problem of scattering of a three-level atom in the field of standing waves in the case of equal detunings of these waves can be reduced to the scattering of an effectively two-level atom. Indeed, the equation for the variable r shows that the given state is not coupled to the other levels of the three-level system. Here finding the solution of this equation for arbitrary (but equal) detunings is easy, and the other two equations in (6) for the variables s and a are well known from the theory of scattering of two-level atoms.[1–5,8]

Note that the possibility of representing a three-level atom in the form of an effectively two-level atom interacting with the field and a separate level not interacting with the field is closely linked to the symmetry properties of the interaction Hamiltonian and has been studied in detail in Ref. 13. Here we directly show how the general principle can be applied to the problem of scattering of a three-level atom in the field of standing waves.

Now let us study in greater detail the case of a zero detuning of the standing waves. The solution of system (6) can easily be obtained if we use the results of Cook and Bernhardt:[8]

$$\begin{aligned}a_1(p,t)=&\frac{1}{2}\sum_n(-1)^n\exp\left(-\frac{ip^2t}{2M\hbar}\right)\{J_{2n}(2\widetilde{G}t)[a_1^0(p\\&-2n\hbar k)+a_2^0(p-2n\hbar k)]\\&+i\sqrt{2}J_{2n+1}(2\widetilde{G}t)a_3^0[p-(2n+1)\hbar k]\}\\&+\frac{1}{2}\exp\left(-\frac{ip^2t}{2M\hbar}\right)[a_1^0(p)-a_2^0(p)],\end{aligned}$$

$$\begin{aligned}a_2(p,t)=&\frac{1}{2}\sum_n(-1)^n-\frac{1}{2}\exp\left(-\frac{ip^2t}{2M\hbar}\right)\{J_{2n}(2\widetilde{G}t)\\&\times[a_1^0(p-2n\hbar k)+a_2^0(p-2n\hbar k)]\\&+i\sqrt{2}J_{2n+1}(2\widetilde{G}t)a_3^0[p-(2n+1)\hbar k]\}\\&-\exp\left(-\frac{ip^2t}{2M\hbar}\right)[a_1^0(p)-a_2^0(p)],\end{aligned} \tag{7}$$

$$\begin{aligned}a_3(p,t)=&\sum_n(-1)^n\exp\left(-\frac{ip^2t}{2M\hbar}\right)\bigg\{J_{2n}(2\widetilde{G}t)a_3^0(p\\&-2n\hbar k)+\frac{i}{\sqrt{2}}J_{2n+1}(2\widetilde{G}t)\\&\times[a_1^0(p-(2n+1)\hbar k)+a_2^0(p-(2n\\&+1)\hbar k)]\bigg\},\end{aligned}$$

where $J_n(x)$ is a Bessel function, the $a_i^0(p)$ are the amplitudes of the states initially, and we have returned to the old notation for the probability amplitudes.

Note that in deriving (7) we used the short–interaction–time approximation determined by the conditions

$$RT_{\text{int}}\ll 1,\quad RT_{\text{int}}p_m/\hbar k\ll 1, \tag{8}$$

where $R=\hbar k^2/2M$ is the frequency related to the recoil energy, p_m is the momentum transferred to the atom in the process of interaction, and T_{int} is the time the atom and the field of the standing waves interact.[8] Here, however, we did not require that there be definite relationships between the recoil frequency R and the Rabi frequencies G of the standing waves. Hence the obtained solutions describe, in principle, both the scattering mode with $G\ll R$ and the opposite scattering mode.

The solutions (7) immediately suggest that when the initial probability amplitudes of the lower levels are

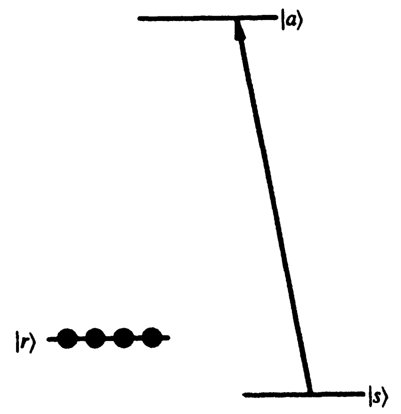

FIG. 2. The interaction diagram for a three-level system in terms of the variables r, s, and a. The initial conditions correspond to $a_1(p)=-a_2(p)$ and $a_3=0$.

$$a_1^0=\frac{1}{\sqrt{2}}F(p), \quad a_2^0=-\frac{1}{\sqrt{2}}F(p), \quad a_3^0=0, \tag{9}$$

with $F(p)$ an arbitrary even function, there can be no scattering of three-level atoms by standing light waves. In other words, the initial values of the probability amplitudes are preserved in time and an atom does not "feel" the presence of resonant optical fields. On the whole such a scattering pattern is caused by the presence in the three-level system of special states (which above have been written explicitly) in which the atom cannot either absorb or emit resonant photons.[13] The important fact is that the amplitudes in (9) have different signs, which corresponds to an atomic beam that is in a definite coherent state.

To make the above reasoning more transparent, in Fig. 2 we depict the three-level interaction diagram of Fig. 1 in terms of the variables r, s, and a for the initial amplitudes (9). We see that when the system is in the state r, which according to (6) is not optically coupled with the field (1), there is no excitation of the three-level system.

Now let us examine the case where the initial amplitudes of the lower levels are

$$a_1^0=\frac{1}{\sqrt{2}}F(p), \quad a_2^0=\frac{1}{\sqrt{2}}F(p), \quad a_3^0=0. \tag{10}$$

In this case initially the entire population of the system is in the state s optically coupled with the upper state of the three-level system. The effectively two-level atom with states s and a actively interacts with the standing-wave field, while the population in the state r is zero. The results of calculations for a velocity distribution with initial width $0.25\hbar k$ are depicted in Fig. 3. For instance, Fig. 3a shows the variation of the total momentum distribution in the atomic beam for different scattering times. We see that even at times of order G^{-1} the scattering pattern resembles that of a two-level atom.[8] Figure 3b depicts the velocity distribution for the part of the atomic population in the state $|3\rangle$. Clearly, a substantial population appears on the third level already at times of order G^{-1}.

Note that the scattering pattern in Fig. 3b demonstrates the possibility of obtaining atoms, with well-defined momenta, in the upper state $|3\rangle$ for interaction times of G^{-1} or $3G^{-1}$. Here, generally, the entire population in the system is distributed among three levels, but the probability of discovering an atom on a common level has well-resolved peaks at odd values of $\hbar k$. We also note that for times longer than $3G^{-1}$ the precise one-resonance structure is transformed into a multiresonance structure (Fig. 3b), and using this interaction scheme for such times as a beam splitter is problematic. In addition, using the upper state in a beam splitter is highly unlikely from the practical viewpoint because of rapid spontaneous decay of the state. Actually, the interaction scheme depicted in Fig. 1 can be inverted: we can assume the common level $|3\rangle$ to be the ground and long-lived state (the V-diagram of levels). Then beam splitting occurs for atoms in the ground state $|3\rangle$ for atom–field interaction times T_{int} equal to G^{-1} or $3G^{-1}$, but the main difficulty here lies in preparing the excited coherent state (10).

We turn to the scattering process realized in a system in which initially the entire population is in the state $|3\rangle$:

$$a_1^0=0, \quad a_2^0=0, \quad a_3^0=F(p). \tag{11}$$

According to (6), the noninteracting state r is not populated,

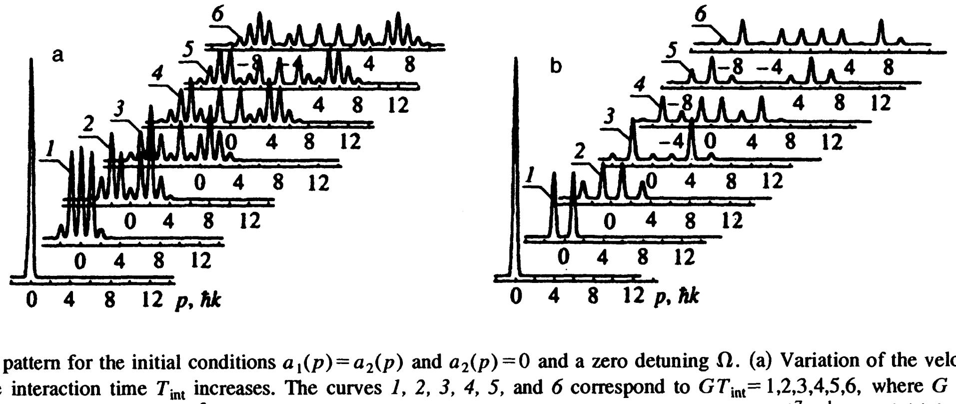

FIG. 3. The scattering pattern for the initial conditions $a_1(p)=a_2(p)$ and $a_2(p)=0$ and a zero detuning Ω. (a) Variation of the velocity distribution of the total probability as the interaction time T_{int} increases. The curves *1, 2, 3, 4, 5*, and *6* correspond to $GT_{\text{int}}=1,2,3,4,5,6$, where G is the Rabi frequency: $G=G_1=G_2$. The recoil frequency is $R=\hbar k^2/2M=0.002G$, which is valid for potassium atoms at $G=2\pi\times10^7\,\text{s}^{-1}$. The initial distribution of the total probability is assumed normal with a width $0.25\hbar k$. (b) The velocity distribution of the atomic population in state $|3\rangle$ with the same parameters as in Fig. 3a.

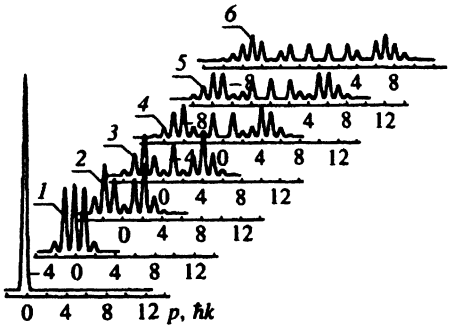

FIG. 4. Temporal evolution of the total population of a three-level system obtained from the initial state $a_3=F(p)$ by the normal distribution of width $0.25\hbar k$ in interaction times $GT_{\text{int}}=1,2,3,4,5,6$ (curves *1, 2, 3, 4, 5*, and *6*). The other parameters are similar to those chosen in Fig. 3.

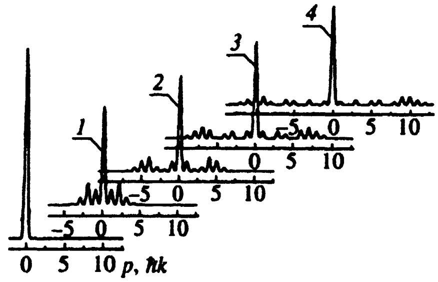

FIG. 5. Variation of the distribution of the total probability for atoms in the initial state with $a_1=F(p)$ and $a_2=a_3=0$. Curves *1, 2, 3*, and *4* correspond to interaction times such that $GT_{\text{int}}=2,4,6,8$.

and we again arrive at the case of scattering of an effectively two-level atom in the field of a standing light wave. Accordingly, the general scattering pattern differs little from the case discussed above.

Figure 4 depicts the shape of the temporal evolution of the total population of the three-level system. We see that effective scattering occurs for all atom–field interaction times. If we again assume that the common level $|3\rangle$ is the ground level (the V-level scheme), beam splitting occurs for atoms in the ground state $|3\rangle$ for an atom–field interaction time $T_{\text{int}}=2G^{-1}$, since there are two dominant peaks in the scattering cross section corresponding to atoms in the state $|3\rangle$ at momentum values equal to $\pm 2\hbar k$.

Finally, we believe that the most interesting case is that in which initially the entire population is in one of the lower states of the three-level system, say in state $|1\rangle$:

$$a_1^0=F(p), \quad a_2^0=0, \quad a_3^0=0. \tag{12}$$

Here, according to (6), half of the population is in the unperturbed state r, while the other half is in the state s, which effectively interacts with the field of the standing waves. Consequently, the fraction of atoms in the state r is not excited by the field, and half of the atoms do not participate in the scattering process, so to speak. On the other hand, the fraction of atoms in the state s actively participates in the scattering process. Figure 5 depicts the distribution of the total probability for this case, with the initial distribution assumed normal with a width $0.25\hbar k$. We see that with the passage of time only a fraction of the population participates in the scattering process, while roughly half the population remains unperturbed.

Above we studied the scattering of a three-level atom in the field of standing waves by employing the solution (7) obtained in an approximation that ignores Doppler frequency

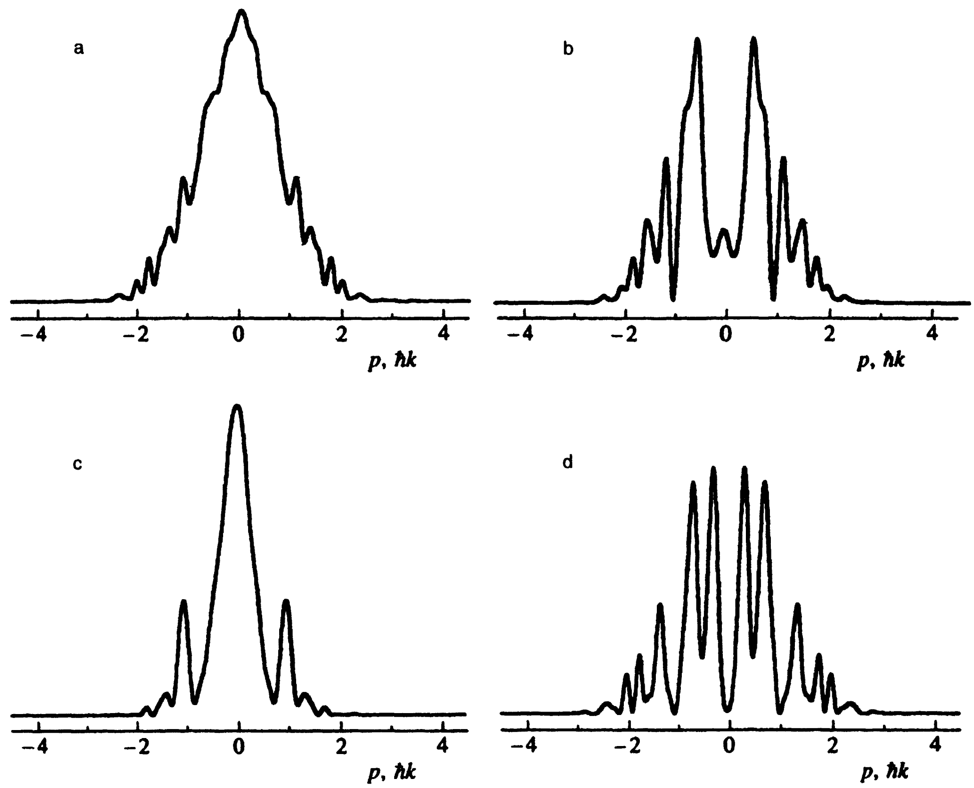

FIG. 6. The result of interaction of a three-level atom with an optical field when the recoil frequency R coincides with the Rabi frequency G. The interaction time is fixed at $T_{\text{int}}=6G^{-1}$, the initial distribution has a width $\hbar k$, and the atom is in the state with $a_1=F(p)$ and $a_2=a_3=0$: (*a*) the distribution of the total probability; and (*b*), (*c*), and (*d*) are the velocity distributions of the probability of discovering the atom in state $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively.

shifts as a result of absorption or emission of a single photon and is valid for small interaction times (8) (see Ref. 8). Lifting the restrictions of this approximation proves especially interesting in the case of low Rabi frequencies (or high recoil frequencies), where the parameter R/G is not small and the interaction time $T_{\rm int}$ is longer than G^{-1}. Here, not having an analytical solution, we integrated the system (6) numerically, for the conditions $|p|<6\hbar k$ and $T_{\rm int}G<12$ at $R=G$. Figure 6*a* depicts the momentum representation of a beam of atoms when the time $T_{\rm int}$ of the interaction with the field of the standing waves is equal to $6G^{-1}$ for the initial state (12). Here the characteristic time it takes the velocity distribution to change is of order G^{-1}. Note that the dependence on p of the total probability density W (Fig. 6a) is smoother than that of $|a_1|^2$ (Fig. 6b), $|a_2|^2$ (Fig. 6c), or $|a_3|^2$ (Fig. 6d), where a rapidly oscillating structure on the $\hbar k$ scale is quite evident. We also observed an increase in the degree of dissection of the velocity distribution with the interaction time $T_{\rm int}$.

The reason for this is the increasing role that $p^2/2M$ plays in the Hamiltonian related to the absorption by atoms of the photon momentum as the interaction time grows. The given term plays the principal role, causing rapid oscillations of the amplitudes, whose frequency depends on the momentum p. This behavior of the atomic system makes it possible to form at the point where the beam exits from the interaction region a "rake" of the probability density with peak widths smaller than $\hbar k$ for the atoms in the given state. Concluding this section, we note once more that such a scattering pattern can be observed only at low light intensities $G\approx R$ (or high recoil frequencies) for interaction times longer than $T_{\rm int}\approx R^{-1}$.

3. SCATTERING OF ATOMS IN THE FIELD OF STANDING WAVES WITH A RELATIVE SPATIAL SHIFT

Now let us discuss the coherent dynamics of a three-level atom (Fig. 1) placed in the field of standing waves characterized by a relative spatial shift of $\pi/2$:

$$\mathbf{E}=\mathbf{e}_1E_1\cos(kz)\exp(i\omega_1 t)+\mathbf{e}_2E_2\cos(kz+\pi/2)\exp(i\omega_2 t). \quad (13)$$

We still assume that the wave with the frequency ω_1 interacts with the $|1\leftrightarrow|3\rangle$ transition in the three-level atom, and that with the frequency ω_2 interacts with the $|2\leftrightarrow|3\rangle$ transition. After substituting (13) into the time-dependent Schrödinger equation with the Hamiltonian (3) for the case of a two-photon resonance (equal detuning frequencies and Rabi frequencies G) we introduce the new variables

$$B_+=(a_1+ia_2)e^{ikz},\quad B_-=(a_1-ia_2)e^{-ikz} \quad (13)$$

and transform the amplitudes B_+ and B_- into the momentum representation (5). The result is a system of equations for the time-dependent probability amplitudes for the states of the three-level atom:

$$i\frac{dB_+(p,t)}{dt}=\frac{(p-\hbar k)^2}{2M\hbar}B_+(p)-Ga_3(p)+\Omega B_+(p),$$

$$i\frac{dB_-(p,t)}{dt}=\frac{(p+\hbar k)^2}{2M\hbar}B_-(p)-Ga_3(p)+\Omega B_-(p), \quad (15)$$

$$i\frac{da_3(p,t)}{dt}=\frac{p^2}{2M\hbar}a_3(p)-\frac{1}{2}GB_+(p)-\frac{1}{2}GB_-(p).$$

We see that the given system has an exact analytical solution for arbitrary interaction times of the atom and the field (13) of the standing waves:

$$a_3(p,t)=\sum_{m=1,2,3}A_m^0(p)\exp[i\lambda_m(p)t],$$

$$B_+(p,t)=\sum_{m=1,2,3}\frac{GA_m^0(p)\exp[i\lambda_m(p)t]}{\lambda_m+R'(p-\hbar k)^2+\Omega}, \quad (16)$$

$$B_-(p,t)=\sum_{m=1,2,3}\frac{GA_m^0(p)\exp[i\lambda_m(p)t]}{\lambda_m+R'(p+\hbar k)^2+\Omega},$$

where the $i\lambda_m$ are the roots of the characteristic equation for (15):

$$\lambda^3+A\lambda^2+B\lambda+C=0, \quad (17)$$

where

$$A=3R'p^2+2R'\hbar^2k^2+2\Omega,$$

$$B=-G^2+3R'^2p^4+R'^2\hbar^2k^4+\Omega^2+\Omega(4R'p^2+2R'\hbar^2k^2),$$

$$C=(R'(p^2+\hbar^2k^2)+\Omega)(-G^2+R'p^2(R'(p^2+\hbar^2k^2))+\Omega R'p^2)-4R'^3p^4\hbar^2k^2,$$

with $R'=1/2M\hbar$, and the constants $A_m^0(p)$ can be determined from the initial distribution of the amplitudes $B_+(p)$, $B_-(p)$, and $a_3(p)$ of the three-level system.

Note that in the case of a two-photon resonance and standing waves shifted in relation to each other by $\pi/2$ the problem of scattering of a three-level atom allows a simple analytical solution without additional restrictions imposed on the times of interaction of the atom and the field of the standing waves and on the value of the Doppler frequency shift.[4,8] More than that, such a solution can be obtained for arbitrary (but equal) detunings and for arbitrary relationships between the frequency detuning and the Rabi frequency of the excitation wave. Here an analysis of Eqs. (15) shows that, irrespective of the width of the initial velocity distribution, scattering occurs only if the atomic momentum changes by $|\Delta p|=0,\hbar k,2\hbar k$. In other words, if initially the atom has a distribution function with a width $\delta p<\hbar k$, after the interaction (under two-photon resonance conditions) with the field of the two standing waves shifted relative to one other by $\pi/2$ there occurs a momentum splitting in the wave packet of the atom, but only up to momentum values $|p|$ equal to $2\hbar k$, while the values of the atomic momentum higher than $2\hbar k$ do not appear in the scattering processes for any interaction time of the atoms and the field of the standing waves. This sets the current case apart from the scattering of a two-level atom as well as from the scattering of a three-level atom in the field of spatially in-phase waves (Sec. 2), where

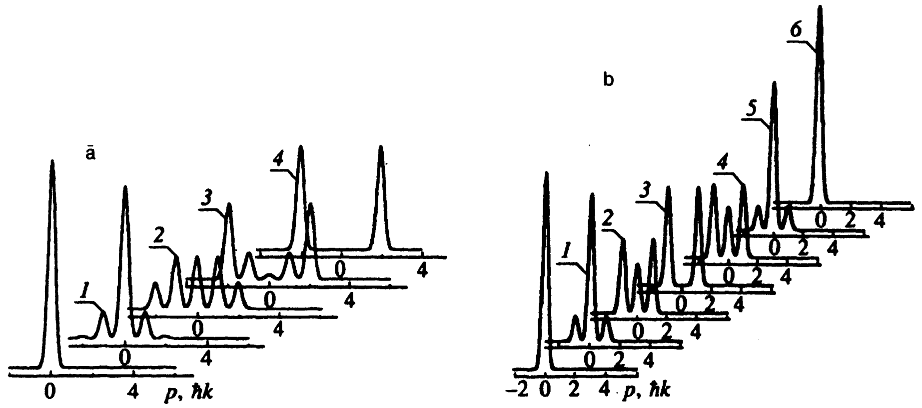

FIG. 7. The velocity distribution of the total probability for the interaction of a three-level atom with the field of two standing waves spatially shifted by $\pi/2$ in relation to each other at zero detunings $\Omega_1=\Omega_2=0$ and a recoil frequency $R=0.002G$. (a) Initially the atom was in the state $|1\rangle$ with a distribution of width $0.25\hbar k$. Curves *1, 2, 3,* and *4* were built for an interaction time such that $GT_{\text{int}}=\pi/4$, $\pi/2$, $3\pi/4$, and π. (b) Initially the atom was in the state $|3\rangle$ with a distribution of width $0.25\hbar k$. Curves *1, 2, 3, 4, 5,* and *6* were derived for an interaction time such that $GT_{\text{int}}=\pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, $5\pi/6$, and π.

the momentum acquired by the atom in the scattering process is directly determined by the time that the atom spends in the light field. This fact can be given a simple and intuitive explanation if we note that introducing the substitutions (14) is equivalent to reducing the three-level system interacting with the field of the standing waves to a three-level system interacting with the field of two oppositely propagative waves (15). Such a system was considered by Cohen-Tannoudji *et al.*[15] who showed that the time it takes the population to be completely transferred from one lower level of the Λ-system to the other is determined by the Rabi frequency. In the process the atomic momenta change by at most $2\hbar k$. The probability amplitudes of these two systems are related through the linear transformation (14) and are shifted in the momentum space by $\pm\hbar k$, so that it is understandable why the increment of the atomic momentum changes by portions that are integral multiples of the recoil momentum.

Now let us study the solution of the system of equations (15) for various types of initial conditions, as we did in the case of in-phase waves. Here we do not solve the characteristic equation for the system (15); rather, we use a code for numerical calculation for interaction times T_{int} up to $12G^{-1}$.

For the initial conditions (9), (10), and (12) the results of scattering are practically the same. For instance, there is no scattering with changes in the atomic momentum larger than $p=2\hbar k$, and the amplitude of the diffraction peaks oscillates for allowed scattering momenta between zero and unity (Fig. 7*a*). The most interesting thing here is the result of scattering for a time T_{int} of the interaction of the atom and the field of shifted standing waves equal to $3G^{-1}$. We see that the result of scattering is the splitting in the initial momentum distribution of the atomic beam into two coherent beams centered at momentum values $p=2\hbar k$. In other words, we have an ideal beam splitter for three-level atoms scattered in the field of two spatially shifted standing waves.

As is known,[5,9,10] the effectiveness of such a device is determined by two factors: the size splitting on the momentum scale, and the number of atoms in the scattered beams. In the present case the splitting amounts to $4\hbar k$, which corresponds to beam splitting on the basis of coherent transfer of populations in a three-level system.[9] But there is an important advantage here: no atomic intensity losses in the coherent beams, since the entire population in the system is distributed only between two diffraction peaks (Fig. 7a). Note that the internal state of the atom at each peak is a superposition. For instance, an atom in one of the diffraction peaks can be in the states $|1\rangle$ and $|2\rangle$ with equal probability (and the state $|3\rangle$ is unpopulated). Thus, the difference in the atomic momenta in the superposition $(|1\rangle+|2\rangle)$ doubles the path of the atom in the field (13) (see Refs. 1 and 2). No less interesting is the fact that for an interaction time $T_{\text{int}}=6G^{-1}$ the momentum distribution function is restored, which can be explained by the periodicity of the general solution of Eq. (17).

For initial conditions of the form (11) the scattering pattern is depicted in Fig. 7b. Clearly, momentum variation does not exceed $\hbar k$, with the system again periodically returning to the initial conditions (11), when the entire population is on the level $|3\rangle$.

Finally, let us examine the case that guarantees the most effective scattering of three-level atoms. We assume that the spatial shift between the standing waves is still equal to $\pi/2$ and that the detunings are equal in absolute value but are opposite in sign: $\Omega_1=-\Omega_2$ (what has become known as mirror detuning). Then the equations for the variables B_- and B_+ assume the form

$$i\frac{dB_+(p,t)}{dt}=\frac{(p-\hbar k)^2}{2M\hbar}B_+(p)-Ga_3(p)+\Omega B_-(p-2\hbar k),$$

$$i\frac{dB_-(p,t)}{dt}=\frac{(p+\hbar k)^2}{2M\hbar}B_-(p)-Ga_3(p)+\Omega B_+(p+2\hbar k), \tag{18}$$

$$i\frac{da_3(p,t)}{dt}=\frac{p^2}{2M\hbar}a_3(p)-\frac{1}{2}GB_+(p)-\frac{1}{2}GB_-(p),$$

where B_+ and B_- have been defined in (14).

We see that these equations, in contrast to (15), comprise a system of equations coupled through different values of the atomic momentum. In analyzing the system we did not employ the short-interaction-time approximation[8] (as we did in Sec. 1); instead we solved system (18) numerically.

The result of the calculations is depicted in Fig. 8. For instance, Fig. 8(b) shows the scattering of the wave packet of a three-level atom under the initial conditions (9), while

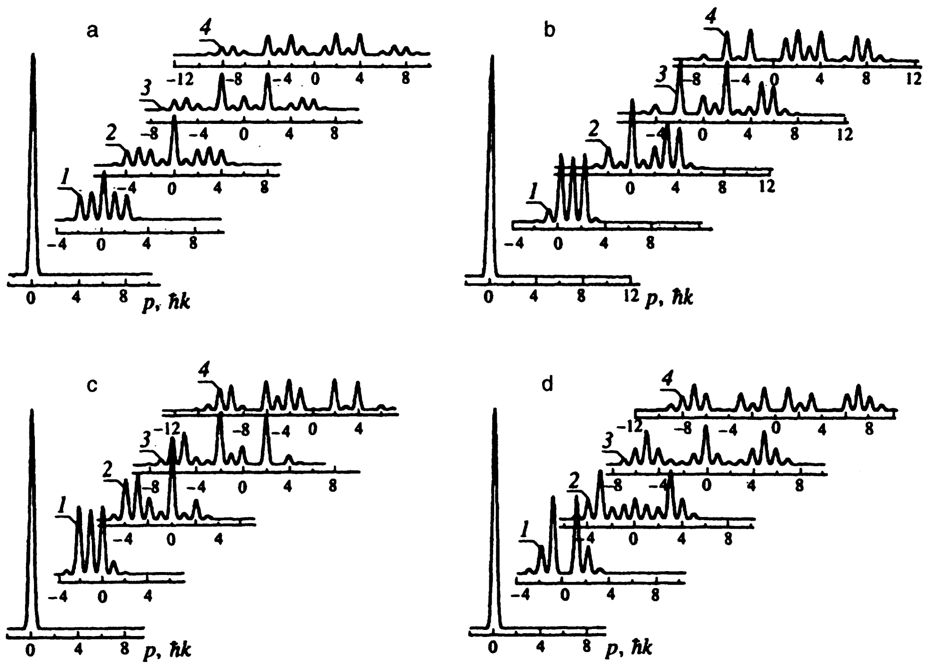

FIG. 8. The velocity distribution of the total probability for the interaction of a three-level atom with the field of two standing waves spatially shifted by $\pi/2$ in relation to each other in the case of mirror detuning $\Omega_1=-\Omega_2=\Omega$ and a recoil frequency $R=0.002G$. Curves *1*, *2*, *3*, and *4* are calculated for interaction times $T_{\text{int}}=2G^{-1}$, $4G^{-1}$, $6G^{-1}$, and $8G^{-1}$. The initial states were considered normal in the total probability with a width $0.25\hbar k$. (a) The initial state is fixed at $a_1=F(p)$ and $a_2=a_3=0$. (b) The initial state is fixed at $a_1(p)=-a_2(p)$ and $a_3=0$. (c) The initial state is fixed at $a_1(p)=a_2(p)$ and $a_3=0$. (d) The initial state is fixed at $a_1=a_2=0$ and $a_3=F(p)$.

Fig. 8c shows the scattering under the initial conditions (10). Clearly, the scattering pattern strongly depends on the sign of the initial amplitudes of the states of the lower levels, being essentially asymmetric for both initial conditions, which leads to a deviation of the atomic beam as a whole in the interaction with the field of the shifted standing waves.

On the other hand, for the initial conditions (11) and (12) we still have a symmetric scattering pattern (Figs. 8d and a). Here, in the case of (12), after interacting with the field of the standing waves over a time $T_{\text{int}}\approx 6G^{-1}$, the atom finds itself on practically the same level on which the process started, i.e., in state $|1\rangle$. In Fig. 8a the peaks in the velocity distribution at odd values of $\hbar k$ corresponds to atoms in the upper state $|3\rangle$, and those at even values of $\hbar k$ correspond to atoms in a lower state, $|1\rangle$ or $|2\rangle$. An atom participates in the scattering processes by starting from the upper level (Fig. 8d), with the result that for such an atom the situation is reversed: odd peaks correspond to the population of state $|1\rangle$ or $|2\rangle$, while even peaks correspond to the population of $|3\rangle$. This scheme guarantees that the scattering is the most effective, but as the interaction time T_{int} increases with the acquired momentum, the relative fraction of atoms at each atomic-density peak decreases, while at small interaction times $T_{\text{int}}<6G^{-1}$ the fraction remains fairly large, which makes it possible to use the scheme in building a splitter for a beam of three-level atoms.[14] Finally, we note that an analytical treatment of the case of an arbitrary phase shift (the field of two standing waves of the form $\mathbf{E}=\mathbf{e}_1E\cos(kz)\exp(i\omega_1t)+\mathbf{e}_2E\cos(kz+\phi)\exp(i\omega_2t)$) has shown that the substitutions $r=a_1+a_2$ and $s=a_1-a_2$ reduce this case to the problem of scattering in the field (13) but with Rabi frequencies that are generally different: $G_1=\tilde{G}\cos(\phi/2)$ and $G_2=-\tilde{G}\sin(\phi/2)$. The equivalent three-level system resembles a system subjected to two oppositely directed traveling waves with equal Rabi frequencies and an additional standing wave with, generally, a different Rabi frequency. This leads in the general case to scattering not limited by the selected momentum, as was the case with in-phase standing waves.

4. CONCLUSION

Here are the main results of our investigation. We studied the coherent scattering of the wave packet of a three-level atom in the field of two standing light waves for two values of the spatial shift. In the case of equal frequency detuning (a two-photon resonance) we found that for a zero spatial shift between the standing waves the problem of the scattering of a three-level atom can always be reduced to that of the scattering of an effectively two-level system. In the event of an exact resonance we explicitly obtained the solution of the time-dependent Schrödinger equation for the probability amplitudes of the states of the three-level atom in the short-interaction-time approximation,[8] which allows, at least in principle, the scattering modes to be studied with a arbitrary relationship between the recoil frequency R and the Rabi frequency G.

At the same time, in the case of standing waves shifted by $\frac{1}{2}\pi$ in relation to each other and zero detuning, the solution of the Schrödinger equation for the probability amplitudes can be found without any additional conditions being imposed on the intensities of the light waves and the interaction times. Note that in the given case a scheme for a highly effective atomic beam splitter can be realized.

Finally, the case of mirror detuning for shifted standing waves was examined. We found that the scattering pattern strongly depends on the signs of the initial amplitudes of the lower levels, which makes it possible not only to effectively scatter the atomic beam but also to deflect the beam as a whole by a definite angle.

APPENDIX

Below we give the expressions for the probability amplitudes of the states of a three-level atom in the case where initially there is a delta-function distribution in the momenta, $\delta(p)$, for zero frequency detuning and a zero spatial shift between the standing waves.

(a) The initial distribution (10) for $F(p)=\delta(p)$:

$$a_{1,2}(p,t)=\frac{1}{\sqrt{2}}\sum_n (-1)^n J_{2n}(2\tilde{G}t) \times \exp\left[-\frac{i(2n\hbar k)t^2}{2M\hbar}\right]\delta(p-2n\hbar k), \tag{A1}$$

$$a_3(p,t)=\sum_n (-1)^n i J_{2n+1}(2\tilde{G}t) \times \exp\left[-\frac{i(2n+1)^2\hbar k^2 t}{2M}\right]\delta(p-(2n+1)\hbar k).$$

(b) The initial distribution (11) for $F(p)=\delta(p)$:

$$a_{1,2}(p,t)=\frac{i}{\sqrt{2}}\sum_n (-1)^n \times \exp\left[-\frac{i(2n+1)^2\hbar k^2 t}{2M}\right]J_{2n+1}(2\tilde{G}t)\delta \times (p-(2n+1)\hbar k), \tag{A2}$$

$$a_3(p,t)=\sum_n (-1)^n \exp\left[-\frac{i(2n\hbar k)^2 t}{2M\hbar}\right]J_{2n}(2\tilde{G}t)\delta(p-2n\hbar k).$$

(c) The initial distribution (12) for $F(p)=\delta(p)$:

$$a_1(p,t)=\frac{1}{2}\left\{\sum_n (-1)^n \times \exp\left[-\frac{i(2n\hbar k)^2 t}{2M\hbar}\right]J_{2n}(2\tilde{G}t)\delta(p-2n\hbar k)\right\} +\frac{1}{2}\delta(p),$$

$$a_2(p,t)=\frac{1}{2}\left\{\sum_n (-1)^n \times \exp\left[-\frac{i(2n\hbar k)^2 t}{2M\hbar}\right]J_{2n}(2\tilde{G}t)\delta(p-2n\hbar k)\right\} -\frac{1}{2}\delta(p), \tag{A3}$$

$$a_3(p,t)=\frac{i}{\sqrt{2}}\sum_n (-1)^n \times \exp\left[-\frac{i(2n+1)^2\hbar k^2 t}{2M}\right]J_{2n+1}(2\tilde{G}t)\delta \times (p-(2n+1)\hbar k).$$

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