Generation of high-power electromagnetic radiation from the development of explosive and high-frequency instabilities in a system consisting of a relativistic ion beam and a nonisothermal plasma

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We have investigated the possibility of generating explosive wave triplets and up-conversion of electromagnetic modes in a system consisting of a relativistic ion beam penetrating a strongly nonisothermal plasma. We derive here conditions for synchronism and the sign of the wave energy of the resonance triplets, and show that in this system it is possible to excite explosive or high-frequency instabilities. Based on an asymptotic method we derive and analyze truncated equations for the complex mode amplitudes. We show that the "explosion" is stabilized by a nonlinear frequency shift, while the high-frequency instability is analogous to the "decay" of low-frequency modes. Unlike the cases treated in well-known papers, we find that the matrix coefficients and the coefficient that characterizes the nonlinear frequency shift are complex, which leads to new solutions to the equations for complex wave amplitudes, even under identical initial conditions. © *1996 American Institute of Physics.* [S1063-7761(96)00803-5]

1. INTRODUCTION

It is known that explosive or high-frequency instabilities are possible in nonequilibrium media, one of which is, e.g., a beam–plasma system (see Refs. 1–7). An explosive instability appears if in the course of a resonant three-wave interaction one wave of higher frequency (or two of lower frequency) has negative energy, while the energies of the two other modes of the wave triplet (or the higher frequency wave) are positive.[1)] The case usually encountered is one where the mode of higher frequency ω_3 has negative energy (for example, in a system consisting of a cold plasma and a single-velocity beam this is the slow beam wave[3,4]). However, studies of the explosive instability of a relativistic high-current circularly polarized wave in both isotropic[8] and magnetically active plasmas[9] have shown that the two lower modes of the resonance triplet have negative energy, while the energy of the higher-frequency mode is positive. The explosive instability is stabilized either by a nonlinear frequency shift[1] or by nonlinear decay.[1,10] The high-frequency instability[6,7] is characteristic only of nonequilibrium media where the wave with intermediate frequency ω_1 or ω_2 has negative energy, while the energy at ω_3 is positive; in this case the lower-frequency wave "decays," i.e., up-conversion takes place, and the modes at the top of the spectrum grow exponentially, as occurs in ordinary decay-like instabilities.[7]

In this paper we study explosive instability in a system made up of a relativistic ion beam penetrating a strongly nonisothermal plasma ($T_e \gg T_i$, where $T_{e,i}$, are the temperatures of the electrons and ions). This problem has wide practical application in plasma physics, astrophysics,[12] and also in problems of current interest such as inertially controlled thermonuclear fusion.[13] Furthermore, it is clearly of interest in the general theory of wave interactions in nonequilibrium media.[2)]

In this paper we investigate the dispersion equation, which describes normal modes of a system, the mode energies, and the conditions for synchronism, and show that in this system both resonance explosive triplets and high-frequency instability are possible. In addition, we use asymptotic methods[14–16] to derive and analyze simplified equations for the complex wave amplitudes. We show that the "explosion" is stabilized by a nonlinear frequency shift, and the solutions to the simplified equations have a form entirely unlike that described in well-known papers (see, e.g., Ref. 1)—the matrix coefficients are complex. Moreover the system supports a nonlinear mode decay, connected with the complex nature of the coefficient that characterizes synchronous wave interactions. We find a new solution to the simplified equations for the amplitudes of modes of the explosive triplet. In view of the cumbersome calculations involved in this problem, many of our results were obtained numerically, which, of course, is not the goal of this paper. These numerical results merely help us interpret the physical effects that appear when we study the system described above.

2. STARTING EQUATIONS. DISPERSION RELATIONS, CONDITIONS FOR SYNCHRONISM, MODE ENERGIES

We start with the dimensionless system of equations that describes the potential motion of relativistic ions penetrating a strongly nonisothermal plasma,[17]

$$\frac{\partial}{\partial t}(\gamma n)+\frac{\partial}{\partial x}(\gamma n v)=0,$$

$$\left(\frac{\partial}{\partial t}+v\,\frac{\partial}{\partial x}\right)P+3\gamma^2 P\left(\frac{\partial}{\partial x}+\frac{1}{c}\,\frac{\partial}{\partial t}\right)v=0,$$

$$\left(\frac{\partial}{\partial t}+v\,\frac{\partial}{\partial x}\right)\gamma v+\frac{\sigma}{n}\,\frac{\partial P}{\partial x}+\frac{\partial \Phi}{\partial x}=0,$$

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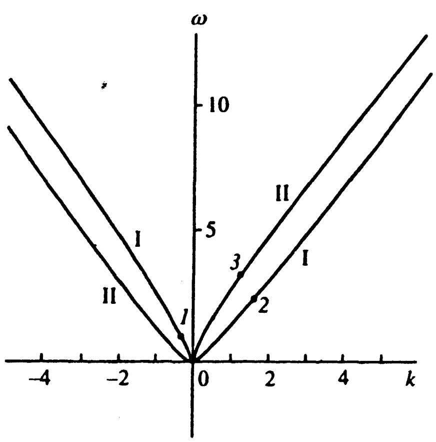

FIG. 1. Dispersion relation $\omega(k)$.

$$\frac{\partial^2\Phi}{\partial x^2}-n_e+n=0, \tag{2.1}$$

where $\gamma=[1-v^2/c^2]^{-1/2}$; $ne=e^{\Phi}$; n_e, n are the dimensionless concentrations of electrons and ions, normalized to the background concentration N_0, v is the velocity of ions moving along the x axis normalized by $c_s=\sqrt{k_BT_e/M}$, M is the ion mass, Φ is a dimensionless electric field potential normalized by the quantity k_BT_e/e (where k_B is Boltzmann's constant, e is the ion charge), P is the pressure normalized by $N_0k_BT_e$; t is normalized by the ion Langmuir frequency $\omega_{pi}=\sqrt{4\pi e^2N_0/M}$, while the coordinate x is normalized by the Debye radius $\lambda_D=\sqrt{k_BT_e/4\pi e^2N_0}$; and $\delta=T_i/T_e$. The third equation in (2.1) describes adiabatic changes in the ion energy, while the fourth is the Poisson equation.

Let us linearize (2.1), including quadratic and cubic terms in the Taylor-series expansion; these terms are omitted here for simplicity. From the linear system with processes of the form $\exp[i(\omega t-kx)]$ (where ω and k are the frequency and wave vector) we obtain the dispersion equation for the system normal modes:

$$\begin{aligned}&\omega^2[\gamma_0(k^2+1)(1+\beta^2\gamma_0^2)]+\omega[k^2\gamma_0^2\gamma_1-2\gamma_0kv_0(1\\&+\beta^2\gamma_0^2)(k^2+1)+3\gamma_0^2\sigma k(k^2+1)\gamma_1]+[-k^2\\&-k^2\gamma_0^2\gamma_1v_0+\gamma_0k^2v_0^2(k^2+1)(1+\beta^2\gamma_0^2)\\&-3\gamma_0^2\sigma k^2(k^2+1)]=0,\end{aligned} \tag{2.2}$$

where $\beta=c_sv_0c^{-1}$; $\gamma_1=\beta c_sc^{-1}$; $\gamma_0=(1-\beta^2)^{-1/2}$; and v_0 is the equilibrium velocity of the ion beam.

Naturally, as $v_0\to 0$ we obtain the dispersion equation for ion sound from (2.2) in the absence of the beam, while as $\gamma\to 0$ we find the well-known (see, e.g., Ref. 18) dispersion relation for a system consisting of a nonrelativistic ion beam and a nonisothermal plasma. We investigated the general form of Eq. (2.1) numerically; Fig. 1 shows a typical plot of the function $\omega(k)$. From this function it is easy to see that the conditions for synchronism are fulfilled:

$$\omega_1+\omega_2=\omega_3,\quad k_1+k_2=k_3 \tag{2.3}$$

(in the figure, the triplets are illustrated by the corresponding points).

Next we investigate the energy of the resonantly interacting modes, using the expression given in Ref. 19. As a result of the analysis of the energy, it is easy to show that branches I in the figure (analogous to the slow beam waves) have negative energy, while branches II have positive energy (the fast beam mode). Thus, it appears that an explosive instability could be realized here. Furthermore, we note that the wave with intermediate frequency ω_2 can have negative energy, while the two other waves at ω_1 and ω_3 have positive, i.e., a high-frequency instability can appear (upconversion in the spectrum). For clarity, we list the resonance triplets in Table I for various system parameters; the fourth row corresponds to the high-frequency instability.

TABLE I. Characteristics of modes that satisfy the synchronism condition for $\sigma=0$.

ω_1	k_1	ω_2	k_2	ω_3	k_3	v_0	γ_1
0.25	−0.02	2	0.22	2.25	0.2	10	0.004
1	−0.4	2.5	1.6	3.5	1.2	2	0.02
0.1	−0.005	1.6	0.085	1.7	0.08	20	0.002
0.5	−0.6	2.25	1.6	2.75	1	2	0.02

TABLE II. Matrix coefficients of the interaction ($\gamma_0>1$).

σ_1	σ_2	σ_3
0.77+0.004i	0.06+0.02i	−0.07−0.02i
9.78+0.3i	21.57−0.15i	−4.47−0.34i
0.2+0.3i	1.97−0.004i	−0.02−0.004i
27.8+0.75i	32.61+0.21i	−4.05−0.18i

3. ANALYSIS OF SIMPLIFIED EQUATIONS FOR THE MODE AMPLITUDES

We seek a solution to system (2.1), taking into account weak quadratic and cubic nonlinearities, of the form

$$\Phi=\sum_{j=1}^{3}a_j(\mu x,\mu t)\exp[i(\omega t-kx)]+\text{c.c.} \tag{3.1}$$

(here $\mu\ll 1$ is a parameter that characterizes the nonlinearity). Then using the asymptotic method,[18] we obtain simplified equations for the complex mode amplitudes in the standard way:

$$\begin{aligned}\frac{\partial a_{1,2}}{\partial t}+v_g^{(1,2)}\frac{\partial a_{1,2}}{\partial x}&=\sigma_{1,2}a_{2,1}^*a_3\\&+\alpha_{1,2}a_{1,2}(|a_1|^2+|a_2|^2+|a_3|^2),\end{aligned}$$

TABLE III. Coefficients that characterize the nonlinear frequency shift.

α_1	α_2	α_3
−3.66+0.045i	−0.0075−1.14i	−0.0027+0.93i
−12.36+0.11i	−1.05−0.09i	−2.4+0.09i
0.00015+1.20i	−0.0017−2.38i	−0.12+2.38i
−7.53+0.38i	−2.49−0.10i	−0.38−0.09i

TABLE IV. Group velocities of modes of the resonance triplet.

No.	$v_q^{(1)}$	$v_q^{(2)}$	$v_q^{(3)}$
1	−0.637	0.521	0.625
2	−2.145	1.054	1.540
3	−0.854	0.754	0.840

$$\frac{\partial a_3}{\partial t}+v_g^{(3)}\frac{\partial a_3}{\partial x}=\sigma_3 a_1 a_2+\alpha_3 a_3(|a_1|^2+|a_2|^2+|a_3|^2). \tag{3.2}$$

The expressions for the matrix coefficients σ_j and α_j are complex and given in Tables II and III; $v_g^{(j)}$ is the group velocity of the mode, and is listed in Table IV.

Since σ_j and α_j are complex, while the nonlinear absorption $\mathrm{Re}(\Sigma_{i=1}^{3}\alpha_i)<0$, the solution to Eq. (3.2) differs fundamentally from that described in Ref. 1. For simplicity let us consider the spatially uniform case ($\partial/\partial x=0$); then for the same initial conditions, system (3.2) has the solution[3)]

$$\frac{1}{u_0}-\frac{1}{u}+\zeta\ \ln\left\{u\left(\frac{1}{\zeta}-u\right)\left[\left(\frac{1}{\zeta}-u_0\right)u_0\right]^{-1}\right\}=t-t_0, \tag{3.3}$$

where

$$u_j(t)=u(t)=|a_j|\sqrt{\sigma_m\sigma_n}\ (m\neq n\neq j;\quad j,m,n=1,2,3),$$

$$\zeta=\sum_{i=1}^{3}\mathrm{Re}\ \alpha_i.$$

From analysis of (3.3) it is easy to verify that the function $u(t)$ increases monotonically from $u(0)=u_0$ up to $1/\zeta$ in a way that differs fundamentally from that given in Ref. 1.

In conclusion, we briefly discuss the high-frequency instability. In the given field of a low-frequency pump, where $|a_2|\gg|a_{1,3}|$, the growth rate of the instability equals

$$\Gamma=|a_2|\sqrt{\mathrm{Re}\ \sigma_1\ \mathrm{Re}\ \sigma_3}, \tag{3.4}$$

i.e., the amplitudes of the high-frequency waves grow in a way that is analogous to the exponential "decay" of the low-frequency mode.[11]

In conclusion, let us estimate the electromagnetic radiation generated in a beam–plasma system with the following parameters: $n_0\sim10^{10}\ \mathrm{cm}^{-3}$; $T\simeq10^{10}$ K; and $c/c_s\simeq0.1$. Then for a characteristic length of $\sim10^{10}$ m, ion-sound waves (with $\lambda\simeq30$ cm) rise to a potential value of ~0.7 V, or up to energies of $3\cdot10^{-10}$ J. Since this amounts to a fraction of a percent of the kinetic energy of the ion beam, the undepleted-beam approach used in this model is completely valid.

Thus, it is possible to generate high-power electromagnetic radiation in a system consisting of a relativistic ion beam and a nonisothermal plasma. The character of the excited modes is completely different from known results presented in Ref. 1. The energy of the radiation is subtracted from the ion beam as long as the signal energy is not comparable to the kinetic energy of the particle beam; however, this problem requires additional investigation, since in this paper we have used the approximation of a weak nonlinearity, where the velocity of the beam is presumed to be given.[4)]

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1)A wave of negative energy has a simple physical meaning (see Ref. 4): the energy of the system+wave is smaller than the energy of the system without the wave, i.e., by increasing the energy of the wave the nonequilibrium medium decreases its own energy. That is, an instability arises that is characteristic of the nonequilibrium medium.

2)In Ref. 17 the author showed that "explosive solitons" are possible in this system; however, he limited himself to numerical investigation of the possibility of an "explosion" of solitons, and did not analyze the alternative possibility of explosive triplets.

3)Note that for the system parameters chosen here the linear growth rate of the instability equals zero.

4)The problem of bunching of a nonrelativistic beam of electrons by potential Langmuir waves was discussed in detail in Ref. 20.

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