Stratification of a liquid-metal current-carrying conductor: experiment and model

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Experiments on the stratification of a liquid-metal conductor with current associated with an electric explosion are discussed. A model of the initial state of the separation of the conductor and current into transverse strata is proposed and investigated. It is shown that the stratification is the result of the generation of large-scale vortex structures in the liquid-metal current-carrying conductor. The stratification process has a threshold, and the type of bifurcation of the solution for the amplitude of the surface perturbation corresponds to subcritical bifurcation. A simplified model for describing the essentially nonlinear stages of stratification of a conductor (division into "particles" on the scale of the diameter) is investigated. © *1996 American Institute of Physics.* [S1063-7761(96)00902-4]

1. INTRODUCTION

The electric explosion of conductors[1,2] is a typical example of a nonequilibrium phase transition, as a result of which a current-carrying conductor breaks up into transverse strata, forming high-temperature plasma jets and shock waves in the surrounding medium. Stratification of an exploding conductor occurs both in the range of microsecond characteristic times[2,3] and in the range of nanosecond times,[4] when there are no grounds for assuming that the strata are formed as the result of the magnetohydrodynamic sausage instability.[5,6]

In the construction in Refs. 7–9 of theoretical models of the stratification of an exploding conductor, the stratification was assumed to be due to a thermal instability initiated by the magnetohydrodynamic sausage instability. The aim of the present paper is to discuss the experiments on the stratification of an exploding conductor with current described in Ref. 2 (Sec. 2) and also to construct and investigate a theoretical model of the stratification that does not invoke the thermal instability (Sec. 3). In the construction of the model, we assume that the liquid-metal current-carrying conductor is incompressible (its density is ρ=const) and that its transport coefficients (electrical conductivity σ and shear viscosity η) are constant. This enables us to concentrate attention on the dynamical nature of the nonequilibrium phase transition, as a result of which the current-carrying conductor is stratified. As is shown in Refs. 10–13, large-scale vortex structures with perturbation wavelength of the order of the radius of the conductor develop in the conductor. In accordance with Refs. 14 and 15, it is these structures that determine the nature of the development of the perturbation of the surface, namely, the surface of the conductor between the vortex rings can be pulled in, as a result of which narrow annular channels form that grow inside the conductor and cause it to stratify. On the other hand, the generation of the vortex structures has a threshold,[10,13] and it is natural to assume that the coefficient of supercriticality depends on the state of the surface of the conductor. The existence of this feedback makes it necessary to construct a self-consistent model of the interaction of the vortex structures with the moving boundary of the current-carrying conductor.

In this paper, we propose and investigate such a model. In Sec. 2, we discuss the experimental results relating to stratification of a conductor. In Sec. 3.1, it is shown that the effect of the perturbation of the boundary on the development of the large-scale vortex structures in the conductor can be taken into account using the Swift–Hohenberg model equation[16] with additional terms associated with the perturbation of the surface. Analysis of the self-consistent model shows that the type of bifurcation (which is subcritical) and also the nature of the near-critical behavior of the system are determined by these terms, i.e., by the state of the conductor surface. We propose and investigate (Sec. 3.2) a very simple model of the growth of a narrow channel within a conductor; this model gives a qualitative description of the nonlinear stages of the stratification of the conductor.

2. THE EXPERIMENT

The electrical circuit of the experiment is shown in Fig. 1. An aluminum wire of diameter $d \simeq 0.58$ mm and length $1 \simeq 10$ cm was exploded in an electrical circuit with the following parameters: storage capacitance $C_0 = 4.2 \cdot 10^{-6}$ F, charging voltage $U_0 = 3 \cdot 10^4$ V, circuit oscillation period $T = 2\pi(LC)^{1/2} \approx 4 \cdot 10^{-5}$ s.

The value of U_0 was maintained with a precision $\delta U_0/U_0 \sim 10^{-3}$ by means of a special control circuit. The

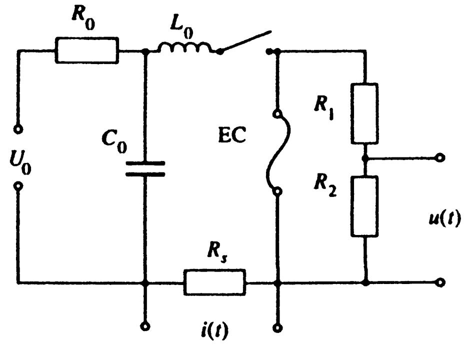

FIG. 1. Electrical circuit of the arrangement for experimental investigation of the electric explosion of conductors (R_1 and R_2 are the resistances of voltage dividers; R_s is the shunt resistance; the conditions of the experiment are described in Sec. 2).

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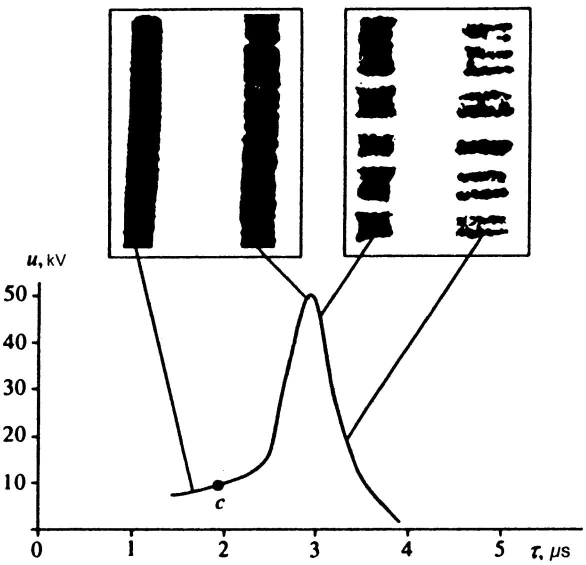

FIG. 2. Curve of the voltage $u(t)$ and two pairs of x-ray images of the same section of an exploding wire of diameter $d=5.8\cdot 10^{-4}$ m and length $l=0.1$ m at different times $\tau=t-(t_b+2.1\cdot 10^{-6})$ [t_b is the time at which the melting of the conductor ends (the units of τ are seconds); the conditions of the experiment are described in Sec. 2].

current $i(t)$ through the conductor and the voltage $u(t)$ across it were measured by a special two-channel circuit with fast (conversion time $\sim 7\cdot 10^{-9}$ s) ADCs possessing an amplitude resolution of $\sim 1\%$. The $i(t)$ and $u(t)$ oscilloscope traces agreed in time to within $\sim 10^{-9}$ s. In the current measuring circuit, a bridge-circuit shunt was used which ensured measurement of the current with a small ($\leq 1\%$) systematic error.

Besides the electrical measurements, a special two-channel scheme of x-ray shadow photography was realized. X-ray photography was necessary because at the onset of the abrupt growth in the voltage ["the initial point" of the explosion, corresponding to the instant of time t_c on the curve $u(t)$ in Fig. 2] a shell is formed around the conductor that is opaque in visible light, on the outer boundary of which there is a shock wave, and the interval between the shock front and the surface of the conductor is filled with opaque metal vapor.

The circuit for the two-channel x-ray photography incorporated two pulsed voltage generators that made it possible to obtain bell-shaped voltage pulses of amplitude $\sim 3\cdot 10^5$ V and width at half-maximum $\sim 2.5\cdot 10^{-8}$ s. The anticathode of the pulsed x-ray tube was a thin copper foil. The cathode was a thin tube with sharp edges placed 1–2 mm from the foil. The radiation passed into the air through a beryllium window. The diameter of the tube focal spot was ~ 0.3 mm, the radiation dose at distance 0.14 m from the anode was ~ 0.2 R, and the effective radiation rigidity was ~ 40 keV.

The radiation beams from the two x-ray tubes were collimated angle so that it was possible to obtain two nonoverlapping images. The smearing of the edge of the photograph in this arrangement is determined by a ratio of distances, namely, that of the focal spot to the conductor (25 cm) and that of the conductor to the x-ray film (1 cm), and also by the diameter of the focal spot of the tube. To shield the film from the effect of the shock wave developed in the air on the explosion of the conductor, a special screen transparent for x rays of the given energy range was placed in the gap between the conductor and the film. By varying the time interval between the times of firing of the two sources, it was possible to obtain two photographs of different phases of the process.

The quality of the images can be characterized by the following numbers: Smearing of the edge of the image $\sim 2\cdot 10^{-5}$ m, number of resolved density gradations >10, minimum resolved thickness $\sim 5\cdot 10^{-5}$ m for the aluminum and a maximum $\sim 10^{-3}$ m for the copper, and accuracy of the matching of the photographs to the voltage and current oscillograms $\sim 2\cdot 10^{-8}$ s.

We discuss the experimental data that, in our view, have a direct bearing on the theoretical model proposed below. We first consider the stage of heating of the liquid metal, which begins on the melting of the current-carrying conductor is complete. During this stage, the transverse dimension of the conductor grows monotonically; this growth can be attributed to the thermal expansion of the metal in the liquid state. In the x-ray photographs, the boundary of the conductor remains fairly sharp; visible perturbations of the conductor surface are not observed during this time interval. Vapor is emitted from the surface, giving rise to a vapor piston and shock wave in the air. There is no way to know when the region of energy content in which the surface reaches the boiling point at atmospheric pressure is crossed using the data of the electrical measurements, and the initial point of the explosion (see below) occurs at significantly higher temperatures. This stage ends ["the initial point of the explosion," corresponding to the time t_c on the curve $u(t)$ in Fig. 2], when smearing of the outer boundary is observed in the x-ray photographs, and one can also begin to see the characteristic modulation of the transverse diameter of the conductor. At this time, the rate of interruption of the current increases sharply, and the voltage begins to grow rapidly (Fig. 2).

The next stage (in which the current ceases abruptly) is bounded on the left by the "initial point of the explosion" t_c. It is shown in Refs. 10 and 13 that a significant cause of such behavior of the electric current in the conductor and decrease of the voltage across it is the formation and development of hydrodynamic and current vortex structures that block the passage of the laminar component of the current (the part of the total current that is short-circuited through the outer electric circuit). In Refs. 14 and 15 it is also conjectured that besides the growth of the "turbulent" resistance due to the increase in the amplitude of the vortices, interruption of the current may result from the breakup of the conductor into transverse strata when the outer surface is "pulled" into it[1)] ("strata" form in accordance with the model proposed in Sec. 3).

We discuss the behavior of the conductor surface in this stage. Figure 2 shows two typical pairs of photographs of the same section of the conductor at different instants of time. The first photograph of the first pair of images was taken near t_c. At this time the conductor maintains its integrity and that the liquid–vapor boundary is smooth. The second photograph of the same pair is taken $\Delta t\sim 10^{-6}$ s later near the maximum of $u(t)$. It can be seen that at certain positions the conductor is broken up by low-density channels. One can make out large-scale structures with longitudinal dimension somewhat greater than the diameter of the conductor in the first photograph. In addition to this spatial harmonic, one can identify a higher harmonic. In the first photograph of the second pair of images, which was taken slightly later [after the $u(t)$ maximum], one can see several dark fragments similar to the one distinguished in the previous photograph. Here there are five of these fragments, and one of them is large. Then (in the second photograph of the second pair of images) a characteristic stratification process takes place: The small fragments (except for one) are transformed into pairs of fragments, and the large fragment is transformed into a pair of fragment pairs. At the same time, a certain symmetry is respected: Three dark fragments in the first photograph form a pair of dark fragments in the second photograph with a relatively small distance between the elements, while the interval between such pairs is greater than the distance between the elements of a pair. In what follows, we shall understand by a stratum a pair of such closely spaced dark fragments of the conductor (see the second photograph of the second pair of images in Fig. 2). The finer structures visible in the second photograph of the first pair of images "die out." The mean characteristic diameter of a stratum is $\lambda \approx 0.65 \pm 0.05$ mm, so that $\lambda d^{-1} \approx 1.2 \pm 0.1$. In accordance with our proposal, such a stratum corresponds to a kind of particle: A pair of hydrodynamic vortex rings (toroids) in which the particles of the liquid counterstream at the conductor axis.

3. THE MODEL

3.1. As the basic mathematical model of the initial stage of the stratification of a current-carrying conductor, we shall, as in Refs. 10 and 13, use the equations of magnetohydrodynamics. At the same time, in order to demonstrate explicitly the dynamical nature of the nonequilibrium phase transition investigated below, we assume that the conducting liquid is incompressible and that its transport coefficients are constant. Such an approach is also justified by the fact that the scenario for the development of the instability is the same for metals with different thermophysical properties.

Bearing in mind that surface perturbations that do not curve the force lines of the magnetic field primarily develop,[17] we specify the surface of the conductor by the expression $r_s = r_0 + r_1(z,t)$, where r_0 is the unperturbed radius of the conductor, and r_1 is an azimuthally symmetric small perturbation of it. We assume that the components of the velocity vector in cylindrical coordinates have the form

$$\mathbf{v} = \{v_r(r,z,t), 0, v_z(r,z,t)\},$$

and that the components of the vector of the magnetic field have the form

$$\mathbf{H} = \{0, H(r,z,t), 0\},$$

which also correspond to the azimuthal symmetry of the problem. It is clear that there exists a potential function ψ such that $v_r = -\partial\psi/\partial z$ and $v_z = \partial\psi/\partial r + \psi/r$. We seek a solution in the form of a sum of perturbations ψ and h that satisfy zero-value boundary conditions, $\psi|_{r=0} = \psi|_{r=r_s} = h|_{r=0} = h|_{r=r_s} = 0$, and an unperturbed (i.e., corresponding to the conditions $r_s = r_0$ and $\psi=0$) distribution of the magnetic field $H_1(r,t)$ over the section of the conductor. In what follows, we shall specify this distribution in the form

$$H_1(r) = \frac{r}{r_0} H_0(f(r,t)+1),$$

where the function f satisfies zero-value boundary conditions, and $H_0 = 2I/cr_0$ (I is the electric current through the conductor). Setting $f \ll 1$, we obtain for the perturbations the system of equations

$$\frac{\partial \hat{D}\psi}{\partial t} = \frac{\partial(\hat{D}\psi/r, \psi r)}{\partial(\mathbf{r},z)} + \frac{H_0}{2\pi\rho r_0}\frac{\partial h}{\partial z} + \frac{h}{2\pi\rho r}\frac{\partial h}{\partial z} + \nu\hat{D}^2\psi, \tag{1}$$

$$\frac{\partial h}{\partial t} = \frac{\partial(h/r, \psi r)}{\partial(r,z)} + \frac{H_0}{r_0}\frac{\partial f}{\partial \ln r}\frac{\partial \psi}{\partial z} + \nu_m \hat{D}h, \tag{2}$$

where ν is the kinematic viscosity, $\nu_m = c^2(4\pi\sigma)^{-1}$ is the magnetic viscosity, and

$$\hat{D}g = \frac{\partial^2 g}{\partial r^2} + \frac{1}{r}\frac{\partial g}{\partial r} - \frac{g}{r^2} + \frac{\partial^2 g}{\partial z^2}$$

and

$$\frac{\partial(f,g)}{\partial(r,z)} = \frac{\partial f}{\partial r}\frac{\partial g}{\partial z} - \frac{\partial f}{\partial z}\frac{\partial g}{\partial r}.$$

We differentiate (2) with respect to z and, using (1), eliminate h from the linearized system (1)–(2). Since in what follows we shall be interested only in near-critical effects, and in the near-critical region ψ is the sum of a rapidly damped part and a part with a small exponent $\lambda = \partial \ln \psi/\partial t$, we can ignore the second derivative of ψ with respect to the time compared with the first, since it is of higher order in λ. Then

$$\frac{\partial \hat{D}^2\psi}{\partial t} = -\frac{s\mathcal{R}}{r_0^2}\frac{\partial f}{\partial \ln r}\frac{\partial^2 \psi}{\partial z^2} + s\hat{D}^3\psi, \tag{3}$$

where $\mathcal{R} \equiv H_0^2 r_0^2 (2\pi\rho\nu\nu_m)^{-1}$ is the magnetic Rayleigh number, and $s = \nu\nu_m/(\nu+\nu_m)$ is the reduced viscosity.

Expanding ψ near $r=r_0$ in a Taylor series, we obtain for the boundary condition

$$\psi|_{r=r_0+r_1} = \psi|_{r=r_0} + r_1\frac{\partial\psi}{\partial r}\bigg|_{r=r_0} + \frac{1}{2}r_1^2\frac{\partial^2\psi}{\partial r^2}\bigg|_{r=r_0} + \ldots,$$

from which it can be seen that ψ is conveniently sought in the form $\psi_0 + \psi_1 + \psi_2 + \ldots$, where

$$\psi_0|_{r=r_0} = 0, \quad \psi_1|_{r=r_0} = -r_1\frac{\partial\psi_0}{\partial r}\bigg|_{r=r_0},$$

$$\psi_2|_{r=r_0} = -r_1\frac{\partial\psi_1}{\partial r}\bigg|_{r=r_0} - \frac{1}{2}r_1^2\frac{\partial^2\psi}{\partial r^2}\bigg|_{r=r_0} \quad \text{etc.}$$

We can write the function ψ_0 in the series form

$$\psi_0(r,z,t)=\sum_{n=1}^{\infty} u_n(z,t)J_1\left(\mu_n \frac{r}{r_0}\right),$$

where μ_n is the nth zero of the Bessel function of first order, J_1, which is an eigenfunction of the operator $m\hat{D}$.

It was shown in Refs. 10–13 that the mode with $n=1$ usually loses stability, and therefore we restrict ourselves for the perturbation ψ_0 to one mode with respect to r: $\psi_0(r,z,t)=u(z,t)J_1(k_1r)$, where we have denoted $k_1=\mu_1/r_0\approx 3.83/r_0$.

To ensure zero-value boundary conditions for the perturbations on the conductor axis, we set $\psi_1\sim r$ and $\psi_2\sim r$. Then, taking into account what was said above, we make an ansatz that satisfies all the required boundary conditions:

$$\psi(r,z,t)=u(z,t)J_1(k_1r)+w(z,t)\frac{r}{r_0},$$

$$w=ucq\left(\frac{3}{2}q-1\right), \tag{4}$$

where $c=\mu_1J_0(\mu_1)$ and $q(z,t)=r_1(z,t)/r_0$. Ignoring the time derivatives of the nonlinear function w compared with the time derivatives of u and retaining dependence on r only through the mode $J_1(k_1r)$, we obtain

$$\left(\frac{\partial^2}{\partial z^2}-k_1^2\right)^2\frac{\partial u}{\partial t}=-\frac{s\mathscr{R}\alpha}{r_0^4}\frac{\partial^2 u}{\partial z^2}+s\left(\frac{\partial^2}{\partial z^2}-k_1^2\right)^3 u$$
$$-\frac{s\mathscr{R}\beta}{r_0^4}\frac{\partial^2 w}{\partial z^2}+s\gamma\frac{\partial^6 w}{\partial z^6}, \tag{5}$$

where we have introduced the coefficients

$$\alpha=2\int_0^1\frac{\partial f}{\partial x}\frac{J_1^2(\mu_1x)}{J_2^2(\mu_1)}x^2dx,$$

$$\beta=2\int_0^1\frac{\partial f}{\partial x}\frac{J_1(\mu_1x)}{J_2^2(\mu_1)}x^3dx \quad \text{and} \quad \gamma=\frac{2}{\mu_1J_2(\mu_1)}.$$

Let g and g_1 be the densities of the Fourier spectra of the functions u and w, respectively; then from (5) we obtain

$$\frac{\partial g}{\partial t}=\frac{s\alpha k^2}{(k^2+k_1^2)^2r_0^4}(\mathscr{R}-\mathscr{R}(k))g+\frac{s\beta k^2}{(k^2+k_1^2)^2r_0^4}$$
$$\times(\mathscr{R}-k^4r_0^4\gamma/\beta)g_1\equiv A(k)g+B(k)g_1, \tag{6}$$

where $\mathscr{R}(k)=(k^2+k_1^2)^3r_0^4\alpha^{-1}k^{-2}$. The value of the function $\mathscr{R}(k)$ has a minimum at $k=k_0\equiv k_1/\sqrt{2}$ and it is equal to $\mathscr{R}_c\equiv 27\mu_1^4/(4\alpha)$. This means that with increasing value of the external control parameter $\mathscr{R}$ it is usually the mode with wave number along the z axis equal to k_0 that loses stability. As is shown in Ref. 11, it corresponds to a pair of vortex rings (Bénard rolls closed into a torus). It is clear that between vortex rings rotating toward the conductor axis the formation of strata with characteristic diameter $2\pi/k_0=2.32r_0$ is possible. This agrees well with the experimental results on the electrical explosion of conductors (see Sec. 2).

Assuming that the coefficient of supercriticality $\varepsilon=(\mathscr{R}-\mathscr{R}_c)/\mathscr{R}_c$ is small, we expand the right-hand side of (6) near the point $k=k_0$ with respect to $(k^2-k_0^2)$. Up to terms of third order, we obtain

$$\frac{\partial g}{\partial r}=(\varepsilon-D(k^2-k_0^2)^2)g+B(k_0)(3k_0^2s)^{-1}$$
$$+O((k^2-k_0^2)^3), \tag{7}$$

where $D=(3k_0^4)^{-1}$, and $\tau=3k_0^2st$ is the dimensionless time. In the expansion of $B(k)$, we have retained only the first term of the series: $B(k_0)$. This approximation is valid if $g_1=O(\varepsilon g)$ holds and is naturally satisfied in the initial stages of the phase transition (see Sec. 3.2).

Going back from the spectra to the functions u and q, we obtain the equation

$$\frac{\partial u}{\partial \tau}=u(p(q)-au^2)-D(k_0^2+\Delta)^2u, \tag{8}$$

where

$$p(q)=\varepsilon+bq(1-3q/2), \tag{9}$$

and we have also denoted $\Delta=\partial^2/\partial z^2$, $b=-B(k_0)(3k_0^2s)^{-1}c$. We have added the cubic term au^3, since it is due to the presence of the nonlinear terms in the original equations (1) and (2) that are invariant with respect to the substitution $\psi\rightarrow-\psi$, $z\rightarrow-z$ and, therefore, ensure "fork" bifurcation type in the problem with unperturbed boundary. Indeed, if $b=0$ holds, then (8) goes over into the well-known Swift–Hohenberg equation for thermal convection in a thin horizontal layer of liquid,[16] and in Ref. 18 it was shown that an analogy exists between the initial stages of the laminar–turbulent transition in a current-carrying medium and thermal convection; this analogy is based on the identity of Eqs. (1)–(2) and Saltzman's system for the theory of the Bénard effect in a liquid.[19] In addition, we shall not dwell in detail on the derivation of the cubic term in Eq. (8) and, in particular, on the determination of the value of the coefficient a, since we shall show below that its contribution is unimportant in the determination of the near-critical behavior of the system as compared with the contribution of the term $buq(1-3q/2)$.

Thus, the initial stage in the generation of the vortex structures in a liquid-metal conductor with current can be investigated by means of the Swift–Hohenberg model equation with an additional term that takes into account the influence of the boundary on the processes in the conductor.

3.2. We shall now construct a self-consistent model of the initial stages of the laminar–turbulent transition in a liquid-metal conductor with current with allowance for the motion of its surface. For this, we represent the surface perturbation q of the conductor in the form

$$q=r_0^{-1}\int_0^{\tau}v_s d\tau,$$

where v_s is its velocity, which we shall seek in the form $v_s=r_0\zeta\partial u/\partial z$, where $\zeta<0$. This representation reflects the fact[14,15] that between hydrodynamic vortices rotating toward the conductor axis its surface will be drawn in with a velocity that depends on the vortex amplitude; moreover, if u is an odd function, then r_1 (and, therefore, q) is an even function. Here we omit the nonlinear terms, since allowance for them would subsequently lead to negligibly small corrections.

Thus, we consider the closed system of equations

$$\frac{\partial u}{\partial \tau}=u\left(\varepsilon+bq\left(1-\frac{3q}{2}\right)-au^2\right)-D(k_0^2+\Delta)^2u. \quad (10)$$

$$\frac{\partial q}{\partial \tau}=\zeta\frac{\partial u}{\partial z}. \quad (11)$$

We now construct from (10) and (11) a few-mode model, using only modes that are multiples of the fundamental k_0. To do this, we represent the perturbations u and q in the form

$$u=X(\tau)\sin(k_0z)+\sum_{n=2}^{\infty}X_n(\tau)\sin(nk_0z),$$

$$q=\zeta k_0Y(\tau)\cos(k_0z)+\zeta k_0\sum_{n=2}^{\infty}nY_n(\tau)\sin(nk_0z).$$

Noting that $\varepsilon\ll D(n^2-1)^2$ for $n>1$ and that therefore the mode with $k=k_0$ is the leading mode, we obtain in the neighborhood of the point $X=Y=X_1=Y_1=\ldots=0$ up to terms of third order.

$$\frac{dX}{d\tau}=\varepsilon X-a_1X^3-b_1XY^2, \quad \frac{dY}{d\tau}=X, \quad (12)$$

$$X_1=\frac{b\zeta}{9k_0^3}XY, \quad Y_1=\frac{b\zeta}{18k_0^3}Y^2 \text{ etc}, \quad (13)$$

where $a_1=3a/4$ and $b_1=3b\zeta^2k_0^4/8$.

It can be seen that the term in (12) corresponding to g_1 in Eq. (6) for the spectrum is cubic and that the corresponding g is linear; therefore, provided X is small the condition $g_1=O(\varepsilon g)$ used in the derivation (7) is valid.

Eliminating the time τ from (12), we obtain the Riccati equation for the phase trajectory:

$$\frac{dX}{dY}=\varepsilon-a_1X^2-b_1Y^2. \quad (14)$$

We expand X in powers of Y: $X=\Sigma_{i=1}^{\infty}c_iY^i$. By substitution in (14), we obtain for the first three coefficients the values $c_1=\varepsilon$, $c_2=0$, and $c_3=-(b_1+a_1\varepsilon^2)/3$. This means that to within terms of higher order the dynamics of the amplitude Y will be determined by the differential equation

$$\frac{dY}{d\tau}=X=\varepsilon Y-\frac{b_1+a_1\varepsilon^2}{3}Y^3. \quad (15)$$

The stationary solution of (15), $Y=0$, is stable for $\varepsilon<0$ and unstable for $\varepsilon>0$. It can be seen from this equation that at small ε the type of bifurcation is entirely determined by the sign of the coefficient b_1. Indeed, for $b_1>0$ we have an ordinary supercritical bifurcation of the "fork" type. If $b_1<0$ holds, then in the neighborhood of the point $\varepsilon=0$, $Y=0$ for positive ε there are no stationary solutions except for the unstable trivial solution (subcritical bifurcation), as a consequence of which the trajectory of the system leaves its neighborhood in a finite time. It should also be noted that if $b_1+a_1\varepsilon^2<0$ holds (or for small ε simply $b_1<0$) then all the coefficients c_i in the expansion of X in powers of Y are equal to zero for even i and are negative for odd $i>3$. This enables us to speak of finiteness of the trajectory of the system (12) if the higher powers of the amplitude Y were taken into account in it.

Thus, we have shown that the type bifurcation and also the nature of the critical behavior of the original system (10)–(11) are entirely determined by the nonlinear term $buq(1-3q/2)$, i.e., by the state of the surface of the conductor. At the same time, the sign of the coefficient b determines the nature of the loss of stability (hard or soft excitation of the instability) of the boundary of the conductor. To establish which mechanism of stability loss actually occurs, and thus identify the sign of b of the model, we consider a simplified model of the growth of a channel. Note that if in (10) and (11) we make the substitution $q\to-q$ and $z\to-z$, then the resulting system of equations is equivalent to the original one except for the one term buq. This means that it is precisely this term that breaks the invariance of the system (10)–(11) with respect to such a substitution and thus determines a difference between the directions "away from the axis" and "toward the axis" in the cylindrical geometry.

We consider the truncated system

$$\frac{\partial u}{\partial \tau}=buq, \quad (16)$$

$$\frac{\partial q}{\partial \tau}=\zeta\frac{\partial u}{\partial z}, \quad (17)$$

obtained by ignoring in Eq. (10) the terms invariant with respect to the substitution $q\to-q$ and $z\to-z$, so that it explicitly expresses the dependence of the nature of the growth of a channel on its direction. Comparing what is given by the system (16)–(17) with the experimental results of Sec. 2 (it can be seen from them that the surface of the conductor is drawn in the direction of its axis, forming narrow channels localized in space), we can determine the sign of b.

We shall be interested in solutions of (16) and (17) that ensure localization of the perturbation of the conductor surface in the neighborhood of a certain point $z=z_0$ and satisfy the property

$$\frac{d}{dt}\int_{-\infty}^{+\infty}q\,dz=0,$$

which corresponds to the continuity equation written in integral form, assuming that the medium is incompressible. These conditions are satisfied by the functions q and u written in the form

$$u=\frac{\varphi(\xi)}{z-z_0}, \quad q=-\zeta\frac{\varphi(\xi)\xi}{z-z_0},$$

where φ is some function of the self-similar variable $\xi=(\tau-\tau^*)/(z-z_0)$. The meaning of τ^* will be indicated below. By substituting in (16) and (17), we obtain

$$\frac{\partial}{\partial\xi}\varphi(\xi)=-b\zeta\varphi^2(\xi)\xi.$$

Solving this equation, we obtain

$$u=\frac{2}{b\zeta}\frac{z-z_0}{c^2(z-z_0)^2+(\tau-\tau^*)^2}, \tag{18}$$

$$q=-\frac{2}{b}\frac{\tau-\tau^*}{c^2(z-z_0)^2+(\tau-\tau^*)^2}, \tag{19}$$

where c is a parameter that characterizes the channel geometry (indeed, the channel volume is proportional to $\int_{-\infty}^{+\infty} q dz = 2\pi/bc$).

The expressions (18) and (19) for u and q describe the time evolution of the surface of the conductor between two vortices: After a finite time, a small perturbation of the surface is transformed into a narrow δ-function channel, and at the same time the vortices are drawn toward each other. Indeed, as $\tau\to\tau^*$ we have

$$u\to\frac{2}{b\zeta}\frac{1}{c^2(z-z_0)}, \quad q\to\frac{2\pi}{bc}\,\delta(z-z_0). \tag{20}$$

It can be seen that the growth of the channel described by our simplified model (16)–(17) with $b<0$ is qualitatively the same as is observed in the experiment: After a finite time, a narrow channel moving toward the conductor axis is formed. At $\tau=\tau^*$, the conductor is broken up into particles with a diameter on the order of the diameter on the conductor (see Sec. 2).

It follows from the fact that $b<0$ that $b_1<0$. Then Eq. (15) describes supercritical bifurcation and, therefore, the loss of stability of the conductor boundary is hard. This is a natural result, since such bifurcation leads to an irreversible (and therefore inexplicable in terms of supercritical bifurcation) change in the state of the conductor.

An additional argument in support of what we have said is provided by the results of Refs. 14 and 15, in which a three-mode model of the stratification of a conductor with subcritical bifurcation is constructed for a special form of the unperturbed magnetic field H_1.

4. CONCLUSIONS

Thus, we have analyzed the results of experiments on the stratification of a liquid-metal conductor with current in the case of an electric explosion, and we have proposed a model of this process. In our model, the large-scale vortex structures that lead to stratification and, therefore, the transverse strata are periodic with respect to the length of the conductor with wavelength $\lambda_0=2.32r_0$. As is shown by the analysis made in Sec. 2 of the sequence of x-ray photographs of the exploding conductor in the stage of rapid interruption of the electric current, the observed stratification process is more complicated than the model process. Namely, in the experiment we observe strata of scale λ_0 as well as $\lambda_0/2$. On the one hand, this confirms the assumption that a liquid-metal "particle" of scale λ_0 consists of a pair of vortices; on the other hand, this fine structure apparently arises because of the pressure from the hot regions formed near the conductor axis which we have ignored. In Refs. 11 and 12 it was shown that these hot regions (so-called hot spots[4]) are localized between two vortex rings in which the particles of the liquid near the conductor axis move in opposite directions. These hot regions are sources of high-velocity plasma jets, which can lead first to the additional formation of channels from within the conductor and, second, to a slowing down or complete stopping of the growth of a channel from the conductor surface as a consequence of "squeezing" by the pressure of the channel growing from the surface within the conductor.

Nevertheless, the model that we have proposed of the stratification of a conductor correctly reflects the main qualitative features of the process. Moreover, since in our model the stratification of the conductor is determined by purely hydrodynamic processes, it can be concluded that a current-carrying conductor can also break up into transverse strata in the case of constant transport coefficients (in our opinion, the thermal instability noted in Refs. 7–9 can accelerate the process of stratification of the conductor at the end of the process [as $r_s\to 0$]). The proposed dynamical approach to modeling the stratification is also supported by the experimental fact that the scenario of the development of the instability is the same for metals with different thermophysical properties.

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[1)]In practice, both mechanisms are probably present; moreover, the first of them—"interruption" without destroying the integrity of the conductor—is realized in liquid-metal current breakers in regimes in which it is possible to obtain multiple interruption and restoration of the current with equal characteristic times (V. T. Shkatov, private communication).

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