The effect of a strong light field on the scattering of an ultrarelativistic electron by a nucleus

S. P. Roshchupkin

Institute of Applied Physics, Ukrainian National Academy of Sciences, 244030 Sumy, Ukraine
(Submitted 24 May 1995)
Zh. Éksp. Teor. Fiz. **109**, 337–344 (February 1996)

This paper presents a theoretical study of the scattering of an ultrarelativistic electron by a nucleus in the field of two codirectional light waves outside the Bunkin–Fedorov region. Highly asymmetric (along the wave vector) large-angle scattering of the electron is predicted. The effect is related to the interference of the two waves. © *1996 American Institute of Physics.* [S1063-7761(96)00102-9]

1. INTRODUCTION

The process of stimulated bremsstrahlung and absorption in the scattering of an electron by a nucleus in the field of a plane electromagnetic wave has been studied for a fairly long time (see, e.g., Refs. 1–6). Recently the case was examined of scattering in the field of two light waves of arbitrary intensities and frequencies propagating in the same direction.[7,8] It was shown that depending on the intensities, frequencies, and polarizations of the waves, the electron can be scattered by the nucleus in different kinematic regions, characterized by different multiphoton parameters: the quantum parameter $\gamma_{1,2}$ in the Bunkin–Fedorov region and the quantum parameters $\beta_{1,2}$ and $\alpha_{\pm}$ outside that region. An interference effect was discovered that manifests itself explicitly in circularly polarized waves with the scattering being highly asymmetric. It is accompanied by correlated emission and absorption of an equal number of photons of both waves. The ultrarelativistic limit of the electron energy was excluded from the picture. The present paper is devoted to a study of precisely this case in the kinematic region in which the Bunkin–Fedorov quantum parameters are small ($\gamma_{1,2} \ll 1$) and multiphoton processes are determined by the quantum parameters $\beta_{1,2}$ and $\alpha_{\pm}$. It is found that in certain conditions ultrarelativistic electrons basically scatter along a preferred direction (except for small-angle scattering), namely, along the wave vector, independent of the angles θ_i between their initial momenta and the wave vector. This sets ultrarelativistic electrons apart from relativistic electrons ($E_i \sim m$; here we use the relativistic system of units: $\hbar = c = 1$) and from nonrelativistic electrons, for which each initial angle θ_i corresponds to a preferred direction of scattering.[8]

2. THE PROBABILITY OF STIMULATED BREMSSTRAHLUNG AND ABSORPTION BY AN ULTRARELATIVISTIC ELECTRON SCATTERED BY A NUCLEUS

As in Ref. 8, we select the 4-vector potential of the external field in the form of the sum of two elliptically polarized electromagnetic waves propagating parallel to the z axis:

$$A = A_1(\varphi_1) + A_2(\varphi_2), \tag{1}$$

where

$$A_j(\varphi_j) = \frac{F_j}{\omega_j}(e_{jx} \cos \varphi_j + \delta_j e_{jy} \sin \varphi_j). \tag{2}$$

Here δ_j is the ellipiticity of a wave, $e_{jx} = (0, \mathbf{e}_{jx})$ and $e_{jy} = (0, \mathbf{e}_{jy})$ are the 4-vectors of wave polarization, F_j and ω_j are the field strength and frequency of the first ($j=1$) and second ($j=2$) waves, and the argument φ_j has the form

$$\varphi_j = \omega_j(t - z), \quad j = 1,2. \tag{3}$$

We assume that the electron energies in the initial and final states, E_i and E_f, are ultrarelativistic, $|\mathbf{P}_{i,f}| \approx E_{i,f} \gg m$, and restrict the intensities of the two waves by the condition

$$\eta_{1,2} \ll E_i/m, \tag{4}$$

where η_1 and η_2 are the classical Lorentz-invariant parameters of the first and second waves:

$$\eta_{1,2} = \frac{eF_{1,2}}{m\omega_{1,2}}. \tag{5}$$

When condition (4) is met, the probability of stimulated bremsstrahlung emission and absorption by an ultrarelativistic electron scattered by a nucleus with the charge Ze in the field of two plane electromagnetic waves propagating in the same direction, Eqs. (1) and (2), assumes the following form (see also Ref. 8):

$$dW_{fi} = \sum_{l=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} dW_{fi}^{(ls)}, \tag{6}$$

where the partial probability of emission ($l,s>0$) or absorption ($l,s<0$) of $|l|$ photons of the first wave and $|s|$ photons of the second is specified as

$$dW_{fi}^{(ls)} = 2(Ze^2)^2 \frac{E_i E_f (1+\cos\theta)}{\tilde{E}_i \tilde{E}_f} |I_{ls}|^2 \times \delta(\tilde{E}_f - \tilde{E}_i + l\omega_1 + s\omega_2) \frac{E_f^2}{\mathbf{q}^4} dE_f d\Omega. \tag{7}$$

Here $\mathbf{q}$ is the momentum transfer, θ is the scattering angle, and $\tilde{E}_i$ and $\tilde{E}_f$ are the electron quasienergies in the initial and final states (the 4-quasimomentum components $\tilde{p}_{i,f} = (\tilde{E}_{i,f}, \tilde{\mathbf{p}}_{i,f})$):

$$\mathbf{q} = \tilde{\mathbf{p}}_f - \tilde{\mathbf{p}}_i + (l\omega_1 + s\omega_2)\mathbf{n}, \tag{8}$$

$$\tilde{p}_{i,f}=p_{i,f}+b_0\frac{m^2}{k_{i,j}},\quad b_0=\frac{1}{4}[(1+\delta_1^2)\eta_1^2+(1+\delta_2^2)\eta_2^2],\tag{9}$$

$$k_{i,f}=E_{i,f}-\mathbf{n}\cdot\mathbf{p}_{i,f}\approx E_{i,f}[1-v_{i,f}+2\sin^2(\theta_{i,f}/2)],\tag{10}$$

$$\theta=\angle(\mathbf{p}_i,\mathbf{p}_f),\quad \theta_{i,f}=\angle(\mathbf{n},\mathbf{p}_{i,f}).\tag{11}$$

In Eqs. (8)–(10), $\mathbf{n}$ is the unit vector specifying the direction of propagation of both waves, $p_{i,f}=(E_{i,f},\mathbf{p}_{i,f})$ is the electron's 4-momentum, and v_i and v_f are the electron velocities in the initial and final states. Note that the quantity $1-v_{i,f}$ in Eq. (10) is much smaller than unity and hence must be retained in (10) only for small angles between the electron momenta (in the initial and final states) and the wave vector ($\theta_{i,f}\ll 1$). In Eq. (7) the functions I_{ls} determine the probability of emission (absorption) by an electron in the field of two waves of l photons of the first wave and s photons of the second.[8] Expanded in a series of Bessel functions J_r of integral order they have the form

$$\begin{aligned}&I_{ls}(\chi_1,\gamma_1,\beta_1;\chi_2,\gamma_2,\beta_2;\alpha_+,\alpha_-)\\&\quad=\sum_{r,r'=-\infty}^{\infty}\exp\{-i(r\tau_-+r'\tau_+)\}J_r(\alpha_+)J_{r'}(\alpha_-)\\&\qquad\times L_\nu(\chi_1,\gamma_1,\beta_1)L_\mu(\chi_2,\gamma_2,\beta_2).\end{aligned}\tag{12}$$

Here the functions L_ν and L_μ ($\nu=l-r'-r$ and $\mu=s+r'-r$) describe multiphoton processes in the field of one wave:[8–10]

$$L_r(\chi,\gamma,\beta)=\exp\{-ir\chi\}\times\sum_{s'=-\infty}^{\infty}\exp(2is'\chi)J_{r-2s'}(\gamma)J_{s'}(\beta).\tag{13}$$

In Eq. (12) the multiphoton quantum parameters $\gamma_{1,2}$ and the quantum parameters $\beta_{1,2}$ and $\alpha_\pm$ have the following form:

$$\gamma_j=\eta_j\frac{m}{\omega_j}\sqrt{(\mathbf{e}_{jx}\cdot\mathbf{g})^2+\delta_j^2(\mathbf{e}_{jy}\cdot\mathbf{g})^2},\tag{14}$$

$$\mathbf{g}=\mathbf{p}_f/k_f-\mathbf{p}_i/k_i,\tag{15}$$

$$\beta_j=(1-\delta_j^2)\eta_j^2\frac{m^2}{8\omega_j}\left(\frac{1}{k_f}-\frac{1}{k_i}\right),\quad j=1,2,\tag{16}$$

$$\alpha_\pm=\eta_1\eta_2\frac{m^2|d_\mp|}{2(\omega_1\pm\omega_2)}\left(\frac{1}{k_f}-\frac{1}{k_i}\right),\tag{17}$$

$$d_\pm=(1\pm\delta_1\delta_2)\cos\Delta+i(\delta_1\pm\delta_2)\sin\Delta,$$
$$\Delta=\angle(\mathbf{e}_{1x},\mathbf{e}_{2x}).\tag{18}$$

From (14) we see that if the electron is scattered in such a way that the vector $\mathbf{g}$ (Eq. (15)) lies outside a narrow cone with its axis parallel to the direction of propagation of both waves, the multiphoton quantum parameters $\gamma_{1,2}$ [Eq. (14)] have the following order of magnitude:

$$\gamma_{1,2}\sim\begin{cases}\gamma'_{1,2} & \text{if } \theta_{i,f}\sim 1,\\ \gamma''_{1,2} & \text{if } \theta_{i,f}\ll 1,\end{cases}\tag{19}$$

where

$$\gamma'_{1,2}=\eta_{1,2}\frac{m}{\omega_{1,2}},\quad \gamma''_{1,2}=\eta_{1,2}\frac{m}{\omega_{1,2}(1-v_{i,f}+\theta_{i,f}^2/2)}.\tag{20}$$

Here the angle ψ between the vector $\mathbf{g}$ (Eq. (15)) and the direction of propagation of both waves satisfies the following condition:

$$\psi\gtrsim\begin{cases}1/\gamma'_{1,2} & \text{if } \theta_{i,f}\sim 1,\\ 1/\gamma''_{1,2} & \text{if } \theta_{i,f}\ll 1,\end{cases}\tag{21}$$

Notwithstanding the ultrarelativistic limit of the electron energy and following the notation of Ref. 8, we call the region specified by (19)–(21) the Bunkin–Fedorov region, since the main multiphoton parameters here are the quantum parameters $\gamma_{1,2}$, which first appeared in the classical paper by Bunkin and Fedorov.[1] Here we assume that the fields are strong: $\gamma'_{1,2}\gtrsim 1$ and $\gamma''_{1,2}\gtrsim 1$, i.e., both one-photon and multiphoton processes are essential. In the opposite case ($\gamma'_{1,2}\ll 1$ and $\gamma''_{1,2}\ll 1$) the processes with the highest probability are the emission (absorption) of one photon of the first or second wave (this specifies the range of applicability of perturbation theory in the external field).

Now suppose that the electron is scattered in such a way that the vector $\mathbf{g}$ lies within a narrow cone whose axis is parallel to the direction of propagation of both waves. Then the angle ψ satisfies the opposite condition:

$$\psi\ll\begin{cases}1/\gamma'_{1,2} & \text{if } \theta_{i,f}\sim 1,\\ 1/\gamma''_{1,2} & \text{if } \theta_{i,f}\ll 1,\end{cases}\tag{22}$$

Here the multiphoton quantum parameters $\gamma_{1,2}$ (14) are much smaller than unity for arbitrary intensities of both waves, and the functions I_{ls} (Eq. (12)) transform into the functions J_{ls} of the following form (see also Ref. 8):

$$\begin{aligned}&J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)\\&\quad\equiv I_{ls}(0,0,\beta_1;0,0,\beta_2;\alpha_+,\alpha_-)\\&\quad=\sum_{r,r'=-\infty}^{\infty}\exp\{-i(r\tau_-+r'\tau_+)\}J_r(\alpha_+)\\&\qquad\times J_{r'}(\alpha_-)J_{\nu'}(\beta_1)J_{\mu'}(\beta_2)\delta_{\nu,2\nu'}\delta_{\mu,2\mu'},\end{aligned}\tag{23}$$

where $\delta_{\nu,\mu}$ is the Kronecker symbol. The formula simplifies considerably when both fields are circularly polarized:

$$J_{l,\pm l}(0,0,0,\alpha_\pm)=\exp\{-il\Delta\}J_l(\alpha_\pm),\tag{24}$$

i.e., there is correlated emission and absorption of an equal number of photons of the first and second waves ($s=\pm l$) and a clear indication of wave interference. Equations (23) and (24) imply that multiphoton processes in the kinematic region (22) are determined by the quantum parameters $\beta_{1,2}$ [Eq. (16)] and $\alpha_\pm$ [Eq. (17)].

3. SCATTERING OF AN ULTRARELATIVISTIC ELECTRON PARALLEL TO THE WAVE VECTOR

Let us examine the scattering of an ultrarelativistic electron by a nucleus in the field of two waves when condition (22) is met, i.e., outside the Bunkin–Fedorov region. In these conditions,

$$\mathbf{g}^2=(\mathbf{n}\cdot\mathbf{g})^2, \tag{25}$$

in view of which we have

$$(\mathbf{e}_{jx}\cdot\mathbf{g})=(\mathbf{e}_{jy}\cdot\mathbf{g})=0, \tag{26}$$

and hence $\gamma_j=0$ [$j=1,2$; see Eq. (14)]. Writing (26) explicitly, we easily see that electron scattering occurs in the plane determined by the initial momentum and the wave vector, where the azimuthal angles of the electron in the initial and final states are the same and the polar angles and velocities are related as follows:

$$\frac{v_f \sin\theta_f}{(1-v_f)+2\sin^2(\theta_f/2)}=\frac{v_i \sin\theta_i}{(1-v_i)+2\sin^2(\theta_i/2)}. \tag{27}$$

We investigate large-angle electron scattering. This means excluding the case with $\theta_i\sim 1$ and $\theta_f\sim 1$. Indeed, in view of condition (4), the electron energies before and after scattering are equal [$E_i=E_f$; see the argument in the delta function in Eq. (7)] and, clearly, Eq. (27) becomes $\cot(\theta_f/2)=\cot(\theta_i/2)$, which can be satisfied only in zero-angle electron scattering. Hence for large-angle scattering Eq. (27) is satisfied only in two cases: $\theta_i\ll 1$ and $\theta_f\sim 1$ or $\theta_i\sim 1$ and $\theta_f\ll 1$. Let us examine the second case. For arbitrary initial angles θ_i the electrons scatter near the direction of propagation of both waves:

$$\theta_f=(1-v_f)\,\frac{v_i}{v_f}\cot\frac{\theta_i}{2}\ll 1,\quad \theta=\theta_i-\theta_f\approx\theta_i\sim 1. \tag{28}$$

In view of this we can put $\tilde{E}_i=E_i$ and $\tilde{E}_f=E_f(1+2b_0)$ in the law of energy conservation, and the electron energy after scattering is given by the following expression:

$$E_f=\rho_{ls}E_i,\quad \rho_{ls}=\frac{1-(l\omega_1+s\omega_2)/E_i}{1+2b_0}. \tag{29}$$

Bearing this in mind, we can write the expressions (28) for the scattering angles as

$$\theta_f=(1-v_i)\,\frac{\cot(\theta_i/2)}{\rho_{ls}^2-(1-v_i)}\ll 1. \tag{30}$$

From Eqs. (28)–(30) for the intensities $\eta_{1,2}^2\ll 1$ we obtain $E_f\approx E_i$ ($\rho_{ls}\approx 1$), and hence the scattering angles (30) are

$$\theta_f=(1-v_i)\cot(\theta_i/2)\sim 1-v_i \tag{31}$$

and do not depend on the number of emitted and absorbed photons of both waves. In the opposite limit of extremely strong waves ($\eta_{1,2}^2\gg 1$), Eq. (29) yields

$$\rho_{ls}\approx\frac{1}{2b_0}\left[1-\frac{l\omega_1+s\omega_2}{E_i}\right]\sim\eta_{1,2}^{-2}\ll 1, \tag{32}$$

and hence for the scattering angles (30) we have the approximate relation $\theta_f\sim\eta_{1,2}^4(1-v_i)\gg 1-v_i$, and the final electron energies (29) are low compared to the initial electron energies ($E_f\ll E_i$).

Since the electrons in the final state are also assumed to be ultrarelativistic, Eq. (32) leads to a more stringent condition for the intensities of both waves than (4):

$$\eta_{1,2}^2\ll E_i/m. \tag{33}$$

The scattering cross section can be found from the probability (7) by dividing the latter by the incident electron flux. Allowing for (33) and integrating over the energies of the final state electrons, we arrive at the final expression for the partial differential cross section for scattering into the solid angle element $d\Omega$:

$$\frac{d\sigma^{(ls)}}{d\Omega}=\frac{\rho_{ls}^2}{(1+2b_0)^2}|J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)|^2\frac{d\sigma_M^{(E\gg m)}}{d\Omega}. \tag{34}$$

Here

$$\frac{d\sigma_M^{(E\gg m)}}{d\Omega}=Z^2r_e^2\left[\frac{\cot(\theta/2)}{2\sin(\theta/2)}\right]^2\left(\frac{m}{E_i}\right)^2 \tag{35}$$

is the ultrarelativistic limit of the Mott scattering cross section,[11] and r_e is the classical electron radius. The functions J_{ls} are defined in Eq. (23), and their arguments $\beta_{1,2}$ [Eq. (16)) and $\alpha_\pm$ (Eq. (17)] are

$$\beta_{1,2}=\frac{1}{4}(1-\delta_{1,2}^2)\rho_{ls}\eta_{1,2}^2\frac{E_i}{\omega_{1,2}},$$

$$\alpha_\pm=|d_\mp|\,\eta_1\eta_2\rho_{ls}\frac{E_i}{\omega_1\pm\omega_2}. \tag{36}$$

We see that for $\eta_{1,2}\gtrsim 1$ the quantum parameters satisfy the conditions $\beta_{1,2}\sim\alpha_\pm\gtrsim E_i/\omega_{1,2}\gg 1$, with the result that the partial cross sections are small compared to the Mott cross section. In view of this we examine the case $\eta_{1,2}\ll 1$ more thoroughly. Here $E_f\approx E_i$, the scattering angles θ_f and the quantum parameters $\beta_{1,2}$ and $\alpha_\pm$ are specified by Eqs. (31) and (36) with $\rho_{ls}=1$, and the partial cross sections (34) assume the following form:

$$\frac{d\sigma^{(ls)}}{d\Omega}=|J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)|^2\frac{d\sigma_M^{(E\gg m)}}{d\Omega}. \tag{37}$$

Note that when both waves are linearly polarized ($\delta_1=\delta_2=0$), with the angle Δ between the polarization vectors close to $\pi/2$, i.e.,

$$|\Delta-\pi/2|\ll(\eta_1\eta_2)^{-1}(\omega_{1,2}/E_i)\lesssim 1, \tag{38}$$

the interference parameters $\alpha_\pm$ are much smaller than unity, and for (23) and (37) we find that in the process of scattering the electron independently emits and absorbs an even number of photons of both waves:

$$\frac{d\sigma^{(ls)}}{d\Omega}=J_{l'}^2(\beta_1)J_{s'}^2(\beta_2)\frac{d\sigma_M^{(E\gg m)}}{d\Omega},\quad l=2l',\quad s=2s'. \tag{39}$$

When both waves are circularly polarized ($\delta_1=1$ and $\delta_2\mp 1$), from Eqs. (24) and (37) we obtain the partial cross section of scattering of an ultrarelativistic electron by a nucleus with correlated emission and absorption of an equal number of photons of the both waves ($s=\pm l$):

$$\frac{d\sigma^{(l,\pm l)}}{d\Omega}=J_l^2(\alpha_\pm)\,\frac{d\sigma_M^{(E\gg m)}}{d\Omega}. \tag{40}$$

Note that $\beta_{1,2}\gtrsim 1$ and $\alpha_\pm\gtrsim 1$ hold for wave intensities satisfying the corresponding conditions:

$$\eta_{1,2}^2 \gtrsim \omega_{1,2}/E_i, \quad \eta_1\eta_2 \gtrsim \omega_{1,2}/E_i. \tag{41}$$

In the Bunkin–Fedorov region (21) the multiphoton quantum parameters are such that $\gamma_{1,2} \gtrsim \eta_{1,2}(m/\omega_{1,2}) \gtrsim m/\sqrt{E_i\omega_{1,2}}$. Hence if

$$m/\sqrt{E_i\omega_{1,2}} \gg 1, \tag{42}$$

then $\gamma_{1,2} \gg 1$. We can easily show[12] that

$$\frac{|I_{ls}(\cdots)|^2}{|J_{ls}(\cdots)|^2} \sim \begin{cases} (\gamma_1\gamma_2)^{-1} \ll 1, & l \ll \gamma_1, \quad s \ll \gamma_2, \\ (\gamma_1\gamma_2)^{-2/3} \ll 1, & l \sim \gamma_1, \quad s \sim \gamma_2. \end{cases} \tag{43}$$

Equations (7) and (37) imply that the ratio of the partial cross sections of electron scattering by a nucleus in the Bunkin–Fedorov region and in the plane of the initial momentum and the wave vector is equal, by order of magnitude, to the ratio of the functions (43). Hence if conditions (41) and (42) are met, the cross section of multiphoton stimulated bremsstrahlung and absorption in the scattering of an ultrarelativistic electron by a nucleus [Eq. (37)] is considerably higher than the respective scattering cross section in any other geometry, i.e., ultrarelativistic electrons are mostly scattered along the direction of propagation of the two waves (31).

Let us estimate the intensities and electron energies satisfying conditions (41) and (42). In the optical range ($\omega_{1,2} \sim 10^{15}\,\mathrm{s}^{-1}$), from Eq. (42) we obtain $\sqrt{E_i/m} \ll 10^2$–10^3. Hence for $E_i/m \sim 10^2$ conditions (41) yield intensities easily achievable in modern lasers: $\eta_{1,2} \sim 10^3$–10^{-4} ($F_{1,2} \sim 10^8$–$10^6\,\mathrm{V\,cm}^{-1}$).

I would like to express my sincere gratitude to V. P. Kraĭnov and M. V. Fedorov for useful discussions.

[1] F. V. Bunkin and M. V. Fedorov, Zh. Éksp. Teor. Fiz. **49**, 1215 (1965) [Sov. Phys. JETP **22**, 844 (1966)].
[2] M. M. Denisov and M. V. Fedorov, Zh. Éksp. Teor. Fiz. **53**, 1340 (1967) [Sov. Phys. JETP **26**, 779 (1968)].
[3] R. Pantell, G. Sencini, H. E. Puthoff *et al.*, IEEE J. Quantum Electron. **QE-4**, 905 (1968).
[4] F. V. Bunkin, A. E. Kazakov, and M. V. Fedorov, Usp. Fiz. Nauk **107**, 559 (1972) [Sov. Phys. Usp. **15**, 416 (1972)].
[5] R. V. Karapetyan and M. V. Fedorov, Kvant. Elektron. (Moscow) **4**, 2203 (1977) [Sov. J. Quantum Electron. **7**, 1260 (1977)].
[6] M. V. Fedorov, *Electron in a Strong Light Field*, Nauka, Moscow (1991) [in Russian].
[7] R. L. Gorodnitskiĭ and S. P. Roshchupkin, Laser. Phys. **2**, 602 (1992).
[8] S. P. Roshchupkin, Zh. Éksp. Teor. Fiz. **106**, 102 (1994) [JETP **79**, 54 (1994)].
[9] S. P. Roshchupkin, Laser. Phys. **3**, 414 (1993).
[10] S. P. Roshchupkin, Laser. Phys. **4**, 139 (1994).
[11] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics*, 2nd ed., Pergamon Press, Oxford (1982).
[12] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series 55, Washington, D.C. (1964).

Translated by Eugene Yankovsky