Theory of dissipative solitons in the light-induced drift under conditions of nonmonochromatic excitation

F. Kh. Gel'mukhanov and T. I. Privalov

Institute of Automation and Electrometry, Siberian Division of the Russian Academy of Sciences, 630090 Novosibirsk, Russia
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The theory of the light-induced drift (LID) is considered for the case of a nonlinear variable-sign dependence of the LID velocity on the optical thickness under conditions of nonmonochromatic excitation. It is shown that such a dependence of the LID velocity leads to the formation of one or two separating dissipative solitons. The problem of the number of particles captured in dissipative solitons from an initial distribution of particles of an absorbing gas is solved. © *1995 American Institute of Physics.*

1. INTRODUCTION

Light-induced drift (LID)[1,2] is one of the strongest and most pronounced manifestations of the interaction of light with matter. The essential feature of the effect is the development of a macroscopic flux of particles of an absorbing gas that interact with monochromatic radiation and undergo collisions with particles of a buffer gas. Because of the Doppler effect, the excitation of particles of the absorbing gas is selective with respect to the velocities, and oppositely directed fluxes of particles of the absorbing gas in the excited and ground states arise. In the buffer gas, these two fluxes are subject to different levels of friction, and they are not compensated for because of the difference between the cross sections for collisions of excited and unexcited particles with the particles of the buffer gas. As a result, a macroscopic flux of the absorbing gas develops. The present status of the problem is reflected in the monograph of Ref. 3.

The macroscopic equations that describe LID are essentially nonlinear,[2,4–6] and there is therefore much interest in the analysis of the nonlinear effects in the theory of LID. There have already been studies of effects such as the "optical piston" (Ref. 2), dissipative solitons,[5,6,9] and the occurrence of autooscillations in a gaseous mixture under the influence of LID under conditions of a variable-sign dependence of the LID velocity on the temperature.[7,8]

The primary reason for the formation of dissipative solitons is the nonlinear dependence of the drift velocity $u(N)$ on the optical thickness N (see the definition of this quantity in Sec. 2). In the earlier studies of Refs. 5, 6, and 9, the LID effect was investigated for $u(N)$ with constant sign: $u(N)>0$ [or $u(N)<0$]. In Ref. 4, a study was made of the excitation by nonmonochromatic radiation of an absorbing gas of only one species of particles with hyperfine splitting of the ground state. It was established that under conditions for which optical pumping of the components of the hyperfine structure arises the dependence of the velocity $u(N)$ on the optical thickness N has an essentially nonlinear nature with variable sign, as shown schematically in Fig. 1. In the region $0\leq N\leq N_2$, the drift velocity is negative and changes sign at the point N_2, after which it remains positive.

In this paper our main goal is to analyze the LID effect in the case of such a variable-sign dependence of the drift velocity of the absorbing gas on the optical thickness. Such a dependence of $u(N)$ leads to a qualitatively new scenario of soliton formation. In this paper we will show that the variable-sign dependence of $u(N)$ on N (Fig. 1) may lead to decay of an original bunch of absorbing particles into two dissipative solitons.

It was shown in Refs. 6 and 9 that under conditions of weak nonlinearity (optically thin gas) all particles from the original bunch are captured in the soliton. The problem of the number of absorbing particles captured in a dissipative soliton was not settled for the case of an arbitrary dependence of $u(N)$ on N. This problem is solved and analyzed in detail in the present paper.

2. BASIC GAS-DYNAMIC EQUATIONS

To analyze the LID effect, we write the gas-dynamic equations for the density $\rho\ (x,t)$ of the absorbing gas. In this paper we restrict the discussion to a one-dimensional description, making the assumption that all quantities depend only on the one longitudinal coordinate x and the time t. We assume the direction of the x axis to coincide with the direction of propagation of the radiation (from the left to the right).

The density $\rho(x,t)$ of the particles of the absorbing gas satisfies the continuity equation

$$\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial j(x,t)}{\partial x}=0, \tag{1}$$

where the flux j of the absorbing gas is described by

$$j(x,t)=u(N)\rho(x,t)-D\,\frac{\partial\rho(x,t)}{\partial x}. \tag{2}$$

Here D is the diffusion coefficient of the absorbing particles. The drift velocity $u(N)$ depends nonlinearly on the radiation intensity, which in turn is uniquely determined by the optical thickness N of the gas:[5,6,9]

$$N(x,t)=\int_{-\infty}^{x}\rho(x',t)dx'. \tag{3}$$

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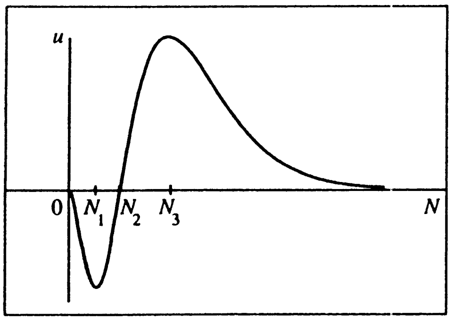

FIG. 1. Dependence of the drift velocity $u(N)$ on the optical thickness: N_1 is the minimum, N_3 the maximum, and N_2 the point of change of sign of the function $u(N)$.

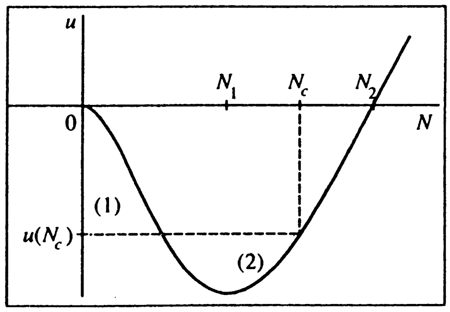

FIG. 2. Geometrical illustration of Eq. (12). The case $u(N)\leqslant 0$.

The definition of the optical thickness N used here differs from the widely adopted concept (σN) (Ref. 4) by the absence in Eq. (3) of the photoabsorption cross section.

Substituting (2) in (1), we obtain the equation

$$\frac{\partial\rho(x,t)}{\partial t}+\frac{\partial}{\partial x}\left[u(N)\rho(x,t)-D\,\frac{\partial\rho(x,t)}{\partial x}\right]=0, \tag{4}$$

or, bearing in mind that in accordance with (3)

$$\rho=\frac{\partial N}{\partial x}, \tag{5}$$

we obtain for the optical thickness $N(x,t)$ the equation

$$\frac{\partial}{\partial x}\left[\frac{\partial N}{\partial t}+u(N)\,\frac{\partial N}{\partial x}-D\,\frac{\partial^2 N}{\partial x^2}\right]=0. \tag{6}$$

Using Eq. (3), we specify the boundary conditions for Eq. (6) in the form

$$N(x,t)|_{x=-\infty}=0,$$
$$N(x,t)|_{x=+\infty}=N_{\rm tot},$$
$$\rho(x,t)|_{x=\pm\infty}=0. \tag{7}$$

The total number of absorbing particles, $N_{\rm tot}=N(\infty,t)=\text{const}$, is an invariant of Eq. (6).

The boundary conditions (7) enable us to rewrite Eq. (6) in the form of the generalized Burgers equation

$$\frac{\partial N}{\partial t}+u(N)\,\frac{\partial N}{\partial x}-D\,\frac{\partial^2 N}{\partial x^2}=0. \tag{8}$$

3. NEGATIVE DRIFT VELOCITY

We begin the analysis of Eq. (8) with the case of a negative drift velocity: $u(N)<0$ [we defer consideration of the region $u(N)>0$ to Sec. 5]. For the time being, we shall consider only N, for which $0<N<N_2$, where N_2 is the point at which $u(N)$ changes sign (Fig. 1). For such a choice of the sign of the drift velocity $u(N)$, the drift of the particles is toward the light beam, i.e., from right to left in the negative direction of the x axis.

Let the original distribution $\rho(x,t)$ of the particles of the absorbing gas have the form of a bunch with bounded longitudinal dimension and total area

$$N_{\rm tot}=\int_{-\infty}^{+\infty}\rho(x',0)dx'.$$

If $N_{\rm tot}\leqslant N_1$, where N_1 is the point of minimum of the function $u(N)$ [see Figs. 1 and 2, in which the region $u(N)<0$ is shown separately], then the absolute magnitude of the drift velocity $|u(N)|$ increases monotonically over the thickness of the bunch, and therefore the trailing edge of the bunch drifts with higher velocity than the leading edge (the bunch as a whole drifts from right to left). In the absence of diffusion spreading, this drift-induced enhancement of the density resulting from the difference between the drift velocities of the leading and trailing edges of the bunch would lead to unbounded compression of the original distribution of the particles of the absorbing gas. However, the competition between the diffusion spreading and the drift compression leads after a certain time to a stabilization of the spatial distribution of the particles and to the formation of a dissipative soliton.[6] The dissipative soliton will be described by a self-similar solution of Eq. (4) of the form $\rho=\rho(x-vt)$, where v is the velocity of the dissipative soliton as a whole. It is to be expected that under our condition $N_{\rm tot}\leqslant N_1$ all the particles from the original bunch will be captured in the dissipative soliton. For $N_1<N_{\rm tot}<N_2$, the absolute magnitude $|u(N)|$ of the drift velocity will increase with increasing N only as long as $N<N_1$; it reaches its maximum at $N=N_1$ and then begins to decrease over the thickness of the bunch for $N>N_1$. We shall see in Sec. 4 that for such a nonmonotonic dependence $u(N)$ a dissipative soliton with area less than $N_{\rm tot}$ can be formed.

We shall seek a self-similar solution of Eq. (8) with the boundary conditions (7) that is formed at sufficiently large times in the form

$$N=N(z),\quad \rho=\rho(z),\quad z=x-vt. \tag{9}$$

Substituting (9) in (8), we obtain the equation

$$\frac{d}{dz}\left[(v(N)-v)N-D\,\frac{dN}{dz}\right]=0, \tag{10}$$

where we have introduced the function

$$v(N)=\frac{1}{N}\int_0^N u(N')dN'. \tag{11}$$

It is easy to show that (10) is equivalent to the equation

$$D\frac{dN}{dz}=N[v(N)-v], \tag{12}$$

or

$$\int_{N(0)}^{N}\frac{dN'}{N'[v(N')-v]}=\frac{z}{D}, \tag{13}$$

where $N(0)$ is the value of $N(z)$ at $z=0$ [in what follows, we take the origin $z=0$ at the point of maximum of $\rho(z)$; at the same time, $N(0)=N_{top}$ is found in accordance with the expression (20), which will be obtained below]. The velocity

$$v=v(N_0)=\frac{1}{N_0}\int_0^{N_0}u(N')dN' \tag{14}$$

of the dissipative soliton is found directly from Eq. (12) and from the boundary condition for the self-similar solution $N(z)\equiv N(x-vt)$:

$$N(z)|_{z=+\infty}=N_0.$$

We must draw particular attention to the fundamental difference between this equation and the boundary condition for $N(x,t)$ [$N(\infty,t)=N_{tot}$] [Eq. (7)]. It will be shown below that, in general, $N_0\leqslant N_{tot}$; i.e., only the fraction

$$N_0=\int_{-\infty}^{\infty}\rho(z)dz$$

of particles out of the total number N_{tot} (7) of absorbing particles is captured in the soliton. The difference between the boundary condition for the self-similar solution $N(z)$ from the boundary condition for the total solution $N(x,t)$ (7) should not give rise to surprise, since

$$N(x,t)=N(z)+\delta N(x,t).$$

The difference $\delta N(x,t)$ between $N(x,t)$ and $N(z)$ describes the "substrate" of particles not captured in the soliton. By virtue of the diffusion spreading, the local concentration

$$\rho(x,t)=\frac{\partial N(x,t)}{\partial x}$$

of the uncaptured particles tends to zero as $t\to\infty$ [but not $\delta N(x,t)$].

Thus, for a concrete dependence $u(N)$ [under the condition $u(N)<0$] Eqs. (11)–(14) and (5) make it possible to find a self-similar solution of Eq. (8) for $N(z)=N(x-vt)$ satisfying the boundary conditions

$$N(-\infty)=0,\quad N(\infty)=N_0.$$

Since the obtained solution of the generalized Burgers equation (8) will have the form of a "smeared step" moving to the left with velocity v [Eq. (9)], the spatial distribution of the density of the absorbing gas found in accordance with (5),

$$\rho(z)=\frac{dN(z)}{dz},$$

will have the form of a solitary wave (dissipative soliton), whose velocity is found in accordance with the same formula (14).

In what follows, we shall consider only the already formed self-similar solution describing the dissipative soliton, and we shall not analyze the actual process of formation of the dissipative soliton from the initial data $\rho(x,0)$.

4. MAXIMUM AREA OF SELF-SIMILAR SOLUTION. AREA OF THE LEADING EDGE

Suppose that at the initial time $t=0$ a distribution $\rho(x,0)$ with total area N_{tot} (initial bunch of particles of the absorbing gas) is specified. What fraction of the particles in the original bunch will be captured in the soliton when $u(N)<0$ (Fig. 2).

To solve this problem, we rewrite Eq. (12) with allowance for (5) and (14) in the form

$$\rho(z)=\frac{N(z)}{D}[v(N)-v(N_0)]. \tag{15}$$

Since the density $\rho(z)$ of the absorbing particles cannot be negative, it follows from Eq. (15) that

$$v(N)\geqslant v(N_0). \tag{16}$$

To elucidate the meaning of this equation, we investigate the function $v(N)$ (11); we therefore rewrite Eq. (11) in the form

$$\frac{dv(N)}{dN}=\frac{u(N)-v(N)}{N}.$$

The "velocity" $v(N)$ has an extremum:

$$\left.\frac{dv(N)}{dN}\right|_{N_c}=0$$

at the point $N=N_c$ if N_c satisfies the equation

$$u(N_c)=v(N_c)\equiv\frac{1}{N_c}\int_0^{N_c}u(N')dN'. \tag{17}$$

Figure 2 is a geometrical illustration of the important equation (17). It can be seen that Eq. (17) has a root N_c lying to the right of the point of minimum of the function $u(N)$ ($N_c>N_1$). In accordance with this equation, N_c can be found from the condition of equality of the areas (1) and (2) in Fig. 2. This means that

$$\left.\frac{du(N)}{dN}\right|_{N_c}>0$$

and, accordingly,

$$\left.\frac{d^2v(N)}{dN^2}\right|_{N_c}=\frac{1}{N_c}\left.\frac{du(N)}{dN}\right|_{N_c}>0.$$

This means that N_c is a minimum of the function $v(N)$. Then if $N_0>N_c$ (we recall that N_0 is the area of the already formed dissipative soliton), then for $N_c<N<N_0$ we find that $v(N)<v(N_0)$, and the condition (16) is not satisfied. It follows that the inequality (16) is equivalent to the inequality

$$N_0\leqslant N_c. \tag{18}$$

The meaning of this important inequality is that the total area N_0 of the self-similar solution obtained in Sec. 3 has an upper bound N_c determined by Eq. (17) and cannot exceed it.

The inequality (18) means that not all the particles in the absorbing cell can be "captured" in the dissipative soliton. Indeed, if at the initial time there were injected into the cell $N_{tot}>N_c$ absorbing particles per unit area of the cross section of the cell, then only N_c of the particles ($N_0=N_c$) would be captured in the dissipative soliton. The remaining particles (to the right of the soliton), having a lower velocity, would remain outside it. In the opposite case, when $N_{tot}\leqslant N_c$, all the particles of the initial distribution $\rho(x,0)$ will be captured in the soliton, i.e., $N_0=N_{tot}$.

What we have said enables us to obtain the final expression for the number N_0 of particles trapped in the soliton:

$$N_0=\begin{cases} N_{tot}, & \text{if } N_{tot}<N_c \\ N_c, & \text{if } N_{tot}\geqslant N_c \end{cases}.$$

We find the area of the leading edge of the dissipative soliton [from $z=-\infty$ to the point of maximum of $\rho(z)$]. Since we have not in any way yet chosen the origin, we choose the origin of x in such a way that the "top" of the dissipative soliton [the maximum in the graph of $\rho(z)$] lies at the point $z=0$ [then in the expression (13) $N(0)=N_{top}$]. Substituting ρ in the form (9) in Eqs. (8) and (4), we obtain

$$D\frac{d\rho}{dz}=[u(N)-v]\rho,$$

$$D\frac{d^2\rho}{dz^2}=\frac{du(N)}{dN}\rho^2+\frac{d\rho}{dz}[u(N)-v]. \quad (19)$$

It is readily noted that the first equation is the condition of absence of a flux:

$$j=[u(N)-v]\rho-D\frac{d\rho}{dz}$$

in the rest frame of the dissipative soliton. Since at the top of the soliton ($z=0$, $N=N_{top}$) $d\rho/dz|_{z=0}=0$ and $d^2\rho/dz^2|_{z=0}<0$, we have in accordance with (19)

$$u(N_{top})=v\equiv\frac{1}{N_0}\int_0^{N_0}u(N')dN', \quad (20)$$

where

$$N_{top}=\int_{-\infty}^{0}\rho(z)dz$$

is the area of the complete leading edge of the dissipative soliton, and $N_{top}<N_1$. For given total area N_0 of the dissipative soliton, Eq. (20) enables us to find the area of its leading edge: $N_{top}=N_{top}(N_0)$.

5. SELF-SIMILAR SOLUTION AND ITS AREA IN THE CASE OF A POSITIVE DRIFT VELOCITY

We now return to Fig. 1, which illustrates the variable-sign dependence $u(N)$. In Secs. 3 and 4, we have considered the case $u(N)\leqslant 0(0\leqslant N\leqslant N_2)$. In this section, we concentrate our attention on the region $u(N)\geqslant 0$. For simplicity and greater clarity of the following discussions, we assume that $u(N)\geqslant 0$ not when $N\geqslant N_2$ [N_2 is the point at which the function $u(N)$ changes sign; see Fig. 1] but when $N\geqslant 0$, and that this dependence has the form shown in Fig. 3. We hope that the identical notation for the maximum of the function $u(N)$ in Figs. 1 and 3 will not lead to confusion [in both figures, N_3 is the maximum of the function $u(N)$]. In the case of a positive drift velocity, the particles of the absorbing gas drift to the right in the positive direction of the x axis, which coincides with the direction of propagation of the radiation.

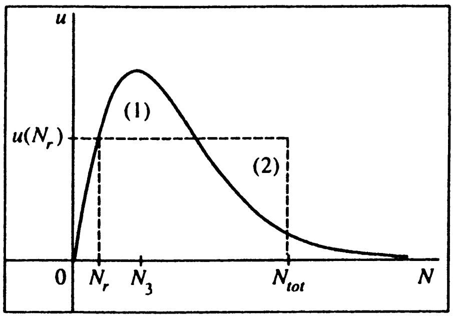

FIG. 3. Geometrical illustration of Eq. (30). The case $u(N)\geqslant 0$.

If the area N_{tot} of the original bunch of particles described by the distribution $\rho(x,0)$ is less than N_3, the drift velocity $u(N)$ increases monotonically over the thickness of the bunch. Since at the same time the trailing edge of the bunch drifts more slowly than its leading edge, the drift leads only to an acceleration of the diffusion spreading of the original bunch of particles in the region $N<N_3$. Thus, for $N_{tot}\leqslant N_3$ there is no "drift compression" and a dissipative soliton is not formed.

A mechanism of drift compression of the bunch of particles arises in the case $N_{tot}\geqslant N_3$, when the drift velocity $u(N)$ decreases across the thickness of the bunch in the part of it in which the optical thickness $N(3)$ satisfies the inequalities $N_3<N\leqslant N_{tot}$. One can say that the point of maximum of the function $u(N)$ (the point N_3) is a kind of threshold for the area of the original distribution of the particles, below which formation of a dissipative soliton does not occur when $u(N)\geqslant 0$.

Using the scheme of arguments from Secs. 3 and 4, we show that the area of the dissipative soliton formed at sufficiently large times is less than N_{tot} by a certain amount N_r. From the original bunch of particles of area N_{tot}, a dissipative soliton of smaller area $N_s=N_{tot}-N_r$ is formed, and the remaining particles lag behind the dissipative soliton, since they move with lower velocity. The area under the "separated" distribution of the particles will be N_r. Indeed, we shall see that the velocity $\bar{v}$ of the dissipative soliton is greater than the drift velocity $u(N)$ of the particles that are not captured in the soliton.

The case $u(N)\geqslant 0$ differs qualitatively from the one considered previously in Secs. 3 and 4. Indeed, in the case $u(N)\geqslant 0$ the particles moving to the left of the dissipative soliton lag behind it. In contrast to drift with $u(N)<0$, a dissipative soliton drifting to the right [$u(N)\geqslant 0$] is illuminated by radiation weakened by the photoabsorption by the particles that are not trapped in it.

We give the simplest method for solving the problem we have posed. We introduce the new function

$$\bar{N}(z)=\int_{z}^{+\infty}\rho(z')dz', \quad (21)$$

where $\rho(z)=\rho(x-vt)$ is the required self-similar solution, and $\bar{N}$ satisfies the boundary conditions

$$\bar{N}(z)|_{z=-\infty}=N_s, \quad \bar{N}(z)|_{z=+\infty}=0, \quad \rho(z)|_{z=\pm\infty}=0,$$

where N_s is the total area of the already formed dissipative soliton. Equation (8) will have the form

$$\frac{\partial\bar{N}}{\partial t}+u(N_{\rm tot}-\bar{N})\frac{\partial\bar{N}}{\partial x}-D\frac{\partial^2\bar{N}}{\partial x^2}=0. \quad (22)$$

We seek the self-similar solution of Eq. (22),

$$\bar{N}=\bar{N}(z), \quad z=x-vt,$$

which is formed at sufficiently large times.

Accordingly, we introduce the function

$$V(\bar{N})=\frac{1}{\bar{N}}\int_0^{\bar{N}}u(N_{\rm tot}-N')dN'. \quad (23)$$

As we did in Sec. 4, we can obtain from Eq. (22) an analog of (12),

$$D\frac{d\bar{N}}{dz}=\bar{N}[V(\bar{N})-\bar{v}], \quad (24)$$

where the velocity of the dissipative soliton is

$$\bar{v}=V(N_s)=\frac{1}{N_s}\int_0^{N_s}u(N_{\rm tot}-N')dN', \quad (25)$$

or

$$\int_{\bar{N}(0)}^{\bar{N}}\frac{dN'}{N'[V(N')-\bar{v}]}=\frac{z}{D}. \quad (26)$$

Since $\rho(z)=-d\bar{N}/dz$ and using expression (25), we rewrite Eq. (24) in the form

$$\rho(z)=-\frac{\bar{N}(z)}{D}[V(\bar{N})-V(N_s)].$$

The density $\rho(z)$ of the absorbing gas cannot be negative, and therefore

$$V(\bar{N})<V(N_s). \quad (27)$$

As we did in Sec. 4, we find the point of extremum $\bar{N}=\bar{N}_c$ of the function $V(\bar{N})$ (23) by expressing Eq. (23) in a different form:

$$\frac{dV(\bar{N})}{d\bar{N}}=\frac{u(N_{\rm tot}-\bar{N})-V(\bar{N})}{\bar{N}}. \quad (28)$$

At the point of extremum we have

$$\left.\frac{dV(\bar{N})}{d\bar{N}}\right|_{\bar{N}=\bar{N}_c}=0.$$

Therefore, $\bar{N}_c$ satisfies the equation

$$u(N_{\rm tot}-\bar{N}_c)=V(\bar{N}_c), \quad (29)$$

or

$$u(N_r)(N_{\rm tot}-N_r)=\int_{N_r}^{N_{\rm tot}}u(N')dN', \quad (30)$$

where we have introduced $N_r=N_{\rm tot}-\bar{N}_c$. The geometrical illustration of Eq. (30) is given in Fig. 3. The value of N_r for given $N_{\rm tot}$ is determined from the condition of equality of the areas of the regions (1) and (2) in Fig. 3. It is easy to see that $N_r<N_3$ and $N_{\rm tot}>N_3$ in Eq. (30), where N_3 is the point of maximum of the function $u(N)$. Since $N_r<N_3$, it follows that

$$\left.\frac{d^2V(\bar{N})}{d\bar{N}^2}\right|_{\bar{N}=\bar{N}_c}=-\frac{1}{\bar{N}_c}\left.\frac{du(N)}{dN}\right|_{N=N_r}<0,$$

and therefore $\bar{N}_c$ is the maximum of the function $V(\bar{N})$ (23). This means that (27) is equivalent to the inequality

$$N_s\leqslant\bar{N}_c,$$

and, thus, $\bar{N}_c$ is formally a lower bound of the area of the dissipative soliton for given $N_{\rm tot}$, and $N_r=N_{\rm tot}-\bar{N}_c$ is the minimum value of the area of the distribution of the particles from the original bunch that do not enter the dissipative soliton. Thus, if at the initial time a bunch of particles $\rho(x,0)$ with area $N_{\rm tot}$ is "injected" into the absorbing cell, then out of this bunch a dissipative soliton with area $N_s=\bar{N}_c$, is formed and the fraction N_r (30) of particles lag behind it.

Thus, Eq. (30) establishes the dependence of the number N_r of the particles that remain outside the dissipative soliton per unit area of the cross section of the absorbing cell on the total "number" of particles. The area of the dissipative soliton is

$$N_s=\bar{N}_c=N_{\rm tot}-N_r(N_{\rm tot}), \quad (31)$$

and the velocity of the dissipative soliton is

$$\bar{v}=u(N_r). \quad (32)$$

As is readily seen from (31) and (30), in the case of unbounded increase of $N_{\rm tot}$ the value of $N_r(N_{\rm tot})$ decreases to zero, and N_s approaches $N_{\rm tot}$. Accordingly, the velocity (32) of the dissipative soliton also decreases with increasing $N_{\rm tot}$. It can be seen that there is a significant difference between this dissipative soliton when $u(N)>0$ from the soliton when $u(N)<0$. For the latter, the area is bounded by N_c [Eq. (17)] and is not changed when $N_{\rm tot}>N_c$.

It follows directly from Fig. 3 and Eqs. (31) and (30) that $u(N)$ for $N_2<N<N_r$ (the region of particles not captured in the dissipative soliton) is less than the soliton velocity $\bar{v}=u(N_r)$; i.e., the soliton moves more rapidly.

6. ASYMPTOTIC EXPRESSIONS FOR THE LEADING EDGE AND "TAIL" OF THE DISSIPATIVE SOLITON

In this section, we find the dependence $\rho(z)$ in the asymptotic regions of the dissipative soliton—in the region of the leading edge ($|z|\gg 1$, $z<0$) and "tail" ($z\gg 1$). We use the expressions of Sec. 4, i.e., we shall consider only the case of negative drift velocity: $u(N)<0$.

We rewrite Eq. (19) in the form

$$u(N)-v=D\frac{d\ln\rho}{dz}. \tag{33}$$

In the region of the "tail" ($z\gg 1$), the area

$$N(z)=\int_{-\infty}^{z}\rho(z')dz'$$

differs from the total area

$$N_0=\int_{-\infty}^{+\infty}\rho(z')dz'$$

of the dissipative soliton by the small amount

$$\delta N=\int_{z}^{+\infty}\rho(z')dz'.$$

This amount decreases with increasing z. This enables us to expand the function $u(N)$ in a series

$$u(N)\equiv u(N_0-\delta N)\simeq u(N_0)-\left.\frac{du(N)}{dN}\right|_{N0}\delta N.$$

We can then rewrite (33) in the form

$$\left[u(N_0)-v-\left.\frac{du(N)}{dN}\right|_{N_0}\delta N\right]=D\frac{d\ln\rho}{dz}. \tag{34}$$

If $v-u(N_0)\neq 0$, then in Eq. (34) we can ignore the term with δN, and then its solution is obviously (within a constant factor)

$$\rho(z)\sim\exp\left[-\left(\frac{v-u(N_0)}{D}\right)z\right], \tag{35}$$

where $v-u(N_0)>0$, $z\gg 1$.

It follows from the results of Sec. 4 that $v-u(N_0)>0$ for $N_0<N_c$ and $v-u(N_0)=0$ for $N_0=N_c$, i.e., for maximum area of the dissipative soliton.

Therefore, for $N_0=N_c$ Eq. (34) will have the form

$$-\left.\frac{du(N)}{dN}\right|_{N_c}\delta N=D\frac{d\ln\rho}{dz}.$$

Differentiating this equation with respect to z, we obtain

$$\gamma\rho(z)=\frac{d^2\ln\rho}{dz^2}, \tag{36}$$

where

$$\gamma=\left.\frac{1}{D}\frac{du(N)}{dN}\right|_{N_c}.$$

It can be shown[10] that Eq. (36) for $v-u(N_0)=0$ ($N_0=N_c$) has the general solution

$$\rho(z)=\frac{2}{\gamma(z-C)^2},$$

where C is an arbitrary constant. For $z\gg 1$, we obtain the required asymptotic expression for the "tail":

$$\rho(z)\simeq\frac{2}{\gamma z^2}. \tag{37}$$

Thus, if the area of the soliton is less than the maximum ($N_0<N_c$), the dependence $\rho(z)$ in the "tail" ($z\gg 1$) is given by (35). If, however, $N_0=N_c$, then the asymptotic behavior for $z\gg 1$ is determined in accordance with expression (37).

We now consider the behavior of $\rho(z)$ for $z<0$, $|z|\gg 1$ at the leading edge of the soliton. In this region, $N\simeq 0$, and therefore we have the expansion

$$u(N)\simeq u(0)+\left.\frac{du(N)}{dN}\right|_{N=0}N+...;$$

since $u(0)=0$ and N in this case is a small quantity, we obtain approximately

$$-v\simeq D\frac{d\ln\rho}{dz}.$$

We thus obtain, within a constant factor,[11]

$$\rho(z)\sim\exp\left(-\frac{vz}{D}\right), \tag{38}$$

where $v<0$ and $z<0$, $|z|\gg 1$.

Comparing (38) with (35) or (37), we see that, in general, there is asymmetry of the dissipative soliton (a difference between the steepness of the leading and trailing edges). However, if $v=u(N_0)/2$ and $N_0\neq N_c$, then expressions (38) and (35) give a symmetric dependence ρ (z) both for $z\gg 1$ and for $z<0$, $|z|\gg 1$.

The above arguments, which lead to expressions (37) and (38), are also valid in the case of negative drift velocity [$u(N)<0$]; it is merely necessary to take into account the fact that in this case the area of the dissipative soliton is always maximal (see Sec. 5) and the dissipative soliton moves to the right.

7. FORMATION OF TWO DISSIPATIVE SOLITONS

In the previous sections of this paper, we have investigated the self-similar solution describing the dissipative soliton separately for $u(N)\leq 0$ and for $u(N)\geq 0$. Combining the results obtained in these sections, we investigate qualitatively the dependence of the self-similar solution on the total area N_{tot} of the original distribution of the gas particles in the general case of a variable-sign dependence of the drift velocity $u(N)$ (Fig. 1).

1) $N_{tot}\leq N_2$

In Secs. 3 and 4, we have considered this case in detail. As we already know, a dissipative soliton that moves to the left with velocity v [Eq. (14)] is formed after a certain time from the original distribution of the particles. If $N_{tot}\geq N_c$ (see Fig. 4), then the area of the formed dissipative soliton is less than N_{tot} and equal to N_c. If, however, $N_{tot}<N_c$, then the area of the dissipative soliton is equal to the area N_{tot} of the original distribution of the particles of the gas.

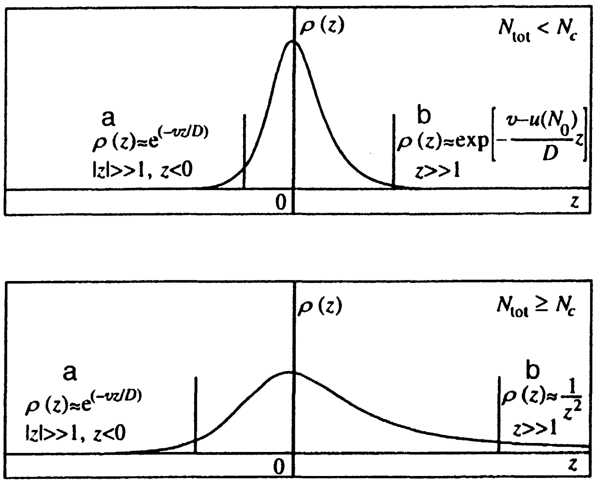

FIG. 4. Asymptotic regions of the dissipative soliton [$u(N)\geqslant 0$]: The leading edge (a) and "tail" (b) respectively, for the cases $N_0<N_c$ and $N_0=N_c$ (or, equivalently, $N_{tot}< N_c$ and $N_{tot}\geqslant N_c$).

2) $N_2<N_{tot}<N_3$

As in the previous case, only one dissipative soliton with maximum area N_c, which moves to the left with velocity v [Eq. (14)] is formed. The only difference is that in the region $N_2<N<N_{tot}$ the drift velocity $u(N)$ is positive, and this portion of the original bunch of the particles will drift to the right. However, as is shown in Sec. 5, this does not lead to the establishment of a self-similar "formation" that moves to the right.

3) $N_3<N_{tot}$

This case is the most interesting one, since two dissipative solitons moving in opposite directions are formed from the original bunch of gas particles. Indeed, the drift velocity $u(N)$ changes sign over the thickness of the original bunch of gas particles. The region $0\leqslant N\leqslant N_2$ [$u(N)\leqslant 0$] was studied in Secs. 3 and 4, and the region $N>N_2$ was considered with a slight simplification in Sec. 5. In our case, a dissipative soliton moving to the right with velocity $\bar{v}$ (25) is formed, and its area N_s is equal to $N_{tot}-N_r$ [Eq. (30)].

In the region of negative drift velocity, a dissipative soliton drifting to the left is formed (see the cases considered above).

Thus, under the influence of the radiation, when

$$N_3<N_{tot}, \tag{39}$$

the original bunch of gas particles is "torn apart" into two parts, from which, after a certain time, two dissipative solitons are formed that move in opposite directions, each with its own velocity.

8. RESULTS OF NUMERICAL CALCULATIONS

To illustrate the process of formation of two dissipative solitons, we solved Eq. (8) numerically, taking into account expression (5), for an absorbing cell with bounded length. We introduce the dimensionless variables

$$\zeta=\frac{x}{x_0}, \quad \tau=\frac{t}{t_0} \tag{40}$$

and dimensionless "density" of particles of absorbing gas and dimensionless optical thickness:

$$P(\zeta,\tau)=\frac{\rho(x,t)}{\rho(L/2,0)}, \quad n(\zeta,\tau)=\frac{N(x,t)}{N_{tot}}, \tag{41}$$

where $x_0=D/u_0$, and $t_0=D/u_0^2$.

We redefine the dependence of the drift velocity on the optical thickness as

$$u(N)=u_0\eta(n), \eta(n)=\alpha n^2(n-n_2)\exp(-\beta n), \tag{42}$$

where u_0 is a factor with the dimensions of a velocity which arises upon transition to (41). We have chosen a model dependence of the dimensionless drift velocity in the form (42), since it gives a qualitatively correct approximation of the variable-sign dependence of the drift velocity $u(N)$ (Ref. 4). In addition, it correctly reflects the quadratic dependence of the drift velocity on the optical thickness in the limiting cases of Doppler or homogeneous broadening even for small values of the optical thickness.

With allowance for (40)–(42), Eq. (8) will have the form

$$\frac{\partial n}{\partial \tau}+\eta(n)\frac{\partial n}{\partial \zeta}=\frac{\partial^2 n}{\partial \zeta^2}. \tag{43}$$

The dimensionless particle density $P(\zeta,\tau)$ of the absorbing gas is found in accordance with the formula

$$P(\zeta,\tau)=c\,\frac{\partial n(\zeta,\tau)}{\partial \zeta}, \tag{44}$$

which is obtained from (5) by going over to the variables (40) and (41). Here $c=N_{tot}/(x_0\rho(L/2,0))$. We augmented Eq. (43) with boundary conditions at the ends of the cell with dimensionless length $l=L/x_0$:

$$n(0,\tau)=0 \qquad n(l,\tau)=1 \tag{45}$$

In choosing the boundary conditions (45), we assume that the dimensionless length l of the cell is sufficiently great for dissipative solitons to form from an initial distribution

$$P(\zeta,0)=\exp\left[-\left(\frac{\zeta-l/2}{\zeta_0}\right)^2\right] \tag{46}$$

with width ζ_0. We assume that $l/2\zeta_0\gg 1$, and the integration in the finding of n_{tot} can be performed between the limits $-\infty$ and $+\infty$ (the error that is introduced is certainly less than the accuracy of the numerical calculation).

The results of the numerical calculations of the dimensionless density $P(\zeta,r)$ of the absorbing gas using Eqs. (44) and (43) with allowance for (42) and (45) are given in Figs. 5 and 6 for the parameter values $\alpha=31250$, $n_2=0.32$, $\beta=6.25,\zeta=1/(10\sqrt{\pi})$.

Figure 5 gives the initial stage in the "breakup" of the original distribution (46) in a cell with $l=1$. Figure 6 shows the evolution of the same initial distribution of the dimensionless particle density of the absorbing gas (46) for the same values of the parameters but for step along the time axis $\Delta\tau=0.01$ and $l=10$. It can be directly seen how two

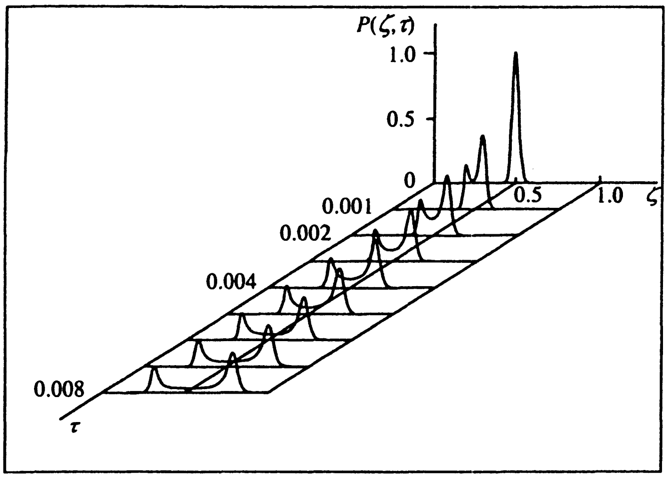

FIG. 5. The formation of two dissipative solitons in the initial stage. The time step is $\Delta\tau=0.001$ and the dimensionless cell length is $l=1$. The notation is given in the text.

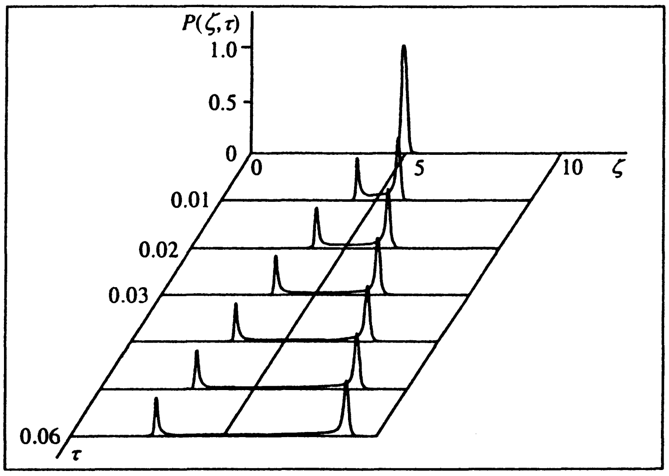

FIG. 6. The formation of two dissipative solitons. The values of the parameters are the same as in Fig. 5 but the time step is $\Delta\tau=0.01$ and the dimensionless cell length is $l=10$.

stable dissipative solitons moving in opposite directions are formed. As long as the boundary conditions (45) do not influence the dissipative solitons by virtue of their bounded spatial size, our numerical solution in the bounded cell models the propagation of dissipative solitons in an unbounded medium.

In conclusion, we briefly discuss the possibility of experimental observation of the breakup of a bunch of particles into two dissipative solitons. The first experimental investigations[12] on the formation of dissipative solitons by light-induced drift have now been made. The absorbing gas in the experiment of Ref. 12 was optically dense sodium atomic vapor. The buffer gas was Xe at pressure 8 torr. Atutov *et al.*,[12] confirmed both the formation of dissipative solitons[5,6,9] and their coalescence (inelastic collision).[6,9] In this paper, it has been shown that the opposite effect—breakup of an initial bunch of particles into two dissipative solitons—is possible.

A necessary condition for the existence of this effect is a variable-sign dependence of the LID velocity $u(N)$ on the optical thickness N (Fig. 1). Such a variable-sign dependence $u(N)$, theoretically predicted in Ref. 4, occurs in the case of illumination of an optically dense gas ($\sigma N>10$) with white light. Here σ is the absorption cross section. As was shown in Ref. 4, a variable-sign dependence $u(N)$ is characteristic of vapors of the alkali metals under conditions of optical pumping.

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[1] F. K. Gel'mukhanov and A. M. Shalagin, Pis'ma Zh. Éksp. Teor. Fiz. **29**, 773 (1979) [JETP Lett. **29**, 711 (1979)].
[2] F. K. Gel'mukhanov and A. M. Shalagin, Zh. Éksp. Teor. Fiz. **78**, 1674 (1980) [Sov. Phys. JETP **51**, 839 (1980)].
[3] S. G. Rautian and A. M. Shalagin, *Kinetic Problems of Nonlinear Spectroscopy* (North-Holland, Amsterdam, 1991).
[4] S. N. Atutov, A. A. Parkhomenko, S. P. Pod'yachev, and A. M. Shalagin, Zh. Éksp. Teor. Fiz. **99**, 378 (1991) [Sov. Phys. JETP **72**, 210 (1991)].
[5] G. Nienhuis, Phys. Rep. **138**, 151 (1986).
[6] F. Kh. Gel'mukhanov and A. A. Parkhomenko, Zh. Éksp. Teor. Fiz. **92**, 3 (1987) [Sov. Phys. JETP **65**, 1 (1987)].
[7] F. Kh. Gel'mukhanov, G. Nienhuis, and T. I. Privalov, Phys. Lett. A **192**, 22 (1994).
[8] F. Kh. Gel'mukhanov, G. Nienhuis, and T. I. Privalov, Phys. Rev. A **50**, 2445 (1994).
[9] F. Kh. Gel'mukhanov, J. E. M. Haverkort, S. W. M. Borst, and J. P. Woerdman, Phys. Rev. A **36**, 164 (1987).
[10] V. F. Zaĭtsev and A. D. Polyanin, *Handbook of Nonlinear Differential Equations: Applications in Mechanics*, Exact Solutions [in Russian] (Fizmatlit, Moscow, 1993), p. 95.
[11] H. G. C. Werij and J. P. Woerdman, Phys. Rep. **169**, 145 (1988).
[12] S. N. Atutov, S. V. Plotnikov, S. P. Pod'yachev, and A. M. Shalagin, Phys. Lett. A **193**, 179 (1994).

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