

Surface magnetic polaritons in antiferromagnets of the easy-plane type

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We study theoretically the behavior of surface magnetic polaritons in antiferromagnets of the easy-plane type in a geometry where the antiferromagnetism vector \mathbf{L} is perpendicular to the sample's surface and to the wave vector \mathbf{k} when magnetic coupling and the magnetic field strength are taken into account. We show that allowing for magnetoelastic coupling leads to polariton generation in a zero magnetic field. We also study the dynamics of the range of existence of polaritons in a magnetic field. As the magnetic field strength increases, at a certain critical value of the field a magnetic polariton is transformed from a virtual polariton into a surface magnetostatic wave with a depth of penetration that depends on the magnetic field strength and the magnetoelastic coupling. Finally, the spectrum of surface magnetic polaritons is found to greatly depend on the dielectric constant of the substance. © 1995 American Institute of Physics.

As is known, the fact that solids have boundaries leads to the emergence of special macroscopic surface electromagnetic waves propagating along the crystal's boundary at optical phonon frequencies, i.e., the propagation of surface phonon polaritons.^{1–3} The properties of such surface phonon polaritons have been studied for a fairly long time, and many theoretical and experimental papers are devoted to this topic (see, e.g., the collection of articles in Ref. 1).

In addition to surface phonon polaritons, magnetically ordered crystals may have surface waves propagating in them at the frequencies of the magnetic subsystem, magnon or magnetic polaritons^{3–5} (by analogy with polaritons they should have been named "magneton"). Up to now there has been no detailed study of magnetic surface polaritons, compared to studies of phonon polaritons, either experimentally or theoretically. A modest number of papers mentioned earlier (Refs. 3–5; see also references cited therein) were devoted to surface polaritons, in addition to the pioneering work of Damon and Eshbach.⁶ In recent years the number of publications devoted to the study of magnetic surface polaritons has increased considerably, both theoretical papers^{7–11} and experimental work.¹² The upsurge of interest in the behavior of polaritons in magnetic substances indicates how important this problem is.

All the above-cited publications on surface magnetic polaritons study the properties of these entities in ferro- and antiferromagnets of the easy-axis type. It appears that the frequency range within which surface magnetic polaritons can exist and the dispersion law that such polaritons obey are determined by the specific form of the permeability tensor. The simplest problem is that of the properties of surface magnetic polaritons with a diagonal permeability tensor.³ Both ordinary polaritons and what is known as virtual polaritons (according to the terminology of Ref. 1; these polaritons are absent in the magnetostatic limit) can be present near the surface of a magnetic substance. In the presence of a mag-

netic field the permeability tensor has finite off-diagonal elements, which leads to the emergence of a nonreciprocity effect in the propagation of surface magnetic polaritons.⁴ For some geometries of the problem (the mutual position of the magnetic field \mathbf{H} , magnetization \mathbf{M} , and the wave vector \mathbf{k}) there may be no magnetic polaritons.⁹ Their existence requires fairly stringent conditions concerning the permeability and the dielectric constant of the crystal.⁹

In this paper we examine the behavior of surface magnetic polaritons in antiferromagnets of the easy-plane type in a geometry where the antiferromagnetism vector \mathbf{L} is perpendicular to the sample's surface and to the wave vector \mathbf{k} , with allowance for the magnetic field and the magnetoelastic interaction. We show that magnetoelastic coupling leads to the emergence of polaritons in cases where there are no such polaritons if the coupling is not accounted for. The dynamics of the variation of the range within which magnetic polaritons exists is studied as a function of the magnetic field (in approaching the orientational phase transition point).

We start with the case of a semi-infinite antiferromagnetic crystal of the easy-plane type filling the space $y > 0$. The xz plane ($y = 0$) is the interface between the crystal and a vacuum. In relation to dielectric properties we assume the crystal to be uniaxial. In this case the dielectric constant tensor of the antiferromagnet can be written as $\varepsilon_{ik} = \varepsilon(\delta_{ix}\delta_{kx} + \delta_{iy}\delta_{ky}) + \varepsilon_2\delta_{iz}\delta_{kz}$ (see Ref. 2), where

$$\varepsilon_{1,2} = \varepsilon_\infty + \frac{(\varepsilon_0 - \varepsilon_\infty)\omega_{T,LO}^2}{\omega_{T,LO}^2 - \omega^2}, \quad (1)$$

with ε_∞ and ε_0 the high-frequency and static dielectric constants, $\omega_{T,LO}$ the frequencies of long-wave (transverse and longitudinal) optical phonons, and ω the frequency of a surface polariton. Equation (1) is written on the assumption that there is no damping or spatial dispersion. We assume that in the antiferromagnet's ground state the antiferromagnetism

vector lies in the easy plane (xy) and is directed along the y axis. A constant external magnetic field is directed along the x axis, is parallel to the ferromagnetism vector \mathbf{M} , and is perpendicular to the wave vector $\mathbf{k} \parallel \mathbf{z}$ (Voigt geometry for vector \mathbf{M}). To find the permeability tensor in such a geometry with allowance for magnetoelastic coupling, we must solve the equations of elasticity theory and Landau–Lifshitz equations. For an antiferromagnet that is isotropic in both elastic and magnetoelastic properties such a problem has been solved, e.g., in Ref. 13. Extending the results of that paper to the problem considered here and neglecting, for the sake of simplicity, decay and spatial dispersion, we arrive at the following expression for the permeability tensor of an antiferromagnet of an easy-plane type:

$$\mu_{ik} = \begin{pmatrix} \mu_{xx} & 0 & 0 \\ 0 & \mu_{yy} & i\mu_{yz} \\ 0 & -i\mu_{yz} & \mu_{zz} \end{pmatrix}, \quad (2)$$

where

$$\begin{aligned} \mu_{xx} &= 1 + \frac{(\mu_0 - 1)\omega_{1s}^2}{\omega_{1s}^2 - \omega^2}, & \mu_{yy} &= 1 + \frac{(\mu_0 - 1)\omega_H^2}{\omega_{2s}^2 - \omega^2}, \\ \mu_{zz} &= 1 + \frac{(\mu_0 - 1)\omega_{2s}^2}{\omega_{2s}^2 - \omega^2}, & \mu_{yz} &= \frac{(\mu_0 - 1)\omega\omega_H}{\omega_{2s}^2 - \omega^2}, \\ \omega_{1s}^2 &= \omega_E\omega_A, & \omega_{2s}^2 &= \omega_H^2 + \omega_E\omega_{me}, \\ \omega_A &= gL|\beta|, & \omega_E &= gL\chi_1^{-1}, & \omega_{me} &= gLb(u_{xx}^0 - u_{yy}^0), \end{aligned} \quad (3)$$

with $\mu_0 = 1 + 4\pi\chi_1$ the static permeability of the antiferromagnet, χ_1 the transverse magnetic susceptibility, g the gyromagnetic ratio, β and b the uniaxial anisotropy and magnetostriction constants, respectively, and u_{ij}^0 the components of the strain tensor in the equilibrium state considered here. Note that at $H=0$ the system undergoes an orientational phase transition from the state with $\mathbf{L} \parallel \mathbf{y}$ to a state with an indifferent orientation of vector \mathbf{L} in the easy plane. At the transition point the component μ_{yy} of the permeability tensor (2) is unity and the component μ_{yz} is zero. In the absence of magnetoelastic coupling ($b=0$), the component μ_{zz} is also unity. In this case the generation of surface magnetic polaritons in an antiferromagnet of the easy-plane type is possible only at the frequency of the high-frequency quasiantiferromagnetic oscillation branch, ω_{1s} and is caused by the component μ_{xx} of the permeability tensor.

When studying the surface polariton spectrum, we seek a solution of the Maxwell equations for plane monochromatic waves,

$$\begin{aligned} \text{curl } \mathbf{e} &= \frac{i\omega}{c} \hat{\mu} \mathbf{h}, & \text{curl } \mathbf{h} &= -i\hat{\epsilon} \frac{\omega}{c} \mathbf{e}, \\ \text{div } \hat{\epsilon} \mathbf{e} &= 0, & \text{div } \hat{\mu} \mathbf{h} &= 0, \end{aligned} \quad (4)$$

with \mathbf{e} and \mathbf{h} the variable electric and magnetic field strengths and c the speed of light in vacuum, in the form

$$\begin{aligned} \mathbf{e}^{(e)}, \mathbf{h}^{(e)} &\propto \exp\{ikz + \kappa_e y\}, & y < 0, \\ \mathbf{e}^{(i)}, \mathbf{h}^{(i)} &\propto \exp\{ikz - \kappa_i y\}, & y > 0. \end{aligned} \quad (5)$$

Here the indices i and e refer to the fields in the interior and exterior of the antiferromagnet. In vacuum ($y < 0$) we put $\mu_{ik} = \delta_{ik}$. For surface waves the quantities κ_e and κ_i must be either real and positive or complex-valued with a finite real part.

Substituting (5) into (4), we arrive at the following dispersion equations:

$$\kappa_e^2 = k^2 - \frac{\omega^2}{c^2}, \quad (6)$$

$$\left[\kappa_i^2 - \frac{\epsilon_2}{\epsilon_1} \left(k^2 - \frac{\omega^2 \epsilon_1 \mu_{xx}}{c^2} \right) \right] \left[\kappa_i^2 - \frac{\mu_{zz}}{\mu_{yy}} k^2 + \frac{\omega^2 \epsilon_1 \mu}{c^2} \right] = 0, \quad (7)$$

where $\mu = (\mu_{yy}\mu_{zz} - \mu_{yz}^2)/\mu_{yy}$. The fact that the expression inside the first pair of square brackets in (7) vanishes means that on both sides of the interface there are waves with nonvanishing e_y , e_z , and $h_x \neq 0$, i.e., TM waves. If the expression inside the second pair of square brackets vanishes, there is a wave with nonvanishing e_x , h_y , and $h_z \neq 0$, i.e., a TE wave propagating near the surface.

To determine the frequency range corresponding to surface polaritons we must ensure that certain boundary conditions are met at $y=0$. For the first type of wave these conditions consist in the normal component of the electric induction, d_y , and the tangential components e_z and h_x being continuous at the boundary (or interface). The boundary conditions lead to the following relationship between κ_i and κ_e :

$$\kappa_i = -\epsilon_2 \kappa_e. \quad (8)$$

From this it follows that TM polaritons are possible only when $\epsilon_2 < 0$. Such a situation occurs only in the range of frequencies $\omega \sim \omega_{T,LO}$. Thus, here TM polaritons are surface phonon polaritons. Their properties are fairly well studied,¹³ so that we do not consider them here.

Let us turn to TE polaritons. The boundary conditions for the normal component of magnetic induction, b_y , and the tangential components h_z and e_x yield the following relationship between the wave numbers κ_i and κ_e of this type of wave:

$$\kappa_i = \frac{k\mu_{yz}}{\mu_{yy}} - \kappa_e \mu. \quad (9)$$

Substituting this into (7) and combining the result with (6), we arrive at the final dispersion equation for TE polaritons:

$$\begin{aligned} c^2 k^2 &= \omega^2 \{ \mu \mu_{yy} [(\mu - \epsilon_1) (\mu_{yz}^2 + \mu^2 \mu_{yy}^2 - \mu_{zz} \mu_{yy}) - 2\mu \mu_{yz}^2] \\ &\quad \pm 2\mu_{yz}^2 \{ \mu^3 [\epsilon_2^2 \mu_{yy}^2 + \epsilon_1 (\mu_{yz}^2 - \mu^2 \mu_{yy}^2 - \mu_{zz} \mu_{yy}) \\ &\quad + \mu \mu_{zz} \mu_{yy}] \}^{1/2} [(\mu_{yz}^2 + \mu^2 \mu_{yy}^2 - \mu_{zz} \mu_{yy})^2 \\ &\quad - 4\mu^2 \mu_{yz}^2 \mu_{yy}^2]^{-1/2} \}. \end{aligned} \quad (10)$$

Generally, this equation has four solutions. Two of these are nonphysical, but the other two correspond to surface polaritons propagating in the positive and negative directions of the z axis, respectively. Equations (9) and (10) imply that the dispersion laws for these waves differ. Thus, for surface polaritons in antiferromagnets of the easy-plane type the non-

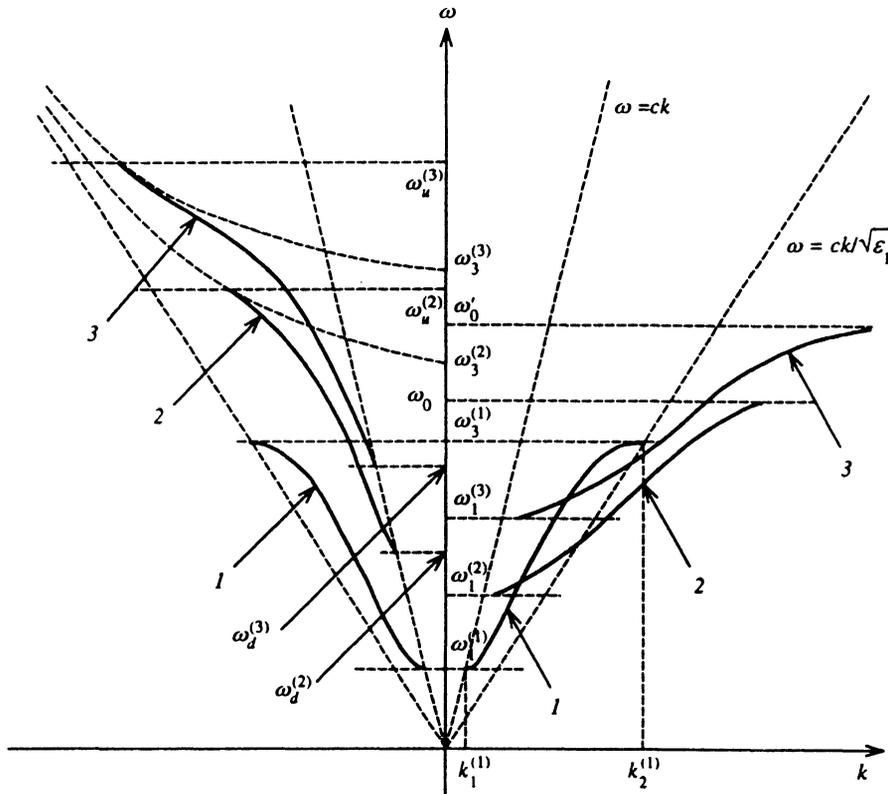


FIG. 2. The schematic of the spectrum of surface magnetic polaritons: curve 1, $H=0$; curve 2, $H=H_2$; and curve 3, $H=H_3$. Here $H_3 > H_2$ and $H_3 > H_0$. The frequencies $\omega_d^{(i)}$, $\omega_u^{(i)}$, ω_0 , ω'_0 , and $\omega_i^{(j)}$, and the magnetic field H_0 are defined in the text. In the range of negative values of k the dashed curves depict the dispersion laws for bulk waves, $\omega = ck(\epsilon_1 \mu_{eff})^{1/2}$, with $\mu_{eff} = \mu \mu_{yy} / \mu_{zz}$ at $H=H_2$ and $H=H_3$.

formula $\kappa_e^{-1} = c(\mu_0 - 1)^{1/2} / \mu_0^{1/2} \omega_H$. Combining this with (13), we find that in weak fields the value of κ_e^{-1} given by (14) greatly depends on the magnitude of the magnetoelastic coupling. In the vicinity of the orientational phase transition point ($H < H_0$), the absolute value of the wave vector at the second edge $\omega = \omega_0$ is

$$k_2 = \frac{\omega \sqrt{\epsilon_1 \mu_{eff}(\omega_0)}}{c}, \quad (15)$$

where $\mu_{eff} = \mu \mu_{yy} / \mu_{zz}$ (curve 2 in Fig. 2). At this edge the depth of penetration of a surface polariton into the magnetic substance becomes infinite ($\kappa_i^{-1} \rightarrow \infty$) but remains finite in the case of a vacuum [Eq. (6) combined with (15)]. If the magnetic field is higher than H_0 , at the second edge of the range within which a surface magnetic polariton can exist, $\omega = \omega'_0$, the absolute value of the wave vector tends to infinity, i.e., the wave transforms into a Damon–Eshbach surface magnetostatic wave⁶ (curve 3 in Fig. 2). However, in contrast to the Damon–Eshbach wave, the depth of polariton penetration into the magnetic substance, κ_i^{-1} , depends on the magnetic field strength and magnetoelastic coupling:

$$\kappa_i^{-1} = k^{-1} \frac{|(\mu_0 - 1)\omega_H^2 - \omega_{me}^2|}{(\mu_0 - 1)\omega_H^2 - \omega_{me}^2}. \quad (16)$$

If $\omega_{me} = 0$ and also if $\omega_{me} \neq 0$ and $\omega_H \gg \omega_{me}(\mu_0 - 1)^{-1/2}$, the depth of penetration coincides with that of the Damon–Eshbach wave. At $H = H'_0 = \omega_{me}(\mu_0 - 1)^{-1/2} / g$ the depth of

penetration of a magnetostatic wave is zero. This is possible only if $H'_0 > H_0$, which is usually the case [see Eq. (20) below].

For negative values of k , the curve representing the dispersion law of surface polaritons, $\omega = \omega(k)$, is monotonic (just as it is for $k > 0$) and lies in the frequency range from ω_d to ω_u . At these frequencies the values of κ_e and κ_i vanish, respectively:

$$\omega_d^2 = \omega_{me}^2 + \frac{(\epsilon_1 \mu_0 - 1)\omega_H^2}{\epsilon_1 - 1}, \quad k_d = \frac{\omega_d}{c},$$

$$\omega_u^2 = \frac{1}{2(\epsilon_1 - 1)} \{ 2\mu_0(\epsilon_1 - 1)\omega_1^2 + (2\mu_0 - 1)(\mu_0 - 1)\omega_H^2 + (\mu_0 - 1)\omega_H[4\mu_0(\epsilon_1 - 1)\omega_1^2 + (2\mu_0 - 1)^2\omega_H^2]^{1/2} \}, \quad (17)$$

$$k_u = \frac{\omega}{c} \sqrt{\epsilon_1 \mu_{eff}}.$$

An interesting problem here is the evolution of the spectrum of surface magnetic polaritons as a function of the dielectric constant ϵ_1 . From the expression (1) for ϵ_1 it follows that if $\omega_{T,LO} \gg \omega_{1s}, \omega_{2s}$, then $\epsilon_1(\omega) \geq 1$ at frequencies of oscillation of the magnetic subsystem.

We start with the case where $\epsilon_1 - 1 \ll 1$. In this case the frequencies ω_0 , ω_d , and ω_u are expressed as

$$\omega_0^2 \approx \frac{\mu_0^2 \omega_1^2}{2\mu_0 - 1}, \quad \omega_d^2 \approx \frac{(\varepsilon_1 \mu_0 - 1) \omega_H^2}{\varepsilon_1 - 1}, \quad (18)$$

$$\omega_u^2 = \frac{(2\mu_0 - 1)(\mu_0 - 1) \omega_H^2}{\varepsilon_1 - 1}.$$

Comparison of ω_0 and ω'_0 of Eqs. (11) shows that ω_0 is always smaller than ω'_0 when $k > 0$, i.e., for $\varepsilon_1 - 1 \ll 1$ there can be no magnetostatic Damon–Eshbach wave, no matter what the value of the magnetic field. From (18) it also follows that for $k < 0$ the region where a surface polariton can exist narrows considerably and is essentially independent of magnetoelastic coupling.

For $\varepsilon_1 \gg 1$ the frequencies ω_0 , ω_d , and ω_u are expressed as

$$\omega_0^2 = \omega_u^2 \approx \mu_0 \omega_1^2, \quad \omega_d^2 = \omega_1^2 \quad (19)$$

and are independent of the dielectric constant, and so are the regions where polaritons can exist when $k > 0$ or $k < 0$. For such values of ε_1 Eq. (12) makes it possible to obtain an explicit analytical expression for the value of H_0 at which there is a transition from a virtual polariton to a Damon–Eshbach wave:

$$H_0 = \frac{\omega_{me}}{g} \sqrt{\frac{2\mu_0^{1/2} - (\mu_0 - 1)^{1/2}}{(\mu_0 - 1)^{1/2}(3\mu_0 + 1)}}. \quad (20)$$

Thus, our study of surface magnetic polaritons in an antiferromagnet of the easy-plane type makes it possible to arrive at the following conclusions.

Allowing for magnetoelastic coupling leads to a situation in which at the orientational phase transition point ($H=0$) there exists a surface magnetic polariton that is a virtual polariton. As we move away from that point (as H increases), the polariton remains virtual when the field is weak. But when the field gets stronger, the polariton is transformed (for $k > 0$) into a magnetostatic Damon–Eshbach wave with a depth of penetration into the substance dependent on magnetoelastic coupling and the magnitude of the magnetic field. If $k < 0$, the polariton remains virtual for all values of H .

The spectrum of a surface magnetic polariton is highly dependent on the dielectric constant ε_1 of the antiferromagnet. When $\varepsilon_1 - 1 \ll 1$, the polariton remains virtual for all values of H , and for $k < 0$ the size of the region where it can exist narrows considerably and for H finite is essentially in-

dependent of magnetoelastic coupling. For $\varepsilon_1 \gg 1$ the size of the region where a polariton can exist is independent of the dielectric constant.

Note that usually the static permeability μ_0 in antiferromagnets is close to unity, with the result that the region where a surface magnetic polariton can exist is fairly narrow in antiferromagnets. In view of this, the polariton examined in our study should be sought for experimentally either in antiferromagnets with a high value of the transverse susceptibility χ_\perp or in ferromagnets of the easy-plane type, where the range of existence of surface magnetic waves is much broader.^{4,8} The latter case, however, requires performing analytic calculations similar to those done in this paper.

In conclusion the following remark is in order. The vanishing of the depth of penetration of a surface wave into a magnetic substance invalidates the macroscopic approach adopted in this paper. The usual condition for waves to be macroscopic is $ak \ll 1$, where a is the lattice constant. Kaganov and Shalaeva⁸ have shown that allowing for the spatial dispersion of the permeability tensor (2) leads to a more stringent condition than that needed for the waves to be macroscopic. Thus, all our results are valid if $ak \ll 1$.

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