Viscous motion of two-dimensional vortices in layered superconducting structures

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The motion of two-dimensional vortices in layered superconducting structures near T_c under the influence of a transport current is considered within the time-dependent Ginzburg–Landau theory. It is shown that when the finite conductivity of the intervening nonsuperconducting layers and the absence of Josephson coupling between the superconducting layers are taken into account, the equation of the dynamics of an individual two-dimensional vortex is significantly nonlinear: the viscosity coefficient diverges logarithmically as the rate of motion of the vortex decreases. © *1995 American Institute of Physics.*

The behavior of the magnetic and transport properties of layered superconducting structures with weak Josephson coupling between the layers has been widely discussed in the literature in recent years. Numerous experimental data confirm that such structures include, for example, Bi–Sr–Ca–Cu–O and Tl–Ba–Ca–Cu–O high-T_c superconductors, as well as superconducting superlattices of the $YBa_2Cu_3O_{7-\delta}/(Pr_xY_{1-x})Ba_2Cu_3O_{7-\delta}$, $DyBa_2Cu_3O_7/(Pr_xY_{1-x})Ba_2Cu_3O_7$, Nb/Cu, Nb/CuMn, Nb/CuGe, and Nb/Al–AlO_x types.[1–6] In these compounds the effective coherence length ξ_c along the **c** axis, which is perpendicular to the layers, is smaller than the distance D between layers over a broad temperature range. This results in the appearance of effects which are not described within the simple phenomenological Ginzburg–Landau theory with an anisotropic effective-mass tensor. This applies, in particular, to the structure of vortices in the mixed state of these superconductors. A vortex line tilted at an angle γ to the **c** axis is a stack of interacting two-dimensional (2D) vortices, each of which has a singularity of the order parameter in only one superconducting layer. Such a model has been widely used, in particular, to describe the oscillations of the vortex lines in the mixed state of high-T_c superconductors.[7–9] We note that 2D vortices can also appear in the internal layers of a superconductor as a result of the thermally activated creation of vortex–antivortex pairs.[10,11] The structure and interaction of 2D vortices have been considered in numerous investigations.[12–18]

The investigation of the features of the viscous motion of vortex structures in highly anisotropic superconductors is also of unquestionable interest. The influence of anisotropy on the dynamics of vortex lines was previously studied in Refs. 19–21 within the time-dependent Ginzburg–Landau theory with an anisotropic effective-mass tensor. Consideration of the layered structure results in the appearance of internal pinning of the vortex lines parallel to the ab planes. The influence of such a pinning mechanism on the viscous flow of the magnetic flux for magnetic fields close to H_{c2} was treated theoretically in, for example, Ref. 22 (see also the references cited therein). A theoretical analysis of the viscous motion of vortex lines in layered superconducting structures for fields significantly smaller than H_{c2} first requires consideration of the problem of the motion of an individual 2D vortex under the influence of a transport current. This question is also important for describing the dynamics of the oscillations of vortex lines and fluctuational vortex–antivortex pairs.

In the present work we studied the viscous motion of a 2D vortex in a layered superconducting structure, whose equilibrium properties can be described within the Lawrence–Doniach model.[23] We consider the case in which superconducting layers of thickness d are separated by intervening layers of a normal metal (or a superconductor with a lower critical temperature) with a thickness $D \gg d$. As will be shown below, consideration of the finite conductivity of the intervening layers has a significant influence on the dissipative processes in the system. The dynamics of vortices near the critical temperature T_c (under the assumption that the condition $\xi_c \ll D$ is satisfied) can be studied by utilizing the time-dependent generalization of the Lawrence–Doniach model (see, for example, Ref. 22), which is patterned after the time-dependent Ginzburg–Landau theory in an ordinary three-dimensional superconductor. As we know, the time-dependent Ginzburg–Landau equations are valid in the case of gapless superconductivity, which can arise either from a high concentration of paramagnetic impurities or from inelastic electron–phonon relaxation (a detailed analysis of this question can be found, for example, in Ref. 24).

We write equations for the order parameter and an expression for the current in the nth superconducting plane with neglect of the Josephson interaction between the superconducting layers (the question of the influence of the finite magnitude of this interaction on the results obtained will be discussed below):

$$\frac{\pi\hbar}{8(T_c-T)}\frac{\partial\rho_n}{\partial t}=\xi^2\nabla_\perp^2\rho_n+\rho_n-\rho_n^3-\xi^2\rho_n\times\left(\nabla_\perp\theta_n-\frac{2e}{\hbar c}\mathbf{A}_{\perp n}\right)^2, \tag{1}$$

$$\frac{\pi\hbar}{8(T_c-T)}\rho_n^2\left(\frac{\partial\theta_n}{\partial t}+\frac{2e}{\hbar}\varphi_n\right)=\xi^2\nabla_\perp\left[\rho_n^2\left(\nabla_\perp\theta_n-\frac{2e}{\hbar c}\mathbf{A}_{\perp n}\right)\right], \tag{2}$$

$$\mathbf{j}_n=\frac{\hbar c^2}{8\pi e\lambda^2}\rho_n^2\left(\nabla_\perp\theta_n-\frac{2e}{\hbar c}\mathbf{A}_{\perp n}\right)+\sigma_0\mathbf{E}_{\perp n}$$
$$=\mathbf{j}_{sn}+\sigma_0\mathbf{E}_{\perp n}. \quad (3)$$

Here xy is the plane of the film; ξ is the coherence length; $\varphi_n(x,y)=\varphi(x,y,z=nD)$ is the scalar potential; $\mathbf{A}_{\perp n}=\mathbf{A}_\perp(x,y,x=nD)$ is the component of the vector potential perpendicular to the z axis; σ_0 is the normal conductivity; λ is the penetration depth of a magnetic field in the superconducting material; ρ_n and θ_n are the amplitude and phase of the superconducting order parameter;

$$\nabla_\perp=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right),\quad \mathbf{E}=-\nabla\varphi-\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t},$$

$$\mathbf{E}_{\perp n}=\mathbf{E}_\perp(x,y,z=nD).$$

We note that we have neglected the variation of the potential φ across the thickness of the film, taking into account the smallness of d in comparison with the characteristic scale of the variation of φ in the superconductor. The expression for the current in the intervening normal-metal layers has the form

$$\mathbf{j}=\sigma_1\mathbf{E}.$$

Using the condition of continuity of the total current, we obtain an equation for the field $\mathbf{E}$ outside the superconducting layers and the boundary condition for the component of the field E_z in the $z=nD$ plane (here we once again assume that the superconducting layers are ideally thin):

$$\operatorname{div}\mathbf{E}=0, \quad (4)$$

$$\frac{\sigma_1}{d}[E_z(z=nD+0)-E_z(z=nD-0)]=-\operatorname{div}\mathbf{j}_n. \quad (5)$$

In the absence of a transport current the distribution of the phase θ_n for a 2D vortex in the $z=0$ plane has the form

$$\nabla_\perp\theta_n=\frac{\mathbf{z}_0\times\mathbf{r}_0}{r}\delta_{n0}, \quad (6)$$

where $\mathbf{z}_0$ and $\mathbf{r}_0$ are the unit vectors of a cylindrical coordinate system whose origin is at the center of the vortex. The stationary problem of the distribution of the magnetic field in this case was considered in Refs. 12–14. Here we present the expression for the vector potential $\mathbf{A}$ obtained in the approximation $\lambda^2/Dd\gg 1$:

$$\mathbf{A}=\frac{\hbar c}{2er}\sqrt{\frac{D}{2\Lambda}}\left[\exp\left(-|z|\sqrt{\frac{2}{\Lambda D}}\right)-\exp\right.$$
$$\left.\times\left(-\sqrt{\frac{2(z^2+r^2)}{\Lambda D}}\right)\right](\mathbf{z}_0\times\mathbf{r}_0). \quad (7)$$

Here $\Lambda=2\lambda^2/d$. Thus, the vector potential $\mathbf{A}$ in the $z=0$ plane is small compared with $(\hbar c/e)\nabla\theta$ in consequence of the smallness of the parameter dD/λ^2. This circumstance leads, in particular, to logarithmic divergence of the energy of one 2D vortex and to a logarithmic dependence of the interaction energy of a vortex and an antivortex on the distance r_{12} between them for an arbitrary value of r_{12} (Refs. 12–14). The condition $dD/\lambda^2\ll 1$ is easily satisfied in real superconducting superlattices, at least in the temperature range close to the critical temperature; therefore, we shall consider just this case below. For a vortex moving under the influence of a Lorentz force, we can neglect its intrinsic magnetic field in Eqs. (1–3). To solve the time-dependent problem we now employ the stationary approach used to obtain the equation of viscous motion of a vortex line in a three-dimensional homogeneous superconductor. We go over to the system of coordinates $\mathbf{R}=\mathbf{r}-\mathbf{V}t$ moving with a vortex velocity $\mathbf{V}$, and we seek the functions ρ_n, θ_n, and φ in the form

$$\rho_n=\rho_n(\mathbf{r}-\mathbf{V}t),\quad \varphi=\varphi(\mathbf{r}-\mathbf{V}t),$$
$$\theta_n=\theta_{nv}(\mathbf{r}-\mathbf{V}t)+\theta_n^t, \quad (8)$$

where θ_n^t is the distribution of the phase corresponding to the transport current

$$\mathbf{j}_t=\frac{\hbar c^2}{8\pi e\lambda^2}\left(\nabla_\perp\theta_n^t-\frac{2e}{\hbar c}\mathbf{A}_{\perp n}^t\right)=\frac{\hbar c^2}{8\pi e\lambda^2}\mathbf{K}_n, \quad (9)$$

and the function θ_{nv} satisfies the conditions $\nabla_\perp\theta_{nv}\to 0$ when $|\mathbf{R}|\to\infty$ and

$$\operatorname{curl}\nabla_\perp\theta_{nv}=2\pi\delta_{n0}\delta(\mathbf{r}-\mathbf{V}t)\mathbf{z}_0. \quad (10)$$

We write the system of equations for ρ_n, θ_{nv}, and φ:

$$-\frac{\pi\hbar}{8(T_c-T)}\mathbf{V}\cdot\nabla_\perp\rho_n=\xi^2\nabla_\perp^2\rho_n+\rho_n-\rho_n^3$$
$$-\xi^2\rho_n(\nabla_\perp\theta_{nv}+\mathbf{K}_n)^2, \quad (11)$$

$$\frac{\pi\hbar}{8(T_c-T)}\rho_n^2\left(-\mathbf{V}\cdot\nabla_\perp\theta_{nv}+\frac{2e}{\hbar}\varphi_n\right)$$
$$=\xi^2\nabla_\perp(\rho_n^2(\nabla_\perp\theta_{nv}+\mathbf{K}_n)), \quad (12)$$

$$\frac{\sigma}{d}[\varphi_z'(z=nD+0)-\varphi_z'(z=nD-0)]$$
$$=-\Delta\varphi_n+\frac{u}{\xi^2}\rho_n^2\left(\varphi_n-\frac{\hbar}{2e}\mathbf{V}\cdot\nabla_\perp\theta_{nv}\right). \quad (13)$$

Here $\sigma=\sigma_1/\sigma_0$. The parameter u is specified by the ratio between the coherence length and the penetration depth of an electric field in the superconducting material. If the gapless situation is provided by electron–phonon relaxation, we have $u=5.79$. For superconductors with a high concentration of magnetic impurities, u must be set equal to 12. In the intervening normal-metal layers we have an equation for the potential φ of the form

$$\Delta\varphi=0. \quad (14)$$

Let us first find the distribution of the phase and the potential φ in the region $|\mathbf{R}|\gg\xi_c$. In this case it can be assumed that $\rho_n\simeq 1$. To solve the system of equations (10), (12)–(14), which is linear with respect to the unknown functions $\mathbf{a}_n=\nabla\theta_{nv}$ and φ, it is convenient to use Fourier transforms:

$$\varphi(\mathbf{k},z)=\int\varphi(\mathbf{R},z)e^{i\mathbf{k}\cdot\mathbf{R}}d^2R, \quad (15)$$

$$\mathbf{a}_n(\mathbf{k})=\int \mathbf{a}_n(\mathbf{R})\exp^{i\mathbf{k}\cdot\mathbf{R}}d^2R. \tag{16}$$

For the Fourier components we have the system of equations

$$-i\xi^2(\mathbf{k}\cdot\mathbf{a}_n)=\frac{\pi\hbar}{8(T-T_c)}\left(\frac{2e}{\hbar}\varphi_n-\mathbf{V}\cdot\mathbf{a}_n\right), \tag{17}$$

$$-i\mathbf{k}\times\mathbf{a}_n=2\pi\delta_{n0}\mathbf{z}_0, \tag{18}$$

$$\frac{\partial^2\varphi}{\partial z^2}-k^2\varphi=0 \tag{19}$$

for $nD<z<(n+1)D$;

$$\frac{\sigma}{d}[\varphi_z'(z=nD+0)-\varphi_z'(z=nD-0)]$$

$$=\left(k^2+\frac{u}{\xi^2}\right)\varphi_n-\frac{\hbar u}{2e\xi^2}\mathbf{V}\cdot\mathbf{a}_n. \tag{20}$$

In the normal-metal layer between the nth and $(n+1)$th superconducting layers the solution of Eq. (19) has the form

$$\varphi=c_ne^{kz}+d_ne^{-kz}. \tag{21}$$

Now using the boundary conditions (20), we can easily obtain an expression for the Fourier component of the potential $\varphi_n(\mathbf{k})=\varphi(\mathbf{k},z=nD)$ which satisfies the requirement $\varphi_n\to 0$ when $|n|\to\infty$:

$$\varphi_n=\frac{\pi\hbar uV_cd}{2e\sigma k\xi}\mathbf{k}\cdot\mathbf{V}\times\mathbf{z}_0\frac{\sinh(kD)}{\sinh(QD)}$$

$$\times(\mathbf{k}\cdot\mathbf{V}-i\xi V_ck^2)^{-1}\exp(-QD|n|), \tag{22}$$

$$V_c=\frac{8(T_c-T)\xi}{\pi\hbar},$$

as well as an expression for the Fourier component of the phase gradient of the order parameter in the nth superconducting layer:

$$\mathbf{a}_n=\frac{2e\varphi_n\mathbf{k}/\hbar-2\pi V_c\xi\delta_{n0}\mathbf{k}\times\mathbf{z}_0-2\pi i\delta_{n0}\mathbf{V}\times\mathbf{z}_0}{\mathbf{k}\cdot\mathbf{V}-i\xi V_ck^2}. \tag{23}$$

Here Q is determined from the equation

$$\cosh(QD)=\cosh(kD)+\frac{\sinh(kD)d}{2k\sigma}$$

$$\times\left(k^2-\frac{iu\xi^{-1}V_ck^2}{\mathbf{k}\cdot\mathbf{V}-i\xi V_ck^2}\right). \tag{24}$$

We consider the range of values of $\mathbf{k}$ satisfying $L_v^{-1}\ll|\mathbf{k}|\ll\xi^{-1}$, where

$$L_v=\frac{8(T_c-T)\xi^2}{\pi\hbar V}=\frac{V_c\xi}{V}.$$

This range of values of $\mathbf{k}$ corresponds to distances $\xi\ll|\mathbf{R}|\ll L_v$ and is sufficiently broad at all $V\ll V_c$ (it will be shown below that the latter condition holds, if the transport current density is considerably smaller than the depairing current density j_c). We write expressions for the distributions for the potential and the phase gradient in the $n=0$ plane (in which a vortex is located) in this range of $\mathbf{k}$ in a linear approximation with respect to V:

$$\varphi_0=\frac{i\pi\hbar}{e}\frac{\mathbf{V}\cdot\mathbf{z}_0\times\mathbf{k}}{k^2F(k)}, \tag{25}$$

$$\mathbf{a}_0=\frac{2\pi i\mathbf{z}_0\times\mathbf{k}}{k^2}+\frac{2\pi\mathbf{k}\cdot(\mathbf{V}\cdot\mathbf{z}_0\times\mathbf{k})}{V_c\xi k^4}\left(1-\frac{1}{F(k)}\right), \tag{26}$$

where

$$F(k)=\sqrt{\left(1+\frac{k^2\xi^2}{u}\right)^2+4k^2D^2s^2+4kDs\left(1+\frac{k^2\xi^2}{u}\right)\coth(kD)},$$

$$s=\frac{\xi^2\sigma}{uDd}.$$

Let us consider the case of sufficiently small velocities V, which satisfy the condition

$$L_v\gg L_m=\max\left[\xi,\frac{\xi}{\sqrt{u}},D,sD\right]. \tag{27}$$

After performing inverse Fourier transformation of the expressions (25) and (26), in the range of distances $L_m\ll|\mathbf{R}|\ll L_v$ we obtain

$$\varphi_0\simeq\frac{\hbar}{2e}\frac{\mathbf{V}\cdot\mathbf{z}_0\times\mathbf{R}_0}{R\sqrt{1+4s}},\quad \nabla_\perp\theta_0\simeq\frac{\mathbf{z}_0\times\mathbf{R}_0}{R} \tag{28}$$

$$+\left(1-\frac{1}{\sqrt{1+4s}}\right)\frac{1}{2V_c\xi}\left(\mathbf{V}\times\mathbf{z}_0\ln\frac{C\xi V_c}{RV}\right.$$

$$\left.-\mathbf{R}_0(\mathbf{R}_0\times\mathbf{V}\cdot\mathbf{z}_0)\right), \tag{29}$$

where C is a constant of the order of unity.

Before proceeding to the direct derivation of the equation of viscous motion of a 2D vortex, let us dwell on some qualitative arguments, which point out some significant differences between the problem under consideration and the problem of the motion of a vortex line in a three-dimensional homogeneous superconductor. The equation of motion for a vortex line parallel to the z axis can be obtained within the time-dependent Ginzburg–Landau theory (see, for example, Ref. 25) and has the form

$$\eta\mathbf{V}=\frac{\phi_0}{c}\mathbf{j}_t\times\mathbf{z}_0, \tag{30}$$

where ϕ_0 is the flux quantum. The viscosity coefficient η can be separated into two terms ($\eta=\eta_1+\eta_2$), which correspond to different dissipation mechanisms: 1) dissipation associated with relaxation of the absolute value of the order parameter in the vortex core; 2) ohmic losses accompanying the flow of normal currents. We present the familiar expressions for η_1 and η_2 here:

$$\eta_1 = \eta_0 \int_0^\infty R(\rho_R')^2 dR = 0.279\eta_0, \quad (31)$$

$$\eta_2 = \eta_0 \alpha = \eta_0 \frac{1}{\pi V^2} \int \rho^2 (\mathbf{V}\cdot\nabla\theta)\left(\mathbf{V}\cdot\nabla\theta - \frac{2e}{\hbar}\varphi\right) d^2R, \quad (32)$$

$$\eta_0 = \frac{\sigma_0 u \phi H_{c2}}{2c^2}.$$

Here φ is the solution of the equation

$$\xi^2 \Delta\varphi = u\rho^2\left(\varphi - \frac{\hbar}{2e}\mathbf{V}\cdot\nabla\theta\right),$$

and ρ and θ are the amplitude and phase of the order parameter for a stationary vortex. For superconductors with a high concentration of paramagnetic impurities, we have $u=12$ and $\alpha=0.159$.

Both dissipation mechanisms just indicated also operate in the case of the motion of a 2D vortex in a multilayer structure. In that case the viscosity η_1, which is associated with relaxation of the absolute value of the order parameter, is specified only by the distribution of $\rho_0(\mathbf{R})$ in the plane of the film and can be found in analogy to (31). The ohmic part of the viscosity η_2 is greatly dependent on the distribution of the gradient-invariant potential

$$\mu = \varphi + \frac{\hbar}{2e}\frac{\partial\theta}{\partial t} = \varphi - \frac{\hbar}{2e}\mathbf{V}\cdot\nabla\theta$$

in the superconductor. In the case of a vortex line, μ does not depend on z, and in the xy plane it drops to zero over a characteristic distance $\sim\xi$ from the vortex axis. As follows from the results obtained above for a 2D vortex [see (28) and (29)], the potential μ determined in the linear approximation with respect to V in the $z=0$ plane (where the vortex is located) decreases as R^{-1} at large distances up to distances of the order of L_v (when the conductivity σ_1 is nonzero). This results in logarithmic divergence of the ohmic viscosity η_2, which terminates at the large distance L_v. The reason for such a slow law for the decrease in $\mu(R)$ with increasing R is the flow of normal currents in the nonsuperconducting layers, which results in penetration of the field $\mathbf{E}$ of the moving vortex into the layered structure along the z axis to a finite depth and, as a consequence, in a decrease in the potential φ in the $z=0$ plane. Thus, in particular, when $s\gg 1$ holds (this is possible in sufficiently close proximity to T_c and when the ratio between the conductivities of the layers σ is not excessively small) and for $R\gg L_m$, the function φ drops to zero along the z axis at a distance $1/Q \simeq D\sqrt{s}$, and $\varphi(z=0) \ll (\hbar/2e)\mathbf{V}\cdot\nabla\theta_0$. Thus, the equation of motion of a 2D vortex must take into account not only the terms which are linear in V, but also terms of the form $V\ln(L_v/\xi) = V\ln(V_c/V)$. In this sense the situation here is formally analogous to that which arises when the motion of a vortex line in an uncharged superfluid is considered.[26,27] We note that the foregoing arguments should probably be valid qualitatively level for other more exact models of real layered structures.

Now, to obtain the equation of motion, we use an approach similar to the approach employed in Refs. 25 and 27 and seek a solution of Eqs. (11) and (12) for $R \ll \min[L_v, L_t]$ (L_t is the characteristic scale for variation of the transport current) in the form

$$\rho_0 = f + g, \quad \theta_{0v} = \chi + \tau, \quad (33)$$

where f and χ are the amplitude and phase of the order parameter in the vortex in the absence of a transport current, and g and τ are small corrections, which appear in response to motion of the vortex and are on the order of V and $V\ln(V_c/V)$. We write down the equations for these corrections:

$$-\frac{\xi}{V_c}\mathbf{V}\cdot\nabla f = \xi^2\Delta g + g - 3gf^2 - \xi^2 g(\nabla\chi)^2 - 2\xi^2 f\nabla\chi\cdot(\nabla\tau + \mathbf{K}_0), \quad (34)$$

$$f^2(2e\varphi_0/\hbar - \mathbf{V}\cdot\nabla\chi) = \xi V_c \nabla\cdot(f^2(\nabla\tau + \mathbf{K}_0) + 2fg\nabla\chi). \quad (35)$$

We note now that for $V=0$, $\mathbf{K}_0=0$, *and* $\varphi_0=0$, *the functions* $\chi_p = (\mathbf{p}\cdot\nabla)\chi$ *and* $f_p = (\mathbf{p}\cdot\nabla)f$ ($\mathbf{p}$ is an arbitrary vector in the xy plane) are solutions of Eqs. (34) and (35) for g and τ. We multiply Eq. (34) by F_p and integrate it over the area S of a circle whose center is at the point $\mathbf{R}=0$ and whose radius $|\mathbf{R}| = R_1$ by varying R_1 so that $L_m \ll R_1 \ll \min[L_v, L_t]$. After some simple transformations and integrations by parts, we use (35) to obtain

$$\int_S f_p \mathbf{V}\cdot\nabla f d^2R - \int_S f^2 \chi_p (2e\varphi_0/\hbar - \mathbf{V}\cdot\nabla\chi) d^2R$$

$$= \xi V_c \oint_L \mathbf{R}_0 \times [\nabla\chi_p(\tau + \mathbf{K}_0\cdot\mathbf{R}) - \chi_p(\nabla\tau + \mathbf{K}_0)] dl. \quad (36)$$

On the right-hand side of this equation the integration is performed along a closed contour encompassing the area S. The correction $\nabla\tau$ to the phase gradient of the order parameter for $R=R_1$ and τ itself are determined from (29), and the potential φ_0 is determined from (28). Calculating the integrals in (36), we obtain the equation of motion of a vortex in the following form:

$$\eta_0\left[\beta + \left(1 - \frac{1}{\sqrt{1+4s}}\right)\ln\frac{V_c}{V}\right]\mathbf{V} = \frac{\phi_0}{c}\mathbf{j}_t \times \mathbf{z}_0. \quad (37)$$

Here β is a constant, which describes the contribution that is not dependent on the velocity V to the viscosity associated with losses due to relaxation of the modulus of the order parameter [see (31)] and the ohmic losses in the range from $R=0$ to $R\sim L_m$. These ohmic losses are significantly dependent on the relationship between the characteristic scales ξ, $\xi/\sqrt{u}$, D, and sD. We note that $\beta\sim 1$ holds for $L_m\sim\xi$. However, if $L_m\gg\xi$, holds, β can be large compared with unity. For example, in the special case of $D\gg\xi$, $sD\gg\xi$, $s<1$, and $u>1$ we have

$$\beta \sim \ln\frac{sD}{\xi}.$$

When the conductivity σ_1 of the intervening layers of the nonsuperconducting metal is not excessively small and V is sufficiently small, the constant β can be neglected in comparison with the second term in the expression for the viscosity, which is proportional to $\ln(V_c/V) \gg 1$. From (37) it is not difficult to see that the condition $V_c/V \gg 1$ is satisfied for all $j_t \ll j_c$:

$$\frac{V_c}{V} = \frac{L_v}{\xi} > \frac{c\eta_0 V_c}{\phi_0 j_t} \sim \frac{j_c}{j_t} \gg 1.$$

If $\sigma_1 \to 0$ holds, then we have $s \to 0$, $\beta \to \alpha + 0.279$, and Eq. (37) takes on the form of (30) with the coefficient η given by (31) and (32). Since the parameter $s = \xi^2 \sigma / uDd$ depends on the temperature as $(T_c - T)^{-1}$, it can be concluded that near T_c (if $s \gg 1$ holds) the equation of motion of a 2D vortex is always significantly nonlinear. This, in turn, can result in nonlinearity of the current–voltage characteristic of the system under consideration. When the temperature is lowered, the term which is nonlinear with respect to the velocity in the expression for the viscosity [see (37)] is the principal term only in the range of velocities

$$V \ll V_c \exp\left[-\beta \left(1 - \frac{1}{\sqrt{1+4s}} \right)^{-1} \right]. \qquad (38)$$

We note that the mechanism proposed here for nonlinearity of the viscosity differs qualitatively from the mechanism previously considered in Ref. 28 (which leads to nonlinear effects when vortex lines move in a three-dimensional homogeneous superconductor and is associated with variation of the distribution function of the quasiparticles).

All the results obtained above apply only to the case of a single 2D vortex. In the real problem we deal with a set of such vortices (which comprise, for example, a lattice of vortex lines tilted at an angle γ to the **c** axis of the multilayer structure). It is significant that if the distances r_{ij} between individual 2D vortices (located in the same layer or neighboring layers) are less than L_v, we can no longer simply use a set of equations like (37) to describe the dynamics of such a system. This is because the presence of other vortices alters the distribution of the potential μ in the range of values of the distance R from the center of the vortex under consideration up to $R \sim L_v$, thereby causing a change in the ohmic losses. We recall that the contribution to the viscosity which is nonlinear with respect to the velocity is associated with just this range. The relative arrangement of the 2D vortices thus greatly determines the dissipation in the system in the presence of a transport current. In the case of a vortex line, for example, this causes the viscosity to depend on the tilt angle γ relative to the **c** axis, since the distance between the 2D vortices increases as γ increases.

Let us dwell in greater detail on a qualitative analysis of this case. Here the viscosity is determined to a significant extent by the projection of the vector joining the centers of 2D vortices located in neighboring layers onto the xy plane ($l = D\tan\gamma$, where D is the distance between the superconducting planes). The angle γ and, therefore, l, depend on the magnitude and direction of the applied magnetic field $\mathbf{H} = \mathbf{z}_0 H\cos\tilde{\gamma} + \mathbf{x}_0 H\sin\tilde{\gamma}$, which determine the energetically favorable configuration of the vortices. In the limit of zero Josephson coupling between the layers, the component of the magnetic field $H_x = H\sin\tilde{\gamma}$ is not shielded and freely penetrates into the multilayer medium, while $H_z = H\cos\tilde{\gamma}$ creates a lattice of vortex lines perpendicular to the layers (see, for example, Ref. 17). Thus, the finite value of the Josephson critical current J_c between the layers must be taken into account to study tilted vortex lines. For this reason, the question of how consideration of the Josephson coupling would influence the results obtained in the present work becomes extremely important. If $J_c \neq 0$ holds, a new characteristic scale appears in the problem, called the Josephson length:[5,6,15]

$$L_j = \sqrt{\frac{d\hbar c^2}{8\pi e \lambda^2 J_c}}.$$

The viscosity of a vortex line will now depend significantly on the relationship between L_j, l, and L_v. In the case of $l \ll L_j$, the Josephson currents between the layers only weakly distort the distribution of the phase of the order parameter in the superconducting planes; therefore, they can be neglected and the approach developed in the present work can be used to obtain an expression for the viscosity η. For $l < L_v$ the logarithmic divergence of the viscosity cuts off at the scale l. Therefore, under the condition $l \gg \xi$ (i.e., when γ is sufficiently large) in the logarithmic approximation we have the viscosity estimate

$$\eta \simeq \eta_0 \left(1 - \frac{1}{\sqrt{1+4s}} \right) \ln\left(\frac{D\tan\gamma}{\xi} \right).$$

But if $l > L_v$ holds, the vortex equation of motion vortex becomes nonlinear, and the expression for the viscosity has the form

$$\eta \simeq \eta_0 \left[\beta + \left(1 - \frac{1}{\sqrt{1+4s}} \right) \ln\frac{L_v}{\xi} \right],$$

as in the case of an isolated 2D vortex [see (37)]. The condition $l > L_v$ corresponds to the velocities $V > V_c \xi / l$ and to the current densities in the superconducting layers

$$j_t > j_c \left(\frac{\xi}{l} \right) \ln\frac{l}{\xi}$$

($s \gtrsim 1$, and j_c is the depairing current density). We note that in this approach the current density j_t should still be considerably smaller than j_c (this is possible, since $l \gg \xi$).

If $l > L_j$ holds, however, consideration of the Josephson coupling becomes significant for calculating the viscosity. Here, of course, it must be taken into account that the 2D vortices forming a vortex line are joined by segments of Josephson vortices, whose dynamics require a separate treatment. As for the contribution of each 2D vortex to the ohmic losses (which is the main contribution at least for $\mathbf{j}_t \perp xz$), it is proportional to $\eta_0 V^2 \ln(L_j/\xi)$, if $L_j < L_v$ holds. Otherwise, i.e., for $L_j > L_v$, this contribution is proportional to $\eta_0 V^2 \ln(L_v/\xi)$. The condition $L_j > L_v$ corresponds to the case of quite large velocities $V > V_c \xi / L_j$ and the range of currents $j_t > j_c(\xi/L_j)\ln(L_j/\xi)$ (for $s \gtrsim 1$ and $\mathbf{j}_t \perp xz$).

The question of the energetically favorable configurations of a lattice of vortex lines in a layered superconductor for various values of $\tilde{\gamma}$ and H has been analyzed, for example, in Refs. 15 and 17. In the case of $L_j < \lambda$ a tilted vortex lattice is energetically favorable over a broad range of values of H and $\tilde{\gamma}$, with the exception of the range

$$H_z < H_j = \frac{\phi_0}{4\pi\lambda^2} \ln \frac{L_j}{\xi},$$

where a lattice of vortices oriented perpendicularly to the **c** axis is realized. The increase in the viscosity η for tilted vortex lines becomes significant in the range $D\tan\gamma \gg \xi$, which corresponds to the condition $D\tan\tilde{\gamma} \gg \xi$, if $H \gg H_{c1}$ holds (in this case $\gamma \simeq \tilde{\gamma}$), and to the condition $\tilde{\gamma} \gg \xi d/L_j^2$, if H is close to H_{c1} (in this case γ can differ strongly from $\tilde{\gamma}$). In order that only one vortex line need be considered to determine η, the condition $l \ll a_x$ (a_x is the distance between 2D vortices which are located in a single plane and belong to neighboring vortex lines) must be satisfied. As was shown in Ref. 17, this occurs for

$$H < \max\left(\frac{\phi_0}{\pi D L_j}, \frac{\phi_0}{\pi D^2 \tan\tilde{\gamma}} \right).$$

These estimates determine the ranges of H and $\tilde{\gamma}$ in which the foregoing results are valid.

The results obtained thus allow us to conclude that in layered systems near T_c consideration of the finite conductivity of the nonsuperconducting intervening layers significantly alters the spatial distribution of the electric field around a moving 2D vortex and the character of the dissipation. This, in turn, can result in a significant increase in the viscosity coefficient and in its logarithmic dependence on the velocity of the vortex.

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