Optical bistability of a thin film of resonant atoms in a phase-sensitive thermostat

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It is shown theoretically that when a thin film of two-level atoms interacting with a resonant coherent electromagnetic wave is additionally illuminated with a squeezed field, a bistable transmission/reflection regime for coherent waves is obtained. This regime depends strongly on the phase difference between the coherent and the squeezed fields. New regimes, including a bistable regime, for the interaction of a coherent field with a film of resonant atoms are predicted based on this phenomenon.

1. INTRODUCTION

Many optical phenomena, for example, absorption bistability,[1] depend strongly on the character of the relaxation of the quantum systems interacting with the electromagnetic field. In Ref. 2, Gardiner showed how strongly the relaxation picture changes in the field of a squeezed light wave. Therefore it is natural to expect that the combined action of coherent and squeezed fields will lead to the appearance of both new effects and new features in already well-known phenomena. For example, in Ref. 3 new echo-type effects accompanying the successive action of resonant light pulses in coherent and squeezed states on a quantum system were investigated. In the present paper the simultaneous effect of coherent and squeezed fields on a thin film of resonant atoms is studied. Because of its simplicity and generality, investigators have long been attracted to the model of a film of resonant atoms with thickness much less than the wavelength of the incident radiation (see, for example, Refs. 4–12). It is well known[5–8] that in the absence of a squeezed field, for certain values of the film parameters, there exists a range of intensities of the incident coherent radiation such that the radiation can pass through the film of resonant atoms in either of two regimes of high and low transmission of the film with two correspondingly different values of the intensity of the transmitted radiation. It will be shown below that this optical bistability effect acquires new features when the film is additionally irradiated with resonant radiation in a squeezed state. In this case the bistable regime of the transmission of the film depends not only on the parameters of the film but also on the state of polarization of the coherent wave (more precisely, on the phase difference between the coherent and the squeezed waves). Moreover, besides hysteresis in the amplitude of the transmitted radiation as a function of the amplitude of the incident radiation, there appears a bistable dependence of the amplitude and phase of the transmitted coherent wave on the phase of the incident coherent wave. In turn, this changes very substantially the character of the transmission of phase-modulated pulses by the thin film. The other name for squeezed light—phase-sensitive thermostat—is thereby strikingly manifested.

This paper is structured as follows. In Sec. 2 a formulation of the problem is given and the basic equations are presented. In Sec. 3 the analytical and numerical results on optical bistability in the absence of a Lorentz field are presented. In Sec. 4 the more complicated case when the difference of the macroscopic field in the field from the microscopic field acting on an atom (the Lorentz field) is taken into account is investigated. Finally, in Sec. 5 the nature of the transmission of phase-modulated pulses through the film and possible practical applications are discussed.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Let a film of two-level atoms occupy the $z=0$ plane. The thickness l of the film is much less than the wavelength of the coherent radiation incident on the film from the half-space $z<0$. The intensity of the electric field of the radiation is

$$E_{\text{coh}}=E_c\exp[i(kz-\omega t]+\text{c.c.} \tag{1}$$

The atoms in the film interact resonantly with both the coherent radiation (1) and the thermostat photons. We shall describe this system by the Hamiltonian

$$H=H_a+H_b+V_{\text{coh.int}}+V_b,$$

where

$$H_a=E_1a_1^+a_1+E_2a_2^+a_2,\quad H_b=\int d\omega\hbar\omega b_\omega^+b_\omega,$$

$$V_{\text{coh.int}}=-E_{\text{atom}}(d_{12}a_1^+a_2+d_{21}a_2^+a_1),$$

$$V_b=-i\int d\omega K(\omega)(d_{12}a_1^+a_2+d_{21}a_2^+a_1)b_\omega+\text{c.c.},$$

H_a is the Hamiltonian of a two-level atom, H_b is the Hamiltonian of the photon thermostat, $V_{\text{coh.int}}$ is the operator describing the interaction of the atom with the coherent electromagnetic field, the field E_{atom} (which is different from E_{coh} because of the boundary conditions for Maxwell's equations at the surface of the film and from the Lorentz field) acts on the atom[7–9], and V_b is the operator describing the interaction of an atom with the thermostat photons. The following additional notation is introduced: a_1 and a_2 are annihilation operators for an atom in states with energies E_1 and E_2, respectively; b_ω is the annihilation operator for photons with frequency ω; $d_{12}=d_{21}^*$ are the nonzero matrix elements of the dipole-moment operator of an atom; and, $K(\omega)$ is a coupling constant, which depends on the geometry of the thermostat.[2] We assume that the in-fields of the thermostat[2,13]

$$b_{\rm in}(t)=(2\pi)^{-1/2}\int d\omega\, \exp[-i\omega(t-t_{\rm in})]b_\omega$$

satisfy the conditions for squeezed white noise

$$\langle b_{\rm in}^+(t)b_{\rm in}(t')\rangle=N\delta(t-t'),$$

$$\langle b_{\rm in}(t)b_{\rm in}^+(t')\rangle=(N+1)\delta(t-t'),$$

$$\langle b_{\rm in}(t)b_{\rm in}(t')\rangle=M\delta(t-t'),$$

$$\langle b_{in}^+(t)b_{in}^+(t')\rangle=M^*\delta(t-t'),$$

where the brackets denote averaging, and M and N are the thermostat parameters, with $|M|\leqslant\sqrt{N(N+1)}$. Setting $K(\omega)=(\varkappa/2\pi)^{1/2}$ and using the apparatus of the Ito quantum stochastic equation (Ref. 12; see also Ref. 13) the following expressions for the density matrix ρ of the atoms, assuming a slowly varying amplitude of the coherent field:

$$\rho_{21}=\tilde\rho_{21}\exp(-i\omega t),$$

$$\frac{d}{dt}\tilde\rho_{21}=i(\omega-(E_2-E_1)/\hbar)\tilde\rho_{21}+\frac{i}{\hbar}E_ad_{21}(\rho_{11}-\rho_{22})-\Gamma_{21}\tilde\rho_{21}-M_{21}\tilde\rho_{12},$$

$$\frac{d}{dt}\rho_{11}=\frac{i}{\hbar}(d_{12}\tilde\rho_{21}E_a^*-\tilde\rho_{21}^*d_{21}E_a)-\Gamma_1\rho_{11}+\Gamma_2\rho_{22}, \tag{2}$$

$$\frac{d}{dt}\rho_{22}=-\frac{i}{\hbar}(d_{12}\tilde\rho_{21}E_a^*-\tilde\rho_{21}^*d_{21}E_a)-\Gamma_2\rho_{22}+\Gamma_1\rho_{11},$$

$$\Gamma_{21}=\varkappa|d_{21}/\hbar|^2\left(N+\frac{1}{2}\right),\quad \Gamma_1=\varkappa|d_{21}/\hbar|^2N,$$

$$\Gamma_2=\varkappa|d_{21}/\hbar|^2(N+1),\quad M_{21}=\varkappa|d_{21}/\hbar|^2|M|,$$

where the complex amplitude E_a of the field acting on the atoms in the film is related to the amplitude E_c of the incident coherent wave (1) by

$$E_a=E_c+2\pi\omega c^{-1}\tilde\rho_{21}d_{12}(i+\xi). \tag{3}$$

Here ξ is a parameter characterizing the difference between the coherent field inside the film

$$E_{\rm film}=E_f\exp(-i\omega t)+\text{c.c.},$$

$$E_f=E_c+2\pi i\omega c^{-1}\tilde\rho_{21}d_{12}$$

and the coherent field $E_{\rm atom}=E_a\exp(-i\omega t)+$ c.c. acting on an atom. The coherent field $E_{\rm film}$ inside the film determines the reflected $E_{\rm ref}$ and transmitted $E_{\rm trans}$ coherent fields according to the formulas

$$E_{\rm ref}=E_{\rm film}-E_{\rm coh},\quad E_{\rm trans}=E_{\rm film}.$$

The parameter ξ originates from the Lorentz field and scales as $\xi=2\lambda\xi_0/3l$, where $\xi_0\approx1$ and $\lambda\!\!\!^{-}=1/k$. For transition-metal films in a quantizing magnetic field, $\xi=0$. It should also be emphasized that for $\xi=0$ the transmission of a coherent wave through low-Q cavities filled with resonant atoms can be described in the mean-field approximation[8] with the aid of Eqs. (2) and (3).

The initial conditions for Eqs. (2), which correspond to the coherent field (1) being switched on at the time $t=0$, are

$$\tilde\rho_{21}|_{t=0}=0,\quad \tilde\rho_{11}|_{t=0}=N_1,\quad \tilde\rho_{22}|_{t=0}=N_2,$$

where N_1 and N_2 are the equilibrium values (in the absence of the wave (1)) of the surface density of the atoms occupying the energy levels E_1 and E_2, respectively.

The basic equations can be written in dimensionless form as

$$\frac{d}{d\tau}p=(i\Delta-\gamma_0)p+i\varepsilon n-\delta p^*,$$

$$\frac{d}{d\tau}n=2i(\varepsilon^*p-\varepsilon p^*)-\gamma n+\gamma,$$

$$\varepsilon=a+(\xi+i)p,\quad p(0)=0,\quad n(0)=1, \tag{4}$$

where

$$n=(\rho_{11}-\rho_{22})N_0^{-1},\quad p=\tilde\rho_{21}d_{12}|d_{12}|^{-1}N_0^{-1},$$

$$\varepsilon=E_a/\varepsilon_0,\quad a=E_c/\varepsilon_0,\quad \varepsilon_0=\hbar/t_0|d_{21}|,$$

$$t_0=\hbar c(2\pi\omega N_0|d_{12}|^2)^{-1},\quad \gamma_0=\Gamma_{21}t_0,\quad \gamma_1=\Gamma_1t_0,$$

$$\gamma_2=\Gamma_2t_0,\quad \gamma=\gamma_1+\gamma_2,\quad \delta=M_{21}t_0,\quad N_0=N_1-N_2,$$

$$\Delta=(\omega-(E_2-E_1)/\hbar)t_0.$$

The quantity ε is the dimensionless amplitude of the local electric field acting on an atom, $\varepsilon_f=a+ip$ is the dimensionless amplitude of the coherent field in the film and is also equal to the amplitude ε_p of the transmitted field, and $ip=\varepsilon_r$ is the dimensionless amplitude of the coherent field reflected from the film. We note that although our result presumes that the relation $\gamma=2\gamma_0$ holds, in what follows we shall treat the relaxation constants γ_0 and γ as being independent of one another so as to cover other possible relaxation mechanisms which are not related to the squeezed thermostat.

It is convenient to write the first equation of the system (4) in the form

$$\frac{d}{d\tau}p=i(\Delta+\xi n)p-(\gamma_0+n)p+ian-\delta p^*, \tag{4a}$$

which reflects clearly the role of the Lorentz field, resulting in a shift of the resonance frequency by an amount $-\xi n$, as well as the role in the film of the radiation field reradiated by the atoms which gives rise to the collective relaxation of polarization (the term $-np$). These terms essentially determine the positive feedback mechanism which is necessary for the formation of bistable regimes.

The equations (4) obtained above differ from the standard equations for a thin film of resonant atoms in the field of a coherent wave by the presence of the relaxation term $-\delta p^*$, which is zero in the absence of squeezing. However, this term changes the character of the relaxation, increasing the rate of relaxation of the real part of the polarization p and decreasing the relaxation of the imaginary part of p. This in turn gives rise to new resonant interactions of the coherent field which depend on the phase of this field. We emphasize that the quantity δ in Eqs. (2) is assumed to be real, so that the phase φ (of the amplitude) of the coherent film (1) $a=|a|e^{i\varphi}$ represents the phase difference between the coherent and squeezed fields. Moreover, it should be noted that $\delta \leqslant (\gamma_1\gamma_2)^{1/2} \approx \gamma_0$.

In concluding this section, we note the following equation, which follows from Eq. (4) and will be helpful for what follows, for the relaxation of the square of the "Bloch vector"

$$\frac{d}{d\tau}\left(|p|^2+\frac{1}{4}n^2\right)=-2\gamma_0|p|^2-\delta(p^2+p^{*2})+\frac{1}{2}\gamma n(1-n).$$

It is obvious that in the stationary state

$$p'^2\frac{\gamma_0+\delta}{\gamma}+p''^2\frac{\gamma_0-\delta}{\gamma}+\frac{1}{4}n^2=\frac{1}{4}n. \quad (5)$$

3. OPTICAL BISTABILITY IN THE ABSENCE OF A LORENTZ FIELD

To see most simply the effect produced by squeezing the electromagnetic field of the thermostat, we confine our attention to the particular case of exact resonance ($\Delta=0$) and neglect the Lorentz field, assuming $\xi=0$. Then the amplitude is $\varepsilon=\varepsilon_f$. The stationary solutions of the system (4) can be easily found in two cases: when the phase of the coherent wave is equal to the phase of the squeezed thermostat and when the phase of the coherent wave is shifted relative to the phase of the squeezed thermostat by $\pi/2$.

When the phase of the coherent wave (1) is equal to the phase of the squeezed field $\varphi=0$ (or $\varphi=\pi$), the field inside the film also has the same phase. The polarization p is then purely imaginary $p^*=-p$. We obtain the stationary solution of the system (4) in the form

$$a=\varepsilon(1+(\gamma_0-\delta+4|\varepsilon|^2/\gamma)^{-1}), \quad (6)$$

which is identical to the classical absorption bistability formula $y=x(1+2C/(1+x^2))$ with the Bonifacio–Lugiato cooperativeness parameter[14] $C=C_0=\frac{1}{2}(\gamma_0-\delta)^{-1}$. We recall that for $C>4$, the amplitude ε of the field in the film (and the amplitude of the transmitted field) exhibit hysteresis as a function of the amplitude a of the incident field. Here, just as in the standard absorption bistability, of the three roots of Eq. (6), which we denote by $\varepsilon_{\min}\leqslant\varepsilon_{\text{unst}}\leqslant\varepsilon_{\max}$, the solution of the system (4) corresponding to the middle root is unstable.

One can see that the cooperativeness parameter C itself depends on the degree of squeezing of the field: The stronger the squeezing, the more favorable the conditions for formation of a bistable regime are.

When the phase of the incident coherent wave is shifted relative to the squeezed field by $\pi/2$ (or $3\pi/2$), the polarization of the film is real. Then it is easily found that

$$a=\varepsilon[1+(\gamma_0+\delta+4|\varepsilon|^2/\gamma)^{-1}].$$

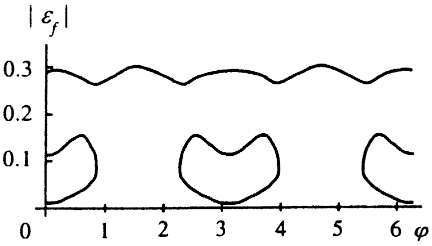

FIG. 1. Magnitude of the field amplitude in the film as a function of the phase of the incident coherent wave. Here $C_0=20$, $C_{\pi/2}=4$, $|a|=0.42$, $\xi=\Delta=0$, $\gamma=2\gamma_0$.

In the case at hand, the cooperativeness parameter is $C=C_{\pi/2}=\frac{1}{2}(\gamma_0+\delta)^{-1}$ and the squeezing of the thermostat makes it more difficult to reach a bistable regime.

In the general case, the coherent field in the film $\varepsilon=|\varepsilon|e^{i\psi}$ can be expressed in terms of the stationary overpopulation n of the energy levels as

$$|\varepsilon|=|a|\left[\left(\frac{\cos\varphi}{1+2C_0n}\right)^2+\left(\frac{\sin\varphi}{1+2C_{\pi/2}n}\right)^2\right]^{1/2},$$

$$\tan\psi=\frac{1+2C_0n}{1+2C_{\pi/2}n}\tan\varphi, \quad (7)$$

where n is the solution of the equation

$$(n-1)(1+2C_0n)^2(1+2C_{\pi/2}n)^2+\frac{8n|a|^2}{\gamma}\times(C_0(1+2C_{\pi/2}n)^2\cos^2\varphi+C_{\pi/2}(1+2C_0n)^2\sin^2\varphi)=0.$$

It is evident from Eq. (7) that the absolute value of the field amplitude in the film varies strongly with the phase φ of the incident field (1), vanishing in the absence of squeezing ($\delta=0$).

The simple situations studied above make it possible to predict the existence of unique bistable regimes for transmission of a coherent wave through a thin film of resonant atoms. The first regime evidently is bistable transmission of a coherent wave through the thin film depending on the phase φ of the wave (more accurately, the phase difference between the coherent and squeezed fields), since for the same values of the parameters γ_0 and δ the system has parameters such that bistability occurs for $\varphi=0$ ($C>4$) and instability occurs for $\varphi=\pi/2$ ($C<4$). Then, if we choose a value of a corresponding to the transmission of a coherent wave through the film for $\varphi=0$ under conditions of low transmission, as φ increases, for some value of $\varphi<\pi/2$ a transition will occur to a regime of high transmission; this is illustrated in Fig. 1. The figure also illustrates the characteristic dependence of the magnitude of the field amplitude on the phase φ in regions where there is no bistability. We also note the bistable dependence of the phase of the field in the film on the absolute value of the amplitude of the incident coherent field (Fig. 2). Obviously, these effects are entirely determined by the presence of the phase-sensitive thermostat.

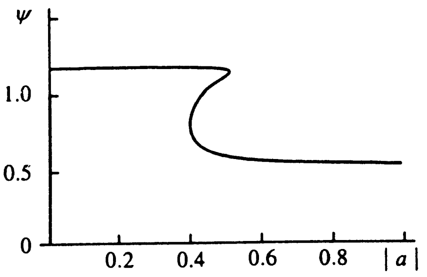

FIG. 2. Bistable behavior of the phase of the field in the film as a function of the absolute value of the amplitude of the incident coherent wave. $C_0=20$, $C_{\pi/2}=4$, $\varphi=0.5$, $\xi=\Delta=0$, $\gamma=2\gamma_0$

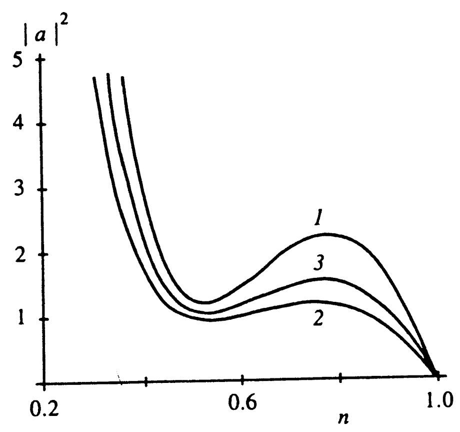

FIG. 4. Shift of the plot of $|a|^2$ versus n caused by the squeezing of the thermostat $\delta=0.4$ in the cases $\varphi=0$ (curve *1*) and $\varphi=\pi/2$ (curve *2*) relative to the curve *3*, characterizing $|a|^2$ as a function of n in the absence of squeezing. Here $\gamma=2\gamma_0=2$, $\Delta=-5$, $\xi=10$.

The other regime is associated with the variation of the degree of squeezing of the additional illumination. Choosing parameters so that for $\delta=0$ we have $C<4$ and the cooperativeness parameter satisfies $C>4$ as δ increases, we obtain a bistable transmission of a coherent wave through the resonant film depending on the degree of squeezing of the illumination (Fig. 3). When these factors act together, the bistability picture becomes even more complicated.

4. OPTICAL BISTABILITY IN THE GENERAL CASE

When the Lorentz field is taken into account ($\xi\neq 0$) and there is detuning from resonance ($\Delta\neq 0$), simple formulas of the form (6), clearly demonstrating the effect of the squeezing of the thermostat on the conditions of formation of a bistable regime, cannot be obtained. However, even here the squeezing of the thermostat gives rise to diametrically opposite effects in the typical cases when the phase of the coherent wave is unshifted ($\varphi=0$) and shifted by $\pi/2$ ($\varphi=\pi/2$) relative to the squeezed thermostat. It is convenient to investigate the stationary solution (4) by means of Eqs. (4) and

$$p'=-\frac{n|a|((\gamma_0+n+\delta)\sin\varphi+(\Delta+\xi n)\cos\varphi)}{(\gamma_0+n)^2-\delta^2+(\Delta+\xi n)^2},$$

$$p''=\frac{n|a|((\gamma_0+n+\delta)\cos\varphi-(\Delta+\xi n)\sin\varphi)}{(\gamma_0+n)^2-\delta^2+(\Delta+\xi n)^2}.$$

When the phase of the coherent wave is equal to the phase of the squeezed field ($\varphi=0$ or $\varphi=\pi$), we have

$$|a|^2=\frac{\gamma(1-n)}{4n}\times\frac{((\gamma_0+\delta+n)(\gamma_0-\delta+n)+(\Delta+\xi n)^2)^2}{(\gamma_0+\delta)(\Delta+\xi n)^2+(\gamma_0-\delta)(\gamma_0+n+\delta)^2}. \quad (8)$$

Making the simultaneous replacements $\gamma_0+\delta\to\gamma_0-\delta$ and $\gamma_0-\delta\to\gamma_0+\delta$, we find that the result for a $\pi/2$ shift of the phase of the coherent wave relative to the phase of the squeezed wave, i.e., $\varphi=\pi/2$ or $\varphi=3\pi/2$, follows from Eq. (6). Figure 4 illustrates the typical changes produced by the squeezing of the thermostat ($\delta\neq 0$), in the plots of $|a|^2$ versus n for $\varphi=0$ (curve *1*) and $\varphi=\pi/2$ (curve *2*), compared with the case of no squeezing ($\delta=0$, curve *3*). Just as in the absence of a Lorentz field, squeezing of the thermostat expands the region of bistability when the phases of the coherent and squeezed fields are equal to one another and it narrows the region of bistability in the case of a $\pi/2$ phase shift. Further numerical analysis of this case will make it possible to draw a conclusion about the presence of bistable regimes, in which the transmission coefficient of the film depends on the phase difference φ and the degree of squeezing δ of the thermostat, like those of Sec. 2. In summary, the characteristic features of the optical bistability of the thin film, which are caused by the additional irradiation of the film with a wave of resonant squeezed light, are largely independent of the existence of the Lorentz field and the detuning from resonance.

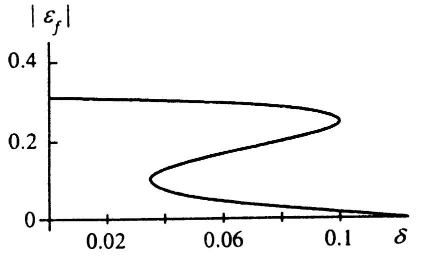

FIG. 3. Magnitude of the amplitude of the coherent field in the film as a function of the degree of squeezing of the thermostat field. Here $\gamma_0=0.128$, $\varphi=0$, $|a|=0.5$, $\xi=\Delta=0$, $\gamma=2\gamma_0$.

5. CONCLUSIONS

If the coherent field (1) is an ultrashort pulse whose duration is much less than the characteristic times $1/\gamma$, then the approach presented above can not reveal how much the illumination influences the resonant interaction of the field (1) with the film, and the problem must be formulated and investigated separately. However, in the opposite case, when the coherent pulse lasts longer than the time $1/\gamma$ of the char-

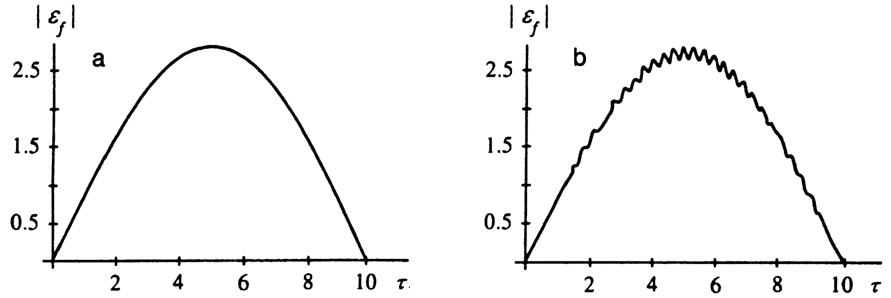

FIG. 5. Profile of the magnitude of the field amplitude in the film $|\varepsilon_f|$ as a function of time for the incident pulse $a(\tau)=|a(\tau)|\exp(i\varphi(\tau))$ with phase modulation $\varphi(\tau)=\varphi_0\tau$ and pulse shape $|a(\tau)|=a_0\sin(\pi\tau/\tau_0)$ in the absence (a) and presence (b) of squeezing of the thermostat. Here $\Delta=0$, $\xi=10$, $\gamma=2\gamma_0=30$, $\delta=15$, $a_0=3$, $\tau_0=10$, $\varphi_0=10$.

acteristic relaxation processes in the medium, the bistable interaction regime predicted above, depending on the phase of the coherent field, makes it possible to predict that the interaction of coherent pulses depends on the presence or absence of phase modulation. The phase modulation of a coherent pulse when the thermostat is squeezed gives rise to amplitude modulation of the transmitted and reflected pulses, while in the absence of squeezing the interaction picture is different. This is illustrated in Fig. 5. This result, like the others, holds with and without a Lorentz field.

In summary, the main result of the present investigation is the discovery of a clear dependence of the regimes of the resonance interaction of a coherent wave on the phase of the wave. This phenomenon found could find application both for measuring of different phase relations and in optical information processing systems, where the basic operations are implemented by means of bistable optical devices.[1] The states of such devices do not depend on the phase of the incident radiation. This will increase the volume of information that can be processed, since an additional degree of freedom—the phase of a light wave—becomes available to encode information. In the present paper, it is proposed to use additional illumination of such devices with a squeezed electromagnetic wave to produce phase-sensitive states of bistable optical devices. A thin film of resonant atoms, with thickness less than the wavelength of the incident radiation, was considered as a model of such devices. This model allowed the different regimes of operation of bistable optical devices to be effectively analyzed. The role of the squeezed electromagnetic wave reduces here, under certain conditions, to the relaxation of resonant atoms in a phase-sensitive thermostat (squeezed vacuum), so that quite simple equations, which can be investigated analytically, are obtained for a thin film, and the phase difference between the coherent wave and the illumination acquired an independent significance.

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