Redistribution function for resonance radiation in a hot dense plasma

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The redistribution function for resonance radiation in the L_α spectral line of hydrogenic ions in a dense hot plasma is calculated. The calculation is based on a self-consistent solution of the equations for the populations of the excited ionic sublevels and for the polarizations of the transitions considered. Nonlinear interference effects due to mixing of atomic states in both static and dynamic ionic fields are thereby taken into account. Molecular dynamics methods are used to account for the evolution of the multiparticle ionic field resulting from thermal motion of the ions. We calculate the L_α line of the hydrogen–like argon ion in a plasma with electron temperature 1 keV and electron density $N_e = 10^{22}-10^{24}$ cm^{-3}. The rescattering function is compared with the approximation provided by complete frequency redistribution. The results demonstrate the limited usefulness of the latter approximation for a plasma consisting of multiply-charged ions. © *1995 American Institute of Physics.*

1. INTRODUCTION

A fundamental characteristic of the theory of resonance radiative transfer in an optically dense medium is the photon frequency redistribution function $R(\omega,\omega')$, which is the joint probability that a photon absorbed at frequency ω' is re-emitted at frequency ω.[1] For a resonance spectral line, $R(\omega,\omega')$ ordinarily contains two types of contributions, corresponding to coherent and incoherent photon scattering. The ratio of these contributions depends on the line broadening mechanisms operating in the medium. For example, when the role played by perturbing particles of the medium reduces to successive collisions of these particles with the emitting atom (collisional broadening), the ratio of coherent to incoherent rescattering mechanisms is determined by A/ϕ, where A is the probability of a radiative decay from the excited state (unaccompanied by frequency redistribution) and ϕ is the rate of line-broadening collisions. Thus, coherent scattering, which in the center of mass system of the atom corresponds to a δ-function relation $\delta(\omega-\omega')$ between the frequencies, dominates for $A/\phi \gg 1$. Note that the Doppler effect in the laboratory frame leads to Doppler frequency redistribution as a result of the differing propagation directions of the absorbed and emitted photons. For $A/\phi \ll 1$, the scattering becomes completely incoherent, and the photon absorption and emission events turn out to be independent, so that the rescattering function $R(\omega,\omega')$ factors into the product $\phi(\omega)\phi(\omega')$ of independent absorption and emission probabilities described by the line profile $\phi(\omega)$, which is determined by all broadening mechanisms, including collisional, radiative, and Doppler. This is known as the total frequency redistribution limit, and is widely used in radiative transfer theory.[1–3] It should be noted that to solve the problem of transfer of resonance radiation, the total frequency redistribution limit can also be effectively invoked for $A/\phi \gg 1$ because of the Doppler mechanism of frequency redistribution, i.e., $R_D(\omega,\omega') \approx \phi_D(\omega)\phi_D(\omega')$, where $\phi_D(\omega)$ is the Doppler line profile, which does not contain the power-law Lorentzian wing produced in the usual line profile $\phi(\omega)$ by the radiative width A (see Refs. 4 and 5).

The traditional objects of study for the various frequency redistribution regimes are astrophysical and low-temperature plasma.[1,6] In the last few years, resonance-radiation transfer problems have garnered new interest in terms of their relation to hot dense plasma containing multiply-charged ions.[7–9] Investigations of such plasma relate to problems of inertial- confinement thermonuclear fusion and x-ray lasers. The peculiarities of line- radiation redistribution in a plasma of multiply-charged ions relate to the sensitive dependence of A/ϕ on ion charge Z. For example, A/ϕ is proportional to Z^6/N_e in hydrogenic ions, where N_e is the electron density.

A fundamental aspect of calculations of the function $R(\omega,\omega')$ in a dense plasma is to account for the plasma ionic microfield, which leads to Stark broadening of the ionic emission lines.

In a dense plasma the photon frequency redistribution in the rest frame of the emitter results from fast "shaking" of the atomic states by electrons and slow shifts of the states by the ions. The action of the electrons can be taken into account quite simply, whereas lengthy numerical calculations are required to take into account the effect of the ions. This is because a large number of ions participate simultaneously in the interaction, and this makes the dynamics of the ionic microfield $\mathbf{F}(t)$ complicated. Two classes of problems arise in the calculation of $R(\omega,\omega')$: 1) modeling of the many-particle microfield $\mathbf{F}(t)$ of the ions, and 2) calculation of the evolution of the atomic states under the action of this field.

For a static ionic field $\mathbf{F}$, the problem can be solved analytically for a model three-level system[10,11] that describes the advent of forbidden components in the emission spectrum of helium-like ions employed for diagnostics of the plasma parameters. In so doing it has been demonstrated that nonlinear interference effects play a role in the formation of the emission spectra and the photon frequency redistribution function. These effects are due to the interference of the atomic states in an external field, which in a plasma with multiply-charged ions leads to a strongly nonequilibrium distribution of the population over the atomic sublevels. The rescattering function $R(\omega,\omega')$ is most sensitive to these effects. Here, first of all, if nonlinear interference effects are neglected, $R(\omega,\omega')$ is no longer positive definite,[11] and secondly, even for $A/\phi \ll 1$, when the contribution of the coherent component is negligibly small, $R(\omega,\omega')$ does not reduce to a product of independent profiles (absence of total frequency redistribution). The latter circumstance does not reduce simply to a trivial discrepancy between the microfield-averaged product of the profiles and the product of the averages; rather, it reflects the interference of atomic states that results from nonlinear interference effects.

The next step in the calculation of $R(\omega,\omega')$ is to take into account the nonstationary ionic field generated by the thermal motion of the ions (ion dynamics). The important role played by ion dynamics in the profiles of spectral lines in a dense plasma of multiply-charged ions was demonstrated in Refs. 12–14. In the present work we make the first attempt to calculate the rescattering function $R(\omega,\omega')$ for the L_α line of hydrogenic ions, taking into account both nonlinear interference effects and ion dynamics. A fundamental point here is the extent to which ion dynamics affect the relationship between the coherent and incoherent components of the rescattering function. As shown in Ref. 15, in the adiabatic perturbation approximation for the radiating atom, the coherent component is always preserved when there is a stationary random process (homogeneity under temporal displacement of dipole-moment correlation functions). The latter fact should obviously also be true for the more realistic model that will be developed below, since the ion microfield dynamics described by the molecular-dynamics method is a stationary random process. Model calculations of $R(\omega,\omega')$, which considers the ion dynamics in the model micropol method, but does not consider the nonlinear interference effects, are reported in Ref. 16. Here it is found, even on the basis of this calculation, that the total frequency redistribution regime is not realized under these conditions. This is important to take into account when interpreting diagnostic measurements of line-radiation yield from impurities added to the compressed thermonuclear target. For these purposes the total frequency redistribution approximation is ordinarily used; see, for example, Ref. 17.

The present paper is organized as follows, taking into consideration the range of problems described above. The basic initial equations are presented in Sec. 2. The methods and results of calculations of the redistribution function $R(\omega,\omega')$ for the L_α line in a static ionic field on the basis of the method of correlation functions for the amplitudes of the states are presented in Sec. 3. This is compared with a previous approach[11] based on the density-matrix formalism. In Sec. 4 the rescattering function is calculated for the L_α line on the basis of the density-matrix formalism, taking ion dynamics into account. It is shown that in the limit of low electron density, these effects strongly influence the relationship between the coherent and incoherent components, despite the fact that at these densities the spontaneous emission spectrum of collisionally excited plasma does not depend on whether ion dynamics is taken into account.[14] The results are discussed in Sec. 5.

2. BASIC EQUATIONS

The rescattering of resonance radiation can be described using various theoretical methods: the compound density matrix method,[11,18] the kinetic Green's function method,[11,19] the method of the correlation functions of the dipole moments of the emitters,[15], and others. The first of these is the principal one used below. This method is based on the equations for the density matrix for the "atom (ion) + spontaneous electromagnetic fields" compound system (the fields describe the absorbed photon with frequency ω' and the emitted photon with frequency ω). These equations take the following general form in the interaction representation with unperturbed Hamiltonian H_0 ($\hbar=1$)[11]:

$$\dot{\rho}=-i[V^{\mathrm{tot}}(t),\rho]+S+R. \tag{1}$$

Here $V^{\mathrm{tot}}(t)$ is the general interaction operator, a sum of the interactions with the plasma (ion) field $\mathbf{F}(t)$

$$V(t)=-e^{-iH_0t}\mathbf{dF}(t)e^{iH_0t} \tag{2}$$

[$\mathbf{d}$ is the dipole moment of the atom (ion)] and the fields of the incident $V_1(t)$ and scattered $V_2(t)$ radiation:

$$V^{\mathrm{tot}}(t)=V(t)+V_1(t)+V_2(t), \tag{3}$$

$$V_1(t)=-G_1e^{-i\Omega_1t}\beta_1\beta_2^\dagger,\quad V_2(t)=-G_2^*e^{i\Omega_2t}\beta_1^\dagger\beta_2, \tag{4}$$

where G_1 and G_2 are coupling constants of the atom with the field amplitudes; $\Omega_1=\omega'-\omega_0$ and $\Omega_2=\omega-\omega_0$ are their frequencies, measured from the unperturbed frequency of the atomic transition; $\beta_{1,2}$ and $\beta_{1,2}^\dagger$ are the annihilation and creation operators of an atom in the corresponding states (1 and 2 represent, in the present case, the ground state and the collection of sublevels of an excited state of a hydrogenic ion). In accordance with the problem of radiation rescattering, the field G_1 is only absorbed and the field G_2 is only emitted. The commutators of ρ and $V_{1,2}$ are therefore defined below to include hermitian conjugation: $[V_{1,2},\rho]=V_{1,2}\rho-\rho V_{1,2}^\dagger$. The operators S and R describe collisional (electronic) and radiative relaxation, respectively. The explicit form of the system of equations (1) in a spherical basis is presented in Ref. 14 for the L_α line of hydrogenic ions.

The system (1) can be solved by perturbation theory in the parameters G_1 and G_2, retaining terms up to second order in G_1 and first order in G_2. The rescattering function is then found by calculating the field G_2 established during evolution of the power (or work done) at frequency ω (see Ref. 11).

Accordingly, we seek the matrix ρ in the form

$$\rho=\rho^{(0)}+\rho^{(1)}. \tag{5}$$

For $\rho^{(0)}$ we have

$$i\frac{\partial}{\partial t}\rho_{ij}^{(0)}=[V,\rho^{(0)}]_{ij}+(V_1)_{ij}\rho_{jj}^{(0)}-\sum_k\rho_{ik}^{(0)}(V_1)_{kj}, \tag{6}$$

$$i\frac{\partial}{\partial t}\rho_{ii'}^{(0)}=[V,\rho^{(0)}]_{ii'}+(V_1)_{ij}\rho_{ji'}^{(0)}-\rho_{ij}^{(0)}(V_1)_{ji'}, \qquad (7)$$

where subscripts i and k refer to the upper state and j refers to the lower state. In solving Eq. (6), we have retained first-order terms in G_1, and in solving Eq. (7), we have retained both first- (for off-diagonal elements) and second-order terms (for diagonal elements). The population of the ground state $\rho_{jj}^{(0)}$ is assumed constant. We note that in the present rescattering problem, Eqs. (6) and (7) are solved for each value of the incident frequency ω' individually.

The equations for $\rho^{(1)}$ are derived to first order in G_2:

$$i\frac{\partial}{\partial t}\rho_{ij}^{(1)}=[V,\rho^{(1)}]_{ij}+[V_1,\rho^{(1)}]_{ij}+[V_2,\rho^{(0)}]_{ij}, \qquad (8)$$

$$i\frac{\partial}{\partial t}\rho_{ii'}^{(1)}=[V,\rho^{(1)}]_{ii'}+[V_1,\rho^{(1)}]_{ii'}+[V_2,\rho^{(0)}]_{ii'}. \qquad (9)$$

Let us examine the second commutator on the right-hand side of Eq. (8):

$$[V_1,\rho^{(1)}]_{ij}=(V_1)_{ij}\rho_{jj}^{(1)}-\sum_k \rho_{ik}^{(1)}(V_1)_{kj}. \qquad (10)$$

The term with $\rho_{jj}^{(1)}$ on the right-hand side of (10) is responsible for the appearance of the coherent (Rayleigh) component of the scattering. Indeed in a static plasma microfield, for example, the equations that govern the evolution of the matrix element $\rho_{jj}^{(1)}$ when there is no relaxation of the ground state take the form[11]

$$-i(\omega-\omega')\rho_{jj}^{(1)}=iG_2\rho_{ij}^{(0)}e^{i\Omega_1 t}. \qquad (11)$$

Clearly, the element $\rho_{jj}^{(1)}$ contains a delta-function singularity $\delta(\omega-\omega')$ associated with coherent scattering. These singularities occur with a nonstationary perturbation $V(t)$ as well if the temporal evolution of the field $\mathbf{F}(t)$ is a stationary random process (see Ref. 15). These singularities can be isolated by ensemble averaging of Eq. (9) for the element $\rho_{jj}^{(1)}$, since the left-hand side of the equation for this element does not contain the operator $V(t)$. The solution of the equaton for $\rho_{jj}^{(1)}$ also contains, along with the δ-function, a principal value integral that must be ascribed to the other terms, which govern incoherent scattering (see Ref. 11).

This separation of the density-matrix element $\rho_{jj}^{(1)}$ into coherent and incoherent terms that can be expressed according to (11) in terms of the elements $\rho_{ij}^{(0)}$, which are first-order terms in G_1^*, also makes it possible to separate the complete matrix $\rho^{(1)}$ in Eq. (8) into coherent and incoherent terms.

Having found the components of the matrix $\rho^{(1)}$, we substitute them into the general expression for the work $P(\omega,\omega')$ done by the field G_2:

$$P(\omega,\omega')-2\omega\ \mathrm{Re}\langle iG_2^* e^{i\Omega_2 t}\rho_{ij}\rangle. \qquad (12)$$

Dividing the result by the energy of the emitted photon ω and the normalization factor $N=\langle\int\int d\omega d\omega' P(\omega,\omega')\rangle$, we obtain the photon frequency redistribution function $R(\omega,\omega')$. The symbol $\langle\ldots\rangle$ denotes averaging over an ensemble of perturbing ions and over the frequencies of the emitting atoms. In optically dense media, the redistribution function $R(\omega,\omega')$ is ordinarily averaged over the angles of the absorbed and emitted photons, and we shall employ it below.

The major aspects of the analysis below are, as already mentioned, its allowance for nonlinear interference and dynamical effects. The nonlinear interference effects are accounted for by the matrix elements $\rho_{ik}^{(0)}$ $(i\neq k)$ and $\rho_{ii'}^{(0)}$ $(i\neq i')$ in Eqs. (6) and (7). Their role has already been demonstrated in Refs. 11 and 14, specifically for the example of L_α line, where they are manifested as a difference between the sublevel populations of the $n=2$ level and the population distribution given by the statistical weights under the influence of the dynamic ion microfield when the radiative decays of the states are taken into account. The effects of ion dynamics are analyzed in Sec. 4. First, we investigate in greater detail the important limiting case of a static microfield.

3. CALCULATION OF THE FREQUENCY REDISTRIBUTION FUNCTION FOR THE L_α LINE IN A STATIC ION MICROFIELD

Equations (6)–(8) for a static microfield obviously simplify, and for a model three-level system the redistribution function $R(\omega,\omega')$ can even be represented analytically.[11] To analyze the general solution for real multilevel systems (specifically, the L_α line) it is then convenient to use the method of correlation functions $\Phi_{ii'}(\tau)$ of the amplitudes b_i and $b_{i'}$ of the emitting upper atomic states.

Since the lower level of the Lyman series of a hydrogenic ion is not subject to plasma microfields and broadening collisions with electrons, we can calculate the radiation frequency redistribution function in the rest frame of the emitting ion, averaged over the direction $\mathbf{n}$ of the radiation and summed over the polarizations, using the relation

$$\bar{R}(\omega,\omega')\sim\mathrm{Re}\ \mathbf{d}_{ij}\mathbf{d}_{ji'}\int_0^\infty e^{i\omega\tau}\Phi_{ii'}(\tau)d\tau$$
$$=\mathrm{Re}\ \mathbf{d}_{ij}\mathbf{d}_{ji'}\tilde{\Phi}_{ii'}(\omega), \qquad (13)$$

which contains the correlation function

$$\Phi_{ii'}(\tau)=\langle b_i(t+\tau)b_{i'}^*(t)\rangle \qquad (14)$$

of the wave functions of the upper state i, i', averaged over electron collisions and states of the lower level. The upper state is excited by monochromatic radiation of frequency ω'. It is assumed that the plasma microfield and the velocity of the emitting ion remain constant during a scattering event, and the redistribution function is then averaged with allowance for the Doppler effect over a Maxwellian velocity distribution $f(\mathbf{v})$ and plasma microfield distribution $P(F)$:

$$R(\omega,\omega')=\frac{1}{(4\pi)^2}\int P(F)f(\mathbf{v})\bar{R}(\omega-\mathbf{nv},\omega'-\mathbf{n}'\mathbf{v})$$
$$\times dF\,d\mathbf{v}d\mathbf{n}d\mathbf{n}'. \qquad (15)$$

Differentiating Eq. (14) with respect to τ and averaging over electron collisions, we obtain

$$\left(\frac{\partial}{\partial\tau}+\frac{1}{2}\Gamma_i\right)\Phi_{ii'}+\frac{i}{\hbar}H_{ik}\Phi_{ki'}=-ie^{-i\omega'\tau}d_{ij}^q(\rho_{i'j}^q)^*, \tag{16}$$

where Γ_i is the spontaneous and collisional relaxation constant of the upper level, and H is the Hamiltonian, which takes into account the energy ε_i of the sublevels and the interaction with the plasma microfield F (the energy ε_i for the L_α line is reckoned from the energy of the $2P_{3/2}$ state):

$$H_{ii'}=\varepsilon_i\delta_{ii'}-Fd_{ii'}^0, \tag{17}$$

and ρ_{ij}^q is the density matrix, calculated via first-order perturbation theory in the intensity of the absorbed radiation

$$\left(\frac{1}{2}\Gamma_i-i\omega'\right)\rho_{ij}^q+iH_{ii'}\rho_{i'j}^q=-id_{ij}^q. \tag{18}$$

In Eqs. (16)–(18), the $d_{\alpha\alpha'}^q$ are the spherical components of the dipole moment operator.

It is easy to see from Eq. (14) that $\Phi_{ii'}(0)=\rho_{ii'}$, where $\rho_{ii'}$ is the density matrix of the upper level, calculated via second- order perturbation theory in the intensity of the absorbed radiation:

$$\frac{1}{2}(\Gamma_i+\Gamma_{i'})\rho_{ii'}+i[H\rho]_{ii'}-S(\hat\rho)_{ii'}=i\rho_{ij}^q(d_{i'j}^q)^* -id_{ij}^q(\rho_{i'j}^q)^*, \tag{19}$$

where the arrival operator $S(\hat\rho)_{ii'}$ takes account of transitions induced by inelastic collisions among sublevels of the upper state.

Performing the integration over τ in Eq. (16) according to Eq. (13), we obtain an equation for the Fourier components $\tilde\Phi_{ii'}(\omega)$:

$$\left(\frac{1}{2}\Gamma_i-i\omega\right)\tilde\Phi_{ii'}+iH_{ik}\tilde\Phi_{ki'}=\frac{d_{ij}^q(\rho_{i'j}^q)^*}{\omega-\omega'+i0}+\rho_{ii'}. \tag{20}$$

For a three-level system, Eqs. (18)–(20) are identical, apart from notation, to Eqs. (4.1)–(4.3) of Ref. 11, for which the starting point was the equations for the density matrix of the "atom (ion) + electromagnetic field" compound system.

It is convenient to separate the redistribution function into coherent and incoherent parts:

$$\bar R(\omega,\omega')=\bar R^{\rm inc}(\omega,\omega')+\bar R^{\rm coh}(\omega,\omega'). \tag{21}$$

To do so, according to Eq. (16), $\Phi_{ii'}(\tau)$ can be represented as the sum

$$\Phi_{ii'}(\tau)=\Phi_{ii'}^{\rm coh}(\tau)+\Phi_{ii'}^{\rm inc}(\tau), \tag{22}$$

where the term $\Phi_{ii'}^{\rm coh}(\tau)$ describes coherent scattering in the ion's rest frame. Making use of (18), we have

$$\Phi_{ii'}^{\rm coh}(\tau)=e^{-i\omega'\tau}\rho_{ij}^q(\rho_{i'j}^q)^*, \tag{23}$$

while the second term, which describes frequency redistribution, can be found by solving the homogeneous equation

$$\left(\frac{\partial}{\partial\tau}+\frac{1}{2}\Gamma_i\right)\Phi_{ii'}^{\rm inc}+\frac{i}{\hbar}H_{ik}\Phi_{ki'}^{\rm inc}=0 \tag{24}$$

with initial conditions

$$\Phi_{ii'}^{\rm inc}(0)=\rho_{ii'}-\rho_{ij}^q(\rho_{i'j}^q)^*. \tag{25}$$

Integrating over τ and introducing the normalization factor

$$A=\frac{3\pi c^3|\mathbf{d}_{ij}|^2}{2(\varepsilon_i-\varepsilon_j)}, \tag{26}$$

where $|\mathbf{d}_{ij}|^2=d_{ij}^q(d_{ij}^q)^*$, we finally obtain for the coherent part

$$\bar R^{\rm coh}(\omega',\omega)=\frac{1}{A}d_{ij}^q\rho_{ij}^{q'}(d_{i'j}^q\rho_{i'j}^{q'})^*\delta(\omega-\omega'), \tag{27}$$

which is obviously real. The incoherent part is

$$\bar R^{\rm inc}(\omega,\omega')=\mathrm{Re}\,\frac{\mathbf{d}_{ij}\mathbf{d}_{ji'}\tilde\Phi_{ii'}^{\rm inc}}{A}, \tag{28}$$

where $\tilde\Phi_{ii'}^{\rm inc}$ can be obtained from the equation

$$\left(\frac{1}{2}\Gamma_i-i\omega\right)\tilde\Phi_{ii'}^{\rm inc}+iH_{ik}\tilde\Phi_{ki'}^{\rm inc}=\rho_{ii'}-\rho_{ij}^q(\rho_{i'j}^q)^*. \tag{29}$$

This normalization ensures that $\int\bar R(\omega,\omega')\,d\omega\,d\omega'=1$ for the L_α line, while for transitions to levels with $n>2$ this integral will equal the effective probability for reemission of a photon in a transition to the ground state, and not to an excited state. In this notation, the absorption line profile normalized to unity is

$$\varphi(\omega')=\mathrm{Re}\,\frac{i\rho_{ij}^q(d_{ij}^q)^*}{\pi|\mathbf{d}_{ij}|^2}. \tag{30}$$

Numerical calculations of the function $R(\omega,\omega')$ for the L_α line were performed for the hydrogenic Ar ion in a hydrogen plasma with a temperature of 1 keV and electron density 10^{22}–10^{24} cm^{-3}. The averaging was performed with a Holtsmark microfield distribution function. The explicit form of the matrix elements in the foregoing equations comes from Ref. 14.

Figure 1 displays results for $N_e=10^{22}$ cm^{-3}. The absorption line profile $\varphi(\omega)$ given by the integral $\int d\omega\,R(\omega,\omega')$ agrees well with the profile obtained in Ref. 14 for an argon plasma with the same temperature. The frequencies in the figures are reckoned from the $2P_{3/2}$ level.The coherent scattering fraction is clearly very large, reaching 70–90% at the different frequencies. Accordingly, the redistribution function is qualitatively similar to the function presented in Ref. 1 for redistribution that is coherent in the rest frame of the atom. The probability $w(\omega)=R(\omega,\omega')/\varphi(\omega')$ that a photon is re-emitted at a given frequency ω (Fig. 1b) under these conditions is substantially different from zero only in a narrow band near the frequency ω' of the incident photon, the width of the band being determined by the Doppler effect.

Figure 2 displays results for $N_e=10^{23}$ cm^{-3}. The coherent scattering fraction is now smaller (30–50%), so that besides coherent scattering, redistribution between lines can be clearly seen in Fig. 2b.

Figure 3 displays results for $N_e=10^{24}$ cm^{-3}. The line profile is very close to that obtained in Ref. 11 under similar

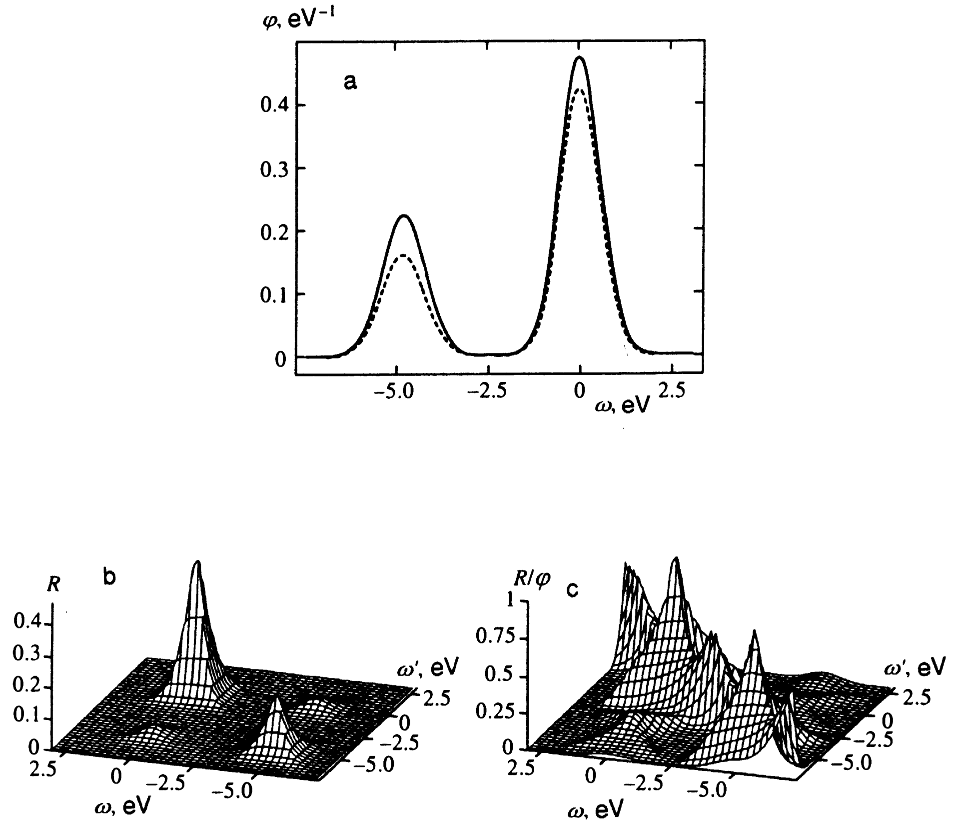

FIG. 1. Total line profile (solid curve) and its coherent part (dashed curve) (a), redistribution function (b), and the probability of photon reemission at a given frequency (c) at a temperature of 1 keV and electron density 10^{22} cm^{-3} in the approximation of a static ion microfield.

conditions. Although the coherent scattering fraction is then small because of frequent electron collisions, the redistribution function is very far from being the total frequency redistribution, which is characterized by a complete lack dependence of the probability of reemission of a photon at a given frequency ω on the frequency ω' of the incident photon, with $R(\omega,\omega')=\varphi(\omega)\varphi(\omega')$ and $w(\omega)=\varphi(\omega)$. This is primarily related to the fact that the Stark mi-crofield is as-

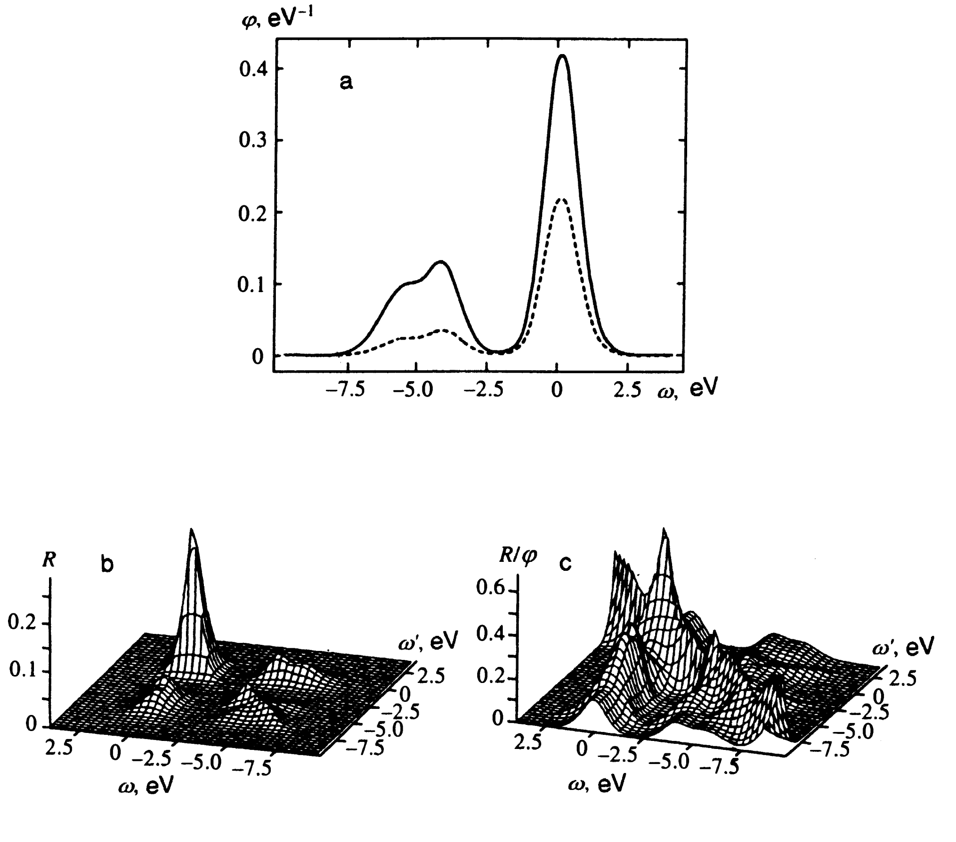

FIG. 2. Total line profile (solid line) and its coherent part (dashed curve) (a), redistribution function (b), and the probability of photon reemission at a given frequency (c) at a temperature of 1 keV and electron density 10^{23} cm^{-3} in the approximation of a static ionic microfield.

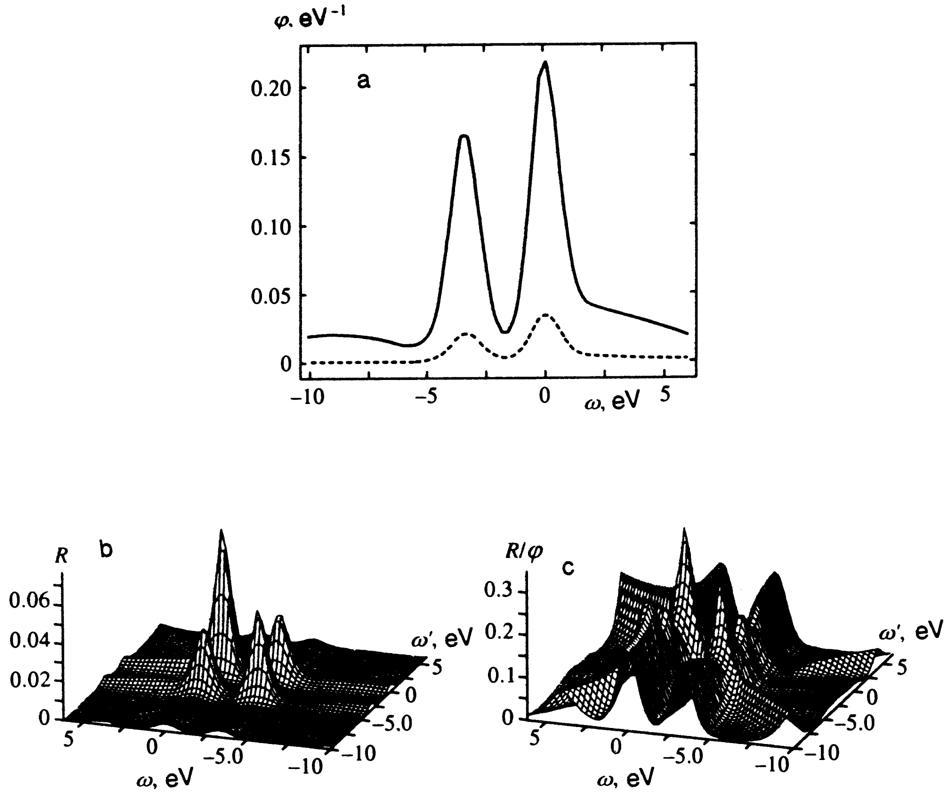

FIG. 3. Total line profile (solid curve) and its coherent part (dashed curve) (a), redistribution function (b), and probability of photon reemission at a given frequency (c) at a temperature of 1 keV and electron density 10^{24} cm^{-3} in the approximation of a static ionic microfield.

sumed to remain unchanged throughout an entire rescattering event. It is evident in Fig. 3b that for large frequency offsets of the incident photon, peaks at the frequency of the absorbed photon are present in the wings of the emission line. The presence of these peaks derives from the fact that absorption in the line wings takes place in a strong microfield whose magnitude is determined by the frequency of the absorbed photon, which remains constant throughout the reemission time, so that with a certain probability, a photon is emitted in the same transition and at a nearby frequency (the frequency shift is determined by collisional broadening and the Doppler effect). In addition, as a result of collision-induced transitions, there is a probability that a photon will be emitted in one of the two unshifted lines or in the opposite wing, which gives for the photons absorbed in the wing a total of four peaks in the probability of reemission of a photon at a given frequency.

4. REDISTRIBUTION FUNCTION FOR THE L_α LINE TAKING ION DYNAMICS INTO ACCOUNT

With ion dynamics taken into account, two classes of problems are encountered; these relate to solving the dynamical system (6)–(8) in a variable stochastic field, and modeling of the dynamics of the ionic field $\mathbf{F}(t)$.

In solving Eqs. (6) and (7), the absorption field was assumed to be classical and small compared to the saturation field. Equations (8) were solved as in Ref. 14, i.e., for the entire set of frequencies ω simultaneously, and the results were averaged, after Fourier transforming, over the initial conditions and over various realizations of the temporal trajectories of the microfield. Allowance for the dependence of $\rho^{(0)}(t)$ on the plasma microfield transforms in this case into the initial conditions for $\rho^{(1)}(t)$.

The ion microfield was modeled, as in Ref. 14, by the molecular-dynamics method (MDM). In accordance with the MDM, the motion of both heavy (ions) and light (electrons) particles in the plasma should be dealt with on the same footing. However, even ignoring the enormous computational times that result from the difference in the masses (and therefore the velocities) of the heavy and light particles, we note that a fundamental difficulty arises here, relating to the mutual attraction of the various charged particles, and leading to a divergence of the partition function for the classical Coulomb system. Thus far, it has not been possible to allow definitively for quantum effects in order to eliminate the divergence; see Ref. 20. In what follows we take a simplified approach, in which we allow for the action of the electrons on the emitting ion by introducing a collisional broadening operator S, and their effect on the inter-ion interaction potential reduces to static Debye screening. The numerical calculation was performed for quasiparticles (ions) moving in a cube with periodic boundary conditions, and with either elastic or isotropic scattering at the boundaries of the cube.[21] Ensembles of 50 and 120 particles were used in the calculations, and the results were accurate to $\sim 10\%$. The size of the modeling region was determined, as usual, starting with a given value of plasma density. The trajectory of the system was calculated over times much longer than the correlation time $\tau_c \sim r_0/v_i$, where $r_0 \sim (N_e/Z)^{-1/3}$ is the mean distance between ions, and v_i is the mean speed of the ions relative to an emitter at rest. Conservation of the total energy of the

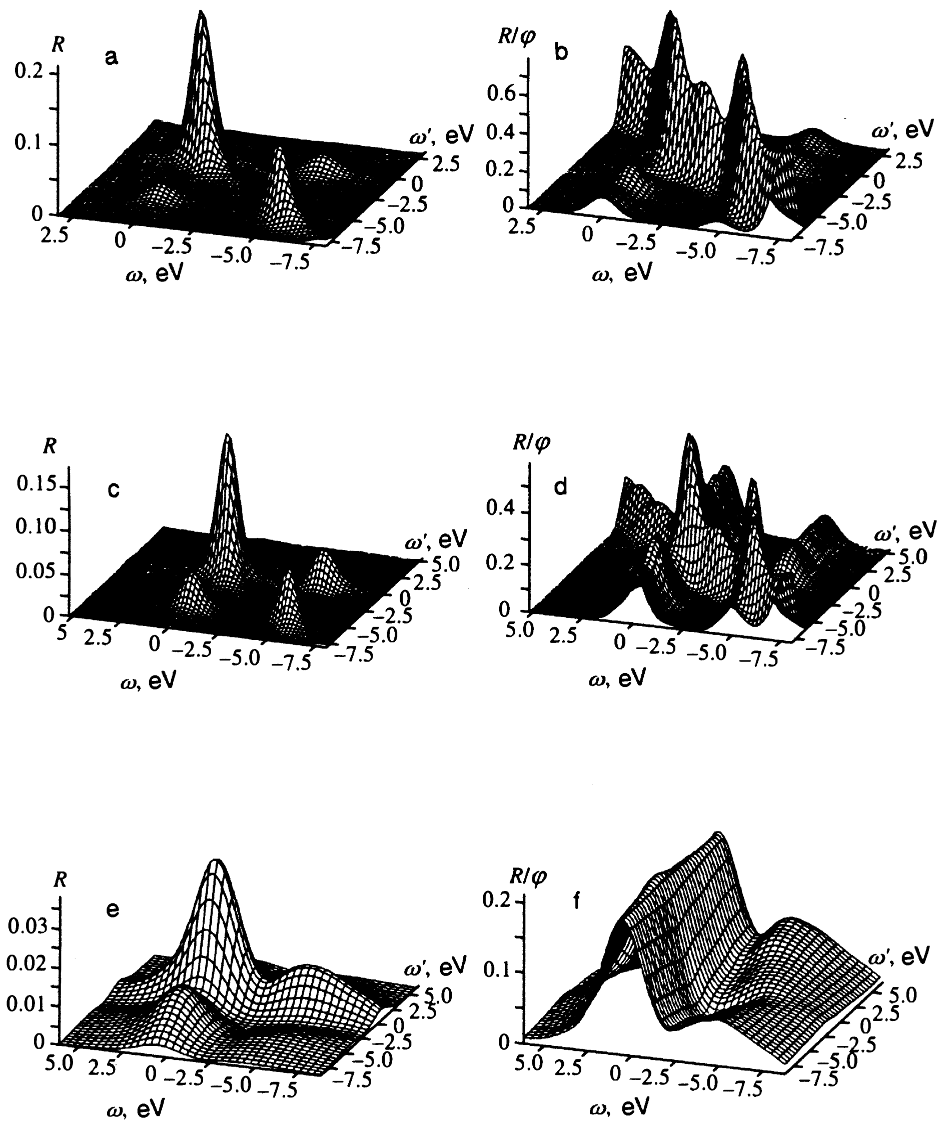

FIG. 4. Redistribution functions (a, c, e) and probabilities of photon reemission at a given frequency (b, d, f) at a temperature of 1 keV and electron densities 10^{22} cm^{-3} (a, b), 10^{23} cm^{-3} (c, d), and 10^{24} cm^{-3} (e, f), taking ion dynamics into account.

system was monitored throughout the integration. Test calculations performed in Ref. 14 agree well with existing results for correlation functions and distribution functions of the ionic microfields. Moreover, for high number densities $N_e \sim 10^{24}$, good quantitative agreement is obtained for the emission spectra calculated for the L_α line by the aforementionec method and those calculated in Ref. 13 (nonlinear interference effects were neglected in the latter work, but for large N_e they turn out to be unimportant compared with ion-dynamics effects).

Figure 4 shows the results of calculations of the redistribution function for the L_α line of the Ar XVIII ion in a hydrogen plasma for various values of the density N_e. Trial calculations showed that in the static approximation, this approach yields results that are close to those obtained in Sec. 3. The small discrepancy between the results is due to the various approximations made to incorporate the Doppler effect: in Sec. 3, the calculation was performed more systematically on the basis of Eq. (15) (this results in the characteristic pyramidal structure of the peaks in Figs. 1a, 1b and 2 as compared with the rounded character of the redistribution function profile in Figs. 4a, 4c, and 4e).

Figure 4a demonstrates the effect of ion dynamics on the redistribution function, as compared with the static approximation (Fig. 1b). The value of the incoherent part of the redistribution function $R(\omega,\omega')$ is given in Fig. 4a. This case is dominated by coherent scattering, just as in the static approximation. This is evident in Fig. 4b, which shows the ratio $R(\omega,\omega')/\varphi(\omega')$.

The enhanced role of ion-dynamics effects with increasing density is shown in Figs. 4c and 4d. The role of incoherent scattering increases (as in the static case of Fig. 2), although the ratio of R^{coh} and R^{inc} is highly distorted by ion motion.

Finally, Figs. 4e and 4f show results for density $N_e = 10^{24}$ cm^{-3}, at which the scattering is almost completely incoherent. Ion- dynamics effects radically distort the redistribution function, as compared to the static case. We note that this range of hydrogen plasma density (with a small admixture of argon) is the main object of study in the compression dynamics of laser fusion targets.[17] For these conditions the function $R(\omega,\omega')$ obviously differs from the total frequency redistribution approximation because of nonlinear interference effects, although it is ion dynamics (rather than statics) that makes this approximation possible.

5. DISCUSSION

We now assess the nature of the broadening and the contributions of the various mechanisms to the rescattering of radiation.

We note that for $N_e \sim 10^{22}$ cm^{-3}, the interaction with the ionic field $V \sim F_0 d$ is small compared to the fine-structure splitting Δ, i.e., for ions, broadening due to the quadratic Stark effect occurs with the constant $C_4 \sim d^2/\Delta$ (see Ref. 22). Then it is easy to conclude that the number of particles $g_4 \equiv N_e C_4 / v_p$ in a sphere with the Weisskopf radius $(C_4/v_p)^{1/3}$ is small ($g_4 \ll 1$), which corresponds to the dominance of collisional (adiabatic) broadening by ions (protons with velocity v_p). For the conditions considered here, however, the collisional (elastic) ion width γ_i turns out to be comparable to the inelastic collisional width Φ_e, and they are both small compared to the radiative width A of the emitting argon ion ($A \approx 6.5 \cdot 10^{13}$ s^{-1}). Under these conditions, as one would expect, the numerical calculations demonstrate that coherent scattering dominates incoherent scat tering. At the same time, calculations that take account of ion dynamics indicate that the deformation of the incoherent component of the redistribution function is more complicated than it is for the collisional mechanism. The characteristic collision rate $\nu_i \sim v_p N_e^{1/3}$ under these conditions is of order $2 \cdot 10^{15}$ s^{-1}, which is much greater than all relaxation constants (including the frequency $F_0 d$), but less than the fine-structure splitting $\Delta \sim 7 \cdot 10^{15}$ s^{-1}. Here the effect of the field is probably adiabatic, whereupon according to Ref. 15 the ratio of the coherent and incoherent components depends on the fluctuation rate of the perturbation.

The role played by ion dynamics for the L_α line increases, on the whole, with density. This is due to the structure of the L_α line itself, which contains unshifted (central) Stark components that do not undergo static broadening, but instead are broadened only by the Doppler mechanism, electron collisions, and ion dynamics, which for $N_e \sim 10^{24}$ cm^{-3} becomes the dominant broadening mechanism.

The character of the radiation rescattering in the wing of the spectral line, which is responsible for radiative transfer in optically dense systems, merits special consideration. In this region the interaction V_{eff} with the ionic field is greater than the characteristic frequency ν_i at which it varies ($V_{\text{eff}} \gg \nu_i$), so broadening here is primarily static. The interaction obviously does not lead to frequency redistribution effects, so that the contribution of coherent scattering can increase in the line wings. One way to resolve this problem is to compare the effective delay times τ^* for reemission of an absorbed photon with the time ν_i^{-1} over which the ionic field changes. Following Ref. 5,

$$\tau^* \sim \frac{1}{\Gamma}\left[1 + \frac{(\Gamma/2)^2 - \Delta\omega^2}{(\Gamma/2)^2 + \Delta\omega^2}\right], \tag{31}$$

where $\Gamma \sim A + \Phi_e$ is the total linewidth and $\Delta\omega$ is the offset from line center (taking into account the Stark shifts). The time τ^* decreases from $2/\Gamma$ (line center, $\Delta\omega = 0$) to $\Gamma/2\Delta\omega^2$ (in the wing) as the system moves farther into the line wing, but it still remains finite (in contrast to the assertion made in Ref. 23 that the scattering is instantaneous). We can estimate the parameter $\nu_i \tau^*$ in the high-density limit. Setting $\Delta\omega \sim Fd \sim N_e^{2/3} d \gg \Gamma \sim \Phi_e \sim N_e d^2 / v_e$, we obtain

$$\nu_i \tau^* \sim \frac{\Gamma v_p}{N_e d^2} \sim \frac{\Phi_e v_p}{N_e d^2} \sim \frac{v_p}{v_e} \ll 1, \tag{32}$$

i.e., in the far static wing, scattering is nearly coherent. Thus, for the Stark rescattering mechanism, emission in the wings of the resonance line is hindered.

These considerations are qualitative, of course, and require more detailed justification.

We now summarize the basic results obtained in the present work.

We have, for the first time, systematically calculated the rescattering function for resonance radiation in the real L_α spectral line of a multiply-charged ion in hot dense plasma, taking nonlinear interference effects and ion dynamics into account.

We have shown that the rescattering function differs from the total frequency redistribution approximation even at high densities, which may have an important influence on the interpretation of diagnostics involving radiation from impurities in fusion targets.

We have shown that ion dynamics strongly influences (in contrast to the emission profiles) the ratio of coherent and incoherent components of rescattering, even at comparatively low plasma densities.

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