An aero-electrodynamic vortex in the atmosphere

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(Submitted 2 February 1995)
Zh. Éksp. Teor. Fiz. **108**, 206–211 (July 1995)

The interaction between an aerodynamic vortex and an electromagnetic vortex is investigated. As a consequence of the vortical motion of the gas and the interaction of the electromagnetic field with the gas particles, a waveguide is formed in the center of the aerodynamic vortex. It is shown that the dimensions of the aero-electrodynamic vortex depend on the structure of the electromagnetic field.

In the present article I consider a vortical formation consisting of an aerodynamic vortex and an electromagnetic vortex in interaction with each other. The intensity of the aerodynamic vortex is so great that a highly rarefied region is formed in its center. Most of the energy of the electromagnetic vortex is concentrated in this region. Thus, in the present article I consider the case in which the confinement of the surrounding gas is due basically to the vortical motion of the particles, while the interaction of the high-frequency field with the medium leads to ionization of the gas and screening of the field. Note that under these conditions, the high-frequency field loses much less energy per unit time than in the case of self-localization (or self-focusing) of the electromagnetic field, in which expulsion of the plasma from the waveguide region is due to interaction of the electrons with the spatially inhomogeneous high-frequency electric field of the wave (see, e.g., Refs. 1–4). Note that the spatial dimensions of the described ring vortex depend on the structure of the electromagnetic field. This makes it possible to obtain an intense aerodynamic vortex with a prescribed size of the vortex ring.

Consider the case in which a wave propagates in the toroidal waveguide which is formed inside the ring vortex along a helical line. This means that the wave vector has components along the major and minor circumferences of the torus. Assuming that the radius of the vortex ring (the major radius of the torus) is significantly greater than the half-width (minor radius of the torus), to investigate the structure of the electromagnetic field of the vortex we will use a local cylindrical coordinate system. This being the case, we represent the electric field of the wave in the form

$$\mathbf{E}(r,\theta,z,t)=\mathbf{E}(r)\cos(k_z z+\kappa\theta-\omega t). \quad (1)$$

Note that the quantities k_z and κ must make $\mathbf{E}(r,\theta,z,t)$ a single-valued function of position:

$$\mathbf{E}(r,\theta,z,t)=\mathbf{E}(r,\theta+2\pi,z,t),$$

$$\mathbf{E}(r,\theta,z,t)=\mathbf{E}(r,\theta,z+2\pi R_0,t),$$

where R_0 is the radius of the vortex ring. We are interested only in those solutions of the electrodynamic equations in which the amplitude of the electric field $\mathbf{E}(r)$ tends to zero in the inner region of the electromagnetic vortex as $r\to 0$, and in the outer region as $r\to\infty$. We will assume in the case under consideration that the radial component of the electric field is significantly smaller than E_z and E_θ. This implies that $\kappa^{1/3}\gg 1$.

Note that a momentum flux due to the spatial inhomogeneity of the electromagnetic field issues from the localization region of the field. The magnitude of this flux is determined by the Maxwell stress tensor (see, e.g., Ref. 5). When the momentum flux of the electromagnetic field is balanced by the pressure of the surrounding plasma, the maximum value of the density of the localized electromagnetic field is proportional to the pressure of the gas at infinity.[6] In Ref. 4 I considered an electromagnetic vortex under conditions in which there is partial charge separation: uncompensated positive charge in the inner region of the electromagnetic vortex, and excess negative charge in the outer region. In this case the momentum flux due to the inhomogeneity of the electric field is balanced not only by the thermal pressure of the plasma, but also by Coulomb attraction between the uncompensated charges in the inner and outer regions of the electromagnetic vortex. For a large enough number of uncompensated charges, the energy of the localized field is proportional to the square of the number of uncompensated charges. It is important to mention that the presence of uncompensated electrons does not lead to additional dissipation of energy of the high-frequency field in the plasma, since these electrons penetrate deeper into the waveguide region than the particles of the quasi-neutral plasma, and are almost completely isolated from the other components of the plasma.

The structure of the vortex can be examined on the basis of the stationary equations of electrodynamics and the equation of charge balance for the ionized gas, neglecting dissipation of energy in the high-frequency field and energy losses of the vortex itself due to the viscosity of the gas:

$$\frac{d^2\mathbf{E}(r)}{dr^2}+\frac{1}{r}\frac{d\mathbf{E}(r)}{dr}-\frac{\kappa^2}{r^2}\mathbf{E}(r)=-\frac{\omega^2}{c^2}\left[1-\left(\frac{k_z c}{\omega}\right)^2-\frac{4\pi e^2}{m\omega^2}n_e\right]\mathbf{E}(r), \quad (2)$$

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\varphi}{dr}\right)=4\pi e(n_e-n_i), \quad (3)$$

$$T_e\frac{d}{dr}n_e-en_e\frac{d}{dr}\varphi+n_e\frac{e^2}{4m\omega^2}\frac{dE^2(r)}{dr}=0, \quad (4)$$

$$T_i \frac{d}{dr} n_i + e n_i \frac{d}{dr} \varphi - \frac{M_i n_i v^2}{r} = 0, \tag{5}$$

where φ is the potential of the constant electric field that arises due to charge separation in the plasma and due to the presence in the volume of uncompensated charges, and v is the velocity of vortical motion of the gas.

In the region where the plasma is highly rarefied and its influence on the spatial distribution of the electromagnetic field can be neglected, we have

$$E(r) \sim J_\kappa \left(r \frac{\omega}{c} \sqrt{1 - \left(\frac{k_2 c}{\omega} \right)^2} \right). \tag{6}$$

The amplitude of the electric field reaches its maximum value at the point corresponding to the first maximum of the Bessel function,

$$r_0 \approx \frac{c}{(\omega^2 - k_z^2 c^2)^{1/2}} (\kappa + 0.8 \kappa^{1/3}). \tag{7}$$

We assume that the field amplitude has only one extremum and that $\kappa^{1/3} \gg 1$. Then, we have for the effective width of the region where the electromagnetic field is concentrated

$$\Delta r \approx \frac{c}{(\omega^2 - k_z^2 c^2)^{1/2}} \kappa^{1/3}. \tag{8}$$

Note that under the present conditions, the characteristic size of a spatial inhomogeneity in the field amplitude is much smaller than r_0, but significantly larger than the wavelength: $r_0 \gg \Delta r \gg \lambda$. It then follows from the equation div $\mathbf{E}=0$ that

$$E_\theta = -\frac{k_z}{k_\theta} E_z, \tag{9}$$

where $k_\theta = \kappa / r_0$.

In the case under consideration, the spatial distribution of the density of the quasi-neutral plasma in the vicinity of the waveguide region is determined mainly by the vortical motion of the gas. Neglecting the last term on the right-hand side of Eq. (4), which characterizes the pressure of the high-frequency field, and allowing for the quasi-neutrality of the plasma ($n_e \approx n_i$), we have from Eqs. (4) and (5) (see, e.g., Ref. 7)

$$(T_e + T_i) \frac{dn_e}{dr} + \frac{M_i n_e v^2}{r} = 0. \tag{10}$$

Hence, in the isothermal case, for the typical size of a spatial inhomogeneity of the plasma density, we obtain

$$\delta r \approx r \frac{v_i^2}{v^2}, \tag{11}$$

where v_i is the thermal velocity of the ion. We assume that near the waveguide region, the vortical velocity of the gas is large enough that $\delta r \ll r_0$. In this case, in the skin-layer region, the high-frequency field supports ionization processes in the gas. Farther from the waveguide region, the plasma density decreases as a result of recombination and diffusion of the plasma into the surrounding neutral gas.

There is a condition under which the pressure of the electromagnetic field due to its spatial inhomogeneity has an insignificant effect on the spatial distribution of the plasma density. When there are no uncompensated charges in either the inner or outer region of the electromagnetic vortex, or their number is so small that the pressure of the electromagnetic field is basically balanced by the thermal pressure of the plasma, we have

$$E_0^2 \ll 16\pi \left(\frac{c}{\omega \Delta r} \right)^2 n_e^* T_e^*, \tag{12}$$

where $E_0 = E(r_0)$ is the maximum value of the electric field amplitude, and n_e^* and T_e^* are the density and temperature of the plasma near the waveguide region, where the high-frequency field is screened by the plasma.

Condition (12) constrains the energy of the electromagnetic field. We are also interested in the case in which the electromagnetic energy is high enough to support the plasma as long as a hydrodynamic vortex exists whose decay is due to the viscosity of the gas. In this regard, note that the vortex can contain much more electromagnetic energy than the limiting value in relation (12) when a positively charged dielectric is confined within the inner region of the electromagnetic field and there are uncompensated electrons in the outer region. The pressure of the high-frequency field is then largely balanced by the Coulomb attraction of the electrons to the dielectric. Equations (2)–(4) allow us to determine the electric field amplitude in this case. First we multiply Eq. (2) by dE/dr and then integrate over r from r_0 to ∞. Invoking the Poisson equation (3) and the equation of charge balance to the electrons (4), we obtain

$$E_0 \approx 2 \frac{\Delta r}{r_0} \frac{e\omega}{c} N, \tag{13}$$

where N is the number of uncompensated electrons per unit length in the z direction. In this case the magnitude of the uncompensated charge and, consequently, the energy density of the electromagnetic field are limited by the breakdown threshold of the dielectric.

In the case under consideration, the spatial distribution of the density of uncompensated electrons and of the electric field amplitude in the waveguide region have qualitatively the same form as in the case investigated in detail in Ref. 4, where it was assumed that the quasi-neutral plasma surrounding the electromagnetic vortex has a high electron temperature and is confined by the pressure of the high-frequency field, and that the wave vector is directed along the minor circumference of the torus ($k_z=0$).

So far, we have discussed the equilibrium conditions for fields and particles in a two-dimensional cylindrical plasma. Now we take into account the curvature of the electric and magnetic field lines associated with the fact that the waveguide formed in the ring vortex is toroidal. Employing the Maxwell stress tensor of the electromagnetic field, we obtain the following formula for the force acting along the major radius of the torus per unit length of the waveguide:

$$F_R = \frac{1}{8\pi} \int \frac{1}{R} (E_\theta^2 \cos^2\theta + B^2 \sin^2\theta - E_z^2) dS, \tag{14}$$

where the integral is taken over the entire transverse cross section of the wave guide and R is the distance from the axis of the torus to the given point. Under the conditions under consideration, the major radius of the torus is significantly greater than the minor radius, $R_0 \gg r_0$, so we can set $R=R_0$ in the integrand in Eq. (14). Noting that $\Delta r \gg \lambda$ and employing relation (9), with the help of formula (14) we obtain

$$F_R=\frac{r_0\Delta_r}{8R_0}\overline{E^2}\left(\frac{k_\theta c}{\omega}\right)^2\left[2\left(\frac{k_z}{k_\theta}\right)^2-1\right], \tag{15}$$

where the bar above E^2 denotes averaging over r in the region Δr.

From (15), it follows that the vortex is in the equilibrium state ($E_k=0$) when

$$k_z=k_\theta/\sqrt{2}. \tag{16}$$

We now show that (16) characterizes stable equilibrium with respect to the radius R_0 of the ring vortex. Note that any change in R_0 should leave the aerodynamic energy of the vortex

$$W=\frac{1}{2}\int \rho v^2 dV, \tag{17}$$

unchanged, where ρ is the density of the gas and the integral is taken over the entire volume occupied by the vortex.

For simplicity, we assume that the density of the gas has a sharp boundary, i.e., we assume that it falls with decreasing r only near the waveguide region. Assuming that $R_0 \gg r_0 \gg \Delta r$, we then obtain the following estimate:

$$W=2\pi^2 R_0 \rho_\infty \int_{r_0}^{R_0} r v^2(r)dr=\text{const.} \tag{18}$$

Formula (11) gives $v(r_0 \propto r_0^{1/2}$ for the vortical velocity of the gas near the waveguide region. Noting that $v(r)\propto 1/r$, we obtain $v^2 \propto r_0^3/r^2$. With the help of (18), we find that

$$R_0 r_0^3 \ln\frac{R_0}{r_0}=\text{const.}$$

Hence, for $\ln(R_0/r_0)\gg 1$ we have

$$\frac{\delta r_0}{r_0}=-\frac{1}{3}\frac{\delta R_0}{R_0}. \tag{19}$$

Now observe that a change in the parameters of the toroidal waveguide should leave the number of waves of the electromagnetic field unchanged along the major and minor circumferences of the torus, i.e., $k_\theta \sim 1/r_0$ and $k_z \sim 1/R_0$. This being the case, with the help of relation (19) we obtain

$$\frac{\delta k_z}{k_z}=-\frac{\delta R_0}{R_0},\quad \frac{\delta k_\theta}{k_\theta}=\frac{1}{3}\frac{\delta R_0}{R_0}. \tag{20}$$

Employing (20) together with (15) near the equilibrium point, which is determined by condition (16), we find that

$$\Delta F_R=-\frac{r_0\Delta r}{12R_0}\overline{E^2}\left[1+5\left(\frac{k_z c}{\omega}\right)^2\right]\frac{\delta R_0}{R}. \tag{21}$$

Thus we have shown that the equilibrium is stable: any change in the radius of the ring vortex results in a force returning the system to the equilibrium state.

Finally, note that an aero-electrodynamic vortex can arise in the interaction of high-power electromagnetic radiation with a target or, in the case of a pulsed electric discharge in a gas, if lasing takes place simultaneously with the formation of the vortex flux of the gas.

[1] V. I. Talanov, Izv. Vyssh. Uchelon. Zaved., (Radiofizika) **7**, 564 (1964).
[2] A. G. Litvak, in *Problems in Plasma Theory* [in Russian], Atomizdat, Moscow (1980), No. 10, p. 74.
[3] Yu. R. Alanakyan, Zh. Éksp. Teor. Fiz. **55**, 1338 (1968) [Sov. Phys. JETP **28**, 700 (1969)].
[4] Yu. R. Alanakyan, Zh. Éksp. Teor. Fiz. **105**, 601 (1994) [JETP **78**, 320 (1994)].
[5] I. E. Tamm, *Foundations of the Theory of Electricity*, Nauka, Moscow (1976).
[6] T. F. Volkov, in *Plasma Physics and the Problem of Controlled Thermonuclear Reactions* [in Russian], Izdatel'stvo Akad. Nauk SSSR, Moscow (1958), Vol. 3, p. 336.
[7] N. Ya. Fabrikant, *Aerodynamics*, Nauka, Moscow (1964).

Translated by Paul F. Schippnick