Longitudinal structure function F_L as function of F_2 and $dF_2/d\ln Q^2$ at small x

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A formula for extracting the longitudinal deep inelastic structure function F_L from the transverse structure function F_2 and its derivative $dF_2/d\ln Q^2$ at small x in the leading order of perturbation theory is derived. A detailed analysis is given for new data of the H1 group from HERA. The values of F_L and the deep inelastic scattering structure function ratio R are found for $10^{-3}\leqslant x\leqslant 2\cdot 10^{-2}$ and $Q^2=20$ GeV^2. © *1995 American Institute of Physics.*

For experimental studies of hadron-hadron processes on the new powerful LHC collider it is necessary to know in detail the values of the parton (quark and gluon) distributions of nucleons, especially at small values of x. The basic information on the quark structure of nucleons is extracted from the process of deep inelastic lepton-hadron scattering. Its differential cross-section has the form

$$\frac{d^2\sigma}{dxdy}=\frac{2\pi\alpha_{em}^2}{xQ^4}\,[(1-y+y^2/2)F_2(x,Q^2)-(y^2/2)F_L(x,Q^2)],$$

where $F_2(x,Q^2)$ and $F_L(x,Q^2)$ are the transverse and longitudinal structure functions, respectively. The longitudinal structure function $F_L(x,Q^2)$ and the ratio

$$R(x,Q^2)=\frac{F_L(x,Q^2)}{F_2(x,Q^2)-F_L(x,Q^2)} \tag{1}$$

are good QCD characteristics because they are equal to zero in the parton model. Moreover, the value of the structure function F_2, whose data are usually deduced from the experiment, depends essentially on the corresponding values of F_L (or R). We note that the value of the structure function F_L (or the ratio R) is very important in the case of polarized structure functions, which are deduced from the measured asymmetry of the cross-sections of polarized leptons and nucleons.

The modern deep inelastic scattering experimental data (see the review of Roberts and Whalley[1)]) are not accurate enough to determine $F_L(x,Q^2)$ [or $R(x,Q^2)$]. In addition, at small values of x the data for the structure function F_L are not yet available, as they require a rather involved procedure (see, e.g., Cooper-Sarkar *et al.*[2)]).

In the present paper we study the behavior of $F_L(x,Q^2)$ at small values of x, using the new H1 data and the method[3] of replacement of the Mellin convolution by ordinary products.

We introduce the standard parametrizations of the singlet quark $s(x,Q^2)$ and gluon $g(x,Q^2)$ parton distribution[1)] (see, Martin *et al.*[4)])

$$\begin{aligned} s(x,Q^2)&=A_s x^{-\delta}(1-x)^{\nu_s}(1+\epsilon_s\sqrt{x}+\gamma_s x)\\ &\equiv x^{-\delta}\tilde{s}(x,Q^2),\\ g(x,Q^2)&=A_g x^{-\delta}(1-x)^{\nu_g}(1+\gamma_g x)\equiv x^{-\delta}\tilde{g}(x,Q^2), \end{aligned} \tag{2}$$

with the Q^2 dependent parameters in the r.h.s. We use a similar small-x behavior for the gluon and sea quark parton distribution that follows from the form of the kernel of Gribov–Lipatov–Altarelli–Parisi (GLAP) equation (see also the recent fits of experimental data[4)]).

The "conventional" choice is $\delta=0$, which leads to nonsingular behavior of the parton distribution (see the D_0' fit in Ref. 4) when $x\to 0$. Another value, $\delta\sim(1/2)$ was obtained[5] as the sum of the leading powers of $\ln(1/x)$ in all orders of perturbation theory (see also the D'_- fit[4)]). Recent NMC data[6] agree with the small values of δ. This choice corresponds to the present experimental data for pp and $\bar{p}p$ total cross-sections (see Ref. 7) and the model of the Landshoff and Nachmann pomeron[8] with the exchange of a pair of nonperturbative gluons, yielding $\delta=0.086$. However, the new H1 data[9] from HERA prefer $\delta\sim 0.5$. Using the GLAP equation some attempts[10] were made to obtain agreement between the results of NMC at small Q^2 and the H1 group at large Q^2.

1. Assuming Regge-like behavior for the gluon and singlet quark parton distribution [see Eq. (2)], we obtain the following equations for the longitudinal structure function F_L and for the Q^2 derivative of the structure function F_2[2)]:

$$\begin{aligned} F_L(x,Q^2)&=\delta_s x^{-\delta}\sum_{p=s,g}(\bar{B}^p_{L,1+\delta}(\alpha)\tilde{p}(0,Q^2)\\ &\quad+\bar{B}^p_{L,\delta}(\alpha)x\tilde{p}'(0,Q^2))+O(x^{2-\delta}),\\ \frac{dF_2(x,Q^2)}{d\ln Q^2}&=-\frac{\alpha(Q^2)}{2}\,\delta_s x^{-\delta}\sum_{p=s,g}(\tilde{\gamma}^{1+\delta}_{sp}(\alpha)\tilde{p}(0,Q^2)\\ &\quad+\tilde{\gamma}^{\delta}_{sp}(\alpha)x\tilde{p}'(0,Q^2))+O(x^{2-\delta}), \end{aligned} \tag{3}$$

where $\bar{B}^p_{L,\eta}(\alpha)$ and $\tilde{\gamma}^\eta_{sp}(\alpha)$ are the longitudinal Wilson coefficients and combinations of the transverse Wilson coefficients and anomalous dimensions, respectively, of the η "moment" of the Wilson operators (i.e., the corresponding variables extended from integer values of the argument to the noninteger values), and

$$\tilde{p}'(0,Q^2)\equiv\frac{d}{dx}\tilde{p}(x,Q^2)\quad\text{at } x=0.$$

Here δ_s is the coefficient which depends on the process and on the number of quarks f: $\delta_s=5/18$ for the ep collision, when $f=4$.

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Further, we restrict the analysis to the leading order of perturbation theory [where $F_2(x,Q^2)\equiv\delta_s s(x,Q^2)$, the $\tilde{B}^p_{L,\eta}(\alpha)$ are the one-loop longitudinal Wilson coefficients $\alpha(Q^2)B^p_{L,\eta}$ and the $\tilde{\gamma}^\eta_{sp}(\alpha)$ are equal to the leading order anomalous dimensions γ^η_{sp}] and to the case δ=0.5 which corresponds to a Lipatov pomeron, which is supported by the H1 data. Taking into account the case δ=0, which corresponds to the standard pomeron and the extension of this analysis to the next order of perturbation theory require additional investigations.

For the gluon parts from the r.h.s. of Eq. (3) to $O(x^2)$ we have the form

$$B^g_{L,3/2}\tilde{g}(x/w_g,Q^2),\quad w_g=B^g_{L,3/2}/B^g_{L,1/2},$$
$$\gamma^{3/2}_{sg}\tilde{g}(x/\xi_{sg},Q^2),\quad \xi_{sg}=\gamma^{3/2}_{sg}/\gamma^{1/2}_{sg}. \tag{4}$$

Equation (3) may be represented in the form

$$F_L(x,Q^2)=\alpha(Q^2)\delta_s x^{-1/2}(B^g_{L,3/2}\tilde{g}(x/w_g,Q^2)+B^s_{L,3/2}\tilde{s}(0,Q^2)+B^s_{L,1/2}x\tilde{s}'(0,Q^2))+O(x^{3/2}), \tag{5}$$

$$\frac{dF_2(x,Q^2)}{d\ln Q^2}=-\frac{\alpha(Q^2)}{2}\delta_s x^{-1/2}(\gamma^{3/2}_{sg}\tilde{g}(x/\xi_{sg},Q^2)+\gamma^{3/2}_{ss}\tilde{s}(0,Q^2)+\gamma^{1/2}_{ss}x\tilde{p}'(0,Q^2))+O(x^{3/2}). \tag{6}$$

Extracting the gluon distribution from Eq. (6) and substituting it into Eq. (5), we obtain the equation

$$F_L(x,Q^2)=-2t_g^{1/2}\frac{dF_2(x/r_g,Q^2)}{d\ln Q^2}+\alpha(Q^2)\delta_s x^{-1/2}(\tilde{B}^s_{L,3/2}\tilde{s}(0,Q^2)+\tilde{B}^s_{L,1/2}x\tilde{s}'(0,Q^2))+O(x^{3/2}), \tag{7}$$

where

$$t_g=\frac{B^q_{L,3/2}B^g_{L,1/2}}{\gamma^{3/2}_{sg}\gamma^{1/2}_{sg}},\quad r_g=\frac{B^g_{L,1/2}\gamma^{3/2}_{sg}}{B^g_{L,3/2}\gamma^{1/2}_{sg}},$$

$$\tilde{B}^s_{L,\eta}=B^s_{L,\eta}-B^g_{L,\eta}\frac{\gamma^\eta_{ss}}{\gamma^\eta_{sg}}.$$

By analogy with Eq. (4) for the quark part from the r.h.s. of Eq. (7), to $O(x^2)$ we have

$$\tilde{B}^s_{L,3/2}\tilde{s}(x/r_s,Q^2),\quad r_s=\tilde{B}^s_{L,3/2}/\tilde{B}^s_{L,1/2},$$

which leads to the following equation for the longitudinal structure function

$$F_L(x,Q^2)=-2t_g^{1/2}\frac{dF_2(x/r_g,Q^2)}{d\ln Q^2}+\alpha(Q^2)t_s^{1/2}F_2(x/r_s,Q^2)+O(x^{3/2}), \tag{8}$$

where $t_s=\tilde{B}^s_{L,1/2}\tilde{B}^s_{L,3/2}$.

Using the exact values of the Wilson coefficients and the anomalous dimensions, we obtain

$$F_L(x,Q^2)=\frac{24}{\sqrt{23\cdot 33}}\left(\frac{dF_2\left(\frac{23}{33}x,Q^2\right)}{d\ln Q^2}+\frac{8}{3}\alpha(Q^2)\sqrt{\left(\frac{13}{3}-4\ln 2\right)(3-4\ln 2)}F_2\times\left(\frac{23}{33}\frac{3-4\ln 2}{\frac{13}{3}-4\ln 2}x,Q^2\right)\right)+O(x^{3/2})$$
$$\approx 0.87\frac{dF_2(0.70x,Q^2)}{d\ln Q^2}+1.39\alpha(Q^2)F_2(0.10x,Q^2)+O(x^{3/2}). \tag{9}$$

Note that the arguments of the transverse structure function and its derivative from the r.h.s. of Eq. (9) are different. This is not convenient because the experimental data are known (see Refs. 9, 11) for both variables in a similar range of x. To overcome this problem we note that to $O(x^2)$ the quark part from the r.h.s. of Eq. (7) may be represented as a sum of two terms like Eq. (4), with some coefficients and shifts of arguments. Choosing the shifts as 1 and r_g^{-1}, we can write the following representation for the quark part:

$$c_1\tilde{s}(x,Q^2)+c_2\tilde{s}(x/\xi_{sg},Q^2),$$

where

$$c_1=\frac{\tilde{B}^s_{L,3/2}\tilde{B}^g_{L,1/2}-\tilde{B}^s_{L,1/2}\tilde{B}^g_{L,3/2}}{\tilde{B}^g_{L,1/2}-\tilde{B}^g_{L,3/2}},$$

$$c_2=\tilde{B}^g_{L,3/2}\frac{\tilde{B}^s_{L,1/2}-\tilde{B}^s_{L,3/2}}{\tilde{B}^g_{L,1/2}-\tilde{B}^g_{L,3/2}} \tag{10}$$

with $\tilde{B}^g_{L,\eta}=B^g_{L,\eta}/\gamma^\eta_{sg}$.

Using the exact values of the Wilson coefficients and the anomalous dimensions, we find the following expression from Eqs. (7)–(10):

$$F_L(x,Q^2)=\frac{24}{\sqrt{23*33}}\frac{dF_2\left(\frac{23}{33}x,Q^2\right)}{d\ln Q^2}+\frac{128}{15}\alpha(Q^2)\times\left(\frac{6}{\sqrt{23\cdot 33}}\left(\frac{8}{5}-\ln 2\right)F_2\left(\frac{23}{33}x,Q^2\right)-F_2(x,Q^2)\right)+O(x^{3/2})$$
$$\approx 0.87\frac{dF_2(0.70x,Q^2)}{d\ln Q^2}+10.29\alpha(Q^2)\times(F_2(0.70x,Q^2)-0.83F_2(x,Q^2))+O(x^{3/2}). \tag{11}$$

2. Let us analyze the predictions inspired by Eq. (11). We use the values of the structure function F_2 and its Q^2

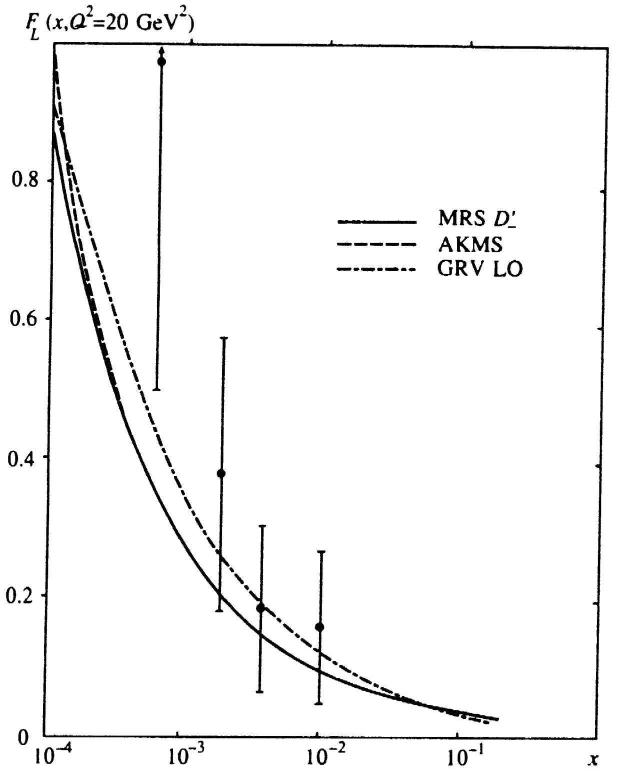

FIG. 1. The longitudinal structure function $F_L(x,Q^2)$ at $Q^2=20\ \mathrm{GeV}^2$. The black circles indicate the values extracted with the help of Eq. (11). Only the statistical errors are presented. The curves represent different parametrizations of $F_L(x,Q^2)$.[4,12,13] The GRV curve is the leading-order parametrization, and the MR parametrization is given in the deep inelastic scattering renormalization scheme. The AKMS curve is the solution of the Lipatov equation and is used at $Q^2=30\ \mathrm{GeV}^2$.

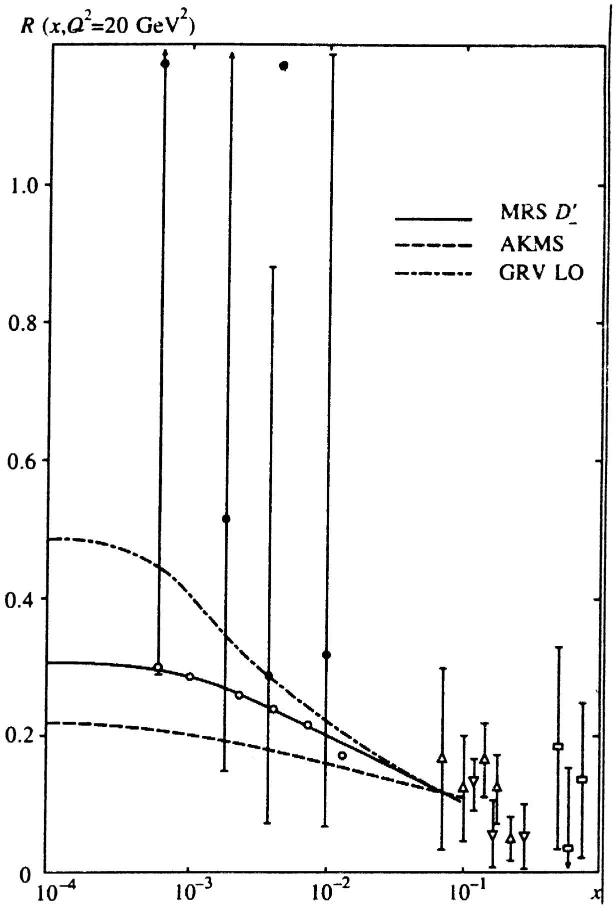

FIG. 2. The same as in Fig. 1, but the $R(x,Q^2)$ values are extracted with the help of Eq. (1). The white circles indicate the $R(x,Q^2)$ values used by H1 group.[11] The symbols Δ, ∇ and $\square$ are represented in the BCDMS (Ref. 15), CDHSW (Ref. 16) and SLAC (Ref. 17) data, respectively

derivative found by the H1 collaboration (see Refs. 9 and 11, respectively). The extracted $F_L(x,Q^2)$ and $R(x,Q^2)$ values are shown in Fig. 1 and Fig. 2, respectively. These values are compared with theoretical predictions. As in Ref. 11, we used the hypothesis concerning the approximate linear $\ln Q^2$ dependence of F_2 at small x as well as the value of QCD scale $\Lambda_{\overline{MS}}^{f=4} = 200$ MeV. As can be seen in the above figures, we found the $R(x,Q^2)$ values to be close to the GRV predictions[12] only.[3)] The predictions of other groups (see Refs. 4, 13) lead to smaller $R(x,Q^2)$ values at $x\sim10^{-4}$. Note that all groups also predict smaller $F_L(x,Q^2)$ values for $x\leqslant10^{-3}$. In addition, as can be seen from Fig. 2, there is also disagreement between the $R(x,Q^2)$ values, which were used by the H1 group in Ref. 11, and the values following from Eqs. (1) and (11). This is due to the very large values of $d(F_2(x,Q^2))/(d\ln Q^2)$ at $x\sim10^{-4}$ found in Ref. 11, which show themselves also in comparison between the gluon distribution at $x\leqslant10^{-3}$ and the corresponding theoretical predictions (see Refs. 11 and 14). This disagreement may be overcome by including the next-to-leading order corrections to our equations, and this requires additional investigations.

Note that the basic contribution to $F_L(x,Q^2)$ [and $R(x,Q^2)$] is given by the $d[F_2(x,Q^2)]/(d\ln Q^2)$ part. However, the contribution from $F_2(x,Q^2)$ increases the values of $F_L(x,Q^2)$ from several percent at $x\approx10^{-4}$ to 30% at $x\approx2\cdot10^{-2}$.

In summary, we have presented Eqs. (9) and (11) to extract the longitudinal structure function F_L at small x from the structure function F_2 and its Q^2 derivative. These equations make it possible to determine F_L (and R) indirectly. This is important since the direct extraction of F_L from experimental data is a cumbersome procedure (see Ref. 2). Moreover, the fulfillment of Eqs. (9) and (11) in deep inelastic scattering experimental data is a check of perturbative QCD at small values of x. The addition of the next-to-leading contribution to Eqs. (9) and (11) can be done by analogy with Ref. 3.

Equation (11) was used in the analysis of H1 data from HERA. The values of $F_L(x,Q^2)$ and $R(x,Q^2)$ were found for small values of x ($10^{-4}\leqslant x\leqslant2\cdot10^{-2}$). The extension of this analysis to the case $\delta\sim0$, which is in agreement with the NMC data and the evaluation of the next-to-leading order contributions, requires additional investigations.

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1)We use the parton distribution multiplied by x and neglect the nonsinglet quark distribution at small x.

2)In contrast to the standard case, below we use $\alpha(Q^2)=\alpha_s(Q^2)/4\pi$.

3)To find the values of $F_L(x,Q^2)$ predicted by the GRV approach, we use the values of $g(x,Q^2)$ and $F_2(x,Q^2)$ from Ref. 12 and then the equation from Ref. 3 to extract F_L from g and F_2. The values of $R(x,Q^2)$ are obtained from the F_L and F_2 via Eq. (1).

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