Giant quantum oscillations of the surface impedance of a compensated metal

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The influence of the quantization of the energy of electrons on Doppler-shifted cyclotron resonance in a compensated metal in an oblique magnetic field has been studied theoretically. Analytic expressions for the elements of the surface impedance tensor of a solid metal under the conditions of strong anisotropy of the transverse conductivity have been obtained for a simple model of the Fermi surface. It has been discovered that the quantum oscillations of the nonlocal doppleron decay caused by magnetic Landau decay are displayed very strongly. It has been shown that these oscillations are large at frequencies comparable to the frequency of electron collisions.

1. INTRODUCTION

Numerous investigations have been devoted to the study of Doppler-shifted cyclotron resonance in metals. Various manifestations of Doppler-shifted cyclotron resonance, particularly in moderate magnetic fields, in which the motion of the conduction electrons in a magnetic field is described satisfactorily by classical means, were studied in these investigations. Naberezhnykh *et al.*[1] investigated the behavior of the radio-frequency impedance of a compensated metal in strong magnetic fields, in which quantum oscillations of the magnetic susceptibility (the de Haas–van Alphen effect) and of the static conductivity (the Shubnikov–de Haas effect) appear. The amplitudes of these oscillations are small (they are inversely proportional to $N^{3/2}$ and $N^{1/2}$, respectively, where N is the number of Landau levels on the Fermi surface of the metal). The latter is attributed to the fact that the electrons in all quantum levels on the Fermi surface contribute to the magnetic susceptibility and the static conductivity, and the oscillations of these quantities are associated with the relative variation in the total number of levels N, which is very large in typical metals. At the same time, effects caused by electrons with a definite longitudinal velocity are known. For example, the absorption of ultrasonic or electromagnetic waves in metals can be effected by electrons moving in phase with these waves. When the magnetic field H varies under the conditions of quantization of the transverse energy of the electrons, the discrete values of the longitudinal velocity on the Fermi surface vary and alternately pass through the value of the phase velocity of the wave. As a result, the efficiency of the absorption of the wave varies sharply, and the absorption coefficient of ultrasound undergoes giant quantum oscillations.[2] The nonlocal decay of helicons (magnetic Landau decay) in uncompensated metals undergoes similar oscillations.[3] The possibility of such oscillations in compensated metals, in which helicons cannot exist, has not been discussed in the literature.

In this paper it will be shown that quantum oscillations of magnetic Landau decay are also possible in compensated metals. Magnetic Landau decay occurs when a constant magnetic field **H** is directed at an angle to the direction of propagation of the wave in the metal or at an angle to the symmetry axis of the Fermi surface. This decay influences the distribution of the radio-frequency field and the surface impedance of the metal in two ways. First, it makes a contribution to the imaginary part of the Doppler root of the dispersion equation and thereby causes nonlocal doppleron decay in the range of magnetic fields in which this propagating mode exists. Second, magnetic Landau decay determines the skin-effect root for one of the linear polarizations of the radio-frequency field.[4] In this case the corresponding quantities determining the spatial distribution of the wave field can undergo quantum oscillations. The field of a doppleron is circularly polarized, and the fields of the skin components are linearly polarized. Therefore, in the case of an oblique magnetic field, magnetic Landau decay influences the impedance in a complex manner, and very lengthy calculations are required to find the elements of the impedance tensor. To simplify these calculations as much as possible, we investigate the quantum oscillations of the magnetic Landau decay and surface impedance of a compensated metal using a simple model, in which the electron Fermi surface has the form of a parabolic lens,[5] and the contribution of holes to the conductivity can be taken into account in a local approximation.

2. MODEL OF THE FERMI SURFACE AND NONLOCAL CONDUCTIVITY

Let the electronic part of the Fermi surface of a metal have the form of a parabolic lens (Fig. 1), and let the spectrum of electrons have the form

$$\varepsilon(\mathbf{p})=\frac{p_x^2+p_{y'}^2}{2m}+v|p_{z'}|,\quad |p_{z'}|\leq p_1, \tag{1}$$

$$\varepsilon(\mathbf{p})=\varepsilon_F\equiv p_1 v, \tag{2}$$

where m, v, and p_1 are constants with the dimensions of mass, velocity, and momentum, respectively, and the $p_{z'}$ axis of the $p_x p_{y'} p_{z'}$ coordinate system is parallel to the axis of the lens. We take a right cylinder with axis parallel to the $p_{z'}$ axis as the hole Fermi surface. We shall assume that a normal to the surface of the metal (the wave propagation vector **k**) is parallel to the z' axis and that the constant magnetic field **H** (the z axis) is inclined at an angle $\theta\ll 1$ to the z' axis.

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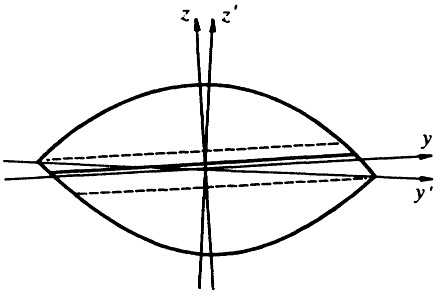

FIG. 1. Orientation of the magnetic field **H** and the coordinate axis relative to a parabolic lens. The layer of p_z=const sections passing through the edge of the lens is bounded by the dashed straight lines.

Let us determine the parameters of motion of electrons in an oblique magnetic field. It can be seen from Fig. 1 that in the case under consideration there are two types of sections of the lens formed by p_z=const planes: sections which do not pass through the edge of the lens ($|p_z|>p\sin\theta$, where $p=\sqrt{2mvp_1}$ is the radius of the lens) and sections which intersect the edge ($|p_z|<p\sin\theta$). The sections of the first type are ellipses, and the longitudinal velocity of electrons in these sections averaged over a cyclotron period does not depend on p_z:

$$\bar{v}_z=-\frac{1}{2\pi m}\frac{\partial S(p_z)}{\partial p_z}=\pm\frac{v}{\cos^2\theta},\quad |p_z|>p\sin\theta$$

(S is the area of the section of the lens formed by the p_z=const plane). Therefore, the electrons of all the sections of the first type contribute to the Doppler-shifted cyclotron resonance and do not contribute to the magnetic Landau decay (their longitudinal velocity is much greater than the phase velocity of the wave). The orbits of the second type consist of parts put together from two different ellipses obtained as a result of sectioning of the upper and lower parabolic portions of the lens. The mean value of the longitudinal velocity of electrons in these sections is given by the formula

$$\bar{v}_z=\frac{v}{\pi\cos^2\theta}\left[\arcsin\left(\frac{p_z}{P_+\sin\theta}-\frac{mv}{P_+}\,\mathrm{tg}\,\theta\right)+\arcsin\left(\frac{p_z}{P_-\sin\theta}+\frac{mv}{P_-}\,\mathrm{tg}\,\theta\right)\right],$$

$$|p_z|<p\sin\theta,\tag{3}$$

where

$$P_\pm=\left[2mv\left(p_z\mp\frac{p_1}{\cos\theta}\right)+(mv\,\mathrm{tg}\,\theta)^2\right]^{1/2}.\tag{4}$$

In the sections of the second type $\bar{v}_z$ varies from $-v/\cos^2\theta$ to $v/\cos^2\theta$. Near the central section of the lens ($|p_z|\ll p\sin\theta$)

$$\bar{v}_z\approx\frac{1}{\pi\cos^2\theta}\left(\frac{2v}{mp_1\sin^2\theta}\right)^{1/2}p_z.\tag{5}$$

Among these electrons there must be electrons which move on the average in phase with the wave and cause magnetic Landau decay. According to Ref. 6, the corresponding part of the nonlocal conductivity tensor $\sigma_{\alpha\beta}^{(L)}$ has the form

$$\sigma_{\alpha\beta}^{(L)}=\frac{e^2m_cm_\parallel}{2\pi\hbar^3|k_z|}W_\alpha W_\beta^*Q(k,\omega,\mathbf{H},T),\tag{6}$$

$$\mathbf{W}=\frac{1}{2\pi}\int_0^{2\pi}d\varphi\mathbf{v}(\varphi)\times\exp\left[-\frac{i}{\omega_c}\int_0^\varphi[\mathbf{kv}(\varphi')]d\varphi'\right]\Bigg|_{p_z=p_0},\tag{7}$$

$$\omega_c=\frac{eH}{m_cc},\quad m_\parallel=\left|\frac{\partial\bar{v}_z}{\partial p_z}\right|^{-1}_{p_z=p_0},\tag{8}$$

where ω is the frequency of the wave, m_c is the electron cyclotron mass, $m_\parallel$ is the longitudinal mass, and p_0 is the value of p_z at which the mean velocity of the electron coincides with the phase velocity of the wave:

$$k_z\bar{v}_z(p_0)=\omega.$$

The function Q describes the effect of quantization of the electron energy on the dissipative part of the nonlocal conductivity. According to Ref. 7, it is defined by the expression

$$Q=\frac{\hbar\omega_c|k_z|}{4k_BTm_\parallel}\int_{-\infty}^{\infty}dp_zD\left(\omega-\frac{\hbar k_z^2}{2m_\parallel}-k_z\bar{v}_z\right)\times\sum_{n=0}^{\infty}\mathrm{ch}^{-2}\left[\frac{\varepsilon_n(p_z)-\varepsilon_F}{2k_BT}\right],\tag{9}$$

where

$$D(\omega)=\frac{1}{\pi}\frac{\nu}{\nu^2+\omega^2},\tag{10}$$

$\varepsilon_n(p_z)$ is the energy of an electron with the quantum number n (n=0,1,2,...) and the longitudinal momentum p_z, k_B is the Boltzmann constant, T is the temperature, and ν is the frequency of electron-impurity collisions. For simplicity, we neglect the spin splitting of the electronic levels. Expression (9) was obtained under the assumption that $\hbar\omega\ll k_BT$.

The value of Q depends in a complex manner on $\hbar\omega_c/k_BT$, on kv/ν, i.e., the ratio of the electron mean free path to the length of the electromagnetic wave, on $N=[\varepsilon_F/\hbar\omega_c]$, i.e., the number of Landau levels on the Fermi surface, and on the frequency of the wave ω. The dependence of Q on these quantities was analyzed in some limiting cases in Ref. 7. The character of the quantum oscillations of the magnetic Landau decay described by Q in the case under consideration will be investigated after we determine the values of k, ω, and H at which the effect is most strongly displayed.

We calculate the values of W_x and W_y in our model. When $\theta\ll 1$, the transverse components of the velocity v_x and v_y in Eq. (7) can be taken in a zeroth approximation with respect to θ in the form

$$v_x=\frac{p}{m}\cos\varphi,\quad v_y=\frac{p}{m}\sin\varphi.\tag{11}$$

The inclination of **H** has a very strong influence on the longitudinal velocity $v_z(\varphi)$: on the upper half of the lens ($z'>0$) $v_z \approx v$, and on the lower half of the lens ($z'<0$) $v_z \approx -v$, i.e., near the central section where $\bar{v}_z \ll v$, the longitudinal velocity is

$$v_z(\varphi) \approx \begin{cases} v, & 0<\varphi<\pi, \\ -v, & \pi<\varphi<2\pi. \end{cases} \tag{12}$$

Substituting (11) and (12) into (7) and calculating the integrals with respect to φ' and φ, we obtain

$$W_x = \frac{iq}{\pi}(1+e^{-i\pi q})(1-q^2)^{-1}\frac{p}{m}, \quad W_y=0, \tag{13}$$

where

$$q = k\frac{mvc}{eH}, \quad p=\sqrt{2mvp_1}. \tag{14}$$

It follows from Eq. (5) and the definition of $m_\parallel$ in Eq. (8) that in our model when $\theta \ll 1$, we have

$$m_\parallel = \pi\left(\frac{p}{2mv}\right)^{1/2} m\theta. \tag{15}$$

At the same time, the cyclotron mass m_c scarcely differs from m: $m_c \approx m$. Thus, for a parabolic lens the longitudinal mass $m_\parallel$ is much smaller than the cyclotron mass (for the electron lens in cadmium $p_1/2mv \approx 0.1$). This fact is important: it will be shown below that the small value of the longitudinal mass favors giant quantum oscillations of the magnetic Landau decay.

Substituting (13) and (15) into (6), we represent the dissipative part of the nonlocal conductivity associated with the magnetic Landau decay in the form

$$\sigma_{xx}^{(L)} = \frac{nec}{H}\kappa(q), \tag{16}$$

$$\kappa(q) = \left(\frac{2mv}{p_1}\right)^{1/2}\theta|1+e^{-i\pi q}|^2\frac{\sqrt{q^2}}{(1-q^2)^2}Q. \tag{17}$$

The remaining elements of the conductivity tensor are specified by electron orbits which do not pass through the edge of the lens and by holes. They have the form

$$\sigma_{\alpha\beta}(\omega,q) = \frac{nec}{H}s_{\alpha\beta}(q), \quad \alpha,\beta=x,y, \tag{18}$$

$$s_{yx} = -s_{xy} = \frac{1}{2i}(s_+ - s_-), \quad s_{yy} = \frac{1}{2}(s_+ + s_-),$$

$$s_{xx} = s_{yy} + \kappa, \tag{19}$$

$$s_\pm(q) = i\left(\frac{I_\pm}{I_\pm^2 - q^2} - J_\pm\right), \quad I_\pm = \pm 1 + i\gamma^*,$$

$$\gamma^* = \frac{\nu - i\omega}{\omega_c}, \quad J_\pm = (\pm 1 - i\gamma_1^*)^{-1},$$

$$\gamma_1^* = \frac{\nu_1 - i\omega}{\omega_{c1}}, \quad \omega_{c1} = \frac{eH}{m_1 c}, \tag{20}$$

m_1 is the hole cyclotron mass, ν_1 is the frequency of collisions of holes with scatterers, $\sigma_{yz}=\sigma_{zy}=0$, and σ_{xz} and σ_{zx} are proportional to $\tan\theta$ and insignificant at small θ.

The pole in the first term of (20) corresponds to the Doppler-shifted cyclotron resonance of electrons whose orbits do not pass through the edge of the lens:

$$\omega - kv = \mp\omega_c.$$

The second term in (20) represents the hole Hall conductivity, which is assumed to be local. Such an approximation can be used to consider the Doppler-shifted cyclotron resonance of electrons, if the longitudinal velocities of the holes are considerably smaller than the velocities of the electrons, as is the case, for example, in cadmium.

3. DISPERSION RELATION (AND SURFACE IMPEDANCE)

The dispersion relation for a radio-frequency wave in a metal

$$\det(k^2\delta_{\alpha\beta} - 4\pi i\omega\sigma_{\alpha\beta}/c^2) = 0 \quad (\alpha,\beta=x,y'), \tag{21}$$

which is written in dimensionless variables in the form

$$\det D_{\alpha\beta}(q) = 0 \quad (\alpha,\beta=x,y'), \tag{22}$$

$$D_{\alpha\beta}(q) = q^2\delta_{\alpha\beta} - i\xi s_{\alpha\beta}(q), \tag{23}$$

$$\xi = 4\pi\omega nm^2v^2/eH^3, \tag{24}$$

can be represented in the form

$$[q^2 - i\xi(\kappa + s_{yy})](q^2 - i\xi s_{yy}) - \xi^2 s_{yx}^2 = 0. \tag{25}$$

In (25) we replaced the tensor index y' by y. Inclination of the magnetic field **H** relative to the symmetry axis of the Fermi surface and the wave propagation vector **k** results in magnetic Landau decay, which is manifested by the appearance of an additional dissipative term κ in s_{xx}, as a result of which the transverse conductivity becomes anisotropic (we are interested in the case $\kappa \gg |s_{yy}(0)|$, which is possible even when $\theta \ll 1$, since $\kappa \sim |q|\theta$ and $s_{yy}(0) \sim |\gamma^* + \gamma_1^*| \ll 1$. The relative differences between $s_{xy'}$ and s_{xy} and between $s_{y'y'}$ and s_{yy} are of the order of θ and can be neglected when $\theta \ll 1$. Therefore, henceforth we shall not distinguish between y' and y or between z' and z.

When Eqs. (19) and (20) for $s_{\alpha\beta}$ are taken into account, Eq. (25) can be written in the form

$$\left(q^2 - i\xi s_+ - \frac{i}{2}\xi\kappa\right)\left(q^2 - i\xi s_- - \frac{i}{2}\xi\kappa\right) + \frac{1}{4}\xi^2\kappa^2 = 0. \tag{26}$$

In the range of magnetic fields corresponding to $\xi \sim 1$, Eq. (26) has two large roots q_+ and q_- of the order of unity and two small roots q_1 and q_a. The large roots q_+ and q_- cause the expressions in the first and second sets of parentheses in (26), respectively, to vanish, since $\kappa \ll 1$. When Eqs. (20) for $s_\pm(q)$ are taken into account, the approximate expressions for these roots have the form

$$q_\pm \approx \pm\sqrt{1\pm\xi} + \frac{i\xi^2}{4(1\pm\xi)^{3/2}}\kappa(\sqrt{1\pm\xi}). \tag{27}$$

In (27) we have neglected terms of order γ and γ_1 specifying the collisional doppleron decay. This approximation becomes invalid at values of H for which $|1-\xi|\sim\gamma$. The root q_- describes the wave vector of a doppleron whose field is circularly polarized and rotates in the same direction as the electrons ("minus" polarization). This mode is a propagating mode in the range of magnetic fields where $\xi<1$. The root q_+ describes the analogous quantity for a mode whose field rotates in the opposite direction. The decay of these dopplerons, which is proportional to κ, is governed by the magnetic Landau decay. Here we neglect the collisional part of the doppleron decay.

To find the small roots q_1 and q_a, it should be taken into account that $s_{yy} \approx \gamma^* + \gamma_1^*$ and $s_{yx}\approx q^2$ for $q^2\ll 1$. As a result, Eq. (25) becomes

$$[q^2-i\xi(\gamma^*+\gamma_1^*)][q^2-i\xi(\gamma^*+\gamma_1^*+\kappa)]-\xi^2q^4=0, \quad (28)$$

whose solution has the form

$$q_1\approx i\xi\left(\frac{2mv}{p_1}\right)^{1/2}\theta\frac{Q(q_1)}{4(1-\xi^2)}, \quad q_a=[i\xi(\gamma^*+\gamma_1^*)]^{1/2}. \quad (29)$$

The root q_1 corresponds to the skin mode specified by Landau decay, whose electric field is polarized along the x axis. The wavelength of this mode increases with the magnetic field proportionally to H^2 (Ref. 4). The mode corresponding to q_a behaves differently at high and low frequencies. At high frequencies, where $\nu\ll\omega\ll\omega_c$, this mode has the form of an Alfvén wave with wave vector $k_a=\omega/v_a$, where $v_a=H/4\pi n(m+m_1)$ is the Alfvén velocity of the electron-hole plasma. In the low-frequency range $\omega<\nu$ this wave becomes a decaying skin mode specified by the collisions of electrons and holes.

To calculate the surface impedance of a metal when carriers are diffusely reflected from its surface, we must solve the system of integrodifferential equations for the electric field of a wave in a metal ($z>0$)

$$\frac{d^2\mathscr{E}_\alpha(z)}{dz^2}+\frac{4\pi i\omega}{c^2}\sum_\beta\int_0^\infty dz_1\sigma_{\alpha\beta}(z-z_1)\mathscr{E}_\beta(z_1)=0$$

$$(\alpha,\beta=x,y). \quad (30)$$

In the case of an oblique magnetic field, in which the transverse conductivity is anisotropic ($\sigma_{xx}\neq\sigma_{yy}$), this system does not separate into two independent equations and cannot be solved by the standard Wiener–Hopf method. An approximate method for solving such a system when the zeros and branch points of the dispersion relation (22) are located in two regions far from one another in the complex plane of q was developed in Refs. 8 and 4. Just such a situation arises in the case under consideration: the field in the metal consists of two short-wavelength components (the Doppler roots $q_\pm^2\sim 1$) and two long-wavelength components (the skin root q_1 and the Alfvén root q_a). The method just cited can be used to separate system (30) into two solvable systems of equations for the long-wavelength and short-wavelength components.

Setting the field $\mathscr{E}_\alpha(z)$ at $z<0$ equal to zero, we write system (30) in the Fourier representation in the form

$$D_{\alpha\beta}(q)\mathscr{E}_{q\beta}=-\mathscr{E}'_\alpha-iq\mathscr{E}_\alpha-\mathscr{F}_{q\alpha}, \quad (31)$$

where

$$\mathscr{E}_\alpha(\zeta)=\frac{1}{2\pi}\int_{-\infty}^\infty\mathscr{E}_{q\alpha}e^{iq\zeta}dq, \quad (32)$$

$\zeta=cmvz/eH$, $\mathscr{E}_\alpha$ and $\mathscr{E}'_\alpha$ are the values of the field and its derivative with respect to ζ on the surface $\zeta=0$, and summation with respect to repeated indices is understood. Here $\mathscr{E}_{q\alpha}$ is a regular function in the lower half-plane of q, and $\mathscr{F}_{q\alpha}$ is such in the upper half-plane.

Following Ref. 8, from the conductivity $s_\pm(q)$ we isolate the zeroth and first terms of the expansion in q^2 and combine them with the magnetic Landau decay κ. This sum forms the long-wavelength part of the conductivity tensor $s_{L\alpha\beta}$. In our model the elements $s_{L\alpha\beta}$ have the form

$$s_{Lxx}\approx\kappa(q)+\gamma^*+\gamma_1^*, \quad s_{Lyy}\approx\gamma^*+\gamma_1^*,$$
$$s_{Lyx}=-s_{Lxy}\approx q^2. \quad (33)$$

The tensor $s_{L\alpha\beta}$ defines the long-wavelength components of the field: the small roots q_1 and q_a are solutions of the equation $\det D_{L\alpha\beta}=0$, where

$$D_{L\alpha\beta}(q)=a_{\alpha\gamma}^{-1}(q^2\delta_{\gamma\beta}-i\xi s_{L\gamma\beta}), \quad a_{\alpha\beta}=\begin{pmatrix}1 & i\xi\\ -i\xi & 1\end{pmatrix}. \quad (34)$$

We introduce the tensor

$$d_{\alpha\beta}(q)=D_{\alpha\gamma}(q)D_{L\gamma\beta}^{-1}(q), \quad (35)$$

which specifies the short-wavelength components of the wave field [the Doppler roots $q_\pm$ are solutions of the equation $\det d_{\alpha\beta}(q)=0$]. The elements of the tensor $d_{\alpha\beta}$ have a pole at $q^2=I_\pm^2$.

The tensor $d_{\alpha\beta}$ can be approximately diagonalized in circular polarizations, to which we can transform via the matrix

$$b=\begin{pmatrix}1 & i\\ 1 & -i\end{pmatrix}.$$

Simple but lengthy calculations lead to the following result for the elements $d_\pm$ of the tensor $\bar{d}=bdb^{-1}$:

$$d_\pm(q)=\frac{q^2-q_\pm^2}{q^2-I_\pm^2}, \quad (36)$$

where the Doppler roots $q_\pm$ are specified by Eqs. (27).

It follows from the results in Refs. 4 and 8 that the high-frequency surface impedance tensor of a semi-infinite metal is given by

$$Z_{\alpha\beta}=\mu(M+N)_{\alpha\beta}^{-1}, \quad \mu=\frac{4\pi\omega v}{\omega_c c^2}. \quad (37)$$

The tensor $M_{\alpha\beta}$ is specified by the contribution of the short-wavelength components:

$$M=b^{-1}\bar{M}b, \quad \bar{M}=-\frac{1}{2\pi i}\int_{-\infty}^\infty dq\ \ln\overline{d(q)}. \quad (38)$$

The tensor $N_{\alpha\beta}$ is specified by the contribution of the long-wavelength components, and, according to Ref. 4, its elements for our model in the case of $|q_1| \gg q_a$ have the form

$$N_{xx} \simeq \eta(1+\sqrt{1-\xi^2}), \quad N_{yy} \simeq \eta(1-\sqrt{1-\xi^2}),$$

$$N_{xy} = -N_{yx} \simeq i\xi\eta, \quad \eta = \frac{iq_1}{2\pi} \ln q_1. \tag{39}$$

Substituting (36) into (38) and integrating with respect to q, we obtain

$$M_{\pm} = q_{\pm} - I_{\pm}, \tag{40}$$

$$M_{xx} = M_{yy} = \tfrac{1}{2}(M_+ + M_-),$$

$$M_{xy} = -M_{yx} = \frac{i}{2}(M_+ - M_-). \tag{41}$$

Now, using expressions (39)–(41), in a linear approximation with respect to η we obtain

$$Z_{xx} \simeq \frac{\mu}{2}\left[\frac{1}{M_+} + \frac{1}{M_-} - \frac{\eta}{\xi^2}(\sqrt{1+\xi} + \sqrt{1-\xi} + 2\sqrt{1-\xi^2})^2\right], \tag{42}$$

$$Z_{yy} \simeq \frac{\mu}{2}\left[\frac{1}{M_+} + \frac{1}{M_-} - \frac{\eta}{\xi^2}(1-\sqrt{1-\xi^2})\right]. \tag{43}$$

In the terms proportional to η we set

$$M_+ \simeq \sqrt{1+\xi} - 1, \quad M_- \simeq 1 - \sqrt{1-\xi}.$$

4. GIANT QUANTUM OSCILLATIONS OF THE IMPEDANCE

The quantity κ defined by (17), which is associated with magnetic Landau decay, appears in the impedance Z in two ways: through the imaginary parts of the Doppler roots $q_{\pm}$ and through the skin-effect root q_1 in η. Here the quantum oscillations of Im $q_{\pm}$ and q_1 can differ significantly. The reason lies in the fact that the function Q, which describes the quantum oscillations, appears in $q_{\pm}$ and in q_1 at appropriate values of q, and the degree of spreading of the giant oscillations can depend strongly on q. Spreading of the quantum oscillations occurs for two reasons: as a consequence of the thermal spreading of the Fermi distribution and as a consequence of collisions between electrons and scatterers. The role of the temperature is defined by the value of the ratio $\hbar\omega_c/k_BT$. In the case of the de Haas–van Alphen and Shubnikov–de Haas effects, under which all the electrons from the spreading region of the Fermi distribution contribute to the physical parameters of the metal, the role of the scattering is characterized by the ratio ν/ω_c, which can easily be rendered sufficiently small. In the case of giant quantum oscillations of the absorption of a wave by electrons moving with it in phase, the degree of spreading of the oscillations due to scattering depends strongly on the wavelength and consequently on q. Since the Doppler roots $q_{\pm}$ are of the order of unity, and the skin-effect root q_1 is small, the conditions for the realization of giant oscillations of q_1 are much more stringent than the conditions for oscillations of $q''_{\pm} = \text{Im}\, q_{\pm}$. Therefore, here we restrict ourselves to consideration of the case in which the oscillations of q_1 are spread strongly by electron collisions and the function $Q(q_1)$ is equal to unity to high accuracy. The oscillations of $q''_{\pm}$ are clear-cut, and below we shall ascertain the optimal conditions for observing them.

Before proceeding to the quantitative theory of the quantum oscillations of magnetic Landau decay, we must find the electron spectrum of the lens $\varepsilon = \varepsilon_n(p_z)$. It is specified by the semiclassical quantization condition

$$S(\varepsilon, p_z) = 2\pi n \frac{\hbar eH}{c}. \tag{44}$$

For sections of the lens near the central section, condition (44) gives the following spectrum:

$$\varepsilon_n(p_z) = \hbar\omega_c n + \frac{p_z^2}{2m_\parallel}. \tag{45}$$

We have omitted all small corrections to the first and second terms in (45).

Let us consider the frequency range bounded by the inequalities $\nu < \omega \ll \omega_c$ under the assumption that the term quadratic in k in the argument of D in (9) is much smaller than ω. Then the maxima of the giant oscillations are found at the values of the magnetic field specified by

$$\hbar\omega_c n + \varepsilon_\parallel = \varepsilon_F, \quad \varepsilon_\parallel = \frac{m_\parallel}{2}\left(\frac{\omega}{k}\right)^2. \tag{46}$$

The corresponding value of Q is given by

$$Q_M = \frac{|k|\nu}{4\pi}\frac{\hbar\omega_c}{k_BT}\int_{-\infty}^{\infty}[\nu^2 + (\omega - kv)^2]^{-1}\,\text{ch}^{-2}\left\{\frac{m_\parallel}{4k_BT}\left(v - \frac{\omega}{k}\right)\left(v + \frac{\omega}{k}\right)\right\}dv. \tag{47}$$

Assuming that $\varepsilon_\parallel \gg k_BT$, we can approximate the factor $v + \omega/k$ $(k>0)$ in the argument of the hyperbolic cosine in (47) by $2\omega/k$. As a result, the formula for Q_M is brought into the form

$$Q_M = \frac{\hbar\omega_c}{4\pi k_BT}\,\alpha\int_{-\infty}^{\infty}\frac{dx}{\text{ch}^2 x}(1+\alpha^2x^2)^{-1},$$

$$\alpha = \frac{2k_BTk^2}{m_\parallel\omega\nu}. \tag{48}$$

The integral is easily calculated in the limiting cases of large and small α. When $\alpha \gg 1$, the region $x \sim 1/\alpha$ is significant in the integral, which is then equal to π/α, so that

$$Q_M^{(T)} \simeq \hbar\omega_c/4k_BT. \tag{49}$$

In this case the collisions of the electrons scarcely influence the giant quantum oscillations, and their amplitude and shape are determined by the thermal spreading of the Fermi distribution. Realization of this case requires fulfillment of the inequalities

$$\varepsilon_\parallel\frac{\nu}{\omega} \ll k_BT \ll \varepsilon_\parallel, \quad k_BT \ll \hbar\omega_c. \tag{50}$$

In the opposite case of $\alpha \ll 1$, the spreading of the quantum absorption maxima is determined by scattering. The integral is then equal to 4, and we obtain

$$Q_M^{(0)} = \frac{2}{\pi} \frac{\hbar \omega_c k^2}{m_\| \omega \nu}. \tag{51}$$

If expression (27) for the wave vector of an electron doppleron with minus polarization is plugged into Eq. (51), it takes on the form

$$Q_M^{(0)} = g(1-\xi)/\xi, \tag{52}$$

where

$$g = 8\hbar \sigma_0 / mc^2, \quad \sigma_0 = ne^2/m_\| \nu, \tag{53}$$

and σ_0 is a quantity of the order of the conductivity of the metal parallel to the axis of the lens in the absence of a magnetic field. It is noteworthy that $Q_M^{(0)}$ depends on ω and H only through the parameter ξ and that g is a constant for a given metallic sample. For example, for cadmium $n \approx 5 \cdot 10^{21}$ cm^{-3}, and in the vicinity of the equator of the lens $m_\| \approx 3 \cdot 10^{-28}$ g. Taking $\nu = 10^9$ s^{-1}, which corresponds to an electron mean free path in the lens equal to 1 mm, we find that $g \approx 30$. According to (27), magnetic Landau decay has a significant influence on doppleron decay over a comparatively narrow range of magnetic fields above the doppleron threshold, where $\xi \lesssim 1$. In this region $Q_M^{(0)} \approx g$; therefore, the amplitude of the quantum oscillations is very large. The high narrow peaks of nonlocal doppleron decay are separated by broad deep minima. The calculations show that at the minima $Q_{\min} \approx 2/Q_M^{(0)} \ll 1$.

The function Q, which undergoes giant quantum oscillations as the field H varies, determines the imaginary part of the doppleron wave vector q_- (27) and, according to Eqs. (40), (42), and (43), appears in the expressions for the elements of the impedance tensor Z. Therefore, the elements of Z should undergo giant quantum oscillations in accordance with these formulas, if $g \gg 1$ for the particular metallic sample, the frequency ω of the exciting field is at least of order ν, and the thermal energy $k_B T$ is much smaller than the distance between the Landau levels of the electrons in the vicinity of the doppleron threshold:

$$\omega \gtrsim \nu, \quad k_B T \ll \hbar \omega_c|_{\xi=1}. \tag{54}$$

In the case of cadmium, when the frequency equals 250 MHz and the field strength $H = 50$ kOe, $(1-\xi)/\xi \approx 1$, and the energy $\hbar \omega_c$ corresponds to a temperature of 6 K. Therefore, at temperatures $T \lesssim 1$ K and angles of the order of several degrees, the impedance of a solid cadmium sample should undergo giant quantum oscillations.

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