Special features of longitudinal electron transport in tunnel-coupled quantum wells with asymmetric scattering

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We develop a theory of longitudinal electron transport in tunnel-coupled quantum wells placed in a nonquantizing transverse magnetic field. The conductivity tensor in a longitudinal variable electric field is calculated using the formulas of the nonequilibrium diagrammatic technique. Such a quantum approach makes it possible to describe the electronic properties of the system under conditions in which the energy of tunneling splitting of a pair of levels is comparable to the energies of collisional broadening and the Boltzmann transport equation breaks down. We derive a formula for the conductivity tensor that cannot be reduced to the Drude formula if the relaxation times differ for different wells (asymmetric scattering). Here the conductivity tensor proves to be a complex function of the parameters of the structure, including the level splitting fixed by an external bias, which gives rise to macroscopic quantum effects in tunnel-coupled quantum wells. We give a detailed description of resistivity resonance, magnetoresistance, and the Hall effect and such magneto-optical phenomena as cyclotron resonance and the Faraday effect. © *1995 American Institute of Physics.*

1. INTRODUCTION

Electronic states in tunnel-coupled double quantum wells are being actively studied by various kinetic methods. We have directly measured[1,2] tunneling electron relaxation by the luminescent spectroscopy method with high temporal resolution, and we have observed submillimeter radiation due to transient oscillations of the dipole moment after femtosecond laser excitation of a double quantum-well structure.[3] IR transitions in such structures have also been studied[4] (recently Faist *et al.*[5] were able to build a monopolar IR laser that uses a double quantum-well structure, which has stimulated interest in studies of transport phenomena in such structures). Longitudinal transport electrons in double quantum wells in the presence of a transverse electric field controlling the splitting of tunnel-coupled levels has been studied[6–11] in connection with electron redistribution (transport in real space) and resistivity resonance. Studies of longitudinal transport in quantizing magnetic fields display the special features of the quantum Hall effect[12–15] and Shubnikov–de Haas oscillations.[16–18] On the other hand, to our knowledge, longitudinal transport in weak (quasiclassical) magnetic fields, both static (magnetoresistance and the Hall effect) and high-frequency (cyclotron resonance and the Faraday effect), has not been specially studied for the case of double quantum wells with asymmetric scattering.

The theoretical description of electrons in double quantum wells is based on the two-level approximation.[19] In this approximation the electron energy spectrum incorporates the two-dimensional (2D) kinetic energy and the transverse energy similar to that in a two-level system, so that the dispersion laws of the states are given by

$$E_{\pm p}=\varepsilon_p\pm\sqrt{(\Delta/2)^2+T^2}\equiv\varepsilon_p\pm\Delta_T/2\ . \tag{1}$$

Here Δ is the splitting of levels in the absence of tunneling, T is the tunneling matrix element, Δ_T determines the splitting of tunnel-coupled levels, $\varepsilon_p=p^2/2m$ (we ignore the difference in effective masses inside a quantum well and in the barriers and the fact that the energy spectrum is not parabolic), and $\mathbf{p}$ is the 2D-momentum. Allowing for longitudinal motion sets the situation apart from the two-level system studied earlier.[20]

References 21 and 22 show that in describing electronic states in double quantum-well structures with asymmetric scattering, one cannot always use the idea of quasiparticles with the dispersion laws (1). The point is that when the energy Δ_T of level splitting is comparable to the difference $\hbar\nu$ in collisional broadening between these levels [here $\nu=\tau_l^{-1}-\tau_r^{-1}$, with τ_l and τ_r the momentum relaxation time in the left (l) and right (r) quantum wells; for the sake of definiteness we assume that $\tau_l<\tau_r$], there is no unitary transformation that diagonalizes the one-particle matrix Green's function of tunnel-coupled states. Hence the quasiparticle description of the energy spectrum is inapplicable for the case in which

$$\Delta_T\leqslant\hbar\nu, \tag{2}$$

which means that describing the kinetic phenomena in double quantum-well structures with asymmetric scattering requires a consistent quantum approach.

Another reason why a quantum approach is required is that the scattering potential leads to inhomogeneous additional terms in the tunneling matrix element T. Usually these terms are not taken into account because they contain the overlap integral of the wave functions of the left and right wells. However, for special cases of selective doping of double quantum wells (e.g., when the middle of the potential barrier is doped with impurities with a short-range potential) these additional terms must be taken into account together with the scattering potentials of the left and right wells. Here the matrix of the scattering potential proves to be nondiago-

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nal, and there is no way in which the matrix Green's function of tunnel-coupled states can be diagonalized.

In this paper we develop a quantum theory of longitudinal electron transport in double quantum-well structures that allows for asymmetric scattering and the above-noted fact that the scattering potential is nondiagonal. Methods of the nonequilibrium diagrammatic technique are used to calculate the linear response of the electron system of the double quantum-well structure in a nonquantizing transverse magnetic field to a variable longitudinal electric field. The general expression for conductivity is used to describe several effects that show up in such systems.

For a vanishing magnetic field and a constant electric field our study generalizes the results of the theory of the resistivity resonance effect, which was recently observed in experiments.[6,7] This resonance occurs at $\Delta \simeq 0$ and is due not to changes in electron concentration in the left and right quantum wells brought about by the variation in Δ (transport in real space) but to changes in the scattering probability caused by tunneling mixing of the states of these quantum wells. When scattering processes suppress the tunneling superposition of quantum-well states, the resistivity resonance peak changes shape[22] in comparison to the classical case studied in Refs. 6, 9, and 10 [where $\Delta_T \gg \hbar/\tau_l, \hbar/\tau_r$ and conductivity is determined by the classical kinetic equation for the two-level system formed by (+) and (−) states; see Eq. (1)].

For a variable electric field, conductivity obeys a frequency dispersion law differing from the Drude formula because of asymmetric scattering and the fact that the scattering potential is nondiagonal. In the presence of a magnetic field, the same effects determine the way in which both the classical magnetoresistance and the classical Hall constant depend on the magnetic field strength,[21] with the possibility that these relationships may be nonmonotonic; in addition, there are negative-magnetoresistance regions.

The asymmetric-scattering effect and the fact that the scattering potential is nondiagonal lead to a non-Drude tensor of high-frequency conductivity of double quantum wells in a magnetic field, which determines the special features of resonant scattering of far-IR or microwave radiation (cyclotron resonance) and rotation of the polarization plane of such radiation (the Faraday effect); these distinguish the system examined here from a single quantum well (see Refs. 23 and 24 on cyclotron resonance and Ref. 25 on the Faraday effect). In Sec. 8 (Conclusion) we analyze the limiting cases in which the classical kinetic approach can be used to describe the conductivity of double quantum wells; we also give numerical estimates of the parameters for the known experiments.

2. MODEL OF A DOUBLE QUANTUM WELL

We start by describing the model considered in this paper, a heterostructure with a pair of tunnel-coupled levels from the left and right quantum wells, and with different scattering in these wells. A simple band diagram of such a structure consists of two rectangular quantum wells with different widths (see Fig. 1, where $d_l < d_r$) separated by a narrow barrier of thickness d_b. The height of the barrier (the energy U_0 in Fig. 1) exceeds all characteristic energies involved in the problem. If these energies are also small compared to the typical localization energy, $(\pi\hbar/d_{l,r})^2/2m$, we can limit our discussion to the tunnel-coupled ground states of the left and right quantum wells and expand the exact wave function in terms of basis functions that contain only the pair of orbitals $|l\rangle$ and $|r\rangle$:

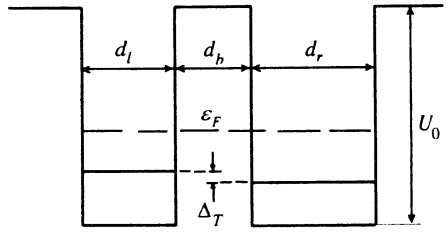

FIG. 1. The band diagram and the position of levels for a model of a double quantum-well structure.

$$\Psi_{l\mathbf{p}}|l\rangle + \Psi_{r\mathbf{p}}|r\rangle. \tag{3}$$

The orbitals in (3) have maxima at the centers of the left and right quantum wells and fall off exponentially under the barriers over distances of order κ^{-1}. These tails of the wave functions determine the small tunneling matrix element, which for the band diagram depicted in Fig. 1 can be estimated to be

$$T \simeq \frac{2\tilde{\varepsilon}}{\kappa\sqrt{d_l d_r}} e^{-\kappa d_b}\ \tilde{\varepsilon} = \frac{(\pi\hbar)^2}{m d_l d_r}, \quad \hbar\kappa \equiv \sqrt{2mU_0}. \tag{4}$$

If $d_r - d_l < d_{l,r}$, we have $\Delta \simeq \tilde{\varepsilon}(d_r^2 - d_l^2)/2d_l d_r$ for the splitting of levels not coupled by tunneling. The expansion (3) is retained for problems with a more complicated band diagram than that depicted in Fig. 1, which emerge when we allow for a self-consistent potential. The only requirement here is that $\tilde{\varepsilon} \gg \Delta_T$. Below we assume that Δ and T are fixed variables of the model whose order of magnitude is estimated by the above formulas (note that the value of Δ can be controlled by a transverse voltage applied to the double quantum-well structure).

In such approximations, also discussed in Refs. 19 and 26, the column matrix whose elements are $\Psi_{l\mathbf{p}}$ and $\Psi_{r\mathbf{p}}$ for an ideal heterostructure is found by solving the eigenvalue problem

$$\begin{vmatrix} \frac{\Delta}{2} + \varepsilon_p & T \\ T & -\frac{\Delta}{2} + \varepsilon_p \end{vmatrix} \begin{vmatrix} \Psi_{l\mathbf{p}}^{\pm} \\ \Psi_{r\mathbf{p}}^{\pm} \end{vmatrix} = E_{\pm p} \begin{vmatrix} \Psi_{l\mathbf{p}}^{\pm} \\ \Psi_{r\mathbf{p}}^{\pm} \end{vmatrix}. \tag{5}$$

The solutions of this system yield the (+) and (−) states (symmetric and antisymmetric at $\Delta = 0$) with an energy spectrum $E_{\pm p}$ defined by Eq. (1). Here it is convenient to use the isospin formalism, where instead of examining 2×2 matrices, the matrix part of the Hamiltonian in (5) is expressed

in terms of the Pauli matrices as $\hat{h} = (\Delta/2)\hat{\sigma}_z + T\hat{\sigma}_x$. Note that this expression can be diagonalized to the form $\Delta_T\hat{\sigma}_z/2$ via the unitary transformation

$$\exp(i\varphi\hat{\sigma}_y), \quad \text{where} \quad \tan 2\varphi = 2T/\Delta\ , \tag{6}$$

which leads to the dispersion law (1).

In a magnetic field $\mathbf{H}$ perpendicular to the plane of the double quantum-well structure, the Hamiltonian in the isospin representation is given by

$$\hat{H}_0(\mathbf{x},t) = \hat{h} + \frac{\Pi^2}{2m} - e\mathbf{E}_t\mathbf{x}, \tag{7}$$

where

$$\Pi = \Pi(\mathbf{x}) = -i\hbar\frac{\partial}{\partial\mathbf{x}} - \frac{e}{2c}[\mathbf{H}\mathbf{x}]$$

is the kinematic-momentum operator, $\mathbf{E}_t = \mathbf{E}_0\exp(i\omega t)$ + c.c. is the variable electric field, the reaction to which is considered below, and $\mathbf{x}$ is the 2D-coordinate. Using the two-level approximation presupposes that the cyclotron frequency $\omega_c = |e|H/mc$ and the frequency ω of the longitudinal electric field are low compared to the frequency $\bar{\varepsilon}/\hbar$ of subband-to-subband transitions. We also assume that the tunneling matrix element (4) does not vary in the weak magnetic fields considered here.

To describe electron scattering we add to the Hamiltonian (7) the potential energy of the interaction with static defects. In terms of the basis functions in (3) we get a 2×2 random-potential matrix whose off-diagonal part is related to the random variations of the tunneling matrix element (4). As a result we arrive at the potential energy

$$\begin{aligned} &U_l(\mathbf{x})\hat{P}_l + U_r(\mathbf{x})\hat{P}_r + U_t(\mathbf{x})\hat{P}_t, \\ &\hat{P}_l = \frac{1+\hat{\sigma}_z}{2}, \quad \hat{P}_r = \frac{1-\hat{\sigma}_z}{2}, \quad \hat{P}_t = \hat{\sigma}_x, \end{aligned} \tag{8}$$

expressed in terms of $\hat{\sigma}_x$ and the projection operators $\hat{P}_l$ and $\hat{P}_r$ on the states of the left and right quantum wells. The potentials $U_l(\mathbf{x})$, $U_r(\mathbf{r})$, and $U_t(\mathbf{x})$ (see Appendix) are generally statistically correlated and described by the Gaussian correlation functions

$$\langle U_j(\mathbf{x})U_{j'}(\mathbf{x}')\rangle = W_{jj'}(|\mathbf{x}-\mathbf{x}'|), \tag{9}$$

where $\langle\cdots\rangle$ denotes statistical averaging over a distribution that is homogeneous and isotropic in the 2D-plane; hereafter j and j' run through the values l, r, and t. In the Appendix we give the explicit expressions for such correlation functions for scattering by heteroboundary roughness and irregularly distributed point defects.

3. THE GREEN'S FUNCTIONS OF ELECTRONS IN A DOUBLE QUANTUM-WELL STRUCTURE

In the Keldysh technique,[27] the one-particle Green's functions $\hat{G}^R_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)$, $\hat{G}^A_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)$, and $\hat{F}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)$ satisfy the symmetrized system of equations

$$\begin{aligned} &i\hbar\left(\frac{\partial}{\partial t_1} - \frac{\partial}{\partial t_2}\right)\hat{G}^R_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2) - \hat{H}_0(\mathbf{x}_1,t_1)\hat{G}^R_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2) \\ &\quad - \hat{G}^R_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)\hat{H}_0(\mathbf{x}_2,t_2) = 2\delta(t_1-t_2)\delta(\mathbf{x}_1-\mathbf{x}_2) \\ &\quad + \int\int d\mathbf{x}'\,dt\,[\hat{\Sigma}^R_{\mathbf{x}_1\mathbf{x}'}(t_1t')\hat{G}^R_{\mathbf{x}'\mathbf{x}_2}(t't_2) \\ &\quad + \hat{G}^R_{\mathbf{x}_1\mathbf{x}'}(t_1t')\hat{\Sigma}^R_{\mathbf{x}'\mathbf{x}_2}(t't_2)], \end{aligned} \tag{10}$$

$$\begin{aligned} &i\hbar\left(\frac{\partial}{\partial t_1} + \frac{\partial}{\partial t_2}\right)\hat{F}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2) - \hat{H}_0(\mathbf{x}_1,t_1)\hat{F}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2) \\ &\quad + \hat{F}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)\hat{H}_0(\mathbf{x}_2,t_2) \\ &\quad = \int\int d\mathbf{x}'\,dt'\,[\hat{\Omega}_{\mathbf{x}_1\mathbf{x}'}(t_1t')\hat{G}^A_{\mathbf{x}'\mathbf{x}_2}(t't_2) \\ &\quad - \hat{G}^R_{\mathbf{x}_1\mathbf{x}'}(t_1t')\hat{\Omega}_{\mathbf{x}'\mathbf{x}_2}(t't_2) + \hat{\Sigma}^R_{\mathbf{x}_1\mathbf{x}'}(t_1t')\hat{F}_{\mathbf{x}'\mathbf{x}_2}(t't_2) \\ &\quad - \hat{F}_{\mathbf{x}_1\mathbf{x}'}(t_1t')\hat{\Sigma}^A_{\mathbf{x}'\mathbf{x}_2}(t't_2)], \end{aligned} \tag{11}$$

where the advanced Green's function $\hat{G}^A_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)$ is determined by an equation that is the Hermitian conjugate of (10). The self-energy functions $\hat{\Sigma}^{R,A}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)$ and $\hat{\Omega}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)$ in the Born approximation can be found by solving the equations

$$\hat{\Sigma}^{R,A}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2) = \sum_{jj'} W_{jj'}(|\mathbf{x}_1-\mathbf{x}_2|)\hat{P}_j\hat{G}^{R,A}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)\hat{P}_{j'}, \tag{12}$$

$$\hat{\Omega}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2) = \sum_{jj'} W_{jj'}(|\mathbf{x}_1-\mathbf{x}_2|)\hat{P}_j\hat{F}_{\mathbf{x}_1\mathbf{x}_2}(t_1t_2)\hat{P}_{j'}. \tag{13}$$

In the case of translation invariance in the 2D-plane, it is convenient to employ the translation-invariant Wigner representation[28] by introducing $\hat{G}^R_{\mathbf{pr}}(\varepsilon,t)$ via the relationship

$$\begin{aligned} \hat{G}^R_{\mathbf{pr}}(\varepsilon,t) = \int d\boldsymbol{\rho}\int d\tau\exp\left[\frac{i}{\hbar}(\varepsilon - e\mathbf{E}_t\mathbf{r})\tau\right] \\ \times\exp\left\{-\frac{i}{\hbar}\left[\mathbf{p} + \frac{e}{2c}[\mathbf{H}\mathbf{r}]\right]\boldsymbol{\rho}\right\} \\ \times\hat{G}^R_{\mathbf{r}+\boldsymbol{\rho}/2,\mathbf{r}-\boldsymbol{\rho}/2}\left(t+\frac{\tau}{2}, t-\frac{\tau}{2}\right), \end{aligned} \tag{14}$$

with $\hat{G}^A_{\mathbf{pr}}(\varepsilon,t)$ and $\hat{F}_{\mathbf{pr}}(\varepsilon,t)$ introduced in a similar manner. Here we assume that the energy quantum $\hbar\omega$ is small compared to the average electron energy $\bar{\varepsilon}$, and that the operator convolutions on the right-hand sides of Eqs. (10) and (11) can be transformed, following Ref. 29, by limiting discussion to leading terms in the expansion in powers of $\hbar$, which corresponds to the quasiclassical approximation for the collision integrals. The equations obtained for $\hat{G}^{R,A}_{\mathbf{pr}}(\varepsilon,t)$ and $\hat{F}_{\mathbf{pr}}(\varepsilon,t)$ reflect the translation invariance of these Green's functions and the lack of any temporal dependence in the advanced and retarded functions: $\hat{G}^{R,A}_{\mathbf{pr}}(\varepsilon,t) = \hat{G}^{R,A}_{\mathbf{p}}(\varepsilon)$ and $\hat{F}_{\mathbf{pr}}(\varepsilon,t) = \hat{F}_{\mathbf{p}}(\varepsilon,t)$. We have

$$\begin{aligned} &\left[\varepsilon - \varepsilon_p + \frac{1}{2m}\left(\frac{e}{c}\mathbf{H}\frac{\partial}{\partial\mathbf{p}}\right)^2\right]\hat{G}^R_{\mathbf{p}}(\varepsilon) \\ &\quad - \tfrac{1}{2}[\hat{h} + \hat{\Sigma}^R_{\mathbf{p}}(\varepsilon), \hat{G}^R_{\mathbf{p}}(\varepsilon)]_+ = 1, \end{aligned} \tag{15}$$

$$i\hbar\left[\frac{\partial}{\partial t}+\left(e\mathbf{E}_t+\frac{e}{mc}[\mathbf{pH}]\right)\frac{\partial}{\partial \mathbf{p}}\right]\hat{F}_{\mathbf{p}}(\varepsilon,t)+[\hat{F}_{\mathbf{p}}(\varepsilon,t),\hat{h}]_-$$
$$=\hat{\Omega}_{\mathbf{p}}(\varepsilon,t)\hat{G}^A_{\mathbf{p}}(\varepsilon)-\hat{G}^R_{\mathbf{p}}(\varepsilon)\hat{\Omega}_{\mathbf{p}}(\varepsilon,t)+\Sigma^R_{\mathbf{p}}(\varepsilon)\hat{F}_{\mathbf{p}}(\varepsilon,t)$$
$$-\hat{F}_{\mathbf{p}}(\varepsilon,t)\hat{\Sigma}^A_{\mathbf{p}}(\varepsilon), \tag{16}$$

where the self-energy functions can be expressed as

$$\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon)=\sum_{jj'}\int\frac{d\mathbf{p}_1}{(2\pi\hbar)^2}W_{jj'}(|\mathbf{p}-\mathbf{p}_1|)\hat{P}_j\hat{G}^R_{\mathbf{p}_1}(\varepsilon)\hat{P}_{j'}, \tag{17}$$

$$\hat{\Omega}_{\mathbf{p}}(\varepsilon,t)=\sum_{jj'}\int\frac{d\mathbf{p}_1}{(2\pi\hbar)^2}W_{jj'}(|\mathbf{p}-\mathbf{p}_1|)\hat{P}_j\hat{F}_{\mathbf{p}_1}(\varepsilon,t)\hat{P}_{j'} \tag{18}$$

in terms of the Fourier transforms $W_{jj'}(|\mathbf{p}-\mathbf{p}_1|)$ of the correlation functions. The $[\cdots]_+$ and $[\cdots]_-$ in Eqs. (15) and (16) stand for an anticommutator and commutator, respectively. In view of the matrix nature of the Hamiltonian, the left-hand side of Eq. (16) contains, in addition to terms common to the classical kinetic equations, the commutator $[\hat{F}_{\mathbf{p}}(\varepsilon,t),\hat{h}]_-$. Note that the electron density matrix $\hat{\rho}_{\mathbf{p}}(t)$ for a double quantum-well structure can be expressed in terms of the functions $\hat{G}^R_{\mathbf{p}}(\varepsilon)$, $\hat{G}^A_{\mathbf{p}}(\varepsilon)$, and $\hat{F}_{\mathbf{p}}(\varepsilon,t)$:

$$\hat{\rho}_{\mathbf{p}}(t)=-i\int\frac{d\varepsilon}{4\pi}[\hat{F}_{\mathbf{p}}(\varepsilon,t)+\hat{G}^A_{\mathbf{p}}(\varepsilon)-\hat{G}^R_{\mathbf{p}}(\varepsilon)].$$

Below, limiting our discussion to the case $\hbar\omega_c/2\ll\bar{\varepsilon}$, we consider only a nonquantizing magnetic field, with the result that in Eq. (15) we can ignore the differential operator, so that $\hat{G}^R$ can be found from a matrix equation. Assuming here that the average energy $\bar{\varepsilon}$ is of the order of the spread in the Fermi distribution, we can ignore not only the quantum Hall effect but also magneto-oscillatory phenomena. Since $\hat{G}^R$ depends only on the 2×2 matrix $\hat{h}+\hat{\Sigma}^R$, the solution of Eq. (15) is

$$\hat{G}^R_{\mathbf{p}}(\varepsilon)=\left[\varepsilon-\varepsilon_p-\frac{\Delta}{2}\hat{\sigma}_z-T\hat{\sigma}_x-\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon)\right]^{-1}. \tag{19}$$

A self-consistent calculation of $\hat{G}^R_{\mathbf{p}}(\varepsilon)$ suggests substituting $\hat{G}^R_{\mathbf{p}}(\varepsilon)$ from Eq. (19) into Eq. (17) and solving the resulting integral matrix equation for $\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon)$. In the case considered here—scattering by a short-range potential—the self-energy functions do not depend on momentum $\mathbf{p}$, and such an equation is no longer an integral equation. However, in general form the solution can be found only numerically. Here we consider high Fermi energies,

$$\varepsilon_F\gg\hbar/\tau_l,\ \ \hbar/\tau_r, \tag{20}$$

for which this solution can be obtained analytically. Since carriers with energies of the order of the Fermi energy ε_F contribute to kinetic phenomena, the self-energy function $\hat{\Sigma}^R$ can be found in the lowest-order scattering approximation ($\delta\to+0$):

$$\hat{\Sigma}^R_{\mathbf{p}}(\varepsilon)\simeq\sum_{jj'}\int\frac{d\mathbf{p}_1}{(2\pi\hbar)^2}W_{jj'}(|\mathbf{p}-\mathbf{p}_1|)\hat{P}_j\left(\varepsilon-\varepsilon_{p_1}-\frac{\Delta}{2}\hat{\sigma}_z\right.$$
$$\left.-T\hat{\sigma}_x+i\delta\right)^{-1}\hat{P}_{j'}$$
$$=\delta\hat{E}(\varepsilon)-\frac{i\hbar}{2}\left[\frac{\hat{P}_l}{\tau_l}\Phi_l(\varepsilon)+\frac{\hat{P}_r}{\tau_r}\Phi_r(\varepsilon)+\bar{\nu}\hat{P}_t\Phi_t(\varepsilon)\right], \tag{21}$$

where we have allowed for the fact that the potential is short-range and have replaced $W_{jj'}(p)$ by constants $w_{jj'}$. The real matrix $\delta\hat{E}(\varepsilon)$ determines the shift of the energy origin, the renormalization of the level-splitting energy Δ, and the renormalization of the tunneling matrix element (in scattering by a short-range potential, the diagonal part $\delta\hat{E}$ is logarithmically divergent, which means that cutoff at small distances is needed; see Ref. 30). When $\varepsilon\gg\Delta_T/2$, the energy-dependence of $\delta\hat{E}(\varepsilon)$ can be ignored. Since the splitting of levels is controlled by a transverse voltage applied to the double quantum-well structure, we do not write the explicit expressions for $\delta\hat{E}$ here and assume Δ to be a given variable. The imaginary part of the self-energy function can be expressed in terms of the departure relaxation frequencies and the energy-dependent functions $\Phi_i(\varepsilon)$:

$$\tau_l^{-1}=\frac{m}{2\hbar^3}(w_{ll}+w_{tt}),\quad \tau_r^{-1}=\frac{m}{2\hbar^3}(w_{rr}+w_{tt}),$$
$$\bar{\nu}=\frac{m}{2\hbar^3}(w_{lt+w_{rt}}), \tag{22}$$

$$\Phi_l(\varepsilon)=\theta\left(\varepsilon-\frac{\Delta_T}{2}\right)+\theta\left(\varepsilon+\frac{\Delta_T}{2}\right)+\left[\frac{\Delta(w_{ll}-w_{tt})}{\Delta_T(w_{ll}+w_{tt})}\right.$$
$$\left.+\frac{4Tw_{lt}}{\Delta_T(w_{ll}+w_{tt})}\right]\left[\theta\left(\varepsilon-\frac{\Delta_T}{2}\right)-\theta\left(\varepsilon+\frac{\Delta_T}{2}\right)\right], \tag{23}$$

$$\Phi_r(\varepsilon)=\theta\left(\varepsilon-\frac{\Delta_T}{2}\right)+\theta\left(\varepsilon+\frac{\Delta_T}{2}\right)+\left[\frac{\Delta(w_{tt}-w_{rr})}{\Delta_T(w_{rr}+w_{tt})}\right.$$
$$\left.+\frac{4Tw_{rt}}{\Delta_T(w_{rr}+w_{tt})}\right]\left[\theta\left(\varepsilon-\frac{\Delta_T}{2}\right)-\theta\left(\varepsilon+\frac{\Delta_T}{2}\right)\right], \tag{24}$$

$$\Phi_t(\varepsilon)=\theta\left(\varepsilon-\frac{\Delta_T}{2}\right)+\theta\left(\varepsilon+\frac{\Delta_T}{2}\right)+\left[\frac{\Delta(w_{lt}-w_{rt})}{\Delta_T(w_{lt}+w_{rt})}\right.$$
$$\left.+\frac{2T(w_{lr}+w_{tt})}{\Delta_T(w_{lt}+w_{rt})}\right]\left[\theta\left(\varepsilon-\frac{\Delta_T}{2}\right)-\theta\left(\varepsilon+\frac{\Delta_T}{2}\right)\right]. \tag{25}$$

The denominator in Eq. (19) incorporates the 2×2 real matrix from the Hamiltonian (7) and the imaginary matrix contribution from (21) describing asymmetric scattering (for $\tau_l\neq\tau_r$). This expression does not commute with its Hermitian conjugate, and hence there is no unitary transformation that can diagonalize the Green's function (19) (see Ref. 31). This simple algebraic result is central and determines the need to use quantum transport theory in this case. If condition (2) is met, we cannot introduce the idea of quasiparticles with a weakly decaying energy spectrum and proceed from the equations of the nonequilibrium diagrammatic technique to a kinetic equation. Therefore, we need a quantum description of transport phenomena even at high Fermi energies

$\varepsilon_F \gg \Delta_T$, i.e., macroscopic quantum phenomena are possible in a double quantum-well structure with asymmetric scattering.

Using the Green's function obtained above, we can write an expression for the density of states

$$\rho(\varepsilon)=\mathrm{Im}\frac{2}{\pi}\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}\mathrm{Tr}\hat{G}_p^R(\varepsilon) \tag{26}$$

that differs from the ordinary expression because of the presence of an additional trace. The behavior of $\rho(\varepsilon)$ over the energy range from $(-\Delta_T/2,\Delta_T/2)$ is fairly complex.[1)] Outside this range, $\rho(\varepsilon)$ varies from $2\rho_{2\,D}$ (here $\rho_{2\,D}$ is the density of 2D-states corresponding to a level of a single quantum well) when the energy is higher than $\Delta_T/2$ to zero when $\varepsilon<-\Delta_T/2$. At high electron concentrations n (when $\varepsilon_F \gg \Delta_T$), the Fermi energy is determined by the relation

$$n=2\rho_{2\,D}\varepsilon_F, \tag{27}$$

which uses the step approximation for $\rho(\varepsilon)$ described above.

4. CALCULATING CONDUCTIVITY

To find the linear response of the electron system to the applied electric field $\mathbf{E}_t$, we linearize the matrix kinetic equation (16) by separating out the nonequilibrium term in the function $\hat{F}_\mathbf{p}(\varepsilon,t)$ via the relationship $\hat{F}_\mathbf{p}(\varepsilon,t)=\hat{F}^{(0)}_\mathbf{p}(\varepsilon)+\delta\hat{F}_\mathbf{p}(\varepsilon,t)$ [and, similarly, $\hat{\Omega}_\mathbf{p}(\varepsilon,t)=\hat{\Omega}^{(0)}_\mathbf{p}(\varepsilon)+\delta\hat{\Omega}_\mathbf{p}(\varepsilon,t)$]. For $\delta F_\mathbf{p}(\varepsilon,t)$ we have

$$\begin{aligned}&i\left(\frac{\partial}{\partial t}+\frac{e}{mc}[\mathbf{pH}]\frac{\partial}{\partial\mathbf{p}}\right)\delta\hat{F}_\mathbf{p}(\varepsilon,t)+\frac{1}{\hbar}[\delta\hat{F}_\mathbf{p}(\varepsilon,t),\hat{h}]\\&\quad+ie\mathbf{E}_t\frac{\partial}{\partial\mathbf{p}}\hat{F}^{(0)}_\mathbf{p}(\varepsilon)\\&\quad=\frac{1}{\hbar}[\delta\hat{\Omega}_\mathbf{p}(\varepsilon,t)\hat{G}^A_\mathbf{p}(\varepsilon)-\hat{G}^R_\mathbf{p}(\varepsilon)\delta\hat{\Omega}_\mathbf{p}(\varepsilon,t)\\&\quad+\hat{\Sigma}^R_\mathbf{p}(\varepsilon)\delta\hat{F}_\mathbf{p}(\varepsilon,t)-\delta\hat{F}_\mathbf{p}(\varepsilon,t)\hat{\Sigma}^A_\mathbf{p}(\varepsilon)].\end{aligned} \tag{28}$$

Note that for the short-range scattering potential considered here, $\hat{\Sigma}^{R,A}_\mathbf{p}(\varepsilon)$ and $\hat{\Omega}_\mathbf{p}(\varepsilon,t)$ are independent of $\mathbf{p}$ and, in view of the obvious asymmetry, i.e., $\delta\hat{F}_{-\mathbf{p}}(\varepsilon,t)=-\delta\hat{F}_\mathbf{p}(\varepsilon,t)$, the terms containing $\delta\hat{\Omega}_\mathbf{p}(\varepsilon,t)$ drop out of this equation.

The current density can be expressed in terms of the nonequilibrium addition to the density matrix:

$$\begin{aligned}&\mathbf{j}(t)=2\frac{e}{m}\int\frac{d\mathbf{p}}{(2\pi\hbar)^2}\mathbf{p}\mathrm{Tr}\delta\hat{\rho}_\mathbf{p}(t),\\&\delta\hat{\rho}_\mathbf{p}(t)=-i\int\frac{d\varepsilon}{4\pi}\delta\hat{F}_\mathbf{p}(\varepsilon,t).\end{aligned} \tag{29}$$

We integrate Eq. (28) with respect to energy, using the expression for the Green's function $\hat{F}^{(0)}_\mathbf{p}(\varepsilon)$ in terms of the equilibrium Fermi distribution function $f(\varepsilon)$:

$$\hat{F}^{(0)}_\mathbf{p}(\varepsilon)=[2f(\varepsilon)-1][\hat{G}^A_\mathbf{p}(\varepsilon)-\hat{G}^R_\mathbf{p}(\varepsilon)].$$

This yields the following equation for $\delta\hat{\rho}_\mathbf{p}(t)$:

$$\begin{aligned}&\left(\frac{\partial}{\partial t}+\frac{e}{mc}[\mathbf{pH}]\frac{\partial}{\partial\mathbf{p}}\right)\delta\hat{\rho}_\mathbf{p}(t)+\frac{1}{i\hbar}[\delta\hat{\rho}_\mathbf{p}(t),\hat{h}]\\&\quad+\int\frac{d\varepsilon}{4\pi\hbar}(\hat{\Sigma}^R(\varepsilon)\,\delta\hat{F}_\mathbf{p}(\varepsilon,t)-\delta\hat{F}_\mathbf{p}(\varepsilon,t)\hat{\Sigma}^A(\varepsilon))\\&\quad=\frac{e\mathbf{p}}{2\pi i}\mathbf{E}_t[\hat{G}^A_\mathbf{p}(\varepsilon_F)-\hat{G}^R_\mathbf{p}(\varepsilon_F)],\end{aligned} \tag{30}$$

where on the right-hand side we have allowed for the high level of degeneracy of the electron gas, due to which the derivative $-df(\varepsilon)/d\varepsilon$ in the integrand was replaced by $\delta(\varepsilon-\varepsilon_F)$. Below we also assume that $\varepsilon_F \gg \Delta_T$, bearing in mind that under the conditions specified by (20), $\varepsilon_F \sim \Delta_T$ corresponds to the situation in which $\Delta_T \gg \hbar/\tau_l, \hbar/\tau_r$, which does not require a quantum description. Here the imaginary parts of $\hat{\Sigma}^R(\varepsilon)$ and $\hat{\Sigma}^A(\varepsilon)$ are energy-independent:

$$\begin{aligned}&\mathrm{Im}\;\hat{\Sigma}^A(\varepsilon)=\frac{\hbar}{2\tau}+\frac{\hbar\nu}{4}\hat{\sigma}_z+\frac{\hbar\bar{\nu}}{2}\hat{\sigma}_x,\\&\mathrm{Im}\;\hat{\Sigma}^R(\varepsilon)=-\mathrm{Im}\;\hat{\Sigma}^A(\varepsilon),\end{aligned} \tag{31}$$

(where $\tau^{-1}=(\tau_l^{-1}+\tau_r^{-1})/2$ and $\nu=\tau_l^{-1}-\tau_r^{-1}$), and the integral term in the kinetic equation is transformed into an algebraic term. After this Eq. (30) can easily be solved by going over to cylindrical coordinates. For the conductivity tensor components $\sigma_d(\omega)=\sigma_{xx}(\omega)=\sigma_{yy}(\omega)$ and $\sigma_\perp(\omega)=\sigma_{xy}(\omega)=-\sigma_{yx}(\omega)$ we then obtain

$$\begin{aligned}\begin{pmatrix}\sigma_d(\omega)\\ \sigma_\perp(\omega)\end{pmatrix}&=\frac{e^2}{\pi h^2}\int_0^\infty d\varepsilon_p\varepsilon_p\int_\infty^0 dt\exp\left(-i\omega t+\frac{t}{\tau}\right)\\&\times\begin{pmatrix}\cos\omega_c t\\ \sin\omega_c t\end{pmatrix}\mathrm{Tr}\left\{\exp\left[\left(\frac{\nu}{4}\hat{\sigma}_z+i\frac{\Delta}{2\hbar}\hat{\sigma}_z+\frac{\bar{\nu}}{2}\hat{\sigma}_x\right.\right.\right.\\&\left.\left.\left.+i\frac{T}{\hbar}\hat{\sigma}_x\right)t\right]\frac{\hat{G}^A_p(\varepsilon_F)-\hat{G}^R_p(\varepsilon_F)}{2\pi i}\\&\times\exp\left[\left(\frac{\nu}{4}\hat{\sigma}_z-i\frac{\Delta}{2\hbar}\hat{\sigma}_z+\frac{\bar{\nu}}{2}\hat{\sigma}_x-i\frac{T}{\hbar}\hat{\sigma}_x\right)t\right]\right\}.\end{aligned} \tag{32}$$

Evaluating the integral with respect to ε_p with allowance for (20) and the condition $\varepsilon_F \gg \Delta_T$ yields

$$\begin{aligned}\begin{pmatrix}\sigma_d(\omega)\\ \sigma_\perp(\omega)\end{pmatrix}&=\frac{\sigma_{\mathrm{res}}}{2\tau}\int_\infty^0 dt\exp\left(-i\omega t+\frac{t}{\tau}\right)\begin{pmatrix}\cos\omega_c t\\ \sin\omega_c t\end{pmatrix}\\&\times\mathrm{Tr}\left\{\exp\left[\left(\frac{\nu}{4}\hat{\sigma}_z-i\frac{\Delta}{2\hbar}\hat{\sigma}_z+\frac{\bar{\nu}}{2}\hat{\sigma}_x+i\frac{T}{\hbar}\hat{\sigma}_x\right)t\right]\right.\\&\left.\times\exp\left[\left(\frac{\nu}{4}\hat{\sigma}_z-i\frac{\Delta}{2\hbar}\hat{\sigma}_z+\frac{\bar{\nu}}{2}\hat{\sigma}_x-i\frac{T}{\hbar}\hat{\sigma}_x\right)t\right]\right\},\end{aligned} \tag{33}$$

where $\sigma_{\mathrm{res}}\equiv e^2 n\tau/m$ is the resonant conductivity expressed in terms of the total electron concentration in the double quantum-well structure. Further calculations of the trace and the integral in (33) can be done directly and yield

$$\begin{pmatrix}\sigma_d\\ \sigma_\perp\end{pmatrix}=\frac{\sigma_{\rm res}}{2}\begin{cases}\Psi(1+i\Omega_c-i\Omega)+\Psi(1-i\Omega_c-i\Omega),\\ [\Psi(1+i\Omega_c-i\Omega)\Psi(1-i\Omega_c-i\Omega)]/i,\end{cases}$$
$$\Psi(s)=s^{-1}\times\left(1+\frac{s^2(\mu^2+\eta^2)+(\mu\delta+\eta\Omega_T)^2}{s^2(s^2+\delta^2+\Omega_T^2)-s^2(\mu^2+\eta^2)-(\mu\delta+\eta\Omega_T)^2}\right). \quad (34)$$

The dimensionless cyclotron and tunneling frequencies, Ω_c and Ω_T, the external-field frequency Ω, the level splitting δ, the degree μ of asymmetry of scattering ($\mu=0$ corresponds to the same scattering in the quantum wells, and $|\mu|=1$ corresponds to scattering in only one quantum well), and the degree η of "nondiagonality" of scattering are introduced in (34) via the relationships

$$\Omega_c=\omega_c\tau,\quad \Omega_T=\frac{2T\tau}{\hbar},\quad \Omega=\omega\tau,\quad \delta=\frac{\tau\Delta}{\hbar},$$
$$\mu=\frac{\nu}{2}\tau=\frac{\tau_r-\tau_l}{\tau_r+\tau_l},\quad \eta=\tilde{\nu}\tau. \quad (35)$$

The μ- and η-dependent additional terms on the right-hand side of the expression for $\Psi(s)$ in (34) determine the deviation of the double quantum-well conductivity tensor from the classical Drude form (where $\Psi(s)=s^{-1}$). Over the frequency range $\Omega_c,\Omega\leqslant 1$, these additional terms can be significant when $\mu^2+\eta^2$ or $(\mu\delta+\eta\Omega_T)^2$ is comparable to $1+\delta^2+\Omega_T^2$. In the event of weak tunneling splitting $\Omega_T^2\leqslant 1$ this condition is equivalent to $\mu^2\sim 1$ or $\eta^2\sim 1+\delta^2$, while for $\Omega_T^2\gg 1$ it is equivalent to $\mu^2\sim 1+\Omega_T^2/(1+\delta^2)$ or $\eta^2\sim 1+\delta^2/(1+\Omega_T^2)$, so that the effects examined here show up under the necessary condition $\mu^2\sim 1$ or $\eta^2\sim 1$ (note that by the very definition of these quantities, $\mu^2<1$ and $\eta^2<1$). This first condition is met in structures studied in resistivity resonance experiments,[6–9] while the second condition has not been realized and, would probably be difficult to implement for technical reasons. Hence, when examining the features of kinetic effects in double quantum-well structures in the following sections, we ignore the fact that the scattering is "nondiagonal" and assume that $\eta=0$.

5. RESISTIVITY RESONANCE

Let us employ Eq. (34) to describe the shape of the resistivity resonance peak,[6–11] i.e., the dependence of the double quantum-well DC resistivity $\rho=1/\sigma$ ($\sigma=\sigma_d$) on the level splitting δ fixed by an external field. Assuming that $\omega=0$ and $\omega_c=0$, we write the formula for the resistivity as

$$\rho(\delta)=\sigma_{\rm res}^{-1}\frac{(1-\mu^2)(1+\delta^2)+\Omega_T^2}{1+\delta^2+\Omega_T^2}. \quad (36)$$

For a finite scattering asymmetry ($\mu\neq 0$), the resistivity is δ-dependent and peaks at $\delta=0$. The "classical" case, in which scattering processes do not suppress the tunneling superposition of the quantum-well states, follows from the above formula in the limit $\Omega_T^2\gg 1$. The relative height of the resonance peak,

$$\frac{\rho(0)-\rho(\infty)}{\rho(\infty)}=\frac{\mu^2}{1-\mu^2}\frac{\Omega_T^2}{1+\Omega_T^2} \quad (37)$$

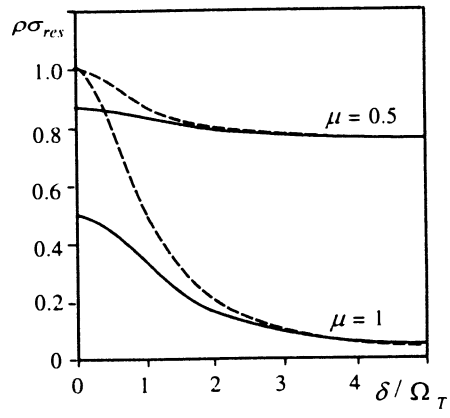

FIG. 2. The shape of the resistivity resonance peak defined by Eq. (36) at $\mu=0.5$ and $\mu=1$ for $\Omega_T=1$ (solid curves) and for $\Omega_T\gg 1$ (dashed curves).

is at its maximum (equal to $\mu^2/(1-\mu^2)$; see Ref. 6) in precisely this case and decreases appreciably when scattering suppresses tunneling ($\Omega_T^2\leqslant 1$). Note that for both structures studied by Palevski *et al.*[6] (for these structures $\Delta_T\simeq 1$ meV, i.e., $\Omega_T^2\sim 1$), the relative heights of the peaks were found to be smaller than $\mu^2/(1-\mu^2)$, which agrees with Eq. (37). If tunnel coupling is entirely absent ($\Omega_T=0$), the resistivity resonance effect vanishes.

Figure 2 depicts an example of the dependence of $\sigma_{\rm res}\rho$ on δ/Ω_T calculated for $\Omega_T=1$ and for $\Omega_T\to\infty$. The curves demonstrate the influence of the degree μ of scattering asymmetry on the shape of the resistivity resonance peak, and the changes in the shape of this peak due to scattering-induced suppression of the tunneling superposition of double quantum-well states.

6. MAGNETORESISTANCE AND THE HALL EFFECT

Let us employ Eq. (34) in describing static magnetotransport in double quantum wells. Expressing the resistivity ρ and the Hall constant R in terms of σ_d and $\sigma_\perp$, we arrive at the following formulas for the dimensionless resistivity $\rho\sigma_{\rm res}$ and Hall constant R/R_0 ($R_0=-1/|e|nc$):

$$\rho\sigma_{\rm res}=\mathrm{Re}[\Psi(1+i\Omega_c)]^{-1}=1-\frac{\mu^2}{1+\Omega_c^2}f_R(\Omega_c|\delta,\Omega_T),$$
$$\frac{R}{R_0}=\frac{1}{\Omega_c}\mathrm{Im}[\Psi(1+i\Omega_c)]^{-1}=1+\frac{\mu^2}{1+\Omega_c^2}f_H(\Omega_c|\delta,\Omega_T), \quad (38)$$

where the modeling factors f_R and f_H are given by

$$f_R(\Omega_c|\delta,\Omega_T)=1-\Omega_T^2\frac{1+\delta^2+\Omega_T^2-3\Omega_c^2}{(1+\delta^2+\Omega_T^2-\Omega_c^2)^2+4\Omega_c^2}, \quad (39)$$

$$f_H(\Omega_c|\delta,\Omega_T)=1-\Omega_T^2\frac{3+\delta^2+\Omega_T^2-\Omega_c^2}{(1+\delta^2-\Omega_T^2-\Omega_c^2)^2+4\Omega_c^2}. \quad (40)$$

When $\Omega_T^2\gg 1$, Ω_c^2 the factors f_R and f_H are small, and neither the resistivity nor the Hall constant depends on the mag-

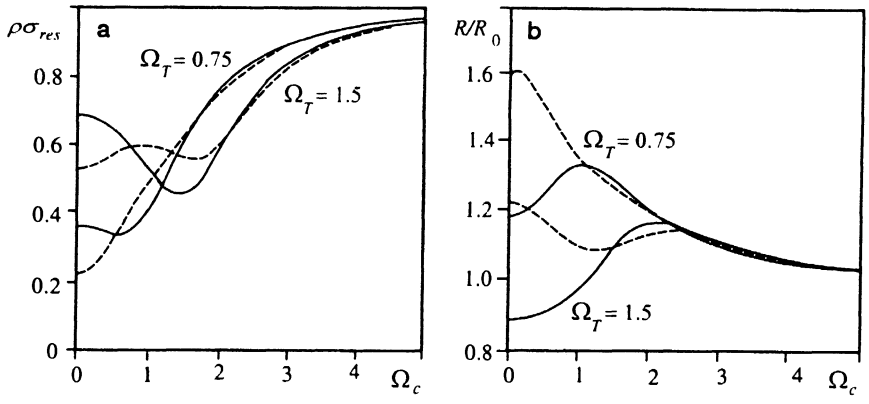

FIG. 3. The magnetic field graphs of the classical magnetoresistance (a) and the Hall constant (b) for the resonant ($\delta=0$, solid curves) and nonresonant ($\delta=1$, dashed curves) cases at two different values of the tunneling matrix element.

netic field. When $\Omega_T^2 \ll 1$, the factors f_R and f_H are equal to unity. In the intermediate region, $\Omega_T^2 \sim 1, \Omega_c^2$, the magnetic-field curves of both $\rho\sigma_{\rm res}$ and R/R_0 are extremely complicated. Examples of such curves are given in Fig. 3. They are not monotonic, and the $\rho\sigma_{\rm res}$ vs Ω_c graphs exhibit regions of negative magnetoresistance ($\rho-\rho|_{\Omega_c=0} < 0$). It can be shown that for $\Omega_c \ll 1$, negative magnetoresistance exists if

$$1+\delta^2 < \Omega_T\sqrt{\Omega_T^2+4}, \tag{41}$$

which for tunneling resonance ($\delta=0$) is equivalent to $\Omega_T > 0.49$.

7. FREQUENCY DISPERSION OF CONDUCTIVITY MAGNETO-OPTICAL PHENOMENA

Now let us examine double quantum-well conductivity in a variable electric field. Ignoring the case of a vanishing magnetic field, which was studied in Ref. 22, we immediately go to the general expression. The real part of the conductivity given by Eq. (34) can be written in the form

$$\begin{aligned}
{\rm Re}\sigma_d &= \frac{\sigma_{\rm res}}{2[1+(\Omega-\Omega_c)^2]}[1+\Phi_d(\Omega-\Omega_c)] \\
&+ \frac{\sigma_{\rm res}}{2[1+(\Omega+\Omega_c)^2]}[1+\Phi_d(\Omega+\Omega_c)], \qquad (42)\\
{\rm Re}\sigma_\perp &= \frac{(\Omega-\Omega_c)\sigma_{\rm res}}{2[1+(\Omega-\Omega_c)^2]}[1+\Phi_\perp(\Omega-\Omega_c)] \\
&- \frac{(\Omega-\Omega_c)\sigma_{\rm res}}{2[1+(\Omega+\Omega_c)^2]}[1+\Phi_\perp(\Omega+\Omega_c)], \qquad (43)
\end{aligned}$$

where the functions $\Phi_d(\Omega\pm\Omega_c)$ and $\Phi_\perp(\Omega\pm\Omega_c)$ describe additional terms ascribable to asymmetric scattering. The expressions for this case in the event of a tunneling resonance ($\delta=0$) are

$$\begin{aligned}
\Phi_d(\Omega) &= \frac{\mu^2(1+\Omega_T^2-\mu^2-3\Omega^2)}{(1+\Omega_T^2-\mu^2-\Omega^2)^2+4\Omega^2}, \\
\Phi_\perp(\Omega) &= \frac{\mu^2(3+\Omega_T^2-\mu^2-\Omega^2)}{(1+\Omega_T^2-\mu^2-\Omega^2)^2+4\Omega^2}
\end{aligned} \tag{44}$$

(the general expressions for $\Phi_d(\Omega)$ and $\Phi_\perp(\Omega)$ are quite cumbersome).

The real part of the conductivity's diagonal component determines the electromagnetic power absorbed by the system, with the absorption maximum for $\Omega^2 > 1$ occurring at $\Omega \simeq \Omega_c$ (the cyclotron resonance condition). Thus, the function Φ_d describes the modification of the classical (Drude) lineshape of a cyclotron resonance in a double quantum-well structure with asymmetric scattering. Figure 4 depicts ${\rm Re}\sigma_d/\sigma_{\rm res}$ as a function of the dimensionless cyclotron frequency Ω_c for $\mu=0.8$ and several different frequencies Ω and Ω_T at $\delta=0$. The discrepancy between these curves and the classical lineshape of a cyclotron resonance is most significant at $\Omega_c \sim \Omega$, where the contribution of $\Phi_d(\Omega-\Omega_c)$ is considerable. Here the cyclotron resonance peak is the nar-

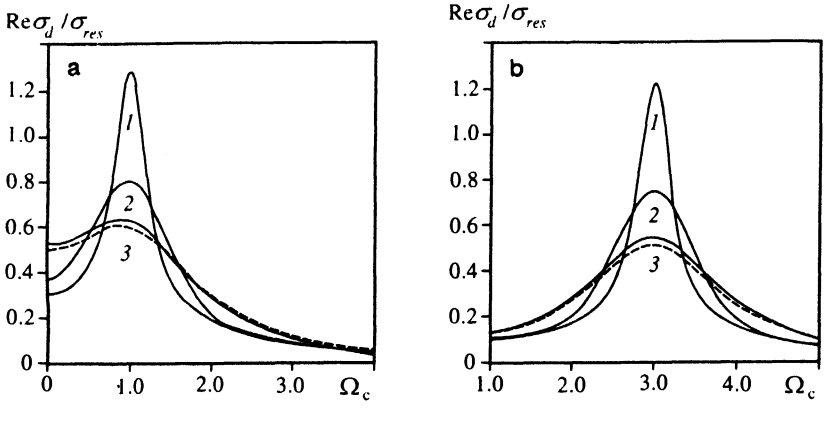

FIG. 4. The shape of the cyclotron absorption peak for the resonant ($\delta=0$) case at $\mu=0.8$ and $\Omega_T=0.3$ (curves *1*), $\Omega_T=1$ (curves *2*), and $\Omega_T=3$ (curves *3*): (a) $\Omega=1$, and (b) $\Omega=3$. The dashed curve corresponds to the classical (Drude) peak specified by the "mean" relaxation time τ.

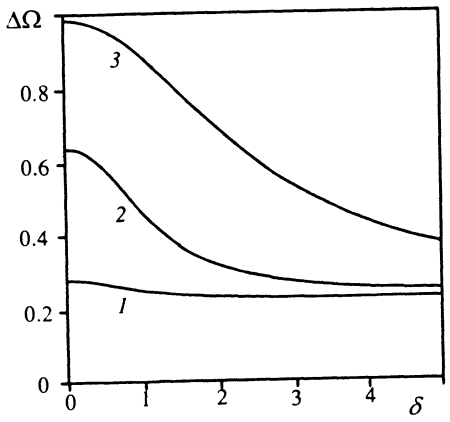

FIG. 5. The halfwidth $\Delta\Omega$ of the cyclotron resonance peak as a function of the splitting energy δ at $\mu=0.8$ and $\Omega_T=0.3$ (curve *1*), $\Omega_T=1$ (curve *2*), and $\Omega_T=3$ (curve *3*).

rowest and highest, which becomes especially evident for small values of Ω_T, at which the tunneling superposition of quantum-well states is suppressed by scattering. The reason is that under highly asymmetric scattering conditions (in which the mobility in one well is much smaller than in the other) and for weakly coupled wells ($\Omega_T^2 \ll 1$), the conductivity is basically determined by electron motion in the well with the weaker scattering, and the broadening of the cyclotron resonance peak is determined by the longer relaxation time $\tau/(1-|\mu|)$. We also note that at $\Omega_T=0$, the double quantum-well conductivity tensor can be written as the sum of the Drude contributions of the two wells, determined by the times $\tau_l=\tau/(1-\mu)$ and $\tau_r=\tau/(1+\mu)$. As tunnel coupling grows ($\Omega_T^2 \gg 1$), the electrons in the well with the weaker scattering begin to "feel" the scattering potential of the well with the stronger scattering and the characteristic relaxation time decreases. In the limit $\Omega_T^2 \gg 1$ (for the case in which $\delta^2 \ll \Omega_T^2$), the function Φ_d tends to zero and the Ω_c dependence of $\mathrm{Re}\sigma_d/\sigma_{\mathrm{res}}$ follows the classical cyclotron resonance lineshape even in highly asymmetric scattering. As the splitting δ grows, tunnel coupling weakens and the cyclotron resonance line narrows.

Let us now discuss in greater detail the case in which $\Omega, \Omega_c \gg 1$. The broadening of the cyclotron resonance line is now much smaller than the cyclotron frequency, the cyclotron resonance line is symmetric, and we can introduce a linewidth that is independent of the frequency of the absorbed radiation. Here the value of $\mathrm{Re}\sigma_d$ at resonance ($\Omega_c=\Omega$) is given by

$$\frac{0.5(1+\delta^2+\Omega_T^2)}{(1+\delta^2)^2(1-\mu^2)^2+\Omega_T^2}\sigma_{\mathrm{res}}, \tag{45}$$

which depends on the parameters δ, Ω_T, and μ, and for finite scattering asymmetry exceeds the classical height $0.5\sigma_{\mathrm{res}}$ of the peak. The halfwidth $\Delta\Omega$ of the cyclotron resonance peak (at half maximum) also depends on these parameters. Figure 5 demonstrates the dependence of the dimensionless halfwidth $\Delta\Omega$ on the level splitting δ for several values of the tunneling matrix element. These are typical resonance curves, with the maximum at $\delta=0$. The "cyclotron absorption linewidth resonance" effect in a double quantum-well structure has the same origin as the resistivity resonance effect considered above. All of the features mentioned, such as the maximum in the relative height of the resonance peak attained as $\Omega_T^2 \to \infty$ (here the halfwidth $\Delta\Omega=1$ at $\delta=0$) and the decrease in the relative height when scattering suppresses tunnel coupling, are present here as well.

The angle of rotation of the polarization plane θ_F and the ellipticity λ of the electromagnetic radiation that has traversed the two-dimensional electron layer can be expressed in terms of, respectively, the real and imaginary parts of the off-diagonal component of the conductivity tensor:[25,32]

$$\tan\theta_F = 2\pi\frac{\mathrm{Re}\,\sigma_\perp}{\sqrt{\varepsilon_0}\,c}, \quad \lambda = 2\pi\frac{\mathrm{Im}\,\sigma_\perp}{\sqrt{\varepsilon_0}\,c}. \tag{46}$$

These equations hold in the limit $2\pi|\sigma_d|/\sqrt{\varepsilon_0}\,c \ll 1$. Here ε_0 is the dielectric constant of the medium surrounding the 2D-layer. Thus, the tangent of the Faraday rotation angle in a double quantum-well structure is determined by the function $\Phi_\perp$. The ellipticity can be expressed in terms of the function Φ_d:

$$\lambda = \frac{\Omega_p/2}{1+(\Omega-\Omega_c)^2}[1+\Phi_d(\Omega+\Omega_c)] - \frac{\Omega_p/2}{1+(\Omega-\Omega_c)^2}[1+\Phi_d(\Omega-\Omega_c)], \tag{47}$$

where we have introduced the dimensionless plasma frequency

$$\Omega_p = \frac{2\pi e^2 n\tau}{\sqrt{\varepsilon}\,cm}. \tag{48}$$

Figure 6 shows the tangent of the Faraday rotation angle (in units of Ω_p) as a function of the dimensionless cyclotron frequency Ω_c at $\delta=0$. As with cyclotron resonance, the discrepancy between this dependence and the Drude dispersion (which is described by the "average" relaxation time τ) shows up most strongly at small Ω_T, while it becomes negligible as the tunneling mixing increases with Ω_T. In describing the Faraday effect, the region of low magnetic fields, in which $\Omega_c \ll 1$, is of interest. Here the dependence of $\tan\theta_F$ on the cyclotron frequency is linear: $\tan\theta_F = B(\Omega)\Omega_c$. The proportionality factor $B(\Omega)$ in this dependence is given by ($\delta=0$):

$$B(\Omega)=B_0\left\{1+\frac{\mu^2}{\Omega^2-1}\left[\frac{16\Omega^2(1+\Omega^2)^2}{[(1+\Omega_T^2-\mu^2-\Omega^2)^2+4\Omega^2]^2} + \frac{2\Omega^2-3-3\Omega^4+(\mu^2-\Omega_T^2)(1-\Omega^2)}{(1+\Omega_T^2-\mu^2-\Omega^2)^2+4\Omega^2}\right]\right\}, \tag{49}$$

where $B_0=\omega_p(\Omega^2-1)/(\Omega^2+1)^2$ is the appropriate proportionality factor for the Drude model. Figure 7 depicts $B(\Omega)$ and compares it with the frequency dispersion of B_0. A large discrepancy (even with a change in sign) occurs at low frequencies Ω for $\Omega_T<1$. But if the tunneling splitting is large, $\Omega_T \gg 1$, there will be an appreciable difference between $B(\Omega)$ and B_0 only within a narrow frequency range

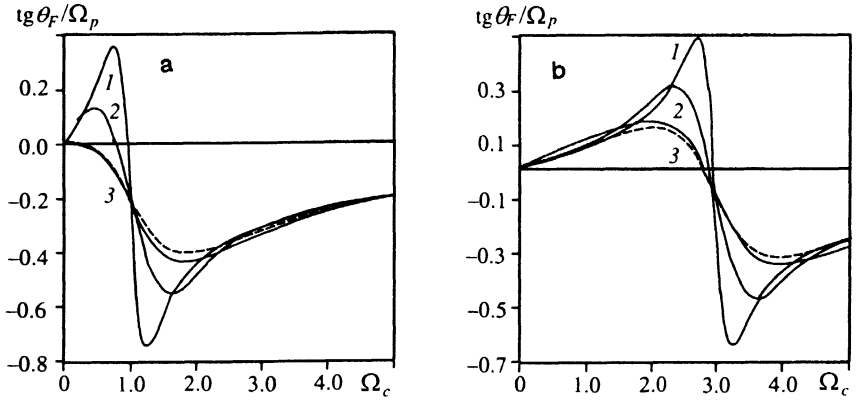

FIG. 6. The magnetic field dependence of the Faraday rotation angle for the resonant ($\delta=0$) case. The values of the parameters are the same as in Fig. figure4. The dashed curves correspond to the Drude dispersion law.

near $\Omega=\Omega_T$, i.e., when the radiation frequency is in resonance with the tunneling splitting energy of the double quantum-well levels. As a result, the Ω-dependence of B will have a resonance peak against the background of the smooth $B_0\simeq\Omega_p/\Omega^2$ curve, as shown in Fig. 8. This feature exists when scattering asymmetry is finite ($\mu \neq 0$).

8. CONCLUSION

The qualitative discrepancy between the longitudinal-conductivity tensor of double quantum wells [Eq. (34)] and the classical (Drude) case can be explained by the presence of tunneling mixing of double quantum-well electronic states and partial suppression of this mixing due to the scattering of electrons by randomly distributed inhomogeneities (impurities, roughness of boundaries, etc.). A necessary condition for the emergence of such a discrepancy is asymmetric scattering, in which the electron mobilities in the left and right wells differ considerably, or the "nondiagonal" nature of the scattering potential, in which scattering has a direct effect on tunneling mixing.

A consistent quantum description within the framework of Keldysh's diagrammatic technique has made it possible to calculate the conductivity under conditions in which the tunneling splitting energy of double quantum-well states is comparable to the collisional broadening energies of these states, $\hbar/\tau_l$ and $\hbar/\tau_r$. Note that there are two opposite limiting cases that also allow for a classical description. The first is encountered at $4T\ll\hbar/\tau_l+\hbar/\tau_r$, where scattering suppresses tunneling superposition of double quantum-well states completely. The conductivity tensor is then independent of the level splitting Δ fixed by the external field (in this limit $\Delta\ll\varepsilon_F$, where variations in population are ignored), and can be expressed by the sum of Drude contributions from the two wells, which are characterized by the relaxation times τ_l and τ_r. This representation corresponds to a model with two independent conducting channels, formed by the left and right wells, that are connected in parallel. Here the finite magnetoresistance and the magnetic field dependence of the Hall constant (see Sec. 6) exist in the first approximation in degeneracy because there are two groups of electrons with different mobilities.

The opposite limiting case, $4T\gg\hbar/\tau_l+\hbar/\tau_r$, presupposes the existence of strong tunneling mixing that cannot be destroyed by scattering. Here the states that are symmetric and antisymmetric at $\Delta=0$ and whose spectrum is specified by Eq. (1) are well-defined. The conductivity can be found by solving the Boltzmann transport equation for the two open channels, connected in parallel, formed by these states. These channels, however, are not independent now, since in

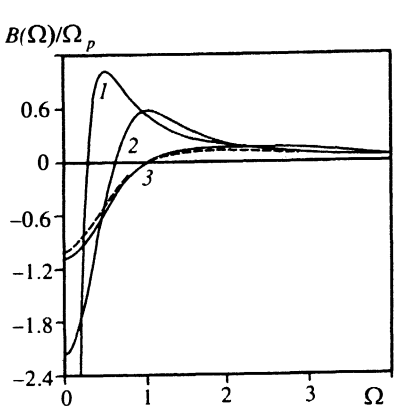

FIG. 7. The proportionality factor $B(\Omega)$ in the formula for the tangent of the Faraday rotation angle ($\tan\theta_F=B(\Omega)\Omega_c$, $\Omega_c\to 0$) as a function of the radiation frequency Ω at $\delta=0$, $\mu=0.8$, $\Omega_T=0.3$ (curve *1*), $\Omega_T=1$ (curve *2*), and $\Omega_T=3$ (curve *3*). The dashed curve describes the $B(\Omega)$ dependence calculated by the Drude formula.

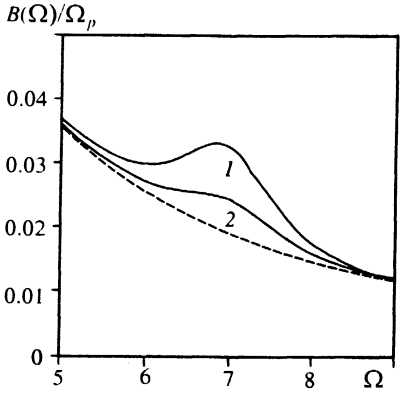

FIG. 8. The resonant behavior of the factor $B(\Omega)$ in the high-frequency range at $\Omega_T=7$, $\delta=0$, $\mu=0.8$ (curve *1*) and $\mu=0.5$ (curve *2*).

the collision integral in the transport equation we must allow for electron transitions between the states (the theory of resistivity resonance for this limiting case is given in Ref. 10). If the splitting Δ is small compared to $2T$, the conductivity tensor is described by the Drude formula with an "average" relaxation time τ. As Δ grows ($\Delta \geqslant 2T$), tunneling hybridization weakens and conductivity increases, since the characteristic relaxation time grows. The dependence of conductivity on the level splitting Δ fixed by the external field brings about such macroscopic quantum effects as resistivity resonance, which has been observed by a number of researchers,[6–8] and resonance broadening of cyclotron resonance lines, which is described in this paper. Note that the widths of the photoluminescence and photoluminescence-excitation spectra in double quantum wells with asymmetric scattering also depend on Δ because of the tunneling hybridization of electronic states.[33] When scattering suppresses tunneling hybridization under conditions where $2T \sim \hbar/\tau$, the relative magnitude of these effects decreases, as described by Eq. (34).

In Secs. 5–7 we used several dimensionless parameters. To estimate realistic values for these parameters, we list typical values of the physical quantities used in the experiments on resistivity resonance in double quantum wells,[6–8] and on cyclotron absorption and Faraday rotation in single quantum wells.[23–25] Palevski *et al.*[6] studied two GaAs/ $Ga_{0.7}Al_{0.3}As$ double quantum-well structures in which scattering asymmetry could be characterized, according to the difference in the mobilities in the wells outside tunneling resonance, by the parameters $\mu = 0.13$ and $\mu = 0.3$. Here $2T$ was approximately 1 meV, with the broadening energy $\hbar/\tau$ of the same order of magnitude (also evaluated via the mobilities), so that $\Omega_T \simeq 1$. Palevski *et al.*[7] studied the structure with $\mu = 0.73$, for which $2T \simeq 3$ meV and $\hbar/\tau$ is also of the order of 1 meV, i.e., $\Omega_T \simeq 3$ (because of the considerable value of μ, the resistivity resonance peak observed by these researchers was quite high). For the structure studied by Okuno *et al.*,[8] $2T \leqslant 1$ meV, $\hbar/\tau \simeq 4$ meV, and $\Omega_T \simeq 0.25$, but $\mu = 0.27$; because of the strong suppression of tunneling coherence by scattering, the resistivity resonance peak was not well-developed. The magnitudes of the magnetic fields used in the experiments whose results are reported in Refs. 23–25 were of the order of 5 T, which corresponds to $\hbar\omega_c \simeq 8.5$ meV for GaAs and $\hbar\omega_c \simeq 3$ meV for silicon. Typical photon energies were the same. For broadening energies $\hbar/\tau \simeq 2$–4 meV we have $\Omega, \Omega_c \simeq 2$–$4$. Typical electron number densities in such experiments were $n \simeq 10^{12}\ \mathrm{cm}^{-3}$, which corresponds to a Fermi energy in GaAs of about 32 meV and a plasma frequency of $2.3 \times 10^{11}\ \mathrm{s}^{-1}$, which at $\hbar/\tau \simeq 3$ meV yields $\Omega_p \simeq 0.05$. Thus, in interpreting the results of the experiments reported in Refs. 6–8, the features of the resistivity resonance described above are essential, while in measurements involving such structures as those used in the experiments reported in Refs. 23–25, one can obtain the relationships of Sec. 7.

Recently Kurobi *et al.*[34] reported having achieved the transition between classical and quantum resistivity resonance modes. Their results agree with the pattern described in Ref. 22 and Sec. 5 of the present paper.[34]

In conclusion we note that the double quantum-well conductivity tensor could have been calculated in a simpler manner using the Kubo formula, as is done in Ref. 22 for a zero magnetic field. However, the equations of Keldysh's diagrammatic technique provided a more convenient setting for calculations involving a finite magnetic field. Moreover, it is a useful technique that generalizes to hot-electron transport in double quantum wells with asymmetric scattering, which becomes important in developing high-speed devices using field control of mobility.[9,10]

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APPENDIX: CORRELATORS OF RANDOM POTENTIALS

1. Scattering by a short-range impurity potential

The components of the matrix potential energy (8) are the matrix elements of the impurity random potential $U(\mathbf{x},z)$ calculated using the left and right orbitals:

$$U_l(\mathbf{x}) = \langle l|U(\mathbf{x},z)|l\rangle, \quad U_r(\mathbf{x}) = \langle r|U(\mathbf{x},z)|r\rangle,$$

$$U_t(\mathbf{x}) = \langle r|U(\mathbf{x},z)|l\rangle = \langle l|U(\mathbf{x},z)|r\rangle.$$

Calculation of the correlators (9) must be done with allowance for the macroscopically inhomogeneous distribution of impurities along the longitudinal coordinate z, described by the function $N_I(z)$. Such a calculation is especially simple for the short-range potential

$$U(\mathbf{x},z) = \sum_i u\,\delta(\mathbf{x}-\mathbf{x}_i)\,\delta)(z-z_i).$$

We have

$$W_{jj'}(|\mathbf{x}-\mathbf{x}'|) = w_{jj'}\,\delta(\mathbf{x}-\mathbf{x}'), \tag{A1}$$

where

$$w_{ll} = u^2 \int dz\, N_I(z) F_l^4(z), \quad w_{rr} = u^2 \int dz\, N_I(z) F_r^4(z), \tag{A2}$$

$$w_{lt} = u^2 \int dz\, N_I(z) F_l^3(z) F_r(z),$$

$$w_{rt} = u^2 \int dz N_I(z) F_r^3(z) F_l(z), \tag{A3}$$

$$w_{lr} = w_{tt} = u^2 \int dz\, N_I(z) F_l^2(z) F_r^2(z), \tag{A4}$$

where $F_l(z)$ and $F_r(z)$ give the explicit expressions for the left and right orbitals. For a more or less even distribution of impurities, in view of the weak overlap of the left and right orbitals, we have $w_{ll}, w_{rr} \gg w_{lt}, w_{rt} \gg w_{tt}, w_{lr}$. But for highly selective doping of the middle of the potential barrier, all the correlators (A2)–(A4) are of the same order of magnitude. Below we also give the expressions for the quantities (A2)–(A4) in the case of uniform doping $N_I(z) = N_I$, using the orbital basis functions of the model depicted in Fig. 1:

$$w_{ll} \simeq \frac{d_r}{d_l} w_{rr} \simeq \frac{3u^2 N_l}{2d_l},$$

$$w_{lr} = w_{tt} \simeq \frac{4\pi^4 u^2 N_l \exp(-2\kappa d_b) d_b}{d_l^3 d_r^3 \kappa^4}, \qquad \text{(A4')}$$

$$w_{lt} \simeq \frac{d_l}{d_r} w_{rt} \simeq \frac{66\pi^4 u^2 N_l e^{-\kappa d_b}}{d_l^{5/2} d_r^{7/2} \kappa^5}.$$

2. Scattering by heteroboundary roughness

Heteroboundary roughness is described by the deviation of the position of the ith boundary from its mean value, $\delta_i(\mathbf{x})$. To describe the effective random potential generated by such deviations, it is convenient to apply a nonlinear coordinate transformation (see Ref. 35 and the literature cited therein) that smooths the rough boundaries to their ideal form. After such a transformation, the calculation of the matrix elements using the orbital basis functions of the model depicted in Fig. 1 yields

$$U_l(\mathbf{x}) \simeq 2\varepsilon_l[\delta_1(\mathbf{x}) - \delta_2(\mathbf{x})]/d_l + U_l^{(1)}(\mathbf{x}), \qquad \text{(A5)}$$

$$U_r(\mathbf{x}) \simeq 2\varepsilon_r[\delta_3(\mathbf{x}) - \delta_4(\mathbf{x})]/d_r + U_r^{(1)}(\mathbf{x}), \qquad \text{(A6)}$$

$$U_t(\mathbf{x}) \simeq \kappa T[\delta_2(\mathbf{x}) - \delta_3(\mathbf{x})], \qquad \text{(A7)}$$

where ε_l and ε_r are the quantization energies in the left and right wells, and the numbers 1–4 label the heteroboundaries of the double quantum-well structure from left to right. The first terms on the right-hand sides of Eqs. (A5) and (A6) describe random variations in the quantization energies due to variations in the widths of the respective wells, and Eq. (A7) describes random variations in the tunneling matrix element due to barrier thickness variations. The additional terms $U_l^{(1)}(\mathbf{x})$ and $U_r^{(1)}(\mathbf{x})$ are proportional[35] to

$$2\frac{\partial \delta_i(\mathbf{x})}{\partial \mathbf{x}} \frac{\partial}{\partial \mathbf{x}} + \frac{\partial^2 \delta_i(\mathbf{x})}{\partial \mathbf{x}^2}.$$

Allowing for these additional terms in calculations of the correlators (9) leads to relative corrections of order $(\varepsilon - \varepsilon_p)^2/\tilde{\varepsilon}^2$ in the self-energy functions. In calculations of the conductivity via Eq. (32), these corrections can be ignored if $2\pi\varepsilon_F \tau l_c^4/d_{l,r}^4 \gg \hbar$, where l_c is the characteristic scale (the correlation length) of the roughness.

In calculating the correlators (9) we can assume that the $\delta_i(\mathbf{x})$ for different boundaries are statistically independent. This means that the deviation correlators are described by the Gaussian functions

$$\langle \delta_i(\mathbf{x}) \delta_{i'}(\mathbf{x}') \rangle = \delta_{ii'} a_i^2 \exp\left[-\frac{(\mathbf{x}-\mathbf{x}')^2}{l_c^2} \right], \qquad \text{(A8)}$$

where the a_i are the rms deviations. The short-range potential considered in this paper corresponds to small correlation lengths $l_c \ll \hbar/p_F$. Here the Fourier transforms of the correlators (9), $w_{jj'}$, are

$$w_{ll} = 4\pi(a_1^2 + a_2^2) l_c^2 \varepsilon_l^2/d_l^2, \quad w_{rr} = 4\pi(a_3^2 + a_4^2) l_c^2 \varepsilon_r^2/d_r^2, \qquad \text{(A9)}$$

$$w_{lt} = -2\pi a_2^2 l_c^2 l_c^2 \kappa \varepsilon_l T/d_l, \quad w_{rt} = -2\pi a_3^2 l_c^2 \kappa \varepsilon_r T/d_r, \qquad \text{(A10)}$$

$$w_{tt} = \pi(a_2^2 + a_3^2) l_c^2 \kappa^2 T^2, \quad w_{lr} = 0. \qquad \text{(A11)}$$

Since the tunneling matrix element T is exponentially small, $w_{ll}, w_{rr} \gg w_{lt}, w_{rt} \gg w_{tt}$. Here $|\tilde{\nu}| \ll \tau_l^{-1}, \tau_r^{-1}$, and the off-diagonal components of the self-energy functions can be ignored.

[1)]These specific features of $\rho(\varepsilon)$ also influence the spectral dependence of the edge of band-to-band optical transitions. Hence special studies of the optical properties of double quantum-well structures with asymmetric scattering would be of interest here.

[1] T. Matsusue, M. Tsuchiya, J. N. Schulman, and H. Sakaki, Phys. Rev. B **42**, 5719 (1990)
[2] B. Deveaurd, A. Chomette, F. Clerot, P. Auvray, A. Regreny, R. Ferreira, and G. Bastard, Phys. Rev. B **42**, 7021 (1990).
[3] H. G. Roskos, M. C. Nuss, J. Shah, K. Leo, D. A. B. Miller, A. M. Fox, S. Schmitt-Rink, and K. Kohler, Phys. Rev. Lett. **68**, 2216 (1992).
[4] A. Lorke, A. D. Wieck, U. Merkt, G. Weimann, and W. Schapp, Superlatt. Microstruct. **5**, 279 (1989); R. Ranganathan, B. D. McComber, N. Nguyen, Y. Zhang, M. L. Rustgiand, and W. J. Schaff, Phys. Rev. B **44**, 1423 (1991).
[5] J. Faist, F. Capasso, D. L. Sivco, C. Sirtari, A. L. Hutchison, and A. Y. Cho, Science **264**, 553 (1994).
[6] A. Palevski, F. Beltram, F. Capasso, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **65**, 1929 (1990).
[7] A. Palevski, S. Luryi, P. L. Gammel, F. Capasso, L. N. Pfeiffer, and K. W. West, Superlatt. Microstruct. **11**, 269 (1991).
[8] E. Okuno, J. Kondo, N. Sawaki, and I. Akasaki, Surf. Sci. **267**, 570 (1992).
[9] B. Vinter and A. Tardella, Appl. Phys. Lett. **50**, 410 (1987).
[10] V. L. Borblik, Z. S. Gribnikov, and B. P. Markevich, Fiz. Tekh. Poluprovodn. **25**, 1302 (1991) [Sov. Phys. Semicond. **25**, 786 (1991)].
[11] R. Lenk, Solid State Commun. **79**, 531 (1991).
[12] S. G. Boebinger, H. W. Jiang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **64**, 1793 (1990).
[13] A. H. MacDonald, P. M. Platzmann, and S. G. Boebinger, Phys. Rev. Lett. **65**, 775 (1990).
[14] L. Brey, Phys. Rev. Lett. **65**, 903 (1990).
[15] N. E. Bonesteel, Phys. Rev. B **48**, 11484 (1993).
[16] S. G. Boebinger, A. Passner, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **43**, 12673 (1991).
[17] F. T. Vas'ko (Vasko) and P. Vasilopoulos, Phys. Rev. B **48**, 1895 (1993).
[18] Y. Ohno, M. Tsuchiya, T. Matsutsue, T. Noda, and H. Sakaki, Surf. Sci. **305**, 322 (1994).
[19] A. Yariv, S. Lindsey, and U. Sivan, J. Appl. Phys. **58**, 3669 (1985).
[20] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms*, Wiley, New York (1975); M. M. Al'perin, Ya. D. Kubis, and A. N. Khizhnyak, *Introduction to the Physics of Two-Level Systems* [in Russian], Naukova Dumka, Kiev (1987).
[21] F. T. Vas'ko, JETP Lett. **56**, 4 (1992).
[22] F. T. Vas'ko (Vasko), Phys. Rev. B **47**, 2410 (1993).
[23] C. S. Ting, S. C. Ying, and J. J. Quinn, Phys. Rev. Lett. **37**, 215 (1976); G. Abstreiter, J. P. Kotthaus, J. F. Koch, and G. Dorda, Phys. Rev. B **14**, 2480 (1976); see also the literature cited in T. Ando, A. B. Fowler, and F. Stern, p **54**, 437 (1982).
[24] H. L. Stormer, R. Dingle, A. C. Gossard, W. Wiegmann, and M. D. Sturge, Solid State Commun. **29**, 705 (1979).
[25] R. F. O'Connell and G. Wallace, Phys. Rev. B **26**, 2231 (1982); **28**, 4643 (1983).

[26] F. T. Vas'ko, Fiz. Tekh. Poluprovodn. **26**, 825 (1992) [Sov. Phys. Semicond. **26**, 464 (1992)].
[27] L. V. Keldysh, Zh. Éksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1965)]; E. M. Lifshitz and Pitaevskiĭ, *Physical Kinetics*, Pergamon Press, Oxford (1981).
[28] I. B. Levinson, Zh. Éksp. Teor. Fiz. **57**, 660 (1969) [Sov. Phys. JETP **30**, 362 (1970)].
[29] F. A. Berezin, Usp. Fiz. Nauk **132**, 497 (1980) [Sov. Phys. Usp. **23**, 763 (1980)].
[30] C. B. Duke, Phys. Rev. **168**, 816 (1968).
[31] F. R. Gantmakher, *The Theory of Matrices* [in Russian], Nauka, Moscow (1988) [Chelsea, New York (1959)].
[32] V. A. Volkov and S. A. Mikhaĭlov, JETP Lett. **41**, 476 (1985).
[33] F. T. Vas'ko (Vasko) and O. E. Raĭchev (Raichev), Phys. Rev. B **51** (1995).
[34] A. Kurobe, I. M. Castleton, E. H. Linfield *et al.*, Phys. Rev. B **50**, 8024 (1994).
[35] F. T. Vasko (Vas'ko) and O. E. Raĭchev (Raichev), Phys. Rev. B **50**, 12195 (1994).

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