Propagation of vector solitons in birefringent fiber lightguides

V. V. Bryksin, M. P. Petrov, and R. V. Kiyan

A. F. Ioffe Physicotechnical Institute of the Russian Academy of Sciences
(Submitted 11 October 1994)
Zh. Éksp. Teor. Fiz. **107**, 732–740 (March 1995)

A general analytical solution is found for the coupled nonlinear Schrödinger equations which correspond to vector solitons linearly polarized at every point. An analytical expression is found for the energy threshold for the existence of vector solitons. It is shown that periodic trains of vector solitons can develop with a repetition frequency that depends on the magnitude of the birefringence and the group velocity dispersion of the fiber lightguide. Quantitative estimates show that the generation of vector solitons by a fiber-optical laser is a realistic possibility.

I. INTRODUCTION

This paper is devoted to the study of the soliton propagation regime of short light pulses in a birefringent fiber lightguide. Recently considerable interest has developed in connection with the problem of the propagation of soliton-like pulses in birefringent optical fibers due to the considerable scientific importance of this problem[1–8] and also to the prospect for practical application of phenomena associated with the propagation of soliton-like pulses in birefringent fibers.[9,10]

The propagation of optical pulses in a birefringent fiber lightguide is described by two coupled nonlinear Schrödinger (NLS) equations.[4–6] This system of equations has two characteristic soliton solutions: solitons polarized along the principal axes of the fiber. The propagation velocity of these solitons is equal to the group velocity for radiation polarized along the corresponding principal axis of the fiber, evaluated at the carrier frequency. Thus, if the central frequencies of these orthogonal solitons are identical, they have different propagation speeds due to the birefringence and can interact only briefly, when they overlap spatially. A number of papers have been devoted to the study of the interaction of such orthogonally polarized solitons and related effects.[9,10]

It is well known that two orthogonally polarized solitons can form a bound state, and that this bound state has an amplitude threshold.[4,5,7,11] The bound state is a two-component optical pulse which propagates at a single velocity. This bound state means that there is a soliton propagation regime for two-component pulses in birefringent fiber lightguides, i.e., that vector solitons can propagate.[12] A vector soliton propagates along the fiber without changing its shape and polarization, but the polarization can vary across the pulse.

In Refs. 12 and 13 particular analytical solutions were obtained of the coupled nonlinear Schrödinger equations, corresponding to "bright" and "mixed" vector solitons (the "bright" component is parallel to the slow axis of the fiber and the "dark" component is parallel to the fast axis). But in these treatments the carrier frequencies of the components of the pulse polarized parallel to the major axes of the fiber were taken to be equal, as were the corresponding group velocities.

The treatment of Malomed[8] took into account the difference in the carrier frequencies of the orthogonal components of the pulse. The coupled NLS equations were solved using perturbation theory[11] for the case when the nonlinear equations are close to the integrable Manakov form. In the adiabatic approximation approximate solutions were found for time-independent vector solitons and periodic chains of solitons, and the interaction of two slightly overlapping vector solitons was treated. A variational technique was used to find the approximate solution of the coupled NLS equations in Ref. 14, using the same assumptions as in Ref. 8.

In the present work we find the general analytical solution for a class of vector solitons which are linearly polarized at every point in the pulse, but in which the direction of polarization varies from point to point. The resulting exact solution contains as a special case the vector solitons described in Refs. 12 and 13.

This general solution contains an infinite set of vector solitons that differ from one another in the parameter δ which characterizes the shift in time of the two pulses that compose the vector soliton; in the final analysis this shift determines how the polarization varies across the vector soliton. The energy of the vector soliton is found to be independent of the parameter δ.

A characteristic feature of these vector solitons is the existence of a lower energy threshold below which there are no vector soliton solutions. In this range of energies the only soliton that exists is polarized along the slow axis of the fiber.

The present treatment takes into account the difference in the carrier frequencies of the components of the vector soliton polarized along the principal axes of the fiber, but we use the condition $\Omega \tau_p \ll 1$ (here τ_p is the duration of the pulse), which implies that the carrier frequency shift of the orthogonally polarized components is much less than the spectral width of the pulse. We discuss the occurrence of periodic sequences of vector solitons separated by an interval

 1063-7761/95/030415-05$10.00

equal to a multiple of $\pi/2\Omega$ due to the difference in the carrier frequencies of the components of the vector soliton.

2. FUNDAMENTAL EQUATIONS OF THE THEORY OF SOLITONS IN A BIREFRINGENT FIBER LIGHTGUIDE

We assume that a light pulse with electric field components $\mathcal{E}_x$ and $\mathcal{E}_y$ directed parallel to the slow (x) and fast (y) axes is propagating in a weakly birefringent fiber. We write $\mathcal{E}_x$ and $\mathcal{E}_y$ in the form

$$\mathcal{E}_{x,y}(t,z)=F(x,y)E_{x,y}(t,z)\exp(i(\beta z-\omega t)), \tag{1}$$

where $F(x,y)$ is a function describing the transverse profile of the field in the fiber, E_x and E_y are complex amplitudes, and we have written $\beta=(\beta_x+\beta_y)/2$, where $\beta_{x,y}$ are the propagation constants for the x and y components of the pulse in the linear approximation. For E_x and E_y we use the familiar nonlinear Schrödinger equations derived under the assumption of slowly varying amplitudes:[15,16]

$$\begin{aligned}
&\frac{\partial E_x}{\partial z}-i\frac{\alpha}{2}\frac{\partial^2 E_x}{\partial \tau^2}-i\kappa E_x+\Delta\beta_1\frac{\partial E_x}{\partial \tau}\\
&\quad=i\gamma\left[\left(|E_x|^2+\frac{2}{3}|E_y|^2\right)E_x+\frac{1}{3}E_y^2E_x^*\right],\\
&\frac{\partial E_y}{\partial z}-i\frac{\alpha}{2}\frac{\partial^2 E_y}{\partial \tau^2}+i\kappa E_y-\Delta\beta_1\frac{\partial E_y}{\partial \tau}\\
&\quad=i\gamma\left[\left(|E_y|^2+\frac{2}{3}|E_x|^2\right)E_y+\frac{1}{3}E_x^2E_y^*\right],
\end{aligned}\tag{2}$$

where α is the group dispersion parameter ($\alpha=-\beta_2$, $\beta_2<0$), $\gamma=2\pi n_2/(\lambda A_{\text{eff}})$, $n_2=3.2\cdot10^{-20}$ m²/W (see Ref. 15 and A_{eff} is the effective cross-sectional area of an optical mode in the fiber, determines by the function $F(x,y)$; we have written $F(x,y)$, $\kappa=\pi/B=\pi(n_x-n_y)/\lambda$, where n_x and n_y are the indices of refraction along the respective axes ($n_x>n_y$), B is the length of the polarization beats in the fiber, and we have written $\Delta\beta_1=(\beta_{1x}-\beta_{1y})/2$, where β_{1x}^{-1}, β_{1y}^{-1} are the group velocities in the x and y directions at frequency ω. In Eqs. (2) we have used the time in the moving coordinate frame, $\tau=t-\beta_1 z$, where $\beta_1=(\beta_{1x}+\beta_{1y})/2$.

The solution of Eqs. (2) will be found in the form

$$E_{x,y}(z,\tau)=A_{x,y}(\tau)\exp[i((q+\Delta\beta_1^2/(2\alpha))z\mp\Omega\tau\pm\phi)], \tag{3}$$

where $A_{x,y}(\tau)$ are complex amplitudes that do not depend on z. Here ϕ specifies the initial difference in the phase of E_x and E_y, and q is the wave vector, which is the same for both components and depends on the energy of the pulse. The parameter Ω is given by

$$\Omega=\Delta\beta_1/\alpha. \tag{4}$$

Equation (3) describes a two-component pulse propagating in a birefringent fiber. The carrier frequencies for the x and y components are different; they are given by $\omega_x=\omega+\Omega$, $\omega_y=\omega-\Omega$, respectively. Condition (4) implies that the effect of birefringence is balanced by the dispersion, and the components of the pulse propagate a long distance without undergoing a relative spatial shift. In this case, when phase synchronization holds, the four-photon parametric interaction between the orthogonally polarized components of the pulse may also be very effective.

Assuming that condition (4) is satisfied and using Eqs. (2) and (3), we find the following equations for the complex amplitudes $A_{x,y}(\tau)$:

$$\begin{aligned}
&(q-\kappa)A_x-\frac{\alpha}{2}\frac{d^2A_x}{d\tau^2}\\
&\quad=\gamma\left[\left(|A_x|^2+\frac{2}{3}|A_y|^2\right)A_x+\frac{1}{3}A_y^2A_x^*\right.\\
&\qquad\left.\times\exp(+4i(\Omega\tau-\phi))\right],\\
&(q+\kappa)A_y-\frac{\alpha}{2}\frac{d^2A_y}{d\tau^2}=\gamma\left[\left(|A_y|^2+\frac{2}{3}|A_x|^2\right)A_y\right.\\
&\qquad\left.+\frac{1}{3}A_x^2A_y^*\exp(-4i(\Omega\tau-\phi))\right].
\end{aligned}\tag{5}$$

Note that Eqs. (5) contain terms that depend explicitly on time. Their structure is such that Eqs. (5) are not invariant under a shift in the origin of the time by an arbitrary amount. However, the system is invariant if τ is replaced by $\tau-\tau_m$, where the displacement is given by $\tau_m=\pi m/2\Omega$ with m an integer. It follows that Eqs. (5) admit periodic solutions such that $A_{x,y}(\tau)=A_{x,y}(\tau-\tau_m)$. We anticipate that these solutions will consist of weakly overlapping pulses displaced periodically in time, i.e., pulses which satisfy the condition $4\Omega\tau_p\ll1$, where τ_p is the pulse duration. Because the overlap is weak we can find the envelope for each pulse by replacing $\exp(\pm4i\Omega\tau)$ by $\exp(\pm4i\Omega\tau_m)$. This replacement is equivalent to neglecting the difference in the group velocities corresponding to the two principal axes of the fiber lightguide.[11–13]

3. LINEARLY POLARIZED VECTOR SOLITONS

Let us find the shape of the envelopes of the solitons located at the points τ_m. Assuming that the condition $4\Omega\tau_p\ll1$ holds we can look for a solution of Eq. (5) in the limit $\phi=0$. Then Eq. (5) admits a solution in the form of solitary waves; in the general case $A_x(\tau)$ and $A_y(\tau)$ are complex. These complex amplitudes can be written in the form

$$A_x(\tau)=C_1(\tau)\cos\varepsilon+C_2(\tau)\exp(+i\psi)\sin\varepsilon,$$

$$A_y(\tau)=C_2(\tau)\cos\varepsilon-C_1(\tau)\exp(-i\psi)\sin\varepsilon.$$

Substituting these expressions reduces the equations to a system of two nonlinear second-order equations for the real quantities $C_{1,2}(\tau)$. These have the form of Newton's laws for the motion of a mass point in a conservative system with a two-dimensional potential (see also Refs. 12 and 17). In the present work we performed the calculation for the case of linearly polarized modes ($\varepsilon=\psi=0$). Then for the dimensionless amplitudes $x(\tau)=A_x(\tau)\sqrt{\gamma/4\kappa}$ and $y(\tau)=A_y(\tau)\sqrt{\gamma/4\kappa}$ we find a system of two nonlinear second-order equations,

$$\nu_- x = \frac{d^2x}{d\theta^2} + (x^2+y^2)x,$$

$$\nu_+ y = \frac{d^2y}{d\theta^2} + (x^2+y^2)y, \qquad (6)$$

where $\nu_\pm = (q \pm \kappa)/4\kappa$, $\theta = \tau\sqrt{8\kappa/\alpha}$.

The system of equations (6) has already been derived in Refs. 12 and 13, where several particular solutions were found. In the present treatment we have obtained the complete solution of these equations, which enables us to find the entire set of soliton states, including those given in Refs. 12 and 13.

In order to find the exact solution of Eqs. (6), it is convenient to use the analogy between these equations and Newton's force laws for a particle with unit mass in the two-dimensional potential

$$U = -\frac{1}{2}\nu_- x^2 - \frac{1}{2}\nu_+ y^2 + \frac{1}{4}(x^2+y^2)^2. \qquad (7)$$

The solution of the equation of motion (6) can be found by separation of variables in the elliptic system of coordinates μ, ξ $(-1<\mu<1,\ 1<\xi<\infty)$:

$$x = \sqrt{(\xi^2-1)(1-\mu^2)}, \quad y = \varepsilon\mu. \qquad (8)$$

In this system of coordinates the Hamiltonian H of the system takes the form

$$H = \frac{(\xi^2-1)p_1^2 + (1-\mu^2)p_2^2}{2(\xi^2-\mu^2)} - \frac{\nu_-}{2}(\xi^2+\mu^2-1)$$

$$+\frac{1}{4}(\xi^4+\mu^4+\xi^2\mu^2-2\xi^2+2\mu^2+1). \qquad (9)$$

Here p_1 and p_2 are the projections of the momentum on the ξ and μ axes, respectively.

The Hamiltonian (9) can be related to the Hamilton–Jacobi equation for the reduced action $S_0(\xi,\mu)$:[18]

$$E(\xi^2-\mu^2) = \frac{1}{2}\left[(\xi^2-1)\left(\frac{\partial S_0}{\partial \xi}\right)^2 + (1-\mu^2)\left(\frac{\partial S_0}{\partial \mu}\right)^2\right]$$

$$-\frac{\nu_-}{2}((\xi^2-1)\xi^2 + (1-\mu^2)\mu^2)$$

$$-\frac{1}{4}((\xi^2-1)^2\cdot\xi^2 - (1-\mu^2)^2\cdot\mu^2), \qquad (10)$$

where E is the particle energy. The variables ξ and μ in (10) are separated, so the action is an additive function $S_0(\xi,\mu) = s_1(\xi) + s_2(\mu)$, which allows us to integrate Eq. (10):

$$s_1 = \pm\int^{\xi} d\xi\left(\xi^2\left(\nu_+ - \frac{1}{2}\xi^2\right) + 2\,\frac{E\xi^2+\lambda}{\xi^2-1}\right)^{1/2},$$

$$s_2 = \pm\int^{\mu} d\mu\left(\mu^2\left(\nu_+ - \frac{1}{2}\mu^2\right) - 2\,\frac{E\mu^2+\lambda}{1-\mu^2}\right)^{1/2}. \qquad (11)$$

Here λ is an integration constant which describes a conservation law associated with the motion in the potential (7). Now we determine the full action $S = S_0 - E\theta$ and find the general solution of the equation of motion (6), setting $\partial S/\partial E = a_1$ and $\partial S/\partial\lambda = a_2$, where a_1 and a_2 are arbitrary constants.[18] Instead of them we can introduce a different set ξ_0 and μ_0, the coordinates of the particle at the initial time (for $\theta=0$). As a result we find the general solution in the form

$$\theta = \pm\int_{\xi_0}^{\xi} d\xi\,\xi^2 f(\xi) \pm \int_{\mu_0}^{\mu} d\mu\,\mu^2 f(\mu),$$

$$\int_{\xi_0}^{\xi} d\xi f(\xi) + \int_{\mu_0}^{\mu} d\mu f(\mu) = 0, \qquad (12)$$

where

$$f(r) = (r^2-1)^{-1}\left(r^2\left(\nu_+ - \frac{1}{2}r^2\right) + 2\,\frac{Er^2+\lambda}{r^2-1}\right)^{-1/2} \qquad (13)$$

The full solution of the equations of motion (12) contains four arbitrary constants: the positions ξ_0 and μ_0 at the initial time, the energy E, and the constant λ. The latter describes the direction of the initial particle velocity. By appropriate adjustment of the arbitrary constants we can specify the trajectories of the motion corresponding to the soliton states.

First let us consider bright solitons, whose amplitude goes to zero in the limit $\theta \to \pm\infty$. In the terminology of classical mechanics this corresponds to trajectories which terminate at the coordinate origin $x=y=0$ ($\xi=1$, $\mu=0$). At this point the potential (7) has an extremum corresponding to an unstable equilibrium. The condition that a particle trajectory terminate at a point $x=y=0$ is that the particle energy E vanish there. Elementary analysis of the general solution (12) shows that the condition $\xi=1$, $\mu=0$ in the limit $\theta\to\pm\infty$ is

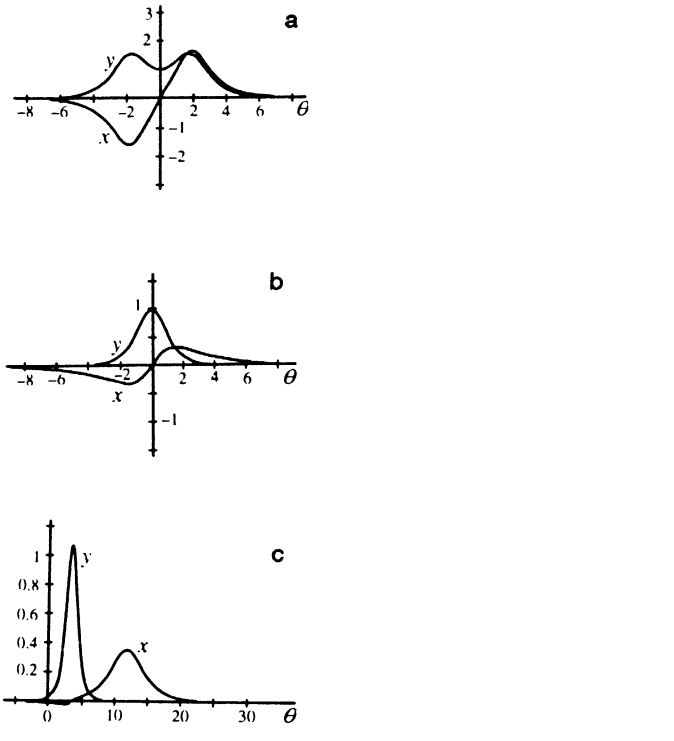

FIG. 1. Examples of the shape of the components of a vector soliton: a) $\delta_+ = \delta_- = 0$, $q/\kappa = 10$; b) $\delta_+ = \delta_- = 0$, $q/\kappa = 1.25$; c) $\delta_+ = \delta_- = 5$, $q/\kappa = 1.25$.

satisfied for $\lambda=0$. In the case $E=0$, $\lambda=0$, the integral in Eq. (12) can be performed by elementary means, as a result of which we find

$$\pm 2\theta\nu_+=\ln\left(\frac{1-v}{1+v}\frac{1+v_0}{1-v_0}\right)+(-1)^m\ln\left(\frac{1-u}{1+u}\frac{1+u_0}{1-u_0}\right),$$
$$\pm 2\theta\nu_-=\ln\left(\frac{a-v}{a+v}\frac{a+v_0}{a-v_0}\right)+(-1)^m\ln\left(\frac{u-a}{u+a}\frac{u_0+a}{u_0-a}\right). \tag{14}$$

Here

$$m=0,1,\ a=\nu_-/\nu_+,\ v\sqrt{=1-\xi^2/2\nu_+},$$

$$u=\sqrt{1-\mu^2/2\nu_+}.$$

Now, using Eq. (8) to solve (14) for x and y and going over to the dimensional time τ, we find the final expression for the components of a bright vector soliton:

$$A_x=\pm 2\sqrt{\frac{\kappa}{\gamma}}\frac{\eta_-\sinh(\eta_+\tau-\delta_+)}{\eta_+\cosh(\eta_+\tau-\delta_+)\cosh(\eta_-\tau-\delta_-)-\eta_-\sinh(\eta_+\tau-\delta_+)\sinh(\eta_-\tau-\delta_-)},$$
$$A_y=2\sqrt{\frac{\kappa}{\gamma}}\frac{\eta_+\cosh(\eta_-\tau-\delta_-)}{\eta_+\cosh(\eta_+\tau-\delta_+)\cosh(\eta_-\tau-\delta_-)-\eta_-\sinh(\eta_+\tau-\delta_+)\sinh(\eta_-\tau-\delta_-)}, \tag{15}$$

where $\eta_\pm^2=2(q\pm\kappa)/\alpha$. In place of the two constants ξ_0 and μ_0 in (15) we have introduced $\delta_\pm$. Without loss of generality we can set $\delta_+=\delta_-=\delta$, which corresponds to a particular choice of the zero of time. Thus, the complete set of solutions contains an arbitrary parameter $-\infty<\delta<\infty$, corresponding to the degeneracy of an ordinary soliton in an isotropic fiber ($\kappa=0$) with respect to the polarization angle. The particular solution of Eqs. (5) obtained in Ref. 12 is derived from (15) for $2\delta=\ln((\nu_++\nu_-)/(\nu_+-\nu_-))$.

It is worth noting some specific properties of these vector solitons which distinguish them from ordinary first-order solitons. Figure 1 shows the time dependence of the components A_x and A_y of the vector soliton, corresponding to the fast and slow axes of the fiber. These functions were calculated using Eqs. (15). From the figure it is clear that, roughly speaking, a vector soliton can be represented as a superposition of two pulses of equal length

$$\tau_\pm=\sqrt{\frac{\alpha}{2(q\pm\kappa)}}. \tag{16}$$

Here "+" corresponds to A_y and "−" corresponds to A_x. These pulses are shifted in time by $\Delta\tau$, which is determined by the parameter δ in Eqs. (15); the separation $\Delta\tau$ increases as a function of δ. Thus, the total length of a vector soliton is either of order τ_- or of order $\delta\tau$ if $\Delta\tau>\tau_-$. In the latter case the vector soliton has a well-defined two-hump structure and, most importantly, breaks up readily in the presence of weak perturbations.

In contrast to an ordinary soliton, the polarization of a vector soliton varies across the pulse. The polarization angle ϑ depends on time according to the relation

$$\mathrm{ctg}\,\vartheta=\frac{A_x}{A_y}=\pm\frac{\eta_-}{\eta_+}\frac{\sinh(\eta_+\tau-\delta)}{\cosh(\eta_-\tau-\delta)}. \tag{17}$$

A curious property of these vector solitons is that, generally speaking, they are asymmetric with respect to inversion $\tau\to-\tau$. At first glance this contradicts the invariance of the equations of motion (6) under time inversion. This contradiction is removed if we note that an arbitrary soliton with a given value of δ corresponds to another soliton with parameter equal to $-\delta$, which transforms into the first under the substitution $\tau\to-\tau$.

The energy of a vector soliton is equal to

$$W=\int_{-\infty}^{+\infty}(A_x^2+A_y^2)d\tau=\frac{\sqrt{8\kappa\alpha}}{\gamma}\times\left[\left(\frac{q}{\kappa}+1\right)^{1/2}+\left(\frac{q}{\kappa}-1\right)^{1/2}\right]. \tag{18}$$

The pulse energy does not depend on the parameter δ, just as the soliton energy in an isotropic fiber does not depend on its polarization.

The parameter q, as in the case of ordinary solitons, determines the power in the pulse. However, in contrast to scalar solitons, there is a lower threshold energy $W_{\min}=2\sqrt{\kappa\alpha}/\gamma$, which is reached when $q=\kappa$ holds. When the pulse energy is less than $W_{\min}$ there are no vector soliton states.

For $q<\kappa$ the nature of the soliton solutions of the equations of motion (6) changes radically. In this range of parameters the extremum of the potential U [cf. Eq. (7)] has a saddle character. The trajectories of the soliton motion become purely one-dimensional parallel to the y axis. The equations of motion can readily be integrated:

$$x=0,\quad y=\sqrt{2\nu_+}\cosh^{-1}(\sqrt{\nu_+}\theta). \tag{19}$$

This solution corresponds to a soliton polarized along the slow axis; it goes over to (15) for $q=\kappa$ (i.e., $\nu_-=0$, $\nu_+=1/2$). As we approach the critical point $q=\kappa$ from the direction of large values of q the amplitude of the components of the vector soliton polarized along the fast axis decreases and its length increases.

Now we briefly discuss the case of a mixed soliton whose amplitude does not vanish in the limit $\tau\to\pm\infty$. This occurs for trajectories which lies between the extrema of the potential U, $x=\pm\sqrt{\nu_-}$, $y=0$. In elliptic coordinates they are $\mu=0$, $\xi=\pm\sqrt{\nu_-+1}$ (these points are saddles; motion along the fast axis x is stable), and the solution passes through the point of absolute equilibrium $x=0$, $y=\pm\sqrt{\nu_+}$. This vector soliton has a dark component along the x axis (the amplitude remains finite in the limit $\tau\rightarrow\pm\infty$) and a bright component along the y axis (the amplitude goes to zero in the limit $\tau\rightarrow\pm\infty$). The exact solution for a mixed soliton was found in Ref. 13.

4. DISCUSSION OF RESULTS

We present some estimates which show that there is a realistic possibility of observing vector solitons under ordinary experimental conditions.

We start from the magnitude of the birefringence $\Delta n=n_x-n_y\cong 10^{-6}$. It follows that for a carrier of wavelength $\lambda=1.5$ μm we have $\kappa=(\pi\Delta n/\lambda)\cong 2$ m^{-1}. For this carrier wavelength the frequency dispersion is $\beta_2\simeq -25$ ps^2/km (Ref. 15) and we have $n_2=3.2\cdot 10^{-16}$ cm^2/W. In accordance with Eq. (18) the threshold energy of the vector soliton, i.e., the energy at $q=\kappa$, is equal to $W_{\rm th} = (4\lambda A_{\rm eff}\sqrt{\kappa|\beta_2|}/2\pi n_2)$, where $A_{\rm eff}$ is the effective cross-sectional area of the fiber. For $A_{\rm eff}=50$ μm^2 the threshold energy of a vector soliton is equal to $W_{\rm th}=350$ pJ. For a pulse of length 1–10 ps the corresponding peak power is equal to 30–300 W. Note that the peak power in fiber lasers which generate solitons reaches hundreds of watts.[19] For the above values of the parameters the soliton repetition frequency is $\Delta\tau=\pi/2\Omega\cong 24$ ps. Thus, both from the standpoint of the required power and from that of being able to achieve a periodic sequence of pulses, the typical parameters are realistic for experimental observation of vector solitons and for producing them with a fiber laser.

We note that there are experimental reports[20,21] in which sequences of short pulses were generated with parameters that agreed in order of magnitude with the estimates we have given. Unfortunately, in Ref. 20 there are no detailed data about the properties of the fibers used and the parameters of the resulting pulses were not studied in detail. Consequently, it is impossible to make an in-depth comparison between the experimental results and the estimates given in our work. In Ref. 21 we were unable to determine the parameters of the fibers used in the experiment with sufficient accuracy to make a rigorous comparison between the experimental and calculated values of the sequence of pulses that were generated.

The second question which requires discussion is the problem of the stability of these solutions. The stability of vector solitons was studied in Refs. 8 and 11. In Ref. 11 it was shown that, neglecting the difference in group velocities corresponding to the principal axes of the birefringent fiber lightguide, a vector soliton is stable if the parameter W^2/κ (where W is the soliton energy) is greater than some threshold value. This conclusion agrees with our result, since, as was shown above, a vector soliton can exist under the condition $W^2/\kappa>16\alpha/\gamma^2$. On the other hand, it is difficult to assert that the exact threshold value for the parameter W^2/κ obtained in our work is the same as that found in Ref. 11, since in Ref. 11 it was assumed that the soliton was almost isotropic. This corresponds to replacing the coefficients 2/3 and 1/3 on the right-hand sides of Eqs. (5) by 1 and 0, respectively. Nevertheless, this comparison shows that there is some threshold value W above which our solution for the vector soliton is stable.

In Ref. 8 the stability of vector solitons was analyzed, taking into account the difference in the group velocities of the orthogonal components. It was shown that in the adiabatic approximation the stability conditions remain qualitatively unchanged, but if the adiabatic approximation fails the vector soliton undergoes radiative decay. However, this analysis does not treat the possibility that a train of vector solitons exists. In the presence of such a train (as proposed in the present work) we can assume that radiative decay of the vector solitons is prevented by the exchange of virtual photons, so that the total energy stored in the pulses and in the radiative background that accompanies them is conserved. Under this condition the total configuration (soliton plus background) becomes stable. However, this question undoubtedly requires further detailed treatment.

Thus, in the present work we have derived a more general analytical solution for vector solitons using previously well-known approximations. We have found an analytical expression for the energy threshold for the existence of these solitons. We have shown that a periodic sequence of vector solitons can occur with a repetition frequency that depends on the magnitude of the birefringence and the group dispersion of the fiber lightguide. Numerical estimates show that the production of vector solitons by a fiber laser is a realistic possibility.

[1] K. J. Blow, N. J. Doran, and D. Wood, Opt. Lett. **12**, 202 (1987).
[2] S. Trillo, S. Wabnitz, E. M. Wright, and G. I. Stegemen, Opt. Lett. **13**, 871 (1988).
[3] M. V. Tratnik and J. E. Sipe, Phys. Rev. A **38**, 2011 (1988).
[4] C. R. Menyuk, Opt. Lett. **12**, 614 (1987).
[5] C. R. Menyuk, J. Opt. Soc. Am. **B5**, 392 (1988).
[6] C. R. Menyuk, IEEE J. Quantum Electron. **25**, 2674 (1989).
[7] Y. S. Kivshar, J. Opt. Soc. Am. **B7**, 2204 (1990).
[8] B. A. Malomed, Phys. Rev. A **43**, 410 (1991).
[9] M. N. Islam, C. E. Soccolich, and D. A. Miller, Opt. Lett. **15**, 909 (1990).
[10] M. N. Islam, C. R. Menyuk, C.-J. Chen, and C. E. Soccolich, Opt. Lett. **16**, 214 (1991).
[11] D. J. Muraki and W. L. Kath, Phys. Lett. A **139**, 379 (1989).
[12] D. N. Christodoulides and R. I. Joseph, Opt. Lett. **13**, 53 (1988).
[13] D. N. Christodoulides, Phys. Lett. A **132**, 451 (1988).
[14] Yu. S. Kivshar' and V. V. Konotop, Kvantovaya Élektron. (Moscow) **17**, 1599 (1990) [Sov. J. Quantum Electron. **14**, 1495 (1990)].
[15] G. P. Agrawal, *Nonlinear Fiber Optics*, Academic Press, Boston (1989).
[16] S. A. Akhmanov, V. A. Bysloukh, and A. S. Chirkin, *Optics of Femtosecond Laser Pulses*, AIP, New York (1992).
[17] V. A. Makarov and K. P. Petrov, Kvantovaya Élektron. (Moscow) **20**, 1011 (1993) [Quantum Electron. **23**, 880 (1993)].
[18] L. D. Landau and E. M. Lifshitz, *Mechanics* (3rd ed.), Pergamon, Oxford (1976).
[19] D. J. Richardson, R. I. Laming, D. N. Payne *et al.*, Electron. Lett. **27**, 730 (1990).
[20] F. Fontana, G. Grasso, M. Manfredim *et al.*, Electron. Lett. **28**, 1291 (1992).
[21] V. V. Bryksin, M. P. Petrov, and R. V. Kiyan, Pis'ma Zh. Tekh. Fiz. **20**(10), 6 (1994) [Tech. Phys. Lett. **20**, 386 (1994)].

Translated by David L. Book