A new possibility for measuring the neutrino mass via the synchrotron mechanism of radiation emission by electrons

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The production of massive neutrinos by electrons moving in an external field is considered. The small neutrino mass is shown to greatly affect the probability of high-energy neutrino production by ultrarelativistic electrons. An entirely new experimental scheme for the discovery of neutrino mass is suggested and analyzed.

1. INTRODUCTION

The existing estimates of the neutrino mass are either based on an analysis of threshold features of the electron spectrum in beta decay or related to considerations of a cosmological nature. As is well known, neither approach has yielded conclusive results, and the possible values of the electron neutrino mass vary from several tens of electron volts to several thousandths of an electron volt, not excluding the possibility of zero mass. This situation is caused, on the one hand, by difficulties in detecting minute threshold variations in the spectrum of ultrarelativistic electrons in beta decay and, on the other, by the fact that the existing neutrino detectors react only to high-energy neutrinos, so that observing so small a mass in experiments is practically impossible. Cosmological ideas of the type of origins of hidden mass in the universe still allow for too many variants to be considered reliable.

This poses the problem of setting up an experiment in which high-energy neutrinos would be generated and detected but the counting rate would nevertheless depend sensitively on the small neutrino mass in the above-mentioned range. This idea was put forward in Ref. 1, which considered a vacuum mechanism for massive neutrino pair production by a variable electromagnetic field that generally allows for mixing effects. Among other things, Ref. 1 discussed a synchrotron vacuum mechanism for neutrino production by a classical charge moving in a circle with a frequency ω. In this mechanism the probability of ultrarelativistic massive neutrino production is found for certain values of the parameters to be proportional to the cutoff factor

$$\exp\left\{-2\sqrt{3}\left(\frac{m_\nu}{\omega}\right)\left(\frac{E_q}{m_q}\right)^2\right\},$$

where E_q and m_q are the energy and mass of the charge, and m_ν is the neutrino mass (in Ref. 1 m_ν denotes the mass of the resulting neutrino pair). In view of the low probability of this effect, Ref. 1 also mentioned the possibility of a process of the type $e^- \to e^- \nu\bar{\nu}$ taking place in a magnetic field. For the goals under consideration the latter process is more acceptable both because of a considerably higher production probability and because of the appropriate characteristics of the electron beams and storage rings in comparison, say, with those of proton devices. The process was studied earlier (see, e.g., Refs. 2–6), but either in connection with massless neutrinos or without allowance for the mass in the present connection. For instance, the first to consider it in the Fermi scheme were Baıer and Katkov.[2] Then Loskutov and Zakhartsev[3] included polarization effects. Ritus[4] studied the process by the invariant technique of a constant crossed field, which in the ultrarelativistic asymptotic limit is equivalent to using a constant field of arbitrary configuration. The contact version of the standard model was investigated in detail in connection with the field of a linearly polarized plane wave by Merenkov.[5] Finally, in Ref. 6 the process was studied for a circularly polarized wave, which led to the results for a crossed field. These papers did not attract experimenters since detecting the synchrotron production of massless neutrinos would only support the presence of the long-known $ee\nu\nu$-interaction with generalization to quantum transitions of an electron in an external field. But in our formulation of the problem such an experiment would be an important step in neutrino physics.

In this paper we obtain the probability of the process $e^- \to e^- \nu\nu$ involving massive neutrinos without any mixing (unimportant in this case) occurring in a circularly polarized plane wave (Sec. 2) and in a constant crossed field (Sec. 3). We also analyze the asymptotic behavior and discuss the possibilities of measuring the neutrino mass by a method suggested in Sec. 4.

2. PROBABILITY OF THE $e^- \to e^- \nu\bar{\nu}$ PROCESS IN THE FIELD OF A CIRCULARLY POLARIZED PLANE WAVE

The material in this section is supplementary because the results for the case of a constant crossed field can be obtained from the equations given below by letting the 4-vector k (or the frequency of the wave) go to zero. Within this sector the effective Lagrangian of the standard model with a left-hand neutrino is

$$L = -\frac{G}{\sqrt{2}}[\bar{\Psi}_e \gamma^\alpha (C_V + C_A\gamma^5)\Psi_e][\bar{\Psi}_\nu \gamma_\alpha (1+\gamma^5)\Psi_\nu],$$
$$C_V = 2\sin^2\theta_W + \tfrac{1}{2}, \quad C_A = \tfrac{1}{2}, \tag{1}$$

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where θ_W is the Weinberg angle. The calculation technique for the case of the field of a circularly polarized plane wave

$$A=a_1\cos kx+a_2\sin kx,$$

$$a_1^2=a_2^2=a^2,\quad a_1a_2=ka_1=ka_2=0 \tag{2}$$

with allowance for the neutrino mass differs little from the one used in Refs. 4 and 6, with the result that the probability per unit time (the rate of the process) is

$$W=w_0\sum_{s=s_0}^{\infty}\int_{u_1}^{u_2}\frac{du}{(1+u)^2}\int_{4\tilde{m}^2}^{\lambda_s}d\lambda\sqrt{1-\frac{4\tilde{m}^2}{\lambda}}[(C_V{}^2-C_A^2)$$

$$\times F_-+(C_V^2+C_A^2)F_+\pm 2C_VC_AF_i], \tag{3}$$

where the following notation has been introduced:

$$w_0=\frac{(Gm^2)^2m^2}{12(2\pi)^3q_0}, \tag{3a}$$

$$F_- -3(\lambda-2\tilde{m}^2)J_s^2+\left(1+\frac{2\tilde{m}^2}{\lambda}\right)\frac{x^2u^2}{2(1+u)}$$

$$\times(2J_s^2-J_{s-1}^2-J_{s+1}^2),$$

$$F_+=[\lambda(1-\lambda)+\tilde{m}^2(\lambda-4)]J_s^2+x^2\left\{-\left(1+\frac{2\tilde{m}^2}{\lambda}\right)\right.$$

$$\left.\times\left[\lambda+\frac{(1+\lambda)u^2}{2(1+u)}\right]+\frac{3}{2}\tilde{m}^2\left(1+u+\frac{1}{1+u}\right)\right\}$$

$$\times(2J_s^2-J_{s-1}^2-J_{s+1}^2), \tag{3b}$$

$$F_i=\frac{(-\lambda+\tilde{m}^2)x^2(2+u)}{z}\left[\frac{2}{y}(\lambda_s-\lambda)-\frac{su}{1+u}\right]$$

$$\times J_s(J_{s+1}-J_{s-1}),$$

$$\arg J_s\equiv z=\frac{2x}{y}\sqrt{(1+u)(\lambda_s-\lambda)},\quad \tilde{m}=\frac{m_\nu}{m},\quad x^2=\frac{e^2(-a^2)}{m^2},$$

$$y=\frac{2kp}{m^2},\quad u_s=sy, \tag{3c}$$

$$s_0=\mathrm{E}\left\{\frac{4\tilde{m}(\tilde{m}+\sqrt{1+x^2})}{y}\right\}+1,\quad \lambda_s=\frac{u}{1+u}(u_s-u-x^2u), \tag{3d}$$

$$u_{1,2}=\frac{u_s-4\tilde{m}^2\mp\sqrt{(u_s-4\tilde{m}^2)^2-16\tilde{m}^2(1+x^2)}}{2(1+x^2)},$$

where $p(q)$ is the momentum (quasimomentum) of the initial electron, m is the electron mass, s is the number of photons captured from the wave, and E$\{\cdots\}$ is the integral part of the number $\{\cdots\}$; the interpretation of the invariant integration variables u and λ is standard.[4,6] The subscripts "$-$" and "$+$" in (3) denote right and left circular polarizations of the wave. For $m_\nu=0$ the result that follows from (3) coincides with that of Ref. 6.

Now to treat the case of a crossed field it is convenient to interchange the summation and integration in the above expressions, i.e., carry out the following substitution:

$$\sum_{s=s_0}^{\infty}\int_{u_1}^{u_2}du\int_{4\tilde{m}^2}^{\lambda_s}d\lambda\cdots\to\int_0^\infty du\int_{4\tilde{m}^2}^{\infty}d\lambda\sum_{s=s_m}^{\infty}\cdots,$$

$$s_m=\mathrm{E}\left\{\frac{x}{2\chi}\left[\lambda\frac{1+u}{u}+u(1+x^2)\right]\right\}+1,\quad \chi=\frac{xy}{2}. \tag{4}$$

3. PROBABILITY OF THE $e^-\to e^-\nu\bar{\nu}$ PROCESS IN A CROSSED FIELD

In the case of a crossed field, we must assume $x\to\infty$ and $y\to 0$ and express the invariant parameter χ in terms of the field-strength tensor and the electron momentum:

$$\chi=\frac{\sqrt{e^2(pF^2p)}}{m^3}. \tag{5}$$

The corresponding transformation of Eq. (3) combined with (4) is described in detail in Ref. 4 and is reduced to the following substitutions:

$$\sum_{s=s_m}^{\infty}\begin{Bmatrix}J_s^2\\ \frac{x^2}{2}(2J_s^2-J_{s-1}^2-J_{s+1}^2)\end{Bmatrix}\to\int_{-\infty}^{\infty}d\tau\begin{Bmatrix}\left(\frac{u}{2\chi}\right)^{1/3}\Phi^2(\tilde{z})\\ -\left(\frac{2\chi}{u}\right)^{1/3}[\tilde{z}\Phi^2(\tilde{z})+\Phi'^2(\tilde{z})]\end{Bmatrix}. \tag{6}$$

The interference term vanishes and the argument of the Airy functions is

$$\tilde{z}=\left(\frac{u}{2\chi}\right)^{2/3}\left(1+\tau^2+\lambda\frac{1+u}{u^2}\right). \tag{6a}$$

Finally, using the nonlinear integral relation for the Airy functions

$$\int_0^\infty d\tau\Phi^2(a+b\tau^2)=\frac{\pi}{4\sqrt{b}}\int_{2^{2/3}a}^{\infty}\Phi(z)dz, \tag{7}$$

we arrive at the following expression for the probability per unit time of the $e^-\to e^-\nu\bar{\nu}$ process in a constant crossed field (w_0 must be replaced by p_0 in Eq. (3a)):

$$W_\perp=w_0\pi\int_0^\infty\frac{du}{(1+u)^2}\int_{4\tilde{m}^2}^{\infty}d\lambda\sqrt{1-\frac{4\tilde{m}^2}{\lambda}}[(C_V^2-C_A^2)\tilde{F}_-$$

$$+(C_V^2+C_A^2)\tilde{F}_+], \tag{8}$$

$$\tilde{F}_-=-3(\lambda-2\tilde{m}^2)\tilde{\Phi}(t_0)+\frac{2}{r}\left(1+\frac{2\tilde{m}^2}{\lambda}\right)\frac{u^2}{1+u}\Phi'(t_0), \tag{8a}$$

$$\tilde{F}_+=[\lambda(1-\lambda)+\tilde{m}^2(\lambda-4)]\tilde{\Phi}(t_0)-\frac{2}{r}\left\{\left(1+\frac{2\tilde{m}^2}{\lambda}\right)\right.$$

$$\left.\times\left[2\lambda+(1+\lambda)\frac{u^2}{1+u}\right]-3\tilde{m}^2\left(1+u+\frac{1}{1+u}\right)\right\}\Phi'(t_0), \tag{8b}$$

$$\tilde{\Phi}(t_0)=\int_{t_0}^{\infty}\Phi(z)dz,\ t_0=r\left(1+\lambda\,\frac{1+u}{u^2}\right),\ r=\left(\frac{u}{\chi}\right)^{2/3},\qquad(8c)$$

where Eq. (8) also determines the distribution over the energy variables u and λ and is also valid in the event of ultrarelativistic motion in constant fields of other configurations, including a uniform magnetic field.

Integration by parts with respect to the variable λ reduces the triple integral (with (8c) taken into account) to a double integral (this, naturally, eradicates all information about the differential distribution):

$$W_\perp=w_0\pi\int_0^\infty du[(C_V^2-C_A^2)f_-+(C_V^2+C_A^2)f_+],\qquad(9)$$

$$f_-=-8\tilde{m}^4\ln(2\tilde{m})\frac{r/u}{1+u}\left[1-\frac{2\tilde{m}^2}{u}\left(1+\frac{4}{u}\right)\right]\Phi(\tilde{t})+3\frac{r/\gamma}{(1+u)^2}\int_{4\tilde{m}^2}^{\infty}d\lambda\left[-\frac{1}{2}(\lambda-6\tilde{m}^2)R-8\tilde{m}^4\frac{(\gamma/r)^3}{\lambda^2R}-2\tilde{m}^4\ln(2R+2\lambda-4\tilde{m}^2)\right]\Phi(t_0),\qquad(9a)$$

$$f_+=\frac{8}{3}\tilde{m}^4(3+\tilde{m}^2)\ln(2\tilde{m})\frac{r/u}{1+u}\left[1-\frac{2\tilde{m}^2}{u}\left(1+\frac{4}{u}\right)\right]\Phi(\tilde{t})+\frac{r/\gamma}{(1+u)^2}\int_{4\tilde{m}^2}^{\infty}d\lambda\left\{R\left[-\frac{\lambda^2}{3}+\frac{\lambda}{6}(3+5\tilde{m}^2)-\tilde{m}^2(5-\tilde{m}^2)\right]+2\tilde{m}^4(3+\tilde{m}^2)\ln(2R+2\lambda-4\tilde{m}^2)+\frac{2\gamma^2}{r^3\lambda R}\left[\left[12\tilde{m}^4\frac{\gamma}{\lambda}+(2+\gamma)(-2\tilde{m}^4-2\tilde{m}^2\lambda+\lambda^2)\right]\right]\right\}\Phi(t_0),\qquad(9b)$$

where the following notation has been introduced:

$$R=\sqrt{\lambda^2-4\tilde{m}^2\lambda},\quad \tilde{t}=r\left(1+4\tilde{m}^2\,\frac{(1+u)}{u^2}\right),\quad \gamma=\frac{u^2}{1+u}.\qquad(10)$$

4. DISCUSSION AND ANALYSIS OF RESULTS

Analyzing the above results requires examining two possible types of asymptotic behavior with respect to the parameters χ and $\tilde{m}$.

(a) $\chi\ll\tilde{m}$ ($\tilde{m}\ll1$). The arguments of the Airy functions, $\tilde{t}$ and t_0, are large since even their minimum values in the range of integration with respect to u and λ,

$$\tilde{t}_{\min}=t_{0\min}\simeq3\left(\frac{\tilde{m}}{\chi}\right)^{2/3},$$

are much larger than unity, with the result that the following asymptotic representation holds:

$$\Phi(t)\simeq\frac{\sqrt{\pi}}{2}t^{-1/4}\exp\left(-\frac{2}{3}t^{3/2}\right).$$

To estimate the resulting integrals it is advisable to use a method employed in a similar situation in Ref. 7. Then to within insignificant factors we obtain[1)]

$$W_\perp\sim\frac{(Gm^2)^2m^2}{p_0}(\tilde{m}\chi)^{5/2}\exp\left\{-2\sqrt{3}\left(\frac{\tilde{m}}{\chi}\right)\right\}.\qquad(11)$$

Thus, in this asymptotic range the probability is found to be exponentially small in a parameter that is virtually identical with the one mentioned in Sec. 1.

In a similar way one can estimate the effective energy of the resulting $\nu\bar{\nu}$ pairs and arrive at the following result (see also Ref. 1):

$$E_{\nu\,\mathrm{eff}}\simeq m_\nu\frac{p_0}{m},\qquad(12)$$

i.e., for $p_0\gg m$ the neutrinos are ultrarelativistic.

(b) $\chi\gg\tilde{m}$. Formally this condition defines the case of massless neutrinos, and the corresponding result, which also follows from Eq. (8) at $\tilde{m}=0$, is given in Ref. 5 and in Ref. 4 for $C_V=C_A=1$. For one thing, for real values $\chi\ll1$,

$$W_\perp=\frac{(Gm^2)^2m^2}{144\sqrt{3}(2\pi)^3p_0}\chi^5(49C_V^2+427C_A^2).\qquad(13)$$

In the case of ultrarelativistic motion the estimate is

$$E_{\nu\,\mathrm{eff}}\simeq p_0\chi,\qquad(14)$$

which is also true of the ordinary problem of the synchrotron radiation emitted by an electron in a magnetic field.

In this connection the following experimental scheme is suggested. The discovery of neutrino synchrotron radiation in experiments with high-energy electron beams would mean that $m_\nu\lesssim m\chi$. It is possible to establish the approximate value of m_ν (provided that m_ν is not zero in general) by varying the experimental conditions so that χ decreases. That is, if at a certain stage the counting rate and/or the neutrino energy become significantly lower than the values specified by Eqs. (13) and (14) (see also Eqs. (11) and (12)), the following estimate is true: $m_\nu\sim m\chi$. Interestingly, with the approximate characteristics of the existing storage rings $p_0\sim10$ GeV and $F\sim10^4$ G, the magnitude of the parameter $m\chi\sim10$ eV (roughly 10 eV) falls in the range of neutrino masses suggested in Sec. 1, with the energies of the emitted neutrinos in the 0.1–1 MeV range. In other words, such an experiment could be very promising. Note also that other versions of the experiment are also possible, such as scattering of electron beams by high-strength magnetic targets.

1)The analytical dependence of the form $\exp\{-2z_0^{3/2}/3\}$ has also been noted in Ref. 4 for the decay process $\pi^\pm\to\pi^0e^\pm\nu$.

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