Effects of longitudinal susceptibility and relaxation on spin- and elastic-wave spectra in antiferromagnets with of spin reorientation

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The spectrum of coupled spin-elastic waves in a two-sublattice antiferromagnet is treated theoretically for the first time with the longitudinal susceptibility and sublattice magnetization relaxation taken into account. In the absence of magnetoelastic coupling, it is shown that the spin mode spectrum, close to orientational phase transitions and in a weak magnetic field, consists of four relaxational branches and one weakly damped precessional branch. The soft mode at a transition is one of the relaxational branches, that corresponding to the transverse relaxation of the antiferromagnet's magnetizations. In a strong field, one further weakly damped precessional branch may appear in the spin wave spectrum. When the magnetoelastic coupling is included, the spectrum of the coupled modes consists of two activation (precessional and relaxational) and two nonactivation (quasielastic) branches. The relaxational mode, which in the absence of coupling was soft, becomes an activation mode, the magnitude of its gap being determined by the magnetoelastic interaction. The soft mode close to an orientational phase transition is a quasielastic mode, which may become nonpropagating when the relaxation parameter of the magnetic subsystem is large.

1. In descriptions of static and time-dependent antiferromagnetic properties, the conditions that the absolute sublattice magnetizations be constant and equal are frequently used.[1] For a two-sublattice antiferromagnet these condition are written as

$$\mathbf{M}_1^2=\mathbf{M}_2^2=M_0^2=\text{const},$$

or equivalently

$$\mathbf{ML}=0,\quad \mathbf{M}^2+\mathbf{L}^2=4\mathbf{M}_0^2, \tag{1}$$

where $\mathbf{M}_i$ is the magnetization of sublattice i, and $\mathbf{M}=\mathbf{M}_1+\mathbf{M}_2$ and $\mathbf{L}=\mathbf{M}_1-\mathbf{M}_2$ are the ferro- and antiferromagnetism vectors. The first of the conditions (1) is fulfilled for all phases of the antiferromagnet in the absence of an external magnetic field $\mathbf{H}$. In a magnetic field, the first of conditions (1) may hold only for phases in which $\mathbf{H}\perp\mathbf{L}$ (Ref. 2). At the same time, in phases in which $\mathbf{L}$ is not perpendicular to the magnetic field, this condition fails to hold. In practically any antiferromagnet placed in an arbitrary magnetic field there is at least one phase in which $\mathbf{ML}\neq 0$. In this case the condition $\mathbf{ML}=0$ is usually satisfied by letting the coefficient of the invariant $(\mathbf{ML})^2$ in the expansion of the free energy density in powers of $\mathbf{M}$ and $\mathbf{L}$ go to infinity. Then the ground-state phases with $\mathbf{ML}\neq 0$ either disappear or become distorted so as to satisfy the first of conditions (1). This approximation is equivalent to the vanishing of the longitudinal magnetic susceptibility $\chi_{\parallel}$. Strictly speaking, the longitudinal susceptibility of an antiferromagnet is zero only at absolute zero of temperature ($T=0$), however, there are magnetic materials in which $\chi_{\parallel}(T=0)\neq 0$ (Ref. 3). Thus, in the general case it is necessary to lift the conditions (1) when describing the static and dynamic behavior of the antiferromagnet.

In the present work, the ground state of the antiferromagnet and the spectra of spin and coupled magnetoelastic waves in it in are analyzed without imposing the conditions (1).

With no loss of generality, we consider a two-sublattice antiferromagnet isotropic in both its elastic and magnetoelastic properties. The free energy density is written in the form

$$F=\frac{1}{2}A\mathbf{L}^2+\frac{1}{4}B\mathbf{L}^4+\frac{1}{2}a\mathbf{M}^2+\frac{1}{2}D(\mathbf{ML})^2+\frac{1}{2}D'\mathbf{M}^2\mathbf{L}^2+\frac{1}{2}\alpha\left(\frac{\partial\mathbf{L}}{\partial x_i}\right)^2+d(M_xL_z-M_zL_x)+\frac{1}{2}\beta_1L_z^2+\frac{1}{2}\beta_2L_y^2+\frac{1}{2}\beta_3L_x^2-\mathbf{MH}+\frac{1}{2}bL_iL_kU_{ik}+\frac{1}{2}\lambda U_{ll}^2+\mu U_{ik}^2. \tag{2}$$

Here the first six terms represent the exchange interaction and the seventh term, the Dzyaloshinskiĭ interaction. The next three terms are related to the anisotropy energy. The last three terms account for the magnetostrictive and elastic energy. The magnetic-field term describes the energy of the antiferromagnet in an external field (Zeeman energy).

We first restrict ourselves to an antiferromagnet free of the Dzyaloshinskiĭ interaction. Then, for example for $\mathbf{H}\|\mathbf{x}$, the following ground-state magnetic phases may occur:

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$$1)\ \mathbf{L}\|\mathbf{x},\quad \mathbf{M}\|\mathbf{x},\quad M=\chi_{\|}H\equiv\chi_{\perp}(1-\eta)H,$$
$$(\beta_3-\beta_1)L^2+\chi_{\perp}\eta H^2\leqslant 0,\quad (\beta_3-\beta_2)L^2+\chi_{\perp}\eta H^2\leqslant 0;$$
$$2)\ \mathbf{L}\|\mathbf{y},\ \mathbf{M}\|\mathbf{x},\quad M=\chi_{\perp}H,$$
$$\beta_1-\beta_2\geqslant 0,\quad (\beta_3-\beta_2)L^2+\chi_{\perp}\eta H^2\geqslant 0; \tag{3}$$
$$3)\ \mathbf{L}\|\mathbf{z},\ \mathbf{M}\|\mathbf{x},\quad M=\chi_{\perp}H,$$
$$\beta_1-\beta_2\leqslant 0,\quad (\beta_3-\beta_1)L^2+\eta\chi_{\perp}H^2\geqslant 0.$$

Here $\chi_{\perp}^{-1}=a+D'L^2$, $\chi_{\|}^{-1}=\chi_{\perp}^{-1}+DL^2$, and $\eta=1-\chi_{\|}/\chi_{\perp}$. For brevity, Eqs. (3) omit the equations for determining the absolute value L of the antiferromagnetism vector in each phase. These equations may be obtained by minimizing F with respect to the components of L. Also, fields much smaller than the exchange field are considered. The equilibrium strains in phases (3) look like follows:

$$U_{ik}^{(0)}=-\frac{b}{4\mu}L_iL_k+\frac{\lambda bL^2}{4\mu(3\lambda+2\mu)}\delta_{ik}. \tag{3'}$$

The condition $\mathbf{ML}=0$ is, we noted earlier, equivalent to the vanishing of the longitudinal magnetic susceptibility, $\chi_{\|}=0$ (since $D\to\infty$). In this case we have $\eta=1$. Thus, the parameter η is, in a sense, a measure of departure from the condition $\mathbf{ML}=0$. Notice also that the parameter η enters the phase equilibrium condition in the product with the magnetic field H. Hence for $H=0$ the condition $\mathbf{ML}=0$ holds for all of the phases (3). And finally, from Eq. (3) it follows that in the first phase $\mathbf{ML}\neq 0$. The equality signs in the phase stability conditions locate the positions of nonhysteretical transitions of the first kind between the phases. In the terminology of Ref. 4, interphase transitions are orientational (spin-flop) transitions.

In describing antiferromagnet dynamics we start with a coupled system of elastic and Landau–Lifshitz equations,

$$\rho\ddot{U}_i=\partial(\partial F/\partial U_{ik})/\partial x_k,$$
$$\dot{\mathbf{M}}=g\{[\mathbf{MH_M}]+[\mathbf{LH_L}]+\lambda_1 L\mathbf{H_M}\}, \tag{4}$$
$$\dot{\mathbf{L}}=g\{[\mathbf{MH_L}]+[\mathbf{LH_M}]+\lambda_2 L\mathbf{H_L}\},$$

where g is the gyromagnetic ratio, $\mathbf{H_x}=-\delta F/\delta\mathbf{x}$ ($\mathbf{x}=\mathbf{M},\mathbf{L}$) are the effective magnetic fields and λ_i are dimensionless relaxation parameters. In Eq. (4), the relaxation terms are taken in their simplest version without loss of generality. Note that including relaxation terms in the Landau-Lifshitz equations is equivalent to lifting the conditions (1) in antiferromagnet dynamics as well.

Let us present the mode spectrum of the antiferromagnet. To this end we express $\mathbf{M}$, $\mathbf{L}$ and $\hat{U}$ in the respective forms $\mathbf{M}^{(0)}+\mathbf{m}$, $\mathbf{L}^{(0)}+\mathbf{l}$, and $\hat{U}^{(0)}+\hat{u}$, where $\mathbf{m}$, $\mathbf{l}$, and $\hat{u}$ are small departures from the equilibrium values (3). We consider, again for the sake of simplicity, only the case of waves propagating along the z axis (the wave vector $\mathbf{k}$ is parallel to the z axis). After this linearization, the system (4) is solved by the Fourier method.

2. We first present the spin wave spectrum for no magnetoelastic coupling ($b\equiv 0$). In phases (3) it looks as follows:

1) Phase $\mathbf{L}\|\mathbf{M}\|\mathbf{x}$

$$\omega_{1,2}=\frac{1}{2(1-\eta)}\{-i[\lambda_1\omega_E+\lambda_2\omega_B(1-\eta)]$$
$$\pm\{4\lambda_1\lambda_2\omega_E\tilde{\omega}_B(1-\eta)-[\lambda_1\omega_E+\lambda_2\omega_B\times(1-\eta)]^2\}^{1/2}\};$$

for $\omega_E\lambda_1^2\ll\omega_{13}\omega_{23}/(\omega_{13}+\omega_{23})$

$$\omega_{3,4}^2=\omega_E(\omega_{13}-i\lambda_1\sqrt{\omega_E\omega_{13}}),\quad \omega_{5,6}^2=\omega_E(\omega_{23}-i\lambda_1\sqrt{\omega_E\omega_{23}}); \tag{5}$$

for $\omega_E\lambda_1^2\gg\omega_{13}\omega_{23}/(\omega_{13}+\omega_{23})$

$$\omega_3=-i\omega_{13}\omega_{23}/[\lambda_1(\omega_{13}+\omega_{23})],\quad \omega_4=-i\lambda_1\omega_E,$$
$$\omega_{5,6}^2=\omega_E[(\omega_{13}+\omega_{23})-i\lambda_1\sqrt{\omega_E(\omega_{13}+\omega_{23})}].$$

Note that the branches ω_2 and ω_1 describe the longitudinal relaxational modes of the vectors $\mathbf{L}$ and $\mathbf{M}$ respectively, and ω_3 and ω_4 for $\omega_E\lambda_1^2\gg\omega_{13}\omega_{23}/(\omega_{13}+\omega_{23})$ describe the transverse relaxation of the same vectors.

2) Phase $\mathbf{L}\|\mathbf{y}$, $\mathbf{M}\|\mathbf{x}$

$$\omega_1=-\frac{i\omega_{32}\omega_E^2\lambda_1}{\lambda_1^2\omega_E^2+(1-\eta)[\omega_E\omega_{32}+\omega_H^2(1-\eta)]},$$
$$\omega_2=-\frac{i\omega_{12}\omega_E\omega_B'\lambda_2}{\lambda_1\lambda_2\omega_E\omega_B'+\omega_{12}[\lambda_1\lambda_2\omega_E+\lambda_2^2\omega_B+\omega_E']},$$
$$\omega_{3,4}=\frac{1}{2}\left\{-i\omega_E\lambda_1\frac{2-\eta}{1-\eta}\pm\left\{4[\omega_E\omega_{32}+\omega_H^2(1-\eta)]-\frac{\omega_E^2\lambda_1^2\eta^2}{(1-\eta)^2}\right\}^{1/2}\right\}, \tag{6}$$
$$\omega_{5,6}=\frac{1}{2}\{-i[\lambda_1\omega_E+\lambda_2(\omega_B+\omega_{12})]\pm\{4[\lambda_1\lambda_2\omega_E(\omega_{12}+\omega_B')+\lambda_2^2\omega_B\omega_{12}+\omega_{12}\omega_E']-[\lambda_1\omega_E+\lambda_2(\omega_B+\omega_{12})]^2\}^{1/2}\}.$$

Here ω_1 and ω_2 are the branches corresponding to the transverse relaxation of the components of the vector $\mathbf{L}$. The transverse relaxation of $\mathbf{M}$ affects the branches ω_3 and ω_4, and the longitudinal relaxation of $\mathbf{M}$ and $\mathbf{L}$ influences the ω_5 and ω_6 branches.

3) Phase $\mathbf{L}\|\mathbf{z}$, $\mathbf{M}\|\mathbf{x}$

The spin wave frequencies ω_i ($i=1,\ldots,6$) can be obtained from Eq. (6) by the replacements $\omega_{12}\to\omega_{21}$ and $\omega_{32}\to\omega_{31}$.

In Eqs. (5) and (6) the following notation has been introduced:

$$\omega_E=gL\chi_{\perp}^{-1},\quad \omega_B=2gL^3B,\quad \tilde{\omega}_B=2gL^3[B-2(D+D')^2\chi_{\|}^3H^2],$$
$$\omega_{ij}=gL(\alpha k^2+\beta_i-\beta_j)+\eta\omega_H^2/\omega_E(-\delta_{i1}\delta_{j3}+\delta_{j1}\delta_{i3}-\delta_{i2}\delta_{j3}+\delta_{i3}\delta_{j2}),\quad (i,j=1,2,3), \tag{7}$$
$$\omega_H=gH,\quad \omega_B'=2gL^3[B-2D'\chi_{\perp}^3H^2],$$
$$\omega_E'=\omega_E[1+2\chi_{\perp}^3H^2(B-2D')],$$

δ_{ij} is the Kronecker symbol. The spin wave frequencies (5) and (6) are written in the approximation $\omega_E \gg \omega_H$, $g\alpha k^2/L$, ω_{ij}, $\lambda_{1,2} \ll 1$.

Let us analyze the results. From Eqs. (5) and (6) it follows that when conditions (1) do not hold, the spin wave spectrum of the antiferromagnet consists of six branches. All mode frequencies are complex. The imaginary part of a frequency gives the damping of the oscillation. Purely imaginary frequencies describe the relaxational oscillations of the vectors **M** and **L**, while frequencies with a nonzero real part correspond to damped spin waves (damped precessional oscillations of **M** and **L**). In the latter case, the damping of the spin waves is due to their interaction with the relaxational oscillations. The amount of spin wave damping depends on the relation between the imaginary part of the relaxational mode (Im ω_r) and the real part of the precessional mode (Re ω_{pr}). If Re $\omega_{pr} \gg$ Im ω_r, the precessional mode is a weakly damped spin wave. For Re $\omega_{pr} \ll$ Im ω_r the precessional motion transforms into relaxational. Usually in magnetic materials the first condition is met. However, close to an orientational phase transition, i.e., as $\omega_{pr} \to 0$, the second condition may also be fulfilled. The same situation may occur for the modes of the paramagnetic subsystem in the field of the magnetized one if the damping of the former is very large (as exemplified by the rare-earth subsystem at high temperatures in rare-earth orthoferrites).

In our case, the positions of orientational phase transitions are determined by the equality signs in the antiferromagnet phase stability conditions, Eq. (3). Using the notation (7), the positions may also be found from the condition for the vanishing of the corresponding frequencies ω_{ij} at $k=0$, $\omega_{ij}(k=0)=0$. We will consider separately the antiferromagnetic mode spectrum away from and close to orientational phase transitions.

Away from a transition, when $\lambda_{1,2} \ll \sqrt{\omega_{ij}/\omega_E}$ for all i and j, the mode spectrum of each antiferromagnetic phase consists of two relaxational ($\omega_{1,2}$) and two weakly damped precessional ($\omega_{3,4}^2$ and $\omega_{5,6}^2$) branches (only the positive values of the frequencies being physically meaningful).

Close to orientational phase transitions, the mode spectrum depends on the particular transition studied. Near the transitions $1\to 2$ [$\omega_{23}(0)\to 0$], $1\to 3$ [$\omega_{13}(0\to 0$], and $2\leftrightarrow 3$ [$\omega_{12,21}(0)\to 0$], in the case when $\lambda_{1,2} \gg \sqrt{\omega_{ij}/\omega_E}$ ($ij=23, 13, 12, 21$ for the transitions $1\to 2$, $1\to 3$, and $2\leftrightarrow 3$, respectively), the mode spectrum of each phase consists of four relaxational branches ($\omega_1-\omega_4$ for phase 1 and $\omega_{1,2}$, $\omega_{5,6}$ for phases 2 and 3; two corresponding to the longitudinal and two to transverse relaxation of **M** and **L**), and one weakly damped precessional branch ($\omega_{5,6}^2$ in phase 1 and $\omega_{3,4}^2$ in phases 2 and 3). For the last of these it is assumed that $\lambda_{1,2} \ll \sqrt{\omega_{ij}/\omega_E}$, where $ij=13, 23, 32, 31$ for the transitions $1\to 2$, $1\to 3$, and $2\leftrightarrow 3$, respectively. On the basis of these modes, the antiferromagnet is far away from an orientational phase transition. One of the relaxational branches listed is soft near an orientational phase transition, its frequency vanishing at precisely the transition point for $k\to 0$. This is the frequency ω_3 in phase 1 near the transitions $1\to 2$ and $1\to 3$, and the frequency ω_2 in phases 2 and 3 near the transitions $2\to 3$ and $3\to 2$, respectively:

$$\omega_{2,3}=ig\alpha k^2/\lambda_1 L\to 0, \quad k\to 0. \tag{8}$$

The precessional branch at the above phase transitions at $k\to 0$ is an activation branch. Its activation (to first order in λ) is determined by the exchange, anisotropy, and magnetic field:

$$\text{Re } \omega_{5,6}^2 = \omega_E\omega_{23,13}(0), \quad \text{phase 1,} \quad \text{transitions } 1\to 2,\ 1\to 3, \tag{9}$$

$$\text{Re } \omega_{3,4}^2=\omega_E\omega_{32,31}(0), \quad \text{phase 2,3, transition } 2\to 3, 3\to 2.$$

A similar spectrum obtains near the orientational phase transitions $2\to 1$ [$\omega_{32}(0)\to 0$] and $3\to 1$ [$\omega_{31}(0)\to 0$] for $\lambda_{1,2} \gg \omega_H(1-\eta)/\omega_E\eta$. However, for $\omega_H(1-\eta)/\omega_E\eta \gg \lambda_{1,2} \gg \sqrt{\omega_{ij}/\omega_E}$ ($ij=32$ and 31 for $2\to 1$ and $3\to 1$, respectively) the spectrum, like that far away from a transition, consists of two relaxational ($\omega_{1,2}$) and two weakly damped ($\omega_{3,4}^2$ and $\omega_{5,6}^2$) precessional branches (as before, it is assumed that the transition-neutral modes obey $\lambda_{1,2} \ll \sqrt{\omega_{ij}/\omega_E}$, where $ij=12$ and 21 for phases 2 and 3, respectively). The relaxational branch ω_1 near orientational phase transitions we consider is soft, its frequency vanishing just at the transition points for $k\to 0$:

$$\omega_1=\frac{ig\alpha k^2\lambda_1\omega_E^2}{\lambda_1^2\omega_E^2+(1-\eta)^2\omega_H^2}\to 0, \quad k\to 0. \tag{10}$$

The branch $\omega_{3,4}^2$ for $\lambda_{1,2} \gg \omega_H(1-\eta)/\omega_E\eta$ is relaxational, while for $\lambda_{1,2} \ll \omega_H(1-\eta)/\omega_E\eta$ it is precessional, its activation being largely determined by the magnetic field strength:

$$\text{Re } \omega_{3,4}^2(0)=\omega_H^2(1-\eta), \quad \lambda_{1,2} \ll \omega_H(1-\eta)/\omega_E\eta. \tag{11}$$

The activation of the precessional branch $\omega_{5,6}^2$ is determined by the anisotropy and exchange:

$$\text{Re } \omega_{5,6}^2(0)=\omega_{12,21}(0)\omega_E' \approx \omega_{12,21}(0)\omega_E. \tag{12}$$

Note that, for $H=0$, one does not find a significant softening for any oscillation branch in phase 1. The explanation is that at the point $H=0$ the candidate for softening is the mode ω_2 corresponding to the longitudinal relaxation of the vector **L**. It can completely soften only at the Néel point, when $\mathbf{L}\to 0$. At the same time, as $H\to 0$, it is found that in phases 2 and 3 the region where the weakly damped precessional mode $\omega_{3,4}^2$ exists gets narrower (in the damping parameter).

Thus, when the conditions (1) do not hold, the spectrum of the magnetic oscillations in the antiferromagnet, when close to an orientational phase transition and in the low-field region, consists of one precessional and four relaxational branches. The precessional branch at the transition has an activation, determined by the exchange, anisotropy, and magnetic field. One of the relaxational branches is soft (its frequency vanishing, for $k\to 0$, at precisely the orientational transition point). In the large-field region, in the phases with $\mathbf{ML}=0$ (phases 2 and 3) near their transition to the phase with $\mathbf{ML}\neq 0$ (phase 1) there are two precessional branches. However, even here the soft mode close to the phase transition is a relaxational mode. This last case, without the condition $\mathbf{ML}=0$ (assuming $\mathbf{L}^2=$const), has been studied theoretically and experimentally in Refs. 5 and 6 for rare-earth orthoferrites. The results correspond to the case $\omega_H(1-\eta)/\omega_E\eta \gg \lambda_{1,2} \gg \sqrt{\omega_{ij}/\omega_E}$.

3. Let us now include the magnetoelastic interaction, $b \neq 0$. Analysis of the coupled system of equations (4) shows that the main features of the magnetoelastic mode spectrum are the same for all of the phases (3). We therefore limit our consideration to the spectrum in phase 3 close to the orientational phase transition $3 \to 1$. In this case the only branch interacting with the magnetic branches ω_1 and $\omega_{3,4}$ is the transverse elastic branch with polarization along the x axis. The other (y-polarized) transverse branch and the longitudinal sound interact with the magnetic branches ω_2 and $\omega_{5,6}$. The interaction of these latter near the transition $3 \to 1$ may be ignored.

The solution of the dispersion equation for coupled magnetic and elastic oscillations in the long-wavelength approximation $\omega_{tk}^2 \ll \omega_E\omega_{me}$ and for $M \ll L$, $\lambda_i \ll 1$ *is of the form*

$$\omega_{1,2}^2=\tilde{\omega}_{sk}^2+\frac{\lambda_1^2\omega_E^2[(6-6\eta+\eta^2)\tilde{\omega}_{sk}^4-\omega_{sk}^4]}{2(1-\eta^2)\tilde{\omega}_{sk}^4}+\frac{\zeta_{tk}\omega_{tk}^2\omega_{sk}^2}{\tilde{\omega}_{sk}^2} -\frac{i\lambda_1\omega_E[(2-\eta)\tilde{\omega}_{sk}^2-\omega_{sk}^2]}{\tilde{\omega}_{sk}(1-\eta)},$$

$$\omega_3=-\frac{i\lambda_1\omega_E\omega_{sk}^2}{(1-\eta)\tilde{\omega}_{sk}^2+\lambda_1^2\omega_E^2}, \tag{13}$$

$$\omega_{4,5}=\frac{1}{2}\left\{\pm\left[4\omega_{tk}^2(1-\zeta_{tk})-\frac{\omega_{tk}^4\Omega_{sk}^4}{\lambda_1^2\omega_E^2\omega_{sk}^4}\right]^{1/2} -\frac{i\omega_{tk}^2\Omega_{sk}^2}{\lambda_1\omega_E\omega_{sk}^2}\right\},$$

where

$$\begin{gathered}\tilde{\omega}_{sk}^2=\omega_{sk}^2+(1-\eta)\omega_H^2, \quad \zeta_{tk}=\omega_E\omega_{me}/\omega_{sk}^2,\\ \omega_{me}=gb^2M_0^2/\mu,\\ \Omega_{sk}^2=(1-\eta)[\tilde{\omega}_{sk}^2-\omega_E\omega_{me}]+\lambda_1^2\omega_E^2,\\ \omega_{sk}^2=\omega_{31}^2+\omega_E\omega_{me},\\ \omega_t=s_tk, \quad s_t^2=\mu/\rho.\end{gathered} \tag{14}$$

From this it is seen that, at the orientational phase transition $[\omega_{31}(0)=0]$ and for $k \to 0$, the spectrum of the coupled oscillations consists of three activation ($\omega_1-\omega_3$) and two nonactivation ($\omega_{4,5}$) branches. The branch $\omega_{1,2}^2$ is a quasispin (weakly damped precessional) branch. Its activation at the orientational phase transition is determined by the magnetoelastic gap and magnetic field (which enter the first term in $\tilde{\omega}_{sk}^2$) and by the relaxational gap (the second term in $\omega_{1,2}^2$). Let us estimate the magnitude of activation of the precessional branch $\omega_{1,2}$ for the orthoferrites $YFeO_3$ and $TmFeO_3$, with known experimental values of $\omega_{1,2}(0)/2\pi=107$ GGz (Ref. 5) and $\omega_{1,2}(0)/2\pi=20$ GGz (Ref. 6), respectively. The constants for Eqs. (13) and (14) will be taken from Refs. 4–7. For $YFeO_3$, at $T=293$ K and $H=71.5$ kOe,

$$\omega_E=1.13\cdot10^{14}\ s^{-1}, \quad \omega_{me}=4\cdot10^6\ s^{-1},$$
$$\omega_H=1.3\cdot10^{12}\ s^{-1}, \quad \eta=0.7.$$

For $TmFeO_3$, at $T=84$ K and $H=10$ kOe,

$$\omega_E=2.25\cdot10^{14}\ s^{-1}, \quad \omega_{me}=1.5\cdot10^8\ s^{-1},$$
$$\omega_H=1.76\cdot10^{11}\ s^{-1}, \quad \eta=0.9.$$

By putting these data into Eq. (13) (for $\lambda_1 \ll 1$) we obtain the following values for the activation at the orientational phase transition: for $YFeO_3$, $\omega_{1,2}(0)/2\pi=110$ GHz, and for $TmFeO_3$, $\omega_{1,2}(0)/2\pi=30$ GHz. The theoretical and experimental results agree well.

The quasirelaxational branch ω_3, which in the absence of the magnetoelastic interaction was soft, now has become an activation branch. At an orientational phase transition its activation is determined by the magnetoelastic interaction. The last two branches for $\lambda_1\omega_E\omega_{sk}^2\sqrt{1-\zeta_{tk}}/\Omega_{sk}^2 \gg \omega_{tk}$ (and in the region of small k) are weakly quasielastic branches with quadratic dispersion,

$$\omega_{4,5}=\pm\omega_{tk}\sqrt{\frac{g\alpha Lk^2}{\omega_E\omega_{me}}}-\frac{i\omega_{tk}^2\Omega_{sk}^2}{2\lambda_1\omega_E\omega_{sk}^2}. \tag{15}$$

In the case $\lambda_1\omega_E\omega_{sk}^2\sqrt{1-\zeta_{tk}}/\Omega_{sk}^2 \ll \omega_{tk} \ll \sqrt{\omega_E\omega_{me}}$ ("middle" k's) the branches $\omega_{4,5}$ become purely relaxational, with quadratic k dependence:

$$\begin{gathered}\omega_4=-i\omega_{tk}^2\Omega_{sk}^2/\lambda_1\omega_E\omega_{sk}^2,\\ \omega_5=-i\lambda_1\omega_E\omega_{sk}^2(1-\zeta_{tk})/\Omega_{sk}^2.\end{gathered} \tag{16}$$

The first of these branches is quasielastic, the second quasirelaxational. In the region of large values of k ($\omega_{tk}^2 \gg \omega_E\omega_{me}$), the spectrum of the coupled oscillations will again be composed of a quasispin (precessional) and a quasielastic branch, both weakly damped, and a quasirelaxational branch.

We note that at an orientational phase transition the condition $\lambda_1\omega_E\omega_{sk}^2\sqrt{1-\zeta_{tk}}/\Omega_{sk}^2 \ll \omega_{tk}$ transforms from a condition on the absolute value of the wave vector k into an inequality restricting the antiferromagnet parameters. In fact, making use of Eqs. (7) and (14), at the transition point $\omega_{31}(0)=0$ and for $k \to 0$ we obtain instead of that condition a new inequality, $\sqrt{g\alpha L\omega_{me}} \ll \tilde{\lambda}_1 s_t$, where $\tilde{\lambda}_1=\lambda_1[1+(1-\eta)^2(\omega_H/\lambda_1\omega_E)^2]$. This is, in fact, a condition on the damping parameter $\tilde{\lambda}_1$. For typical antiferromagnetic properties ($g\approx2\cdot10^7$ $Oe^{-1}s^{-1}$, $\alpha\approx10^{-12}$ cm^2, $L\approx10^2$ Oe, $\omega_{me}\approx10^7$ erg/cm^3, $s_t\approx3\cdot10^5$ cm/s) it is found that the nonactivation branches will be purely relaxational for a damping parameter $\tilde{\lambda}_1 \gg 10^{-4}$. This condition is quite easy to fulfill, particularly if one takes into account that close to an orientational phase transition there is a marked increase in the damping rate of coupled magnetoelastic waves.[8] Presumably, it is the fulfillment of this condition—and hence the transformation of the quasielastic wave into a purely relaxational one—which explains the lack, thus far, of experimental evidence for the 100% reduction in the velocity of transverse quasielastic waves at the orientational phase transition.

When the Dzyaloshinskiĭ interaction is included, then for an antiferromagnet in equilibrium in a field $\mathbf{H}\|\mathbf{x}$, three magnetic phases will also occur: 1) M_x, M_z, L_x, L_z; 2) M_x, L_y, L_z, and 3) M_x, L_z. The first two of these are angular phases. The condition $\mathbf{ML}=0$ holds only for phases 2 and 3. Analysis of the equations of motion (4) shows that the spectrum of the coupled oscillations in phase 3 will be given by formulas analogous to Eqs. (6) and (13), but with different expressions for the characteristic frequencies (7) and (14). The behavior of the coupling branches near the second-order phase transitions 3→1 and 3→2 remains in this phase just as in the absence of the Dzyaloshinskiĭ interaction. The spectrum in the angular phases 1 and 2 is given by more complicated formulas. However, even in those phases the behavior of the spectrum near the orientational transitions of the second kind, 1→3, 2→3, and of the first kind, 1→2, will be similar to that discussed above for phases 1 and 2 with no Dzyaloshinskiĭ interaction.

4. The following conclusions can be drawn from the above analysis of the antiferromagnetic mode spectrum without imposing the constancy and equality conditions on absolute sublattice magnetizations in either static or dynamical problems.

Lifting the condition $\mathbf{ML}=0$ in statics leads to the appearance of a phase with $\mathbf{M}\|\mathbf{L}$, and hence also $\mathbf{ML}\neq 0$, in an antiferromagnet in a magnetic field.

Also lifting the conditions $\mathbf{ML}=0$ and $\mathbf{M}^2+\mathbf{L}^2=\text{const}$ in dynamics has a consequence that, in addition to the precessional motion of the vectors $\mathbf{M}$ and $\mathbf{L}$, also their (transverse and longitudinal) relaxational motions become possible. In the absence of magnetoelastic coupling, the soft mode in orientational phase transition region is represented by a relaxational mode corresponding to the transverse relaxation of the antiferromagnetism vector. Near the orientational phase transition there exists only one branch, corresponding to weakly damped precessional oscillations.

When the magnetoelastic coupling is included, the soft relaxational mode at the orientational phase transition becomes activated, the activation being determined by the magnetoelastic interaction. In this case the soft mode is a quasielastic mode. Its dispersion at the transition point is quadratic. When the damping parameter is sufficiently large, the quasielastic branch may become purely relaxational.

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