Nonlinear transition waves associated with ponderomotive self-focusing of radiation in a plasma

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The dynamics of self-focusing of electromagnetic radiation is studied by simultaneously solving the quasioptical equation for a wave beam and the hydrodynamic equations for the density and velocity of the plasma. It is shown that the transition process is characterized by the appearance of a nonlinear wave propagating along the beam axis from the boundary into the interior of the plasma with a velocity much greater than the sound speed. The physical reasons for the occurrence of this wave are discussed.

The usual theory of steady or quasisteady self-focusing of electromagnetic beams is based on the assumption that the nonlinear response develops over a time much shorter than the time scale on which the intensity changes (see, e.g., Refs. 1 and 2). However, many types of nonlinearity in plasmas (e.g., thermal, ionization, and ponderomotive) are determined by relatively slow processes, so that the time for them to develop can be much longer than the time scale on which the intensity in the beam changes. In this case the behavior of the self-focusing process depends substantially on the dynamics of the nonlinear response.

For the plasma parameters considered in the present work the principal mechanism determining the nonlinear response is redistribution of the plasma density under the action of the average ponderomotive force. Previously we have shown[3] that in the transition process associated with ponderomotive self-focusing a nonlinear wave of density and radiation intensity is excited, which propagates along the axis of the beam from the boundary into the interior of the plasma with a velocity much greater than the speed of sound. Hence Andreev *et al.*[3] assumed that the perturbations in the plasma density were small. By virtue of this approximation the time-independent self-focusing is described by the nonlinear Schrödinger equation with a cubic nonlinearity, whose axisymmetric solution is well known (see, e.g., Refs. 2 and 4) to have a singularity (focus). In order to avoid this singularity in Ref. 3 a nonlinear dissipation in the electromagnetic field was introduced, which actually corresponds to multiphoton absorption in gases,[1,5] but has no physical basis in connection with a fully ionized plasma.

In the plasma the physical reason for the absence of a singularity in the steady state is the nonlinearity of the density perturbations produced by the ponderomotive forces. In order to describe the time-dependent process of the interaction of the radiation with itself it is necessary to use the nonlinear hydrodynamic equations both for the plasma density and for the velocity with which it moves in the electromagnetic field. These equations are the basis for the study of the dynamics of self-focusing of an axisymmetric beam in the present work.

In contrast to the previous work,[3] according to which the transitional nonlinear wave propagates only in the part of the plasma near the boundary until a time-independent focus is established, in the present work it is shown that the transition wave passes continuously into the plasma and is a characteristic feature of the self-focusing in media without nonlinear dissipation, in which the nonlinear response takes a relatively long time to become established. Following the nonlinear wave a steady state develops, consisting of a sequence of maxima in the radiation intensity on the beam axis.[6,7] Increasing the time over which the radiation intensity on the plasma boundary reaches its steady value increases the distance on the boundary at which the transient nonlinear wave begins to be excited.

We also calculate the transition process using a simplified model to describe the dynamics of the nonlinear plasma response. In this model the linear acoustic equation is used for the natural logarithm of the density contrast,[8,9] not for the plasma density perturbations. Although this equation cannot be justified on the basis of the hydrodynamic equations, it yields on the one hand the correct dependence of the plasma density on the radiation intensity (with an exponential saturation of the nonlinearity) in the steady state, and on the other hand, the correct dynamic equation for small density perturbations. Our calculations reveal that the simplified model of the nonlinear focusing dynamics agrees reasonably

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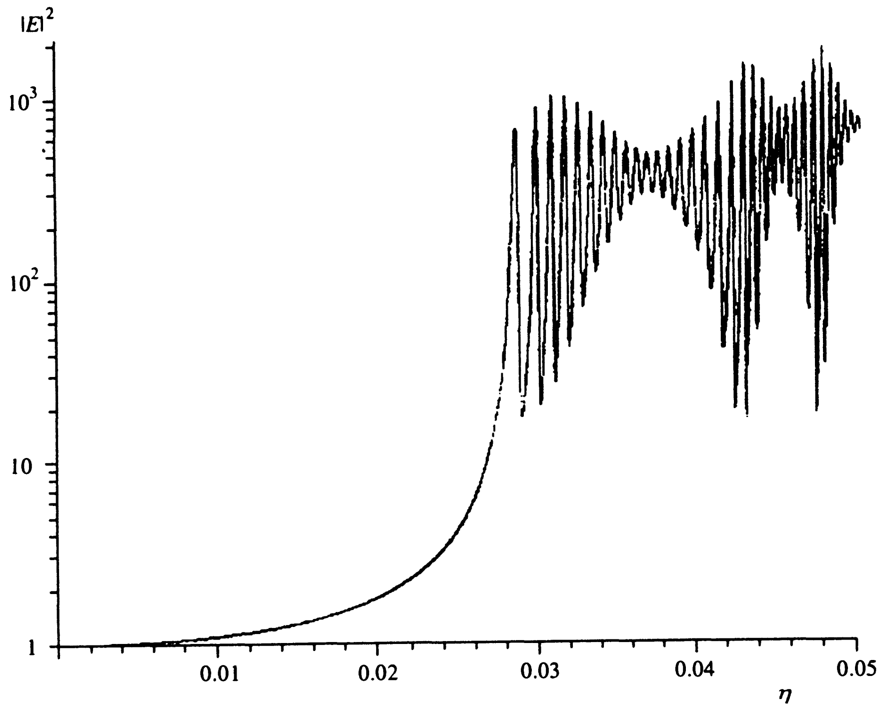

FIG. 1. Square of the electric field amplitude of a Gaussian beam on axis as a function of the longitudinal position $\eta = z/2k_0a^2$ for the parameters used in Ref. 7.

well with the results based on the full system of hydrodynamic equations.

1. BASIC EQUATIONS

With a view to averaging over the fast time dependence, we use for the beam electric field the expression

$$\mathbf{E}(\mathbf{r},t) = \tfrac{1}{2}(\mathbf{E}_0(\mathbf{r},t)e^{-i\omega_0 t} + \mathbf{E}_0^*(\mathbf{r},t)e^{i\omega_0 t}), \tag{1}$$

in which $\mathbf{E}_0$ is a complex amplitude, assumed to be a slowly varying function of time over the interval ω_0^{-1}, where ω_0 is the radiation frequency.

The hydrodynamics equations describing a quasineutral plasma in a high-frequency field take the form[10]

$$\frac{\partial N}{\partial t} + \mathrm{div}(N\mathbf{V}) = 0, \tag{2}$$

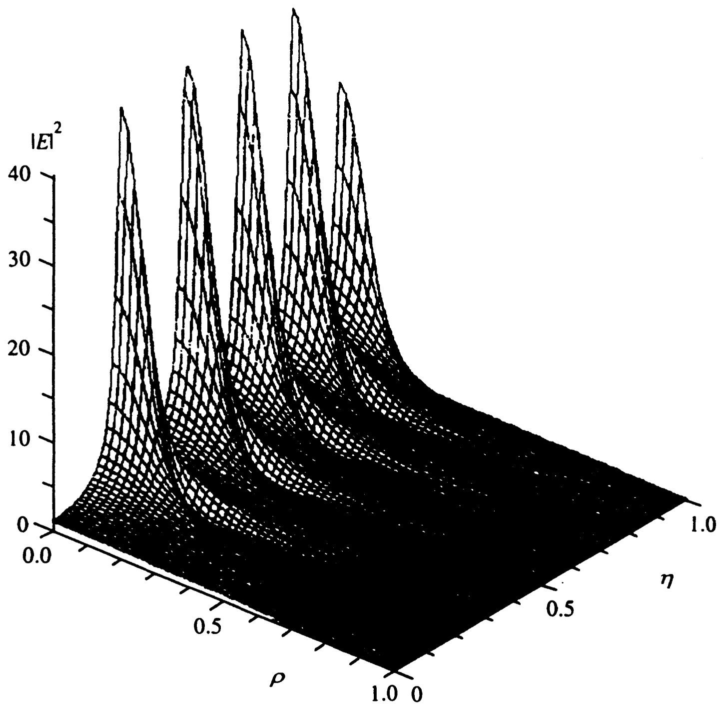

FIG. 2. Stationary profile of the square of the electric field amplitude as a function of ρ and η for $\alpha A_0 = 12$ and $A_0 = 500$.

$$\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}=-\frac{Z_i e^2}{4mM_i\omega_0^2}\nabla|\mathbf{E}_0|^2-v_s^2\nabla \ln N, \tag{3}$$

where N is the electron density, e and m are the electron charge and mass, $\mathbf{V}$ is the plasma flow velocity, Z_i and M_i are the ion charge number and mass respectively, $v_s=\sqrt{T/M_i}$ is the sound speed expressed in terms of the temperature $T=T_i=Z_iT_e$, and T_i and T_e are the ion and electron temperatures. To determine the amplitude $\mathbf{E}_0$ from the Maxwell equations we have

$$\text{curl curl } \mathbf{E}_0-2i\frac{\omega_0}{c^2}\frac{\partial \mathbf{E}_0}{\partial t}+\frac{\omega_0^2}{c^2}\mathbf{E}_0+\frac{4\pi e^2}{mc^2}N\mathbf{E}_0=0. \tag{4}$$

In Eq. (4) we have discarded a small term proportional to the second derivative with respect to time of the slowly varying amplitude $\mathbf{E}_0$. We will assume that the axisymmetric beam propagates in the z direction and the electric field strength takes the form $\mathbf{E}_0=\mathbf{e}_x E_b(t,z,r)e^{ik_0 z}$, where $\mathbf{e}_x$ is the polarization vector, k_0 is the longitudinal wave number, and E_b changes slowly over the radiation wavelength. We further assumed that the characteristic transverse dimension of the beam is much greater than the wavelength. The characteristic scale on which the field varies in the direction of the beam axis, which is comparable with the diffraction scale, is much larger than the transverse dimension, and the ponderomotive forces, which act mainly radially, cause the plasma to move in this direction. As a result we find from Eqs. (2)–(4) our fundamental set of equations:

$$\frac{\partial N}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rNV)=0, \tag{5}$$

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial r}=-\frac{ze^2}{4mm_i\omega_0^2}\frac{\partial}{\partial r}|E_b|^2-v_s^2\frac{\partial \ln N}{\partial r}, \tag{6}$$

$$\left[2i\frac{\omega_0}{c^2}\frac{\partial}{\partial t}+2ik_0\frac{\partial}{\partial z}+\Delta_\perp+\frac{\omega_{p_0}^2}{c^2}\left(1-\frac{N}{N_0}\right)\right]E_b=0, \tag{7}$$

where $V=V_r$ is the radial plasma velocity and $\omega_{p0}=\sqrt{4\pi e^2N_0/m}$ is the plasma frequency expressed in terms of the unperturbed electron density N_0 outside the beam. It is assumed that the quantities ω_0 and k_0 are related by the dispersion relation $\omega_0^2=k_0^2c^2+\omega_{p_0}^2$. We introduce the dimensionless dependent variables

$$W=\frac{V}{v_s},\quad A=\frac{N}{N_0}\left(\frac{\omega_{p_0}a}{c}\right)^2,\quad E=\frac{E_b}{E_{b0}}, \tag{8}$$

where E_{b0} and a respectively are the maximum value of the amplitude E_b and the characteristic beam width at the plasma boundary at the point $z=0$. We use the dimensionless independent variables

$$\rho=\frac{r}{a},\quad \tau=\frac{v_s t}{a},\quad \eta=\frac{z}{2k_0a^2}. \tag{9}$$

As a result, Eqs. (5)–(7) are transformed to

$$\frac{\partial A}{\partial \tau}+\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A W)=0, \tag{10}$$

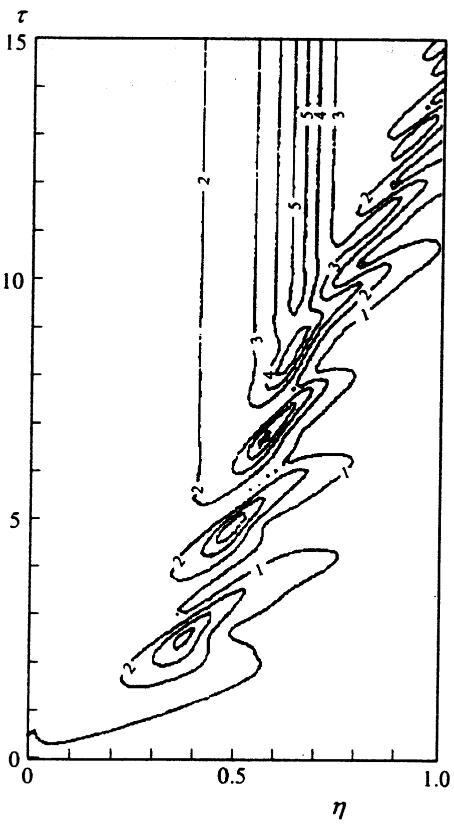

FIG. 3. Contours of constant $|E|$ in the η–τ plane ($\alpha A_0=8.4$, $A_0=500$, $\beta=1$, $\gamma=0.05$).

$$\frac{\partial W}{\partial \tau}+W\frac{\partial W}{\partial \rho}=-\alpha\frac{\partial}{\partial \rho}|E|^2-\frac{\partial \ln A}{\partial \rho}-\gamma W, \tag{11}$$

$$\left[i\left(\beta\frac{\partial}{\partial \tau}+\frac{\partial}{\partial \eta}\right)+\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right)+A_0-A\right]E=0, \tag{12}$$

where the constant $\alpha=Z_ie^2E_{b0}^2/4m\omega_0^2T$ characterizes the ratio of the so-called high-frequency potential to the temperature, the constant $\beta=2a\omega_0v_s/c^2$ determines the magnitude of effects resulting from the finite propagation speed of the electromagnetic radiation, and the quantity $A_0=a^2\omega_{p_0}^2/c^2$ is the dimensionless plasma density outside the beam. In Eq. (11) an additional term has been introduced to take into account dissipation. The quantity $\gamma=\nu a/v_s$ is related to the effective collision frequency ν. For the amplitude of the electric field on the plasma boundary ($\eta=0$) we have used the expression

$$E(\rho,\eta=0,\tau)=f(\tau)\exp[-\rho^2+i\varphi(\rho)], \tag{13}$$

where the function $f(\tau)$ characterizes the increase in the field at the boundary and varies from 0 to 1 as τ goes from 0 to ∞. The function $\varphi(\rho)$ determines the shape of the beam phase front at the boundary. The condition $f(0)=0$ enables us to use as initial conditions for the plasma

$$A(\tau=0)=A_0,\quad W(\tau=0)=0. \tag{14}$$

The boundary conditions are given on the beam axis:

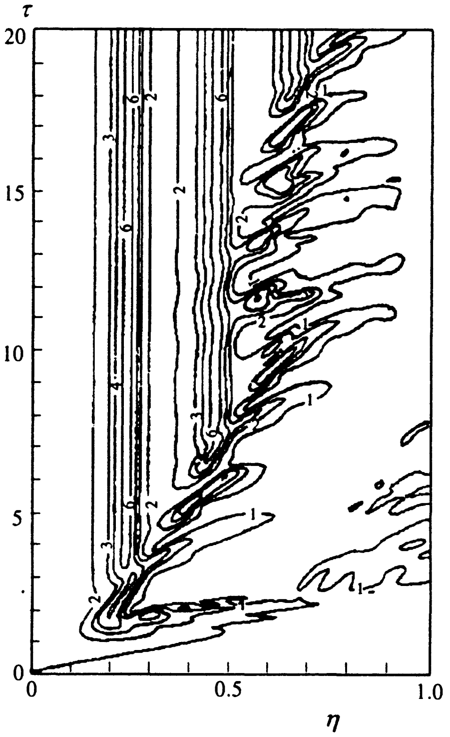

FIG. 4. Contours of constant $|E|$ in the η–τ plane ($\alpha A_0=12, A_0=500, \beta=1, \gamma=0.05$) when the beam is switched on instantaneously [$f(\tau)=1$ for $\tau>0$].

$$W(\rho=0)=0, \quad \left.\frac{\partial A}{\partial \rho}\right|_{\rho=0}=0, \quad \left.\frac{\partial E}{\partial \rho}\right|_{\rho=0}=0, \tag{15}$$

and also at a large distance $\rho_{max} \gg 1$ from the beam axis:

$$E(\rho_{max})=0, \quad A(\rho_{max})=A_0, \quad W(\rho_{max})=0, \tag{16}$$

or

$$\left.\frac{\partial A}{\partial \rho}\right|_{\rho_{max}}=\left.\frac{\partial E}{\partial \rho}\right|_{\rho_{max}}=\left.\frac{\partial W}{\partial \rho}\right|_{\rho_{max}}=0. \tag{17}$$

The system of equations (10)–(12) with the initial and boundary conditions (13)–(17) has been solved numerically. The mathematical details of the calculations are presented in Ref. 11.

2. STEADY SELF-FOCUSING

Before investigating the transition process, we study the steady state to which it leads. We set $W=0$ in Eqs. (10) and (11) and, dropping the derivative with respect to τ and Eq. (12) we write it in the form

$$\left[i\frac{\partial}{\partial \eta}+\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial}{\partial \rho}\right)+A_0(1-e^{-\alpha|E|^2})\right]E=0. \tag{18}$$

The solutions of this equation have been studied both analytically[6,12–15] and numerically.[7] In order to check our numerical treatment we have performed a calculation with the same parameters as were used in Ref. 7: $\sqrt{z}eE_{b0}/\sqrt{m}\omega_0\sqrt{T} = 0.1$, $k_0a=700$, $N_0/N_c=0.1$, where $N_c=m\omega_0^2/4\pi e^2$ is the critical electron density. In our notation these parameters correspond to $\alpha=2.5\cdot 10^{-3}$ and $A_0=4.9\cdot 10^4$.

Figure 1 shows the intensity $|E|^2$ as a function of the longitudinal position on the beam axis ($\rho=0$) when there is a plane wave front on the boundary [$\varphi(\rho)=0$]. In Ref. 7 the calculation was carried out in the interval $0\leq\eta\leq 3.57\cdot 10^{-2}$. Our results are in complete agreement on this interval with those of Ref. 7. For $\eta\geq 3.57\cdot 10^{-2}$, as can be seen, the intensity on the beam axis becomes modulated and the oscillation period decreases as a function of η. This implies that the results of the numerical calculations differ from those of the analytical studies carried out in the aberrationless approximation[6] not only qualitatively but also quantitatively. This was noted previously in Ref. 7.

Figure 2 shows the variation of the radiation intensity $|E|^2$ as a function of ρ and η for $\alpha A_0=12$ and $A_0=500$, corresponding to the parameters of the time-dependent calculations whose results were presented above. As can be seen, for those intensities the steady state consists of an essentially periodic sequence of maxima on the beam axis. In the aberrationless approximation[6] these oscillations appear when a condition related to the values of the parameters A_0 and α holds:

$$A_0(1-e^{-\alpha})>2. \tag{19}$$

It is easy to see that in our calculations the inequality (19) was satisfied.

In the limit $\alpha<1$, which corresponds to a cubic nonlinearity, Eq. (19) yields an expression for the critical beam power $(\alpha A_0)_{cr}=2$. Analysis of the exact Schrödinger equation[1,2] shows that $(\alpha A_0)_{cr}=7.54$.

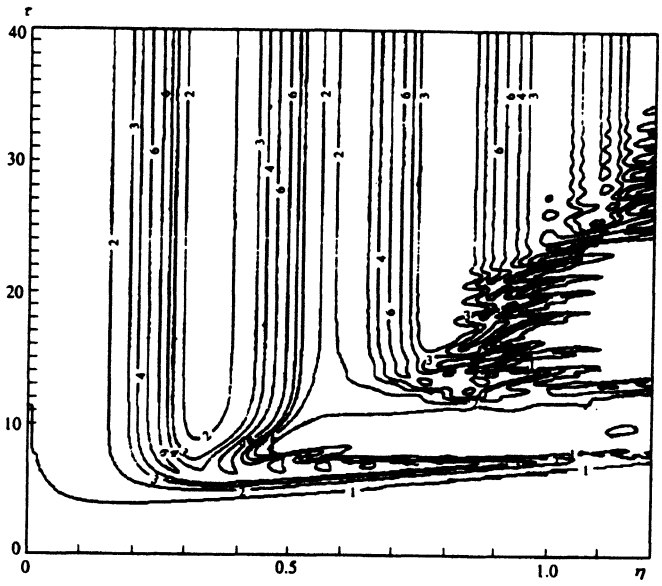

FIG. 5. Contours of constant $|E|$ in the η–τ plane ($\alpha A_0=12$, $A_0=500$, $\beta=1$, $\gamma=0.05$) when the beam is turned on smoothly at the boundary ($f(\tau)=1$ for $\tau>0$).

3. THE TRANSITION PROCESS

In order to make a comparison with the results of Ref. 3, we begin by doing calculations with the same beam power as was used there ($\alpha A_0=8.4$). In addition, we used the values $A_0=500$, $\beta=1$, and $\gamma=0.05$ and assumed that the beam was turned on instantaneously at the boundary $[f(r>0)=1]$. Figure 3 shows contours of constant $|E|$ as a function of τ and η for $\rho=0$. As can be seen, the nonlinear transition wave excited near the boundary propagates into the interior of the plasma. A steady state is established behind the wave. In contrast with Ref. 3, when this process ended in the formation of a single time-independent maximum where a substantial portion of the beam energy was absorbed, in the present case the process continues beyond the first maximum. With increasing time the nonlinear wave passes deeper and deeper into the plasma, leaving behind it new stationary intensity maxima on the beam axis.

In order to further elucidate this process we have also carried out calculations with the parameters $\gamma=0.05$, $\alpha A_0=12$, $A_0=500$, and $\beta=1$ (Fig. 4). The steady state shown in Fig. 2 was obtained for precisely these parameters.

It is natural to assume that for the transition process an important role is played by the form of the function $f(\tau)$, which determines the rise in the beam amplitude at the boundary. In Fig. 5 results are displayed from calculations carried out with the same parameters as in Fig. 4, but for a function $f(\tau)$ of the form

$$f(\tau)=[\text{th}(\tau/\tau_0)]^2, \tag{20}$$

where $\tau_0=2.5$. The function (20) reaches the value 1 for $\tau\approx 10$. As can be seen, the nonlinear wave begins to be excited later and at a larger distance from the plasma boundary. The first intensity maximum approaches its limiting value monotonically as a function of time. A wavelike process develops at greater depths, associated with the establishment of subsequent maxima.

A number of studies[8,9] used a simplified model to describe the nonlinear plasma dynamics:

$$\left(\frac{\partial^2}{\partial\tau^2}+2\gamma\frac{\partial}{\partial\tau}-\Delta_\perp\right)\ln\frac{A}{A_0}=\alpha\Delta_\perp|E|^2. \tag{21}$$

In the limit of small density perturbations, when we have $A=A_0+\delta A$, $|\delta A|\ll A_0$, Eq. (21) yields the acoustic equation used in Ref. 3. In the steady state ($\partial/\delta\tau=0$) Eq. (18) follows

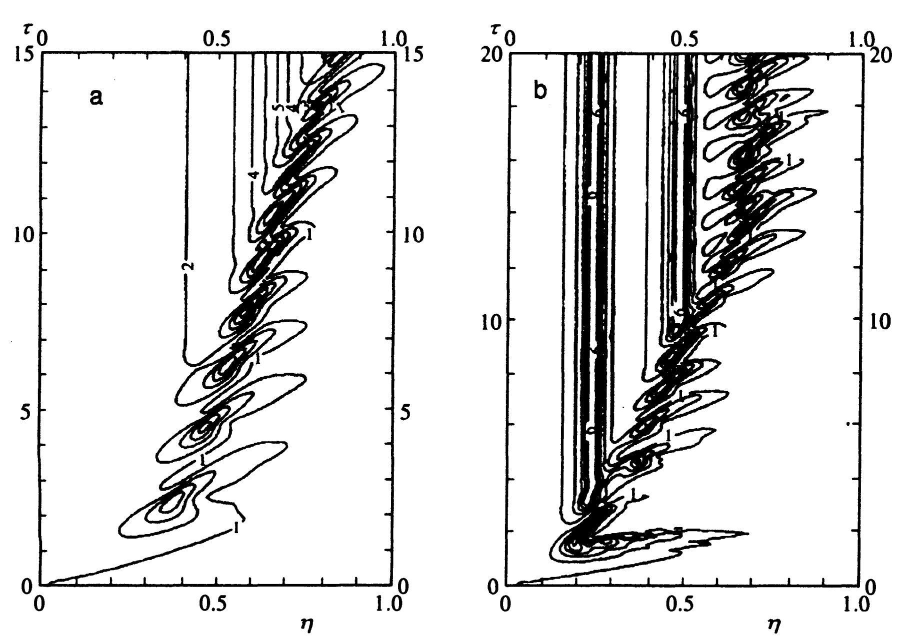

FIG. 6. The same as in Fig. 3a and Fig. 4b, but using the simplified model to describe the plasma dynamics.

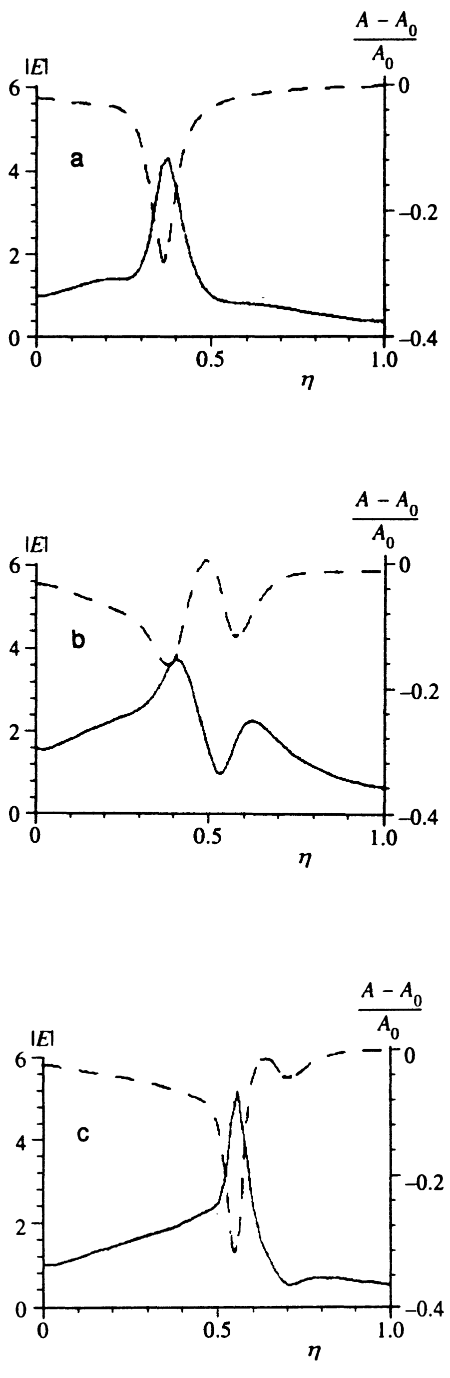

FIG. 7. Relative perturbation in the plasma density $(A-A_0)/A_0$ (broken trace) and the amplitude $|E|$ on the beam axis at three times $\tau=2.5$ (a), 4.0 (b), and 6.5 (c) $[\alpha A_0=8.4, A_0=750, \beta=1, \gamma=0.05, f(\tau>0)=1]$.

from Eqs. (12) and (21). In order to clarify when it is valid to use Eq. (21) to calculate the time dependence of highly nonlinear processes, we carried out calculations with the same parameters as those used above. Figures 6a and 6b display the same function as in Figs. 3 and 4, derived using Eqs. (12) and (21). Comparison of the figures shows that the calculations using the simplified model (21) and those with the full hydrodynamic model (10), (11) agree not only qualitatively but also quantitatively.

4. CONCLUSION

Let us briefly discuss the physical processes that determine the origin of the nonlinear transition waves for ponderomotive self-focusing. In the time during which the beam penetrates into the plasma, which occurs with a velocity close to that of light, the plasma density is unable to vary and the plasma remains essentially uniform. The excess high-frequency pressure begins to expel the plasma from the region occupied by the beam. The ponderomotive forces (due to the high-frequency pressure) are greatest where the beam is narrowest. Since a beam with a planar phase front spreads out in a uniform plasma due to refraction, its width is initially smallest near the plasma boundary. It is there that the expulsion of the plasma occurs fastest and a region forms with reduced density.[16] This region, which acts as a focusing lens, transports the region of maximum intensity deep into the plasma. There the expulsion of the plasma occurs faster and a region of reduced density forms all over again, as a result of which the radiation is focused still deeper into the plasma. This process goes on continuously. The intensity maxima of the radiation and the region with reduced plasma density begin to move from the boundary along the beam axis, reconstituting and maintaining one another (Fig. 7a).

After an intensity maximum leaves the region, the plasma expelled by the high-frequency pressure forces, being an elastic medium, begins to oscillate with a frequency of order v_s/a. Consequently, after a time of order $a/2v_s$ a density maximum develops in place of the minimum on the beam axis (Fig. 7b). In contrast to a density minimum, it acts as a defocusing lens and reduces the fraction of the radiation that reaches the first intensity maximum. As a result, the latter begins to decrease. However, the next density minimum, beyond which there is still no maximum, acts as a focusing lens and begins to create a new intensity maximum (Fig. 7c). This process by which the first maximum is quenched and another one grows behind it, separated from the first minimum, is repeated over and over.

In the time the nonlinear wave propagates along the beam axis the expelled plasma spreads out in the form of sound waves propagating from the beam axis. As a result, equilibrium is established behind it, in which the high-frequency pressure is balanced by the thermal pressure of the plasma.

A moving intensity maximum can develop only if the diffractive divergence of the beam is dominated by the refraction that develops due to plasma nonuniformity. Hence the focal length l of the lens that develops as a result of the expulsion of the plasma by the beam can be taken equal in order of magnitude to the diffraction length k_0a^2. Multiplying this length by the period a/v_s of the density oscillations, we find that the velocity with which the maxima (or minima) in intensity propagate is on the order of $V_m \simeq v_s k_0 a$. Since it is assumed that the beam width a is much greater than the radiation wavelength, the velocity V_m is much greater than the sound speed v_s. In the range of beam powers in question it does not depend on the radiation intensity (i.e., the parameter α) which is confirmed by comparison of Figs. 3 and 4.

Each maximum that develops on the trailing edge of the nonlinear wave moves forward in time to the leading edge and disappears. Consequently, its velocity is greater than that of the boundary of the steady state, and the latter is somewhat less than V_m. This effect is clearly displayed in Figs. 3–6.

As an example illustrating the physical conditions under which the basic calculations apply, we consider a laser beam with frequency $\omega_0=4\cdot 10^{15}$ s^{-1} (the second harmonic of a neodymium laser) with a characteristic width $a=20$ μm propagating in a hydrogen plasma with density $N_0=5\cdot 10^{19}$ cm^{-3} and electron temperature $T_e=100$ eV. If the maximum beam intensity on the plasma boundary is $3\cdot 10^{14}$ W/cm^2, the transition waves have a characteristic length $\sim$0.3 cm and propagation speed $\sim 10^9$ cm/s. In a plasma with dimensions of order 1 cm such waves can arise for pulses whose rise time is less than 1 ns.

Thus, the results of this investigation show that in media in which the nonlinear response takes a relatively long time to become established the development of a regime in which the radiation propagates in a steady fashion is accompanied by excitation of a nonlinear transition wave, which propagates from the boundary. This transient process is a consequence of the hydrodynamic mechanism for saturation of the ponderomotive nonlinearity of the plasma described by Eqs. (5) and (6), together with the conservation of electromagnetic energy flux, which follows from Eq. (7).

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