# Pulse propagation and gain dynamics in two-component media

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(Submitted 6 July 1994; resubmitted 27 July 1994)
Zh. Eksp. Teor. Fiz. **106**, 1343–1359 (November 1994)

We study certain aspects of coherent gain and pulse propagation in two-component media. We show that in the subthreshold gain regime, a pulse whose area exceeds a critical value will develop into a $\pi$ pulse; as it propagates in a two-component medium, the latter will be continuously amplified in intensity and compressed in width. We also study the propagation of large-area pulses in two-component media, and show that such a medium can give rise to bound pulse pairs that periodically overtake one another. A class of soliton solutions has been identified for two-component media. © *1994 American Institute of Physics.*

## 1. INTRODUCTION

Self-induced transparency is one of the most obvious effects of the coherent interaction of radiation with resonant media.[1] The theory underlying this phenomenon is presently well understood not just for two-level media—it has also been generalized to degenerate levels and multilevel media.[2–5] Efficient mathematical methods have been developed for solving this class of problem.[6–7]

The advent of femtosecond pulsed lasers has engendered a new wave of interest in studying the coherent interaction of light with resonant media. This interest derives both from the possibility that new methods for the spectroscopy of matter may be developed, and that we may be able to do without a number of familiar theoretical approximations. It may also be possible to develop control techniques for the shape and parameters of femtosecond pulses.

The generation of solitons with self-induced transparency is, on the one hand, one of the fundamental phenomena of quantum electronics, while on the other, it governs the extent to which pulses can be amplified in two-component resonant media. Further pulse amplification and narrowing is feasible only if one moves to denser media, which then increases the transverse relaxation rate and thereby destroys the coherence of the atomic subsystem.

In this paper, we address the coherent interaction dynamics of light with two-component media that consist of resonant components with differing transition dipole moments. We show that the formation dynamics of $n\pi$ pulses in such media is much richer than, and qualitatively different from, processes in one-component media. A $2\pi$ pulse relative to one of the components can be amplified by virtue of the population inversion of the second component while its area is conserved, and the pulse is therefore compressed. Furthermore, a bound propagation regime is possible in which two $2\pi$ pulses (relative to one of the components) alternately invert the population of the second component, and thus periodically overtake one another, thereby forming a bound pulse pair.

## 2. STATEMENT OF THE PROBLEM. BASIC EQUATIONS

Consider the amplification of a short pulse in a medium that consists of two types of atoms, $a$ and $b$, with differing transition dipole moments $d_a<d_b$. We use the two-level atomic approximation to describe the medium. Assume that the two types of atoms have the same transition frequency $\omega$ between the ground and excited states. Since the Rabi frequency is proportional to the dipole moment ($\Omega_{a,b}\propto d_{a,b}$), we have $\Omega_a<\Omega_b$, and we can thus refer to type $a$ atoms as "slow" and type $b$ atoms as "fast."

We work with a specimen of a two-component medium of volume $V$ containing $N_a$ slow atoms and $N_b$ fast ones. We use dimensionless coordinates and time

$$x'=x/L,\quad t'=t/\tau, \tag{1}$$

where $L=c\tau$ and $\tau$ is some characteristic pulse width. Henceforth we omit the primes.

For exact resonance, the combined Maxwell and optical Bloch equations for the slowly varying wave amplitudes ($\alpha$), the polarization of the fast ($p$) and slow ($P$) atoms, and the population differences of the fast ($r$) and slow ($R$) atoms, which describe pulse propagation in a two-component homogeneously broadened medium, are

$$\frac{\partial a}{\partial t}+\frac{\partial a}{\partial x}=P+p,\quad \frac{\partial P}{\partial t}+\alpha_a P=\beta_a aR,$$
$$\frac{\partial p}{\partial t}+\alpha_b p=\beta_b ar,\quad \frac{\partial R}{\partial t}=-aP,\quad \frac{\partial r}{\partial t}=-ap\ . \tag{2}$$

The notation in these equations is

$$\beta_{a,b}=\frac{2\pi\omega|d_{a,b}|^2}{\hbar}\frac{N_a}{V}\tau^2,\quad \alpha_{a,b}=\frac{\tau}{T_2^{(a,b)}}, \tag{3}$$

where $T_z^{(a)}$ and $T_z^{(b)}$ are the transverse relaxation times of the slow and fast atoms. In deriving (2), we have neglected inhomogeneous line broadening in either type of atom. In actual experiments, however, the latter can substantially relax the resonance requirements on the transitions, thanks to the resonance of the various components of the inhomogeneously broadened spectrum.

We assume that the pulse width is such that $\tau_p\ll T_1^{(a)},T_1^{(b)}$, so that we can neglect the relaxation terms in the dynamical equations for the population differences in (2).

The field amplitude $\alpha$ has been normalized in (2) such that $n(x,t)=|a(x,t)|^2$ is the photon number density expressed in units of the slow-atom density $n_a=N_a/V$.

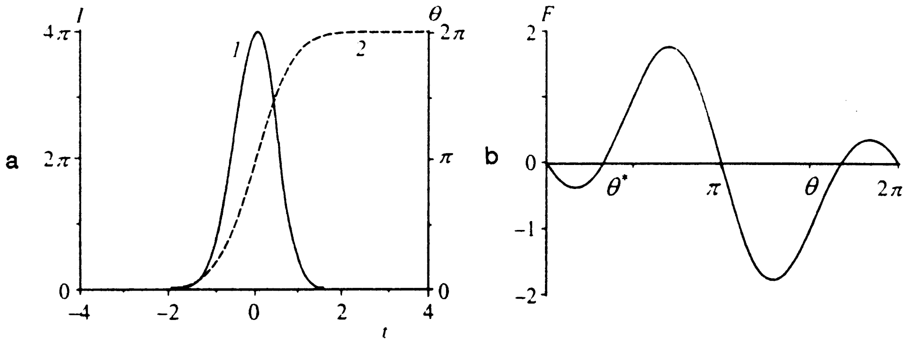


FIG. 1. a) Profile (*1*) and Bloch angle (*2*) for a pulse with amplitude $A_0=2$ and width $\tau_p=1$. b) Dependence of $F$ on the Bloch angle for a pulse with $\gamma=2$.

We further assume that at the initial time, the fast atoms in the two-component medium are in the ground state, and the slow atoms are in the excited state. The medium is therefore made up of amplifying (slow) and damping (fast) atoms. The initial conditions are

$$a(x,0)=0, \quad P(x,0)=p(x,0)=0,$$
$$R(x,0)=1, \quad r(x,0)=-r_0. \tag{4}$$

Thus, $r_0=N_b/N_a$ is the concentration of fast atoms in the two-component medium, and the concentration of slow atoms is taken to be unity ($R_0=1$).

It is well known[1,3,5] that pulse evolution in propagation problems depends on the area under the input pulse, both in resonantly amplifying and resonantly damping media. Using (2), it can be shown that the pulse area for the slow ($a$) and fast ($b$) components is

$$\sum_{a,b}(x)=\theta_{a,b}(x,t=\infty),$$

where the Bloch angle $\theta_{a,b}(x,t)$ is given by

$$\theta_{a,b}(x,t)=\sqrt{\beta_{a,b}}\int_{-\infty}^{t}a(x,\xi)d\xi. \tag{5}$$

Under conditions in which the amplification of the incident pulse is purely coherent, i.e., $\alpha_a=\alpha_b=0$, we can derive the equation for the evolution of the Bloch angle $\theta(x,t) = \theta_a(x,t) = \sqrt{(\beta_a/\beta_b)}\theta_b(x,t)$ in a two-component medium:[8]

$$\frac{\partial^2\theta}{\partial t^2}+\frac{\partial^2\theta}{\partial x\partial t}=F, \tag{6}$$

where

$$F=R_0\beta_a \sin\theta+r_0\sqrt{\beta_a\beta_b}\sin\left(\sqrt{\frac{\beta_b}{\beta_a}}\,\theta\right). \tag{7}$$

In the special case $\sqrt{\beta_b/\beta_a} = 2$, $R_0=1$, $r_0=0.5$, this is equivalent to the double sine-Gordon equation,[2,5,7] which governs the dynamics of self-induced transparency in degenerate transitions with angular momentum $J=2$ and $\Delta J=0$, $\Delta m=0$.[9]

Analysis of Eqs. (6) and (7) shows that a two-component medium will have a threshold. For the initial stages of small-area incident pulses ($\Sigma\ll\pi$), at which time the populations vary slowly ($R\approx R_0, r\approx -r_0$), the right-hand side of Eq. (6) becomes

$$F=(R_0\beta_a-r_0\beta_b)\theta. \tag{8}$$

The area of an incident pulse will increase as it propagates if $R_0\beta_a>r_0\beta_b$. The requirement for coherent amplification of a weak pulse in a two-component medium is therefore

$$R_0\beta_a>r_0\beta_b. \tag{9}$$

If this condition is met, the two-component medium will have an above-threshold gain regime for incident pulses; if not, it will have a subthreshold regime.

For numerical modeling, we assume that the incident pulse has a Gaussian profile:

$$a(0,t)=\sqrt{\pi}A_0\exp\left\{-\frac{(t-t_0)^2}{\tau_p^2}\right\}. \tag{10}$$

Thus, the area of the incident Gaussian pulse can be easily calculated:

$$\Sigma_0=A_0\tau_p\pi. \tag{11}$$

## 3. CRITICAL AREA OF A PULSE

Equation (6) dictates the Bloch-angle dynamics of a pulse propagating through a two-component medium. The right-hand side contains the function $F$, which can have either sign. When $F<0$, the area under an incident pulse will decrease as it propagates, and the pulse will be attenuated; conversely, if $F>0$, the pulse area will increase and the pulse will be amplified. It can be seen from Eq. (7) that in the subthreshold gain regime ($R_0\beta_a<r_0\beta_b$), $F$ will have the form shown in Fig. 1b, in which case a Gaussian pulse with amplitude $A_0=2$ and width $\tau_p=1$ (Fig. 1a) will have area $2\pi$ in accordance with the definition (11). The profile of the incident pulse is given by Eq. (10), so the total pulse amplitude is $A=A_0\sqrt{\pi}$, and its peak intensity is $I=\pi|A_0|^2$.

It can be seen from Fig. 1 that the function $F$ is negative when the Bloch angle $\theta(t)$ at the leading edge of the pulse is less than the critical angle $\theta^*$, so energy is absorbed from the leading edge by the fast atoms in the medium. But if the area of the pulse is large enough and the Bloch angle $\theta(t)$ can exceed $\theta^*$, $F$ will become positive and the pulse area will start to increase, i.e., the pulse will be amplified, extracting energy stored in the medium.

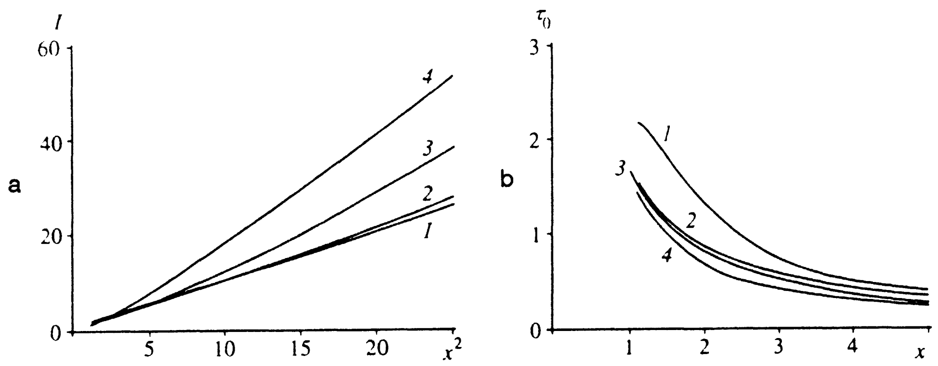


FIG. 2. Pulse intensity as a function of $x^2$ (a) and pulse width as a function of $x$ (b). The concentration of fast-component atoms is $r_0=0.2501$ (*1*), 0.35 (*2*), 0.4 (*3*), 0.45 (*4*).

This scenario clarifies what we mean by a critical pulse area $\Sigma^*=\theta^*$. For weak pulses with less than the critical area ($\Sigma<\Sigma^*$), the right-hand side of Eq. (6) is always negative. Consequently, the area of a weak pulse propagating through a two-component medium will steadily decrease, and the pulse itself will be damped.[8]

This property of two-component media—the absorption of small-area pulses in the subthreshold gain regime—is an important consideration in high-gain systems.

The thresholding nature of pulse amplification is a distinctive feature of two-component media. The expression for the critical pulse area $\Sigma^*$ when $\beta_b=4\beta_a$ can be derived from Eq. (6):

$$\Sigma^*=\text{arc}(1/\gamma), \tag{12}$$

where $\gamma=4r_0/R_0$.

The critical pulse area clearly depends on the properties of the medium, and will have different values for the fast and slow components. Thus, if the area of a pulse incident upon a two-component medium in the subthreshold gain regime is less than the critical area $\Sigma^*$, the pulse will be damped.

If the area of an incident pulse is greater than $\Sigma^*$, it will increase up to the point at which $F>0$. From Fig. 1b, $F$ is clearly positive over the interval $\theta^*<\theta<\pi$, so the pulse area must increase until it reaches $\pi$—in other words, a $\pi$ pulse must form in a two-component medium. Note therefore that $\theta=\theta_a$ by virtue of the $\pi$ pulse relative to the slow component.

## 4. SUBTHRESHOLD AMPLIFICATION OF WEAK PULSES. CRITICAL CONCENTRATION OF THE FAST COMPONENT

We consider first the subthreshold amplification of a pulse with area $\Sigma^*<\Sigma<\pi$. Figure 2 shows the results of numerically modeling the propagation of a low-amplitude pulse in two-component media containing a variety of concentrations of the fast component. A Gaussian pulse with amplitude $A_0=0.7$, width $\tau_p=0.5$, and area $\Sigma_0=0.35\pi$ is incident upon the medium. The critical areas for the media indicated in the figure are $\Sigma^*=0.009\pi$ (*1*), $0.25\pi$ (*2*), $0.29\pi$ (*3*), and $0.32\pi$ (*4*). For curve *1* we have $\Sigma_0\gg\Sigma^*$, and for the medium with properties corresponding to curve *4*, $\Sigma_0\approx\Sigma^*$. It should be clear from (12) that the critical concentration of fast-component atoms is $r^*=1/(4\cos\Sigma_0)=0.55$ for a pulse with area $\Sigma_0=0.35\pi$, implying that such a pulse will be amplified in a two-component medium only if $r_0<r^*$. The numerical models show that as a weak pulse propagates through a two-component medium, its area tends to $\pi$.

Since the fast and slow atoms in a two-component medium have different transition dipole moments $d$, we see from (3) that for a medium in which $d_b=2d_a$, the pulse area relative to the fast component will be twice the area relative to the slow component. A weak pulse will thus be amplified into a $\pi$ pulse relative to the slow component, and at the same time it will become a $2\pi$ pulse relative to the fast component.

Figure 2 shows the intensity of an amplified weak pulse as a function of $x^2$. Two stages can be identified in the gain dynamics of a weak pulse: an initial stage in which the two-component medium takes the incident pulse and produces a characteristic $\pi$ pulse relative to the slow atoms of the medium, and a second stage in which that pulse is amplified. Note that the first, in which the pulse grows in area, displays more rapid growth (than $x^2$) in the peak pulse intensity. As soon as the pulse area reaches $\pi$, however, the intensity rises less rapidly, becoming proportional to the square of the distance that it has traveled in the medium. We see from Fig. 2a that $I\propto x^2$ for $x>2$, and it also shows that the peak pulse intensity rises more rapidly in a two-component medium in which the fast-atom component has the higher concentration. A possible explanation stems from the fact that the pulse energy in the range $\theta\leqslant\theta^*$ goes into redistributing the populations of the components. At $\theta=\pi/2$, the second (fast) component turns out to be fully excited, and since that component amplifies pulses more efficiently, the pulse amplitude rises more rapidly as $\theta^*$ increases, which can be seen in Fig. 2a.

Figure 2b shows the width of an amplified weak pulse as a function of distance traveled in the medium, and makes it clear that the width of the amplified pulse steadily decreases with distance. At a given distance, the narrowest pulse is produced in the two-component medium with the highest concentration of fast atoms. Figure 2b enables us to characterize the pulse-narrowing behavior: the width $\tau_0$ of the amplified pulses decreases as $\propto 1/x$. Plotting $\tau_0$ against $1/x$, it can easily be shown that the pulse width tends to zero, rather than to a finite value as in the soliton case.[5,7]

This work thus demonstrates that the most efficient amplifying medium for a weak pulse with area $\Sigma$ is a two-component medium in which the fast-atom concentration is

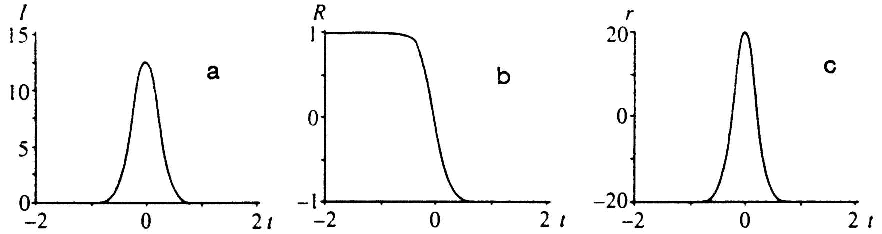


FIG. 3. Profile of a $\pi$ pulse (a), and population inversion dynamics for slow (b) and fast (c) atoms in a two-component medium.

as close as possible to the critical value, $r_0 \approx r^*$. If $r_0 > r^*$, however, an incident pulse will be damped rather than amplified. In the course of amplification, the area of a weak pulse will increase to $\pi$. As it propagates through the two-component medium, the $\pi$ pulse thus formed will steadily grow in peak intensity, proportional to the square of the distance traversed in the medium ($I \propto x^2$), and its width will steadily decrease inversely as the distance traveled ($\tau_0 \propto 1/x$).

### 5. PULSE AMPLIFICATION IN TWO-COMPONENT MEDIA

We established above that a weak pulse in a two-component medium will increase in area and turn into a $\pi$ pulse. In Fig. 3a we have plotted the profile of a $\pi$ pulse emerging from the medium; the concentration of the two components of the medium correspond to $\gamma=80$. Figure 3b shows that after formation of a $\pi$ pulse, the population of slow-component atoms ranges from $+1$ to $-1$. This pulse gathers up all of the energy stored in the medium as it inverts the population of slow atoms, so its energy $E=\int_{-\infty}^{\infty}|a(x,t)|^2 dt$ steadily rises as it propagates through the two-component medium, and is proportional to the distance traversed ($E \propto x$).

The fast-atom population dynamics plotted in Fig. 3c show that the pulse area is twice as great in the fast component, i.e., the two-component medium produces a $2\pi$ pulse in the fast component. The slow- and fast-atom dynamics illustrated in Fig. 3 shows that a narrow pulse interacts with a two-component medium coherently, since after a pulse has passed, both the fast and slow atoms of the medium are in the ground state, rather than a saturated state ($R \approx r \approx 0$), as would be the case for incoherent interaction.

We know from soliton theory[2,5,7] that if a narrow incident pulse has area $2\pi$, it will give rise to a $2\pi$ soliton in both resonantly damping and resonantly amplifying media. The pulse in the example considered above had area $2\pi$ relative to the fast atoms; we now consider the propagation of a narrow pulse with area $2\pi$ relative to the slow atoms. Figure 4 shows the profile dynamics for an incident Gaussian pulse with amplitude $A_0=4$ and width $\tau_p=0.5$ in a two-component medium with fast-atom concentration $r_0=20$. The incident pulse clearly splits in two, and the resulting pulses rapidly separate. The first of the two is steadily amplified as it propagates through the medium, and it becomes narrower and narrower; the second, in contrast, becomes steadily weaker and broader.

To make some sense of this situation, we turn to Fig. 5, where we have plotted the profiles of the incident ($x=0$) and dissociated ($x=6$) pulses along with the fast- and slow-atom population dynamics. Figures 5a–c show that a Gaussian pulse whose area is $2\pi$ relative to the slow component and $4\pi$ relative to the fast component is incident upon the two-component medium. That Gaussian pulse, for $\gamma<1$, could produce a $2\pi$ pulse relative to the slow component and a $4\pi$ relative to the fast component, but in the present situation such a pulse is unstable and splits. In Figs. 5d–f, each of the resulting two pulses has an area equal to $\pi$. Propagating through the two-component medium, the first inverts the

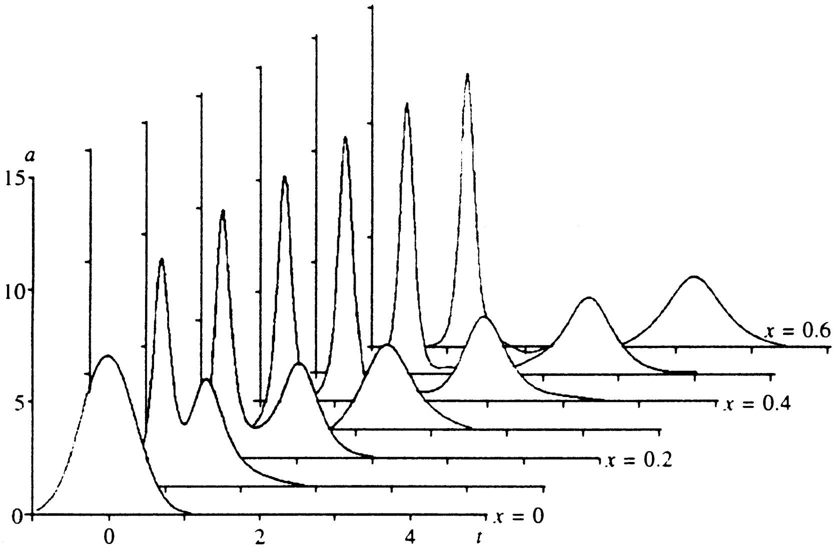


FIG. 4. Spatial and temporal dynamics of a pulse with area $\Sigma_0=2\pi$ in a two-component medium with $\gamma=80$.

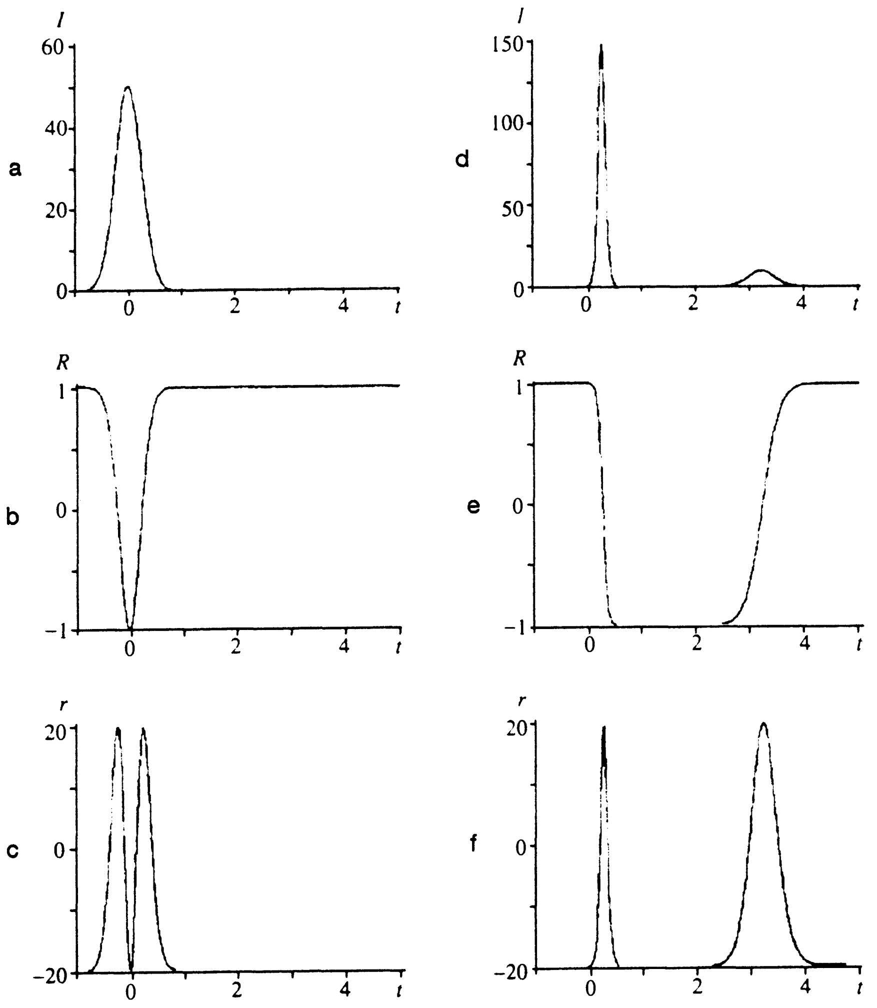


FIG. 5. Pulse profile and slow- and fast-atom population inversion dynamics for a two-component medium with $x=0$ (a,b,c) and $x=0.6$ (d,e,f). The area under the incident pulse is $\Sigma_0=2\pi$.

slow-atom population, taking them from the excited state to the ground state. Although the energy of the first pulse steadily increases, its area remains invariant, so the pulse intensity steadily increases and the pulse narrows. The second pulse raises the slow atoms of the two-component medium from the ground state to the excited state, and its energy therefore decreases. Just as for the first pulse, however, the area of the second pulse is also invariant, so the pulse amplitude falls and the pulse broadens. The two-component medium is thus an amplifying medium for the first pulse and a damping medium for the second. It is well known that the pulse velocity in an amplifying medium exceeds the speed of light ($c$), and it is less than $c$ in a damping medium. Since the first pulse propagates in the former and the second in the latter, the first pulse travels faster than the second and they separate rapidly, as shown in Fig. 5.

We also see in Fig. 5 that as the two pulses traverse the two-component medium, the intensity of the first steadily increases. This intensity is shown in Fig. 6a as a function of distance traveled for various fast-atom concentrations. The abrupt jump in intensity at short distances (curve *5*) is due to the splitting of the incident Gaussian $2\pi$ pulse into two $\pi$ pulses. It is clear that as $\gamma$ decreases (low fast-atom concentration), the splitting process becomes more gradual, and the initial jump vanishes. It is also clear that the higher the concentration of fast-component atoms, the higher the intensity of the first pulse; the highest intensity reached among the media that were investigated here occurred in a two-component medium with fast-atom concentration $r_0=20$.

In Fig. 6b we have plotted the width of the first pulse against distance traveled in the two-component medium. Here it is evident that the pulse steadily narrows as it propagates. In this figure, we can clearly distinguish three stages in the evolution of a $2\pi$ pulse in such a medium: an initial stage, a stage in which the original $2\pi$ pulse splits into two $\pi$ pulses, and a stage in which the first $\pi$ pulse is amplified. We see from Fig. 6 that as the fast-atom concentration increases (and therefore so does $\gamma$), the initial segment is curtailed, while the splitting region of the incident $2\pi$ pulse becomes more distinct. Note that the larger the value of $\gamma$, the narrower the first pulse derived from the splitting.

Figure 6c shows the pulse width as a function of $1/x$. It is clear that when the $2\pi$ pulse splits, the width of the first $\pi$ pulse decreases in a different way from that of a weak pulse ($\tau_0 \propto 1/x$). There exists a range at large $\gamma$ ($\gamma=20, \gamma=80$) over which the pulse width remains approximately constant, but this is followed by even faster pulse narrowing ($\tau_0 \sim 1/x^{\nu}$, where $0<\nu<1$).

We have now considered the gain regime for a weak pulse and the process whereby a $\pi$ pulse is derived from it in a two-component medium; we have also examined the splitting of a $2\pi$ pulse incident upon a two-component medium.

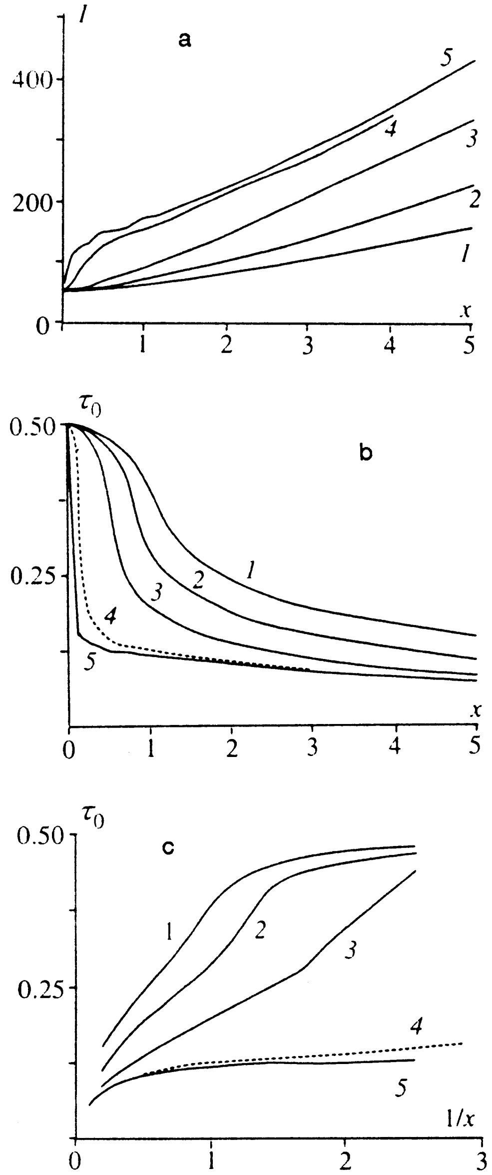


FIG. 6. Pulse intensity as a function of $x$ (a), and pulse width as a function of $x$ (b) and $1/x$ (c). The fast-component atomic concentration is $r_0=0.3$ (*1*), 0.5 (*2*), 1 (*3*), 5 (*4*), 20 (*5*).

Since the right-hand side of Eq. (6) has a period of $2\pi$ (in the Bloch angle $\theta$), we expect one of the aforementioned regimes to pertain to large-area pulses ($\Sigma>2\pi$).

We now consider the amplification of an incident pulse with area $2\pi+\delta\Sigma$. Since we are in a subthreshold gain regime, we expect that the area excess $\delta\Sigma$ will result in the original pulse splitting into a $2\pi$ pulse and a pulse with area $\delta\Sigma$. Numerical modeling shows that if the excess $\delta\Sigma$ is less than the critical area ($\delta\Sigma<\Sigma^*$), the incident pulse will evolve as in Fig. 5, splitting into a $2\pi$ pulse and a pulse with area $\delta\Sigma$ which, as in the weak-pulse, low-amplitude subthreshold gain regime, will be damped by the two-component medium. The newly-formed $2\pi$ pulse will then split into two $\pi$ pulses as it propagates through the medium. Consequently, an incident pulse with area $2\pi+\delta\Sigma$ ($\delta\Sigma<\Sigma^*$) will evolve in the same way as the split $2\pi$ pulse considered above.

## 6. LARGE-AREA PULSE PROPAGATION. BOUND PULSES IN TWO-COMPONENT MEDIA

Consider now the propagation of a pulse with area $2\pi+\delta\Sigma$ ($\delta\Sigma>\Sigma^*$). In Fig. 7, we show the pulse dynamics for an amplitude $A_0=6$ and width $\tau_p=0.5$. The original pulse in the two-component medium first separates into two pulses, the second of which begins to distance itself from the (bimodal) first. As it continues to separate from the bimodal pulse, it steadily rises in intensity and becomes narrower and narrower. The first (bimodal) pulse, however, is not stable (as witness Fig. 7), and it again splits into two new pulses that rapidly draw apart. The first pulse undergoes continual amplification and narrowing, while the second steadily broadens and loses amplitude. The two pulses spawned by the first subsequently become bound into a single entity.

This sort of pulse evolution can be accounted for by Fig. 8, which shows the population dynamics of the two components of the medium. In Fig. 8a, the incident Gaussian pulse is a $3\pi$ pulse for the slow atoms and a $6\pi$ pulse for the fast. Referring back to the various situations discussed above, the present one is analogous to the amplification of a pulse whose area $\Sigma$ is above the critical value ($\Sigma>\Sigma^*$). As shown above, a $\pi$ pulse is then generated by the two-component medium, and this is precisely the pulse that departs from the bimodal pulse in the initial phase of the process (Fig. 7).

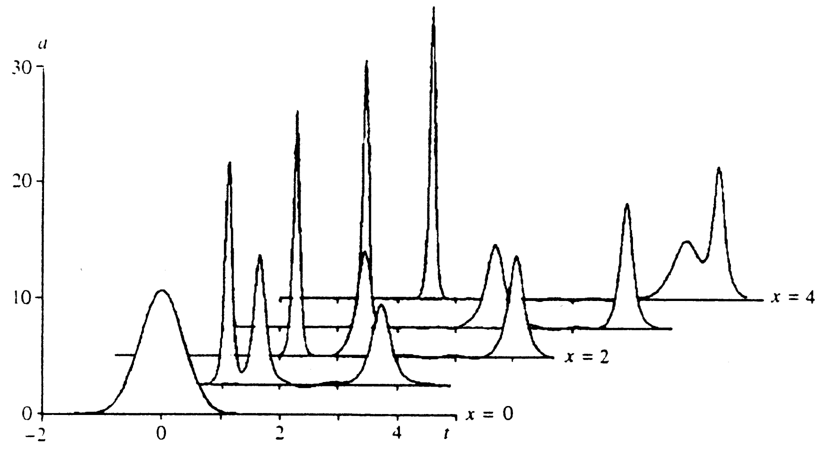


FIG. 7. Spatial and temporal dynamics of a pulse with area $\Sigma_0=3\pi$ in a two-component medium with $\gamma=80$.

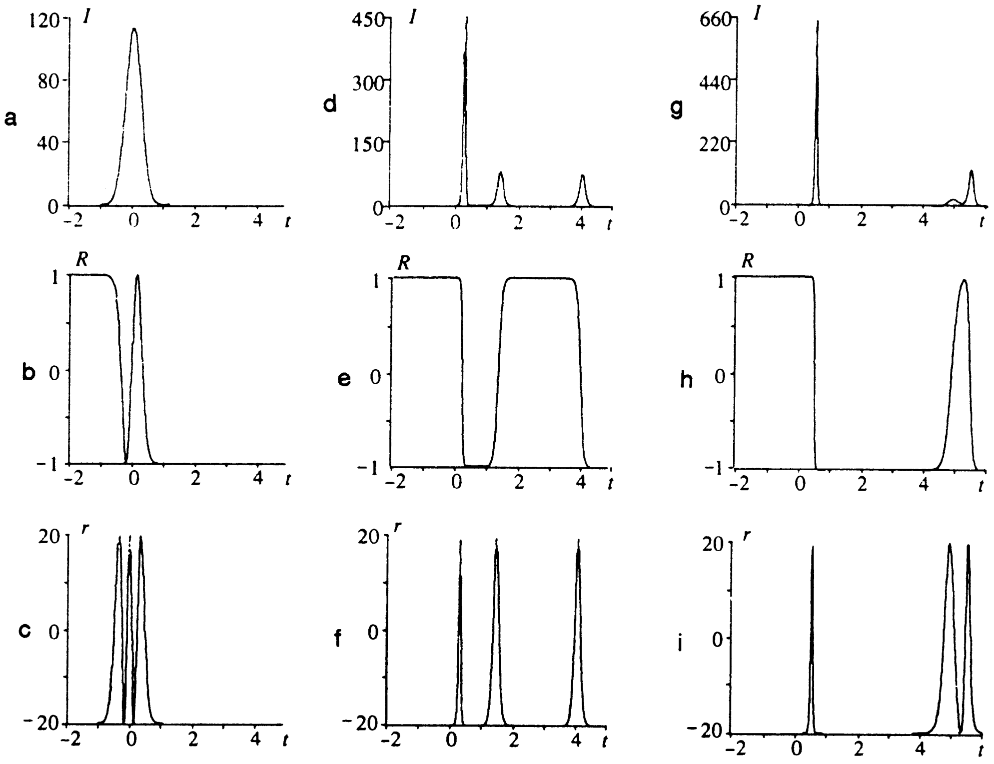


FIG. 8. Pulse profile and slow- and fast-atom population inversion dynamics for a two-component medium with $x=0$ (a,b,c), $x=2$ (d,e,f), and $x=4$ (g,h,i). The area under the incident pulse is $\Sigma_0=3\pi$.

We see from the population dynamics plotted in Figs. 8e and 8f that the pulse in question propagates through the two-component amplifying medium, acquiring energy from the slow atoms and transferring them to the ground state, so that the pulse intensity steadily rises and its width steadily falls. Likewise, we see that the first (bimodal) pulse is a $2\pi$ pulse, which, as in the case depicted in Fig. 5, is unstable and breaks up in the two-component medium into two $\pi$ pulses.

Figures 8d–f show that the original $3\pi$ pulse in the two-component medium evolves at $x=2$ into three $\pi$ pulses: the first and third of these (Fig. 8d) are shown by the atomic population dynamics to propagate in an amplifying medium, while the second propagates in a damping medium. Since the pulse speed in the latter is lower than in the former, the third $\pi$ pulse will overtake the second (Figs. 8g–i). We also see from this figure that the second and third $\pi$ pulses approach and bind to one another, thereby giving rise to a single $2\pi$ pulse. The upshot is that the original $3\pi$ pulse that decayed into three $\pi$ pulses at $x\approx2$ winds up producing a $\pi$ pulse and a $2\pi$ pulse.

In the present case, the area $\delta\Sigma$ is equal to $\pi$. Numerical modeling shows, however, that if $\Sigma^*<\delta\Sigma<\pi$, the tail of the $2\pi+\delta\Sigma$ pulse will contribute its area ($\delta\Sigma$) to that of the $\pi$ pulse, and the pulse dynamics of the $2\pi+\delta\Sigma$ pulse ($\Sigma^*<\delta\Sigma<\pi$) will be analogous to the behavior shown in Fig. 7.

In the situation depicted in Fig. 7, the area of the incident pulse was not large enough for the two-component medium to generate a $2\pi$ pulse immediately after having produced the first $\pi$ pulse.

The opposite case is shown in Figs. 9 and 10. A incident pulse with area $\Sigma=3.5\pi$ in a two-component medium with $\gamma=80$ breaks up immediately into a $\pi$ pulse and a $2\pi$ pulse. In Fig. 9, we see that the $\pi$ pulse propagates faster than the $2\pi$ pulse, so they quickly separate. As it propagates through the two-component medium, the $\pi$ pulse is steadily amplified and narrowed, while the profile of the $2\pi$ pulse continually varies. The pulse immediately following the first $\pi$ pulse is in fact a bound pair of $\pi$-solitons.

Previously, we studied the properties of a two-component medium in the subthreshold gain regime, where only the slow atoms were initially in the excited state ($\gamma>1$). In the present case, with the original pulse splitting into a $\pi$ pulse and a $2\pi$ pulse, the first (the $\pi$ pulse) causes the population of slow atoms (Fig. 10) to revert from $R=1$ to $R=-1$ as it propagates, thereby changing the state of the two-component medium. Thereafter, both the fast and slow atoms will be in the ground state. Since the state of the two-component medium will then have changed, the net result will be that a $2\pi$ pulse that would otherwise have been unstable in the excited two-component medium ($R=1$) will be stable in the fully de-excited medium ($R=-1$, $r=-r_0$).

As we noted above, the $2\pi$ pulse produced by the two-component medium can in fact be represented as two bound $\pi$ pulses. In Fig. 9, the amplitude and width of these bound pulses are seen to vary constantly. The two-component medium is damping for the first of the two bound pulses, so its intensity decreases and its width increases; for the second, it is an amplifying medium, and the intensity increases while the width decreases. Since the pulse speed is higher in a gain medium than in a damping medium, the second of the two bound pulses moves faster than the first and overtakes it; having overtaken it, the second pulse enters the unperturbed medium. The propagation conditions are then interchanged: what had been the second pulse propagates through a damping medium, and the former first pulse propagates through an

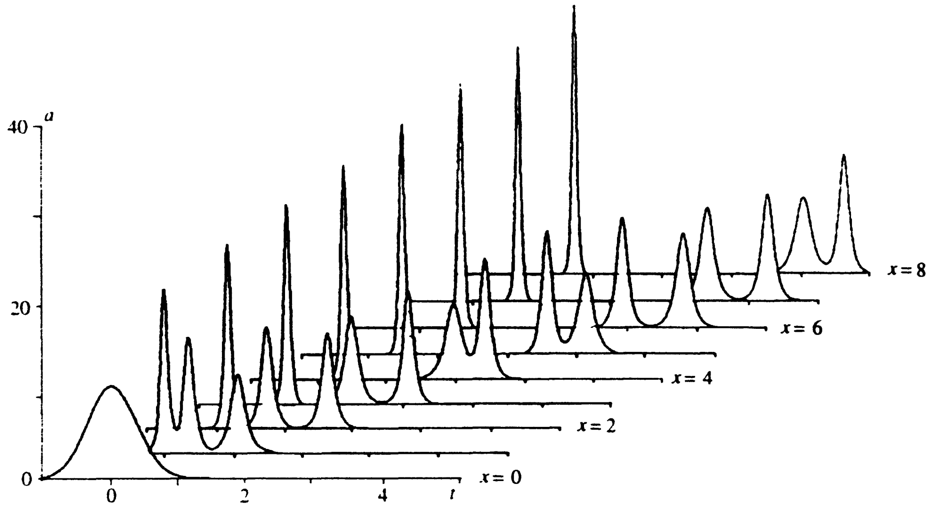


FIG. 9. Spatial and temporal dynamics of a pulse with area $\Sigma_0=3.5\pi$ in a two-component medium with $\gamma=80$.

amplifying medium. The bound pulses thus switch places, and the evolution of the system repeats itself.

Thus, large-area ($\Sigma$) incident pulses in a two-component medium can generate an initial $\pi$ pulse followed by an integral number $\Sigma-\pi/2\pi$ of bound $2\pi$ pulses. This potential for the existence of bound pulses is a characteristic feature of two-component media.

## 7. SOLITON SOLUTIONS

Thus far, we have dealt with boundary value problems associated with the incidence of external pulses upon a semi-infinite or bounded medium. If $\alpha_a=\alpha_b=0$, however, the system of equations (2) has soliton solutions that correspond to the Cauchy problem with vanishing boundary conditions at $\pm\infty$.

Self-consistent solutions of (2) can be represented in the form $a(x,t)=a(t-x/v)$, where $v$ is the propagation velocity of a solitary wave, which depends on the parameters of the particular problem.

For self-consistent solutions in the case $\beta_b=4\beta_a$, we obtain the following equation for the Bloch angle $\theta$:

$$\frac{\partial\theta}{\partial\eta}=\sqrt{2[U(\theta_0)-U(\theta)]}, \tag{13}$$

where

$$U(\theta)=\frac{\beta_a v}{v-1}(R_0\cos\theta+r_0\cos 2\theta), \quad \eta=t-(x/v). \tag{14}$$

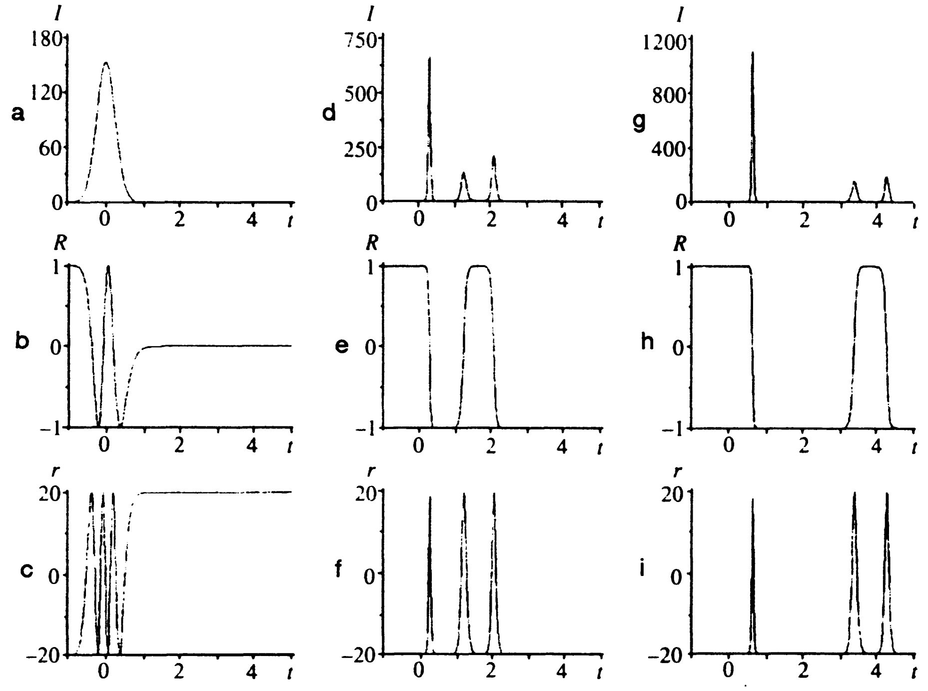


FIG. 10. Pulse profile and slow- and fast-atom population inversion dynamics for a two-component medium with $x=0$ (a,b,c), $x=3$ (d,e,f), and $x=7$ (g,h,i). The area under the incident pulse is $\Sigma_0=3.5\pi$.

For $\gamma=4|r_0|/|R_0|<1$, the extrema of the potential $U(\theta)$ come at $\theta_1=0$, $\theta_{2,3}=\pm\pi$. The soliton solutions of Eq. (13) associated with these extrema are

a) $R(-\infty)=\pm|R_0|$, $r(-\infty)=\mp|r_0|$:

$$a(\eta)=\frac{2}{\tau_1}\left(\frac{1-\gamma}{\beta_a}\right)^{1/2}\frac{\cosh(\eta/\tau_1)}{\cosh^2(\eta/\tau_1)-\gamma};\qquad(15)$$

b) $R(-\infty)=\pm|R_0|$, $r(-\infty)=\pm|r_0|$:

$$a(\eta)=\frac{2}{\tau_2}\left(\frac{1+\gamma}{\beta_a}\right)^{1/2}\frac{\cosh(\eta/\tau_2)}{\cosh^2(\eta/\tau_2)+\gamma},\qquad(16)$$

where

$$\frac{1}{\tau_1}=\left[\frac{\beta_aR_0v}{v-1}(1-\gamma)\right]^{1/2},\quad\frac{1}{\tau_2}=\left[\frac{\beta_aR_0v}{v-1}(1+\gamma)\right]^{1/2}.$$

For $\gamma<1$ and $R_0>0$, the two-component medium is an amplifying medium, and the soliton propagation speed $v$ exceeds the speed of light; for $R_0<0$, $v<1$.

When $\gamma>1$, new extrema appear at $\theta_{4,5}=\pm\text{arc}(1/\gamma)$. However, the form of the soliton solutions associated with the peaks at $\theta_{1,2,3}$ also changes:

c) $R(-\infty)=\pm|R_0|$, $r(-\infty)=\mp|r_0|$:

$$\alpha(\eta)=\frac{2}{\tau_3}\left(\frac{\gamma-1}{\beta_a}\right)^{1/2}\frac{\sinh(\eta/\tau_3)}{\gamma+\sinh^2(\eta/\tau_3)},\qquad(17)$$

where

$$\frac{1}{\tau_3}=2\left[\frac{\beta_br_0v}{v-1}(\gamma-1)\right]^{1/2}.$$

While (15) and (16) are $2\pi$-solitons relative to the slow component, the solution (17) represents a $0\pi$-soliton.

For $R(-\infty)=\pm|R_0|$, $r(-\infty)=\pm|r_0|$ and $\gamma>1$, the soliton solution is given as before by (16), although the soliton itself is bimodal.

The soliton solutions associated with the peaks at $\theta_{4,5}$ take the following form:

d) $-\arccos^{-1}(1/\gamma)\leqslant\theta\leqslant\arccos^{-1}(1/\gamma)$:

$$a(\eta)=\frac{2}{\tau_4}\left(\frac{\gamma^2-1}{\beta_a}\right)^{1/2}\frac{1}{\gamma\cosh(\eta/\tau_4)+1},\qquad(18)$$

e) $\arccos^{-1}(1/\gamma)\leqslant\theta\leqslant2\pi-\arccos^{-1}(1/\gamma)$:

$$a(\eta)=\frac{1}{\tau_4}\left(\frac{\gamma^2-1}{\beta_a}\right)^{1/2}\frac{1}{\gamma\cosh(\eta/\tau_4)-1},\qquad(19)$$

where

$$\frac{1}{\tau_4}=2\left[\frac{\beta_br_0v}{v-1}\left(1-\frac{1}{\gamma^2}\right)\right]^{1/2}.$$

The solution (18) is a soliton with area

$$\Sigma_1=\pi-2\arccos^{-1}(1/\gamma),\qquad(20)$$

and the area of (19) is

$$\Sigma_2=\pi+2\cos^{-1}(1/\gamma).\qquad(21)$$

## 8. CONCLUSION

We have studied certain features of the coherent amplification of narrow pulses by two-component media. This work has shown that in the subthreshold gain regime, a weak pulse whose area exceeds the critical value evolves into a $\pi$ pulse relative to the slow component and a $2\pi$ pulse relative to the fast component. While propagating in the two-component medium, the pulse is amplified in intensity ($I\propto x^2$) and compressed in width ($\tau_0\propto x^{-1}$).

In the subthreshold regime of a two-component medium, a $2\pi$ pulse is unstable and decays into two pulses, the first of which steadily rises in intensity and narrows, and the second of which spreads and is damped.

We have investigated the propagation of large-area pulses in a two-component medium, and shown that an initial pulse with area $\Sigma$ in such a medium will produce a single $\pi$ pulse followed immediately by an integral number $(\Sigma-\pi)/2\pi$ of bound $2\pi$ pulses.

We have shown that two pulses bound to one another can propagate in a two-component medium, leapfrogging along and alternately being amplified and damped.

Finally, the form of soliton solutions has been determined for this system.

This work has demonstrated that the dynamics of coherent interactions in two-component media is substantially richer than in one-component media; it would be genuinely interesting to conduct actual physical experiments. One example of such a medium might be a vapor mixture of Ca and Tl. The $7S\rightarrow6P$ transition in Tl (in which superradiance has been observed a number of times; see, for example, Ref. 10) is resonant with the $4F\rightarrow3D$ transition in Ca. The $7S$ level of Tl can be excited by a pump pulse from the ground state; the $7S\rightarrow6P$ transition in Tl has an oscillator strength $g_{\text{Tl}}=0.151$, and can be considered slow. The $3D$ level of Ca is metastable, and the oscillator strength in the $4F\rightarrow3D$ transition is $g_{\text{Ca}}=0.97$.

This work was carried out with financial support from the Russian Fund for Fundamental Research (Project No. 93-02-03415).

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Translated by Marc Damashek