Electrical response to time-varying plastic deformation in a metal

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The excitation of an electric potential at the surface of a metal sample subjected to nonuniform, time-varying plastic deformation is analyzed. The conditions assumed here correspond to those under which an electrical response to abrupt deformation has been observed experimentally [V. S. Bobrov and M. A. Lebedkin, JETP Lett. **38**, 400 (1983); Sov. Phys. Solid State **31**(6), 982 (1989)]. The excited potential is derived as a function of the flux density of plasticity carriers. The qualitative behavior of the electrical signals is analyzed under some fairly general assumptions regarding the properties of these fluxes. The validity of applying the statistical results on the electrical response found experimentally {V. S. Bobrov *et al.*, in *Abstracts, Fifth All-Union Seminar on Structure of Dislocations and Mechanical Properties of Metals and Alloys* (Sverdlovsk, 1989) [in Russian] (Nauka, Moscow, 1990); Physica B **165–166**, 267 (1990)} to the behavior of the plasticity carriers is discussed. Some experiments are proposed which would make it possible to test conclusions regarding the universal nature of the processes involved in abrupt plastic deformation. © *1994 American Institute of Physics.*

1. INTRODUCTION

When a metal sample is subjected to plastic deformation, and fluxes of plasticity carriers are set up, the electron system of the metal is driven away from equilibrium and must react to the perturbation through an electrical response. This problem was taken up in Ref. 1, where the electrical response to a steady-state flux of dislocations was derived. Voltage signals which arise between contacts on the surface of a sample have been observed[2,3] experimentally during abrupt deformation, under conditions such that substantial flux densities of dislocations are set up briefly and in a spatially local way. The response of the electron system was detected as a series of brief voltage pulses accompanying abrupt changes in the load during the deformation. It was found that the number of pulses corresponds to the number of instances in which individual clusters reach a surface. It may be possible to learn about the behavior of the system of plastic-deformation carriers by studying the electrical response. Interest has been attracted to the development of this method by some unexpected results of statistical analysis of data on series of microsecond pulses. Specifically, it was shown that the normalized distribution functions of the pulse heights during abrupt deformation are the same in Al and Nb (Refs. 4 and 5). This agreement, in the face of some extremely different microscopic deformation processes (the detachment of dislocation clusters in Al versus the propagation of twins in Nb), apparently indicates that the behavior of the complex, nonequilibrium systems of plasticity carriers is universal in nature.[6,7]

Of interest in this connection is a possible correspondence with a state of self-organizing criticality,[8,9] as is suggested by the approximate power-law distribution function found through an analysis of the experimental data. It is important to note that the same shape has been observed for the distribution of time intervals between neighboring pulses. However, it turns out that the lengths of the pulses themselves have a different distribution: an exponential one,[6] which is not characteristic of a state of self-organizing criticality. Accordingly, to find firm support for the electrical-response method, and to justify serious applications of this method, it is necessary to determine in more detail than in Ref. 1 the interrelation between the characteristics of the pulses which are detected and the source perturbing them: the flux of plasticity carriers. This is the problem taken up in the present paper. Formulating the problem in a fairly general way, we focus on determining qualitative characteristics of the perturbed potential at a contact without invoking any specific model for the dynamics of clusters. Along this path we analyze mechanisms for the formation of the pulses which are detected, and we analyze how purely electronic processes determine the shape and length of these pulses.

2. BASIC EQUATIONS

We begin with a kinetic equation for the electron distribution function $f(\mathbf{p},\mathbf{r},t)$:

$$\frac{\partial f}{\partial t}+\frac{\partial \varepsilon}{\partial \mathbf{p}}\frac{\partial f}{\partial \mathbf{r}}+(e\mathbf{E}-\nabla\varepsilon)\frac{\partial f}{\partial \mathbf{p}}+\hat{\nu}f=0. \tag{2.1}$$

We take the effect of plastic deformation into account in (2.1) through changes in the dispersion relation for the electrons due to the field of elastic deformations (by analogy with the way it is taken into account during the application of ultrasound[10]):

$$\varepsilon(\mathbf{p},\mathbf{r},t)=\varepsilon_0(\mathbf{p})+\delta\varepsilon(\mathbf{p},\mathbf{r},t),$$

$$\delta\varepsilon=\left(\lambda_{ik}(\mathbf{p})+\frac{\partial\varepsilon_0}{\partial p_i}p_k\right)w_{ik}(\mathbf{r},t)+\left(\mathbf{p}-m_0\frac{\partial\varepsilon_0}{\partial\mathbf{p}}\right)\mathbf{V}(\mathbf{r},t). \tag{2.2}$$

Here ε_0 is the spectrum of the undeformed metal, $\hat{\lambda}$ and $\hat{w}$ are the deformation-potential tensor and the elastic-distortion tensor, $\mathbf{V}$ is the velocity of the displacement of the elastic medium, and m_0 is the mass of a free electron. In the distribution function we distinguish a locally equilibrium part f_0:

$$f=f_0(\varepsilon_0+\delta\varepsilon-\mathbf{pV}-\delta\mu)+\frac{\partial f_0}{\partial\varepsilon}\chi(\mathbf{p},\mathbf{r},t). \tag{2.3}$$

This part is normalized to the local electron density.[10] The shift of the chemical potential, $\delta\mu$, is

$$\delta\mu=\frac{\langle\lambda_{ik}\rangle}{\langle 1\rangle}w_{ik}(\mathbf{r},t), \tag{2.4}$$

where

$$\langle\ldots\rangle=-\frac{2}{h^3}\int d^3p\,\frac{\partial f_0}{\partial\varepsilon}(\ldots). \tag{2.5}$$

In the linear approximation, the known equilibrium part of the distribution, χ, satisfies the equation

$$\left(\frac{\partial}{\partial t}+\frac{\partial\varepsilon_0}{\partial\mathbf{p}}\nabla+\hat{\nu}\right)\chi=-\lambda_{ik}\dot{w}_{ik}+\delta\dot{\mu}+\frac{\partial\varepsilon_0}{\partial\mathbf{p}}\left(e\nabla\phi+\frac{e}{c}\dot{\mathbf{a}}\right). \tag{2.6}$$

Here we have introduced electric potentials,

$$\mathbf{E}=-\nabla\varphi-\frac{1}{c}\dot{\mathbf{A}}, \tag{2.7}$$

and an electrochemical potential Φ,

$$\Phi=\varphi+\frac{1}{e}\delta\mu, \tag{2.8}$$

and we have introduced the notation

$$\mathbf{a}=\mathbf{A}+\frac{cm_0}{e}\mathbf{V}. \tag{2.9}$$

The potentials are found from the electrical-neutrality condition

$$\langle\chi\rangle=0, \tag{2.10}$$

which replaces the Poisson equation in the case of a metal, and from Maxwell's equation

$$\Delta\mathbf{A}=-\frac{4\pi}{c}\mathbf{j} \tag{2.11}$$

(we have chosen the gauge div $\mathbf{A}=0$), where the current density is

$$\mathbf{j}=-e\left\langle\frac{\partial\varepsilon}{\partial\mathbf{p}}\chi\right\rangle. \tag{2.12}$$

The electrical response to the plastic deformation is sensed at the surface of the sample, so the scattering of electrons by the surface must be taken into account in this problem. This scattering has a substantial effect on the nature of the nonequilibrium distribution and the electric field near the surface (Refs. 11 and 12, for example). We consider the case of diffuse scattering and the corresponding boundary condition (there is no current across the surface)

$$\chi^{>}(\mathbf{v},\mathbf{r}_s)=-\frac{1}{\langle v_n\rangle_+}\langle v_n\chi(\mathbf{r}_s)\rangle_-. \tag{2.13}$$

The superscript $>$ here means $\partial\varepsilon_0/\partial p_n=v_n>0$, $\mathbf{n}$ is the direction of the inward normal to the surface at the point $\mathbf{r}_s$; and the $\pm$ on the angle brackets mean integration regions with $v_n\gtrless 0$, respectively.

Let us analyze the situation in samples which are large in comparison with the mean free path l. In this case the scattering at one face does not affect the scattering at another, so in analyzing one of the contacts it is sufficient to consider only the scattering at the corresponding surface, imposing condition (2.13) at that surface and formulating the other conditions as for a half-space.

Let us consider the case of a very nonuniform deformation, in which fairly isolated clusters are formed. We restrict the discussion to the plane problem: The clusters and the corresponding fluxes of plasticity carriers are uniform along one coordinate. In other words, these fluxes are set up by a system of parallel rectilinear dislocations. We also assume that the changes in the plasticity processes occur over times far longer than the electron relaxation time. We can then omit the term $\partial\chi/\partial t$ from (2.6), on the basis that it is small in comparison with $\hat{\nu}\chi=(1/\tau)\chi$ (below we use the approximation of a relaxation time τ). These limitations, which are not of fundamental importance, are fairly realistic for the case of an abrupt deformation, and they do simplify the calculations.

Let us assume that the contact is at the $x=0$ surface. We consider an individual cluster (or twin) in which the density and the flux are localized in the xy plane (the dislocations are directed along the z axis). We take Fourier transforms in the coordinate y, which is parallel to the surface, and we write (2.6) in the form

$$\left(1+i\mathbf{ql}+l_x\frac{\partial}{\partial x}\right)\chi^q(\mathbf{v},x)=\Psi^q(\mathbf{v},x),$$

$$\Psi^q=-\tau\lambda_{ik}\dot{w}^q_{ik}(x)+\tau\delta\dot{\mu}^q(x)+e\left(i\mathbf{ql}+l_x\frac{\partial}{\partial x}\right)\Phi^q(x)+\frac{e}{c}\mathbf{l}\dot{\mathbf{a}}^q(x). \tag{2.14}$$

Here

$$\mathbf{l}=\mathbf{v}\tau,\quad\text{and}\quad F^q(x)=\int_{-\infty}^{\infty}dy\,e^{-iqy}F(x,y) \tag{2.15}$$

is the Fourier transform of $F(\mathbf{r},t)$. The solution of (2.14) under condition (2.13) is

$$\chi^q_{>}(\mathbf{v},x)=\left\{\int_0^\infty\frac{dx'}{\langle l_x\rangle_+}\left\langle\Psi^q(-v_x,v_y,v_z,x')\times\exp\left[-\frac{x'}{l_x}(1+i\mathbf{ql})\right]\right\rangle_+ +\int_0^x\frac{dx'}{l_x}\Psi^q(\mathbf{v},x')\exp\left[\frac{x'}{l_x}(1+i\mathbf{ql})\right]\right\}$$

$$\times\exp\left[-\frac{x}{l_x}(1+i\mathbf{ql})\right],$$

$$\chi^q_<(\mathbf{v},x)=\int_\infty^x\frac{dx'}{l_x}\Psi^q(\mathbf{v},x')\exp\left[\frac{x'-x}{l_x}(1+i\mathbf{ql})\right].\tag{2.16}$$

To pursue the calculations we assume an isotropic spectrum:

$$\varepsilon_0(\mathbf{p})=\frac{p^2}{2m},\quad \lambda_{ik}(\mathbf{p})=\lambda\varepsilon_F\frac{p_ip_k}{p^2}\tag{2.17}$$

(in typical metals the constant λ satisfies $\lambda\sim1$).

We can write an expression for the current j_x which appears in Eq. (2.11) for A_x:

$$j_x^q(x)=-\frac{3}{2}\sigma_0\frac{l'}{l^2}\left\{G(x)-2\left(\frac{l'}{l}\right)^2G(0)\left[K_{03}(x)-\frac{q^2l^2}{2}\left(K_{03}(x)-3\left(\frac{l'}{l}\right)^2K_{05}(x)\right)\right]\right\}.\tag{2.18}$$

Here

$$G(x)=G_\Phi(x)+\frac{\lambda\,p_Fl'}{4e}G_\lambda(x)+\frac{l}{2c}G_a(x),$$

$$G_\Phi(x)=\int_0^\infty\frac{dx'}{l}\Phi^q(x')K_{13}(|x-x'|)\text{sign}(x'-x),$$

$$G_\lambda(x)=2iql'u^q_{xy}(0)\left[3K_{15}(x)-5\left(\frac{l'}{l}\right)^2K_{17}(x)\right]+\int_0^\infty\frac{dx'}{l'}\left\{\dot w^q(x')\left[K_{13}(|x-x'|)-3\left(\frac{l'}{l}\right)^2K_{15}(|x-x'|)\right]\text{sign}(x-x')+2iql'\dot u^q_{xy}(x')\left[K_{03}(|x-x'|)-\left(\frac{l'}{l}\right)^2K_{05}(|x-x'|)\right]-(l')^2D(\dot w)\left[3K_{15}(|x-x'|)-5\left(\frac{l'}{l}\right)^2K_{17}(|x-x'|)\right]\text{sign}(x-x')\right\},$$

$$G_a(x)=\dot a^q_x(0)\left[K_{13}(x)-3\left(\frac{l'}{l}\right)^2K_{15}(x)\right]+\int_0^\infty\frac{dx'}{l'}\left\{\dot a^q_x(x')\left[K_{01}(|x-x'|)-\left(\frac{l'}{l}\right)^2K_{03}(|x-x'|)\right]+l'\text{ div }\mathbf{a}^q(x')\left[K_{13}(|x-x'|)-3\left(\frac{l'}{l}\right)^2K_{15}(|x-x'|)\right]\text{sign}(x-x').\right.\tag{2.19}$$

Electrical-neutrality condition (2.10) can be written as the integral equation

$$\Phi^q(x)-\int_0^\infty\frac{dx'}{2l}\Phi^q(x')K_{01}(|x-x'|)=\left(\frac{l'}{l}\right)^2G(0)K_{13}(x)+F(x),\tag{2.20}$$

where

$$F(x)=\frac{(l')^2}{2lc}F_a(x)-\lambda\frac{p_Fl'}{8e}F_\lambda(x),$$

$$F_a=\dot a^q_x(0)K_{03}(x)+\int_0^\infty dx'\text{ div }\dot{\mathbf{a}}^q(x')K_{03}(|x-x'|),$$

$$F_\lambda=2iql'\dot u^q_{x'}(0)\left[K_{03}(x)-3\left(\frac{l'}{l}\right)^2K_{05}(x)\right]+\int_0^\infty\frac{dx'}{l'}\left\{\dot w^q(x')\left[K_{01}(|x-x'|)-3\left(\frac{l'}{l}\right)^2K_{03}(|x-x'|)\right]-(l')^2D(\dot w)\left[K_{03}(|x-x'|)-3\left(\frac{l'}{l}\right)^2K_{05}(|x-x'|)\right]\right\}.\tag{2.21}$$

In (2.18)–(2.21) we have used

$$\sigma_0=\frac{e^2n\tau}{m}\quad l'=l(1+q^2l^2)^{-1/2},\quad w=w_{xx}-\frac{1}{3}w_{ii},$$

$$u_{xy}=\frac{1}{2}(w_{xy}+w_{yx}),\quad D(w)=\Delta w_{xx}-\frac{\partial^2w_{jk}}{\partial x_j\partial x_k}.$$

$$K_{nm}(x)=\int_1^\infty dz\,\frac{z^n\exp(-xz/l')}{(z^2-q^2l'^2)^{m/2}}.\tag{2.22}$$

Equations (2.11) and (2.20) for the potentials contain terms which depend on Fourier components of the velocities of the elastic distortion, $\dot w^q(x,t)$, and the acceleration of the medium, $\dot{\mathbf{V}}^q(x,t)$, accompanying the plastic deformation. These functions serve in this problem as given sources, which generate the electrical response.

3. CALCULATION OF POTENTIALS

We write a solution of Eq. (2.20) as follows:

$$\Phi^q(x)=\left(\frac{l'}{l}\right)^2G(0)\left[K_{13}(x)+\int_0^\infty dx'\Gamma(x|x')K_{13}(x')\right]+F(x)+\int_0^\infty dx'\Gamma(x|x')F(x').\tag{3.1}$$

Here $\Gamma(x|y)$ is a Green's function, given by

$$\Gamma(x|y)-\int_0^\infty\frac{dx'}{2l}K_{01}(|x-x'|)\Gamma(x'|y)=\frac{1}{2l}K_{01}(|x-y|).\tag{3.2}$$

It can be written in the following way:[13,14]

$$\Gamma(x|y)=\gamma(|x-y|)+\int_0^{\min(x,y)} du\,\gamma(x-u)\gamma(y-u). \tag{3.3}$$

The function $\gamma(x)$ is a solution of Eq. (3.2) which is nonincreasing as $x\to\infty$; in it we have set $y=0$. This solution is made up of a linear combination of the solution of the homogeneous equation,

$$M(x)-\int_0^\infty \frac{dx'}{2l} K_{01}(|x-x'|)M(x')=0, \tag{3.4}$$

and its derivative $M'(x)$.

In the limit $ql\to 0$, in which the kernels $K_{nm}(x)$ in (2.22) become the known functions $E_{n-m}(x)$, where

$$E_n(x)=\int_1^\infty dz/z^n \exp(-xz/l), \tag{3.5}$$

Eq. (3.4) becomes the Milne equation, which describes the transmission of radiation or a flux of particles with independent velocities (e.g., neutrons) through an isotropically scattering medium.[14,15] As in the Milne problem, Eq. (3.4) can be analyzed by the Wiener–Hopf method, and we can determine the asymptotic behavior of the function $M(x)$. Skipping over the details of the analysis, we write the result:

$$M(x)=M_{as}(x)+M_1(x),\quad M(x)\xrightarrow[x\gg l]{} M_{as}(x),$$

$$M_{as}(x)=\frac{\sqrt{3}}{ql}M(0)\operatorname{sh} q(x+z_0),$$

$$z_0=\frac{1}{|q|}\ln(|q|l+l/l')-\frac{l}{\pi}\int_0^\infty\frac{dz}{q^2+z^2}$$

$$\times\ln\left\{3\,\frac{z^2+(l/l')^2}{z^2+q^2l^2}\left[1-\frac{\operatorname{arctg}\sqrt{z^2+q^2l^2}}{\sqrt{z^2+q^2l^2}}\right]\right\}. \tag{3.6}$$

In the Milne problem, i.e., with $ql=0$, we have a length $z_0\simeq 0.71l$. Without any loss of generality we can set $M(0)=1$. The function $M_1(x)$ satisfies an equation which differs from (3.4) by virtue of its right side, which contains the functions $K_{03}(x)$ and $K_{13}(x)$, and which decays exponentially at $x\gg l$. The function $\gamma(x)$ is given by

$$\gamma(x)=M'(x)-|q|M(x)=M_1'(x)-|q|M_1(x)+\frac{\sqrt{3}}{l}$$

$$\times\exp[-|q|(x+z_0)]. \tag{3.7}$$

The right side of (3.1) depends on the functions describing the source, on the function Φ^q itself [the quantity $G_\Phi(0)$ in (2.19)], and on the potential A_x^q. In turn, the potential A_x^q must be found from Eq. (2.11), whose right side contains the integral term $G_\Phi(x)$ according to (2.18) and (2.19). This integral term is also determined by the function $\Phi^q(x)$. This term can be calculated by multiplying (3.1) by $(1/l)K_{13}(|x-x'|)\operatorname{sign}(x-x')$ and integrating over x. Using several integral relations (given in the Appendix), we find

$$G_\Phi(x)=-2G(0)\left\{\frac{2}{3}\left|q\right|l\operatorname{ch} qz_0\exp[-|q|(x+z_0)]\right.$$

$$\left.-\left(\frac{l'}{l}\right)^2[K_{03}(x)+q^2l'^2K_{05}(x)]\right\}$$

$$+2\int_0^\infty\frac{dy}{l'}\operatorname{ch} qyF(x+y)-\frac{2|q|l}{\sqrt{3}l'}$$

$$\times\exp[-|q|(x+z_0)]\int_0^\infty dyF(y)M(y). \tag{3.8}$$

We can now determine the quantity $G_\Phi(0)$ which appears in the expression for the potential Φ^q in (3.1) and also in the equation for the current, (2.18), as one of the terms in $G(0)$. The contribution described by $G_\Phi(0)$ stems from a redistribution of the electron density near the surface as the result of diffuse scattering by the surface. Finding $G_\Phi(0)$ from (3.8), and substituting the result into $G(0)$, we find

$$G(0)=\frac{3}{4|q|l}\left[\operatorname{ch} qz_0\cdot\exp(-|q|z_0)\right.$$

$$\left.-\frac{|q|l(1+2l/l')}{4(1+l/l')^2}\right]^{-1}\left\{\frac{l}{2c}G_a(0)\right.$$

$$+\frac{\lambda p_F l'}{4e}G_\lambda(0)+\frac{2}{l'}\exp(-|q|z_0)\int_0^\infty dyF^q(y)$$

$$\left.\times\left[\operatorname{ch} qz_0\exp(-|q|y)-\frac{|q|l}{\sqrt{3}}M_1(y)\right]\right\}. \tag{3.9}$$

Incorporating $G_\Phi(0)$ results in a substantial renormalization of the parts of $G(0)$ associated with the sources. In the case of interest here, $ql<1$, the denominator in (3.9) is small; this circumstance is reflected in a substantial way in the values of the potentials at the surface.

Using (3.8) and (3.9) to eliminate the potential Φ from expression (2.18) for j_x, we find from (2.11) an inhomogeneous integrodifferential equation for A_x. The inhomogeneous part contains contributions which depend on x in various ways. Some of the terms $[\sim K_{nm}(x)]$ are significant only at distances on the order of the electron mean free path l. The behavior of the other terms is governed by the spatial variation of the deformations. Let us assume that these variations occur over distances $\gg l$, so we can simplify the integral expressions in (2.19), (2.21), (3.8), and (3.9), retaining the leading terms. In the variation of $A_x(x)$ we should again see a different behavior, dictated by the behavior of the inhomogeneous part of the equation and also by the form of the operators which act on A_x, and which introduce yet another dimensional parameter: the skin depth $\delta\sim c(\sigma_0\omega)^{-1/2}$ (ω is the characteristic frequency). We also assume $l\ll\delta$ (the normal skin effect). We distinguish two parts in j_x and A_x: a rapidly varying part (which varies over a distance $\sim l$) and a smoother part. It is not difficult to see that the rapidly varying part of A_x can be ignored in comparison with the smooth part, so the equation for A_x can be written in the approximation of the normal skin effect. It becomes an inhomogeneous differential equation. We will write this equation, without reproducing the corresponding calculations, whose content is described above:

$$\frac{d^2A_x(x)}{dx^2}-Q^2A_x(x)=\delta^{-2}\left[-A_x(0)\exp(-|q|x)+\frac{cm_0}{e}\mathscr{H}(x)\right], \tag{3.10}$$

where

$$\delta^2=\frac{c^2}{4\pi\sigma_0 p}, \quad Q^2=q^2+\delta^{-2},$$

$$\mathscr{H}(x)=V_x(x)+\int_0^\infty dx' \text{ ch } qx' \text{div } \mathbf{V}(x+x')+\frac{\lambda p_F l}{5m_0}iqu_{xy}(x)-\exp(-|q|x)\left[V_x(0)-\frac{3}{16}l \text{ div } \mathbf{V}(0)+\int_0^\infty dx' \text{ ch } qx' \text{ div } \mathbf{V}(x')+\frac{3\lambda p_F l}{10m_0}iqu_{xy}(0)\right]. \tag{3.11}$$

Here we have used the Laplace time transform

$$F^p=\int_0^\infty dte^{-pt}F(t), \tag{3.12}$$

where p is the Laplace argument. Everywhere in (3.10)–(3.11) we have written the Fourier and Laplace transforms of the functions (for brevity, we are omitting the indices q and p).

Finding a solution of (3.10),

$$A_x(x)=\frac{1}{2Q\delta^2}\int_0^\infty dx' \exp(-Q|x-x'|)\left[A_x(0)\exp\times(-|q|x')-\frac{cm_0}{e}H(x)\right], \tag{3.13}$$

and determining $A_x(0)$,

$$A_x(0)=-\frac{cm_0}{e}(Q-|q|)\int_0^\infty dx' \exp(-Qx')\mathscr{H}(x'), \tag{3.14}$$

which appears on the right in (3.13) and which enters $\Phi(x)$ in (3.1) [through expression (3.9) for $G(0)$], we have all the equations we need to write explicit expressions for the electric potentials in terms of functions describing the deformation.

In this problem we need to find effective values W_{eff} of the potentials at the surface contacts such that the differences between these potentials specify the voltage in the external measurement circuit connected to these contacts. It is not difficult to show for the general case, by analyzing the expression for the average rate of change of the electron pulse over the electron distribution, that this voltage is

$$\Delta W_{\text{eff}}=W_{\text{eff}}(\mathbf{r}_2,t)-W_{\text{eff}}(\mathbf{r}_1,t)=\int_{\mathbf{r}_1}^{\mathbf{r}_2}d\mathbf{r}\left\{\nabla\Phi(\mathbf{r})+\frac{1}{c}\dot{A}(\mathbf{r})\right\}, \tag{3.15}$$

where $\mathbf{r}_i$ are the coordinates of the contacts at the $x=0$ surface. Fourier transforming in y in (3.15), and using the gauge condition div $\mathbf{A}=0$ adopted above, we find a Fourier component of the effective potential:

$$W_{\text{eff}}^q|_{x=0}=\left[\Phi^q+\frac{1}{cq^2}\frac{d\dot{A}_x^q}{dx}\right]_{x=0}. \tag{3.16}$$

Using (3.9) and (3.14); the expressions for F, H, and Γ; and several integral equations given in the Appendix [involving the functions M in (3.6)], we now substitute Φ from (3.1) and A_x from (3.13) into (3.16). After some lengthy calculations, we find the leading part (in the small parameter ql and the other small length ratios) of the effective potential:

$$W_{\text{eff}}^{qp}(0)=\frac{m_0}{eq^2}\left[g_V^{qp}+\frac{3}{5}\lambda\frac{p_F l}{m_0}iqg_\lambda^{qp}\right],$$

$$g_V=Q\left[\dot{V}_x(0)+\int_0^\infty dx \text{ div } \dot{\mathbf{V}}(x')\exp(-Qx')\right]-\delta^{-2}\int_0^\infty dx'\dot{V}_x(x')e^{-Qx'},$$

$$g_\lambda=Q\dot{u}_{xy}(0)-\frac{2}{3\delta^2}\int_0^\infty dx'\dot{u}_{xy}(x')\exp(-Qx'). \tag{3.17}$$

This expression contains Fourier and Laplace transforms of the functions $\dot{\mathbf{V}}$ and $\dot{u}_{xy}$. Taking inverse transforms, we find the value of W_{eff} at the time t at the point ($x=0$, y) on a contact:

$$W_{\text{eff}}(0,y,t)=\frac{1}{(2\pi)^2 i}\int_{-\infty}^\infty dqe^{iqy}\int_{Cp}dpe^{pt}W_{\text{eff}}^{qp}(0). \tag{3.18}$$

The Laplace contour C_p runs parallel to the imaginary axis in the (Re p, Im p) plane, to the right of all the singularities of the function W_{eff}^{qp}.

4. RESPONSE TO NONUNIFORM TIME-VARYING PLASTIC FLUXES

To pursue the analysis we need to invoke a description of the plastic flow. Using Hooke's law for an isotropic material in the equation of motion of the medium,

$$\rho\dot{V}_i=\frac{\partial\sigma_{ik}}{\partial x_k} \tag{4.1}$$

and expressing the stress $\hat{\sigma}$ in terms of the distortion $\hat{w}$, we eliminate $\hat{w}$ from (4.1) with the help of an equation for the dynamic plasticity,[16]

$$\frac{\partial}{\partial t}w_{ik}=\frac{\partial V_k}{\partial x_l}+j_{ik}. \tag{4.2}$$

We find equations for the components of the velocity of the medium, V. The inhomogeneous parts of these equations contain components of the dislocation flux density j_{ik}. As we have already mentioned, we are assuming that this flux is

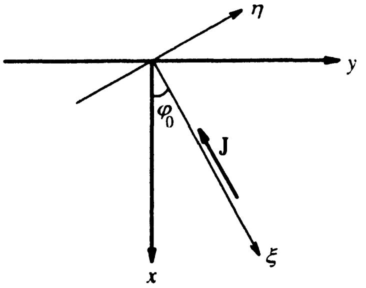

FIG. 1.

spatially quite nonuniform, which makes it possible to distinguish individual clusters, and we are assuming a rapid variation in time (departure from the point of attachment and stopping upon attainment of the surface). Within the framework of our analysis, these time and length scales should be substantially greater than the electron scales (τ and l).

In describing continuous clusters of parallel dislocations or plane twins, it is convenient to use the representation[17]

$$j_{ik}=\mathbf{b}_k[\nu \mathbf{J}]_i, \tag{4.3}$$

where $\mathbf{b}$ is the Burgers vector, ν is the vector tangent to the dislocation lines, and $\mathbf{J}$ is the vector flux density, which satisfies the continuity equation

$$\frac{\partial \rho_d}{\partial t}+\operatorname{div}\mathbf{J}=0. \tag{4.4}$$

The dislocation density ρ_d is normalized to N, which is the number of dislocations in a cluster. We assume that the flux density $\mathbf{J}$ corresponding to the cluster under consideration is directed at an angle φ_0 from the x axis, parallel to the ξ axis, like the vector $\mathbf{b}$ (Fig. 1):

$$\mathbf{J}=-\mathbf{1}_\xi J(x,y,t),\quad \mathbf{J}(x,y,t)=\mathbf{J}(\xi,\eta,t),$$
$$\xi=x\cos\varphi_0+y\sin\varphi_0,\quad \eta=-x\sin\varphi_0+y\cos\varphi_0. \tag{4.5}$$

After we carry out a Fourier expansion in y and a Laplace expansion in t, the equations for the quantities V_x and $\operatorname{div}\mathbf{V}$, which we will need below, take the following form:

$$\mathscr{P}''(x)-\kappa_l^2\mathscr{P}(x)=-2b\gamma_s^2\left\{iq\cos 2\varphi_0 J'(x)-\frac{\sin 2\varphi_0}{2}[J''(x)+q^2J(x)]\right\},$$
$$V_x''(x)-\kappa_t^2V_x(x)=(1-\gamma_s^{-2})\mathscr{P}'(x)+b[\sin 2\varphi_0 J'(x)-iq\cos 2\varphi_0 J(x)],$$
$$\kappa_{t,l}^2=q^2+p^2/s_{t,l}^2,\quad \gamma_s^2=s_t^2/s_l^2,\quad \mathscr{P}=\operatorname{div}\mathbf{V}, \tag{4.6}$$

and $S_{t,l}$ is the transverse or longitudinal sound velocity. The boundary conditions corresponding to the absence of a stress σ_{ix} $(i=x,y)$ at the $x=0$ surface are

$$(1-2\gamma_s^2)\mathscr{P}(0)+2\gamma_s^2V_x'(0)-b\gamma_s^2\sin 2\varphi_0 J(0)=0$$
$$iqV_x(0)+V_y^-(0)+b\cos 2\varphi_0 J(0)=0. \tag{4.7}$$

In the case of interest here, in which the flux $\mathbf{J}$ is directed toward the $x=0$ surface, the solutions of (4.6) must satisfy the condition $\mathscr{P}(x)$, $\mathbf{V}(x)\to 0$ as $x\to\infty$. Writing the solutions of (4.6), determining $\dot{u}_{xy}$, and substituting the necessary quantities into the equation for the effective potential, (3.17), we find a rather lengthy expression which contains integrals of the type $\int_0^\infty dx\dot{J}(x)\exp(-\alpha x)$, where $\alpha=Q$, κ_t or κ_l, and $\dot{J}=\dot{J}^{qp}$ is a Fourier and Laplace transform of the time derivative of the flux J. In this step we draw on some qualitative considerations which follow from the physical picture of abrupt plastic deformation: The rate of change of the flux, $\dot{J}(x,t)$, must be greatest where dislocations are detached and where they stop. In the case at hand, the predominant contribution to the integral with $\dot{J}$ should come from the vicinity of the point at which the surface is reached. (In particular, this point is closer to the contacts at which the effect is detected. Furthermore, surface perturbations, which decay to a lesser extent than bulk perturbations with distance from the source, should propagate away from the point at which the surface is reached.) Under the assumption that the size of this surface region satisfies $x_0<\alpha^{-1}$ $(\alpha=Q,\ \kappa_t,\ \kappa_l)$, we omit the exponential factors from the integrals:

$$\int_0^\infty dx\dot{J}^{pq}(x)\exp(-\alpha x)\to\int_0^{x_0}dx\dot{J}^{pq}(x)\equiv\dot{Y}_{pq},$$

$$Y_{pq}=\int_0^\infty dte^{-pt}Y_q(t),\quad Y_q(t)=\int_0^{x_0}dxJ_q(x,t). \tag{4.8}$$

Under approximation (4.8) the expressions for $q_{v,\lambda}$ from (3.17) simplify, and W_{eff}^{pq} becomes

$$W_{\mathrm{eff}}^{pq}=2\sin 2\varphi_0(1-\gamma_s^2)\frac{bm_0\dot{Y}_{pq}}{eB(p,q)}\left\{\frac{p^2(q^2-Q\kappa_t)}{s_t^2(Q+\kappa_t)}+2\Lambda\left[\frac{(q^2+\kappa_t^2)^2}{Q+\kappa_t}-\frac{4q^2\kappa_t\kappa_l}{Q+\kappa_l}\right]\right\}. \tag{4.9}$$

Here

$$\Lambda=\frac{4\pi\lambda p_F l\sigma_0}{5m_0c^2},\quad B(p,q)=(q^2+\kappa_t^2)^2-4q^2\kappa_t\kappa_l, \tag{4.10}$$

with

$$p=|q|s_t\vartheta\quad B(p,q)=q^4B(\vartheta),$$

$$B(\vartheta)=\frac{\vartheta^2\prod_{j=1,2,3}(\vartheta^2+u_j^2)}{(2+\vartheta^2)^2+4\sqrt{(1+\vartheta^2)(1+\gamma_s^2\vartheta^2)}}.$$

Here u_j are the roots of the dispersion relation for surface waves,[16] with $u_1<1$ and $u_{2,3}>\gamma_s^{-1}$. Substituting (4.9) into (3.18), and carrying out some manipulations, we find

$$W_{\text{eff}}(0,\zeta,T)=\frac{\sin 2\varphi_0}{i\pi^2 e}(1-\gamma_s^2)bs_t m_0\beta\int_0^\infty dkk \times\cos k\zeta\int_0^T dT' Y_k(T')\int_{C_\vartheta}\frac{d\vartheta}{B(\vartheta)} \times\exp(\vartheta k\Delta T)\left\{\frac{\vartheta^2}{1-k\vartheta}[(2+\vartheta^2)\sqrt{k^2+k\vartheta} -(2k+\vartheta)\sqrt{1+\vartheta^2}]+\Lambda \times\left[\sqrt{k^2+k\vartheta}\left(\frac{(2+\vartheta^2)^2}{1-k\vartheta} -\frac{4\sqrt{(1+\vartheta^2)(1+\gamma_s^2\vartheta^2)}}{1-\gamma_s^2 k\vartheta}\right) -k\sqrt{1+\vartheta^2}\left(\frac{(2+\vartheta^2)^2}{1-k\vartheta}-\frac{4(1+\gamma_s^2\vartheta^2)}{1-\gamma_s^2 k\vartheta}\right)\right]\right\}. \tag{4.11}$$

Here we have introduced

$$y=\frac{\zeta}{\beta},\quad q=\beta k,\quad t=\frac{T}{\beta s_t},\quad \beta=\frac{4\pi\sigma_0 s_t}{c^2},\quad \Delta T=T-T', \tag{4.12}$$

and the dimensionless variables ζ, k, T, and ϑ. The contour C_ϑ passes to the right of the singularities of the integrand, parallel to the imaginary u axis; being closed on the left, it sets an upper limit on the integration over time. The integrand, as a function of ϑ, has a pair of surface-wave poles, $\vartheta=\pm iu$, and branch points $\vartheta=\pm i$, $\pm i/\gamma_s$, $-k$. Taking a cut between i/γ_s and $-i/\gamma_s$, and also between $-\infty$ and $-k$, and closing the contour of the integration over ϑ, we find the pole contributions and the integrals along the banks of the cuts in (4.11). The pole contributions, which describe the electrical response accompanying the surface acoustic waves which are excited, and also the contribution from the $(-\infty, -k)$ cut, which is related to the "skin" behavior of the electrical spike, characteristic of a transverse field, are

$$W'_{\text{eff}}=\frac{2\sin 2\varphi_0}{\pi^2 e}(1-\gamma_s^2)bm_0 s_t\beta\frac{\partial}{\partial\zeta}\int_0^T dT' \times\int_0^\infty dk Y_k(T')\sin k\zeta(Z^{(1)}+Z^{(2)}), \tag{4.13}$$

$$Z^{(1)}=\frac{\pi(2-u_1^2)^3}{u_1^2\prod_{j=2,3}(u_j^2-u_1^2)}\left\{\exp(iku_1\Delta T)\frac{\sqrt{k(k+iu_1)}}{k+i/u_1} \times\left[1-\Lambda(1-\gamma_s^{-2})(2u_1^{-2}-1)\left(1-\frac{1}{1-iu_1\gamma_s^2 k}\right)\right] +\text{c.c.}\right\}, \tag{4.14}$$

$$Z_2^{(2)}=\int_k^\infty d\vartheta\exp(-\vartheta k\Delta T)\frac{\sqrt{k(\vartheta-k)}}{1+\vartheta k}\left\{1-\frac{2}{B(\vartheta)}[2 +\vartheta^2-2\sqrt{(1+\vartheta^2)(1+\gamma_s^2\vartheta^2)}]+\Lambda\left[1 -\frac{4(1-\gamma_s^2)\vartheta k\sqrt{(1+\vartheta^2)(1+\gamma_s^2\vartheta^2)}}{B(\vartheta)(1+\gamma_s^2 k\vartheta)}\right]\right\}. \tag{4.15}$$

The other parts of W_{eff}, associated with cuts on the imaginary ϑ axis, are not as representative. They vary more smoothly, and to a lesser extent, as a function of y and t. The analysis which follows is accordingly conducted for expression (4.13).

5. ANALYSIS OF THE INTEGRAL RELATIONS

To analyze the integrals in (4.13) we need to specify the k dependence of Y_k, i.e., the behavior of the dislocation flux density J in (5.5) as a function of y. We have already mentioned that we are considering narrow clusters. For the discussion below we assume that the dimensions of the contacts at which their potentials are detected are far larger than the width of the clusters transverse with respect to the glide plane. The electrical pulse, which varies in the y direction, is received by the entire area of the contact, so the detected signal should be compared with the average of expression (4.13) over the width of the contact:

$$\bar{W}(T)=\frac{1}{\beta d}\int_{\zeta_1}^{\zeta_2}d\zeta W_{\text{eff}}(0,\zeta,T),\quad d=y_2-y_1=\frac{\zeta_2-\zeta_1}{\beta}, \tag{5.1}$$

where d is the width of the contact, and $y_2>y_1$ are the coordinates of the edges of the contact ($y=0$ is the point at which the cluster reaches the surface). Without specifying the details of the model of the cluster, we can now approximate the behavior of the transverse direction by a δ-function:

$$J(x,y,t)=J(\xi,\eta,T)=a\delta(\eta)J(\xi,T),$$
$$J_a(x,t)=\frac{a}{\cos\varphi_0}J\left(\frac{x}{\cos\varphi_0}t\right), \tag{5.2}$$

where the constant a is on the order of the width of the cluster. According to (4.8), we should thus set

$$Y_k(T)=aY(T),\quad Y(T)=\int_0^{x_0/\cos\varphi_0}dxJ(x,t) \tag{5.3}$$

in (4.13). Let us consider the first term in (4.13) (with $Z^{(1)}$). Expressing the integral over k in terms of an integral along a closed contour (which includes the real semiaxis; the imaginary semiaxis, positive or negative, depending on the sign of the argument of the exponential function, with the distance between the branch points; and the arc connecting the semiaxes), we find

$$\int_0^\infty dkY_k(T')\sin k\zeta Z^{(1)}=\frac{\pi a(2-u_1^2)^3\,\text{sign}\,\zeta}{u_1^3\prod_{j=2,3}(u_j^2-u_1^2)}Y(T')$$

$$\times\Bigg\{[1-\Lambda(2u_1^{-2}-1)(\gamma_s^{-2}-1)]$$

$$\times\frac{\partial}{\partial T'}[R^{(-)}-R^{(+)}]+\mathscr{P}\Bigg\}. \quad (5.4)$$

Here

$$R^{\mp}=\exp(\varphi^{\mp})K_0(|\varphi^{\mp}|),$$

$$\mathscr{P}=\int_0^\infty \frac{dx}{x+u_1^{-2}}\sqrt{\frac{x}{x+1}}\,[\exp(-2x\varphi^{(+)})$$

$$+\theta\langle\varphi^{(-)}\rangle\exp(-2x\varphi^{(-)})]\Bigg\{u_1^{-2}-1-\Lambda(2u_1^{-2}-1)$$

$$\times(\gamma_s^{-2}-1)\,\frac{u_1^{-2}+x[1+\gamma_s^2(1-u_1^2)]}{1+u_1^2\gamma_s^2x}\Bigg\}$$

$$+\theta(-\varphi^{(-)})\int_1^\infty \frac{dx}{x-u_1^{-2}}\sqrt{\frac{x}{x-1}}\exp(2x\varphi^{(-)})$$

$$\times\Bigg\{u_1^{-2}-1-\Lambda(2u_1^{-2}-1)(\gamma_s^{-2}-1)$$

$$\times\frac{u_1^{-2}-x[1+\gamma_s^2(1-u_1^2)]}{1-u_1^2\gamma_s^2x}\Bigg\},$$

$$\varphi^{(\mp)}=\frac{u_1^2}{2}\left(\Delta T\mp\frac{|\zeta|}{u_1}\right), \quad (5.5)$$

where K_0 is the modified Bessel function, and θ is the unit step function.

The most important part of (5.4) is the part with the function $R^{(-)}$, which describes the pulsed response as a singularity propagating in the y direction at the surface-wave velocity u_1s_t. The moving amplitude of the spike does not decay, because of the planar situation (the cluster is extended along the z direction) and because we are ignoring the decay of the surface waves. The divergence in the pulsed singularity is generated by the approximations made above [e.g., the limit of small values of ql in the derivation of (3.17), the replacement of (4.8) etc.]. This divergence must be removed by means of a cutoff in a final step. The term in (5.4) with $R^{(+)}$ also has a singularity, but only if the conditions $\Delta T=0$ and $\zeta=0$ hold simultaneously. This situation corresponds to a central spike at the point at which the cluster arrives at the surface; this spike decays over time and also with distance from the point $\zeta=0$. The other parts of (5.4) are less important. For the estimates below we will simply use the term with $R^{(-)}$.

After an average is taken over the width of the contact [in accordance with (5.1)], the part of the effective potential associated with the propagation of the surface-wave singularity becomes

$$\bar{W}^{(1)}(T)+B_1\,bm_0s_t\,\frac{a}{ed}\int_0^T dT'Y(T')[S_2^{(1)}-S_1^{(1)}],$$

$$S_i^{(1)}=\text{sign}\,\zeta_i\,\exp(\varphi_i^{(-)})[K_0(|\varphi_i^{(-)}|)-\text{sign}\,\varphi_i^{(-)}$$

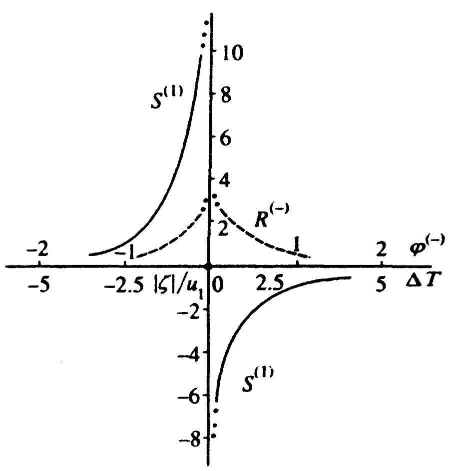

FIG. 2. The convolution function $S^{(1)}$ in (5.6) and the function $R^{(-)}$ in (5.5) (dashed curves) versus $\varphi^{(-)}$ and $\Delta T(u_1=0.9)$.

$$\times K_1(|\varphi_i^{(-)}|)],$$

$$B_1=\frac{\sin 2\varphi_0(1-\gamma_s^2)(2-u_1^3)}{\pi u_1\prod_{j=2,3}(u_j^2-u_1^2)}[1-\Lambda(2u_1^{-2}-1)(\gamma_s^{-2}-1)],$$

$$\varphi_i^{(-)}=\frac{u_1^2}{2}[\Delta T-|\zeta_i|/u_1], \quad i=1,2. \quad (5.6)$$

The singularities in the functions $S_i^{(1)}$ are of the following nature:

$$S_i^{(1)}|_{\Delta T\to|\zeta_i|/u_1}\sim-\text{sign}\,\zeta_i\Bigg\{\ln\left|\Delta T-\frac{|\zeta_i|}{u_1}\right|+\frac{2}{u_1^2\left(\Delta T-\frac{|\zeta_i|}{u_1}\right)}\Bigg\}. \quad (5.7)$$

With distance from the singularity, the amplitude spike decreases, and the decrease is a symmetric: For $|\varphi_i^{(-)}|\gg1$, the decay is exponential $[\propto|\varphi_i^{(-)}|^{-1/2}\exp(-2|\varphi_i^{(-)}|)]$ on the left (at $\varphi_i^{(-)}<0$), while it is a power law $[\propto(\varphi_i^{(-)})^{-3/2}]$ on the right (at $\varphi_i^{(-)}>0$). The shape of the spike near one of the singularities is shown schematically in Fig. 2 [the singularities are "cut off" at the argument $(|\varphi^-|=0.1)$]. The length of the left branch can be estimated to be $\Delta T\simeq1$; the right branch of the spike is longer. The convolution function in (5.6) contains two such bursts, which are separated by a time interval $(1/u_1)[|\zeta_2|-|\zeta_1|]$. The signs of the pulses are determined by the positions of the edges of the contact, i.e., by[1)] sign ζ_i.

In analyzing the second part of (4.13) (with $Z^{(2)}$), we simplify expression (4.15), retaining only the ones in square brackets in the integral. The other parts do not contain any new singularities on the integration interval, and they have only a slight effect on the value of the integral over ϑ. Skipping over the intermediate calculations, we write the result, found through the use of (5.3) and (5.1):

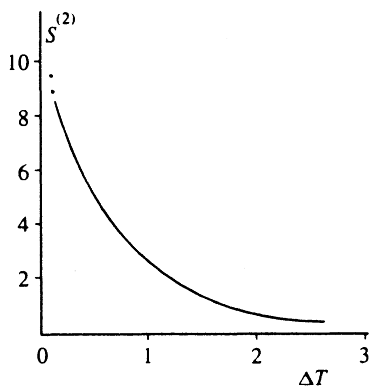

FIG. 3. The convolution function $S^{(2)}$ in (5.8) versus $\Delta T(\eta=3)$.

$$\bar{W}^{(2)}(T)=B_2\, bm_0 s_t \frac{a}{ed}\int_0^T dT' Y(T')[S_2^{(2)}-S_1^{(2)}],$$

$$S_i^{(2)}=\frac{\text{sign } \zeta_i}{\sqrt{\Delta T}}\int_0^1 dz \exp[\Delta T(1-z^{-2})]\text{erf}(\kappa_i z),$$

$$B_2=\frac{1+\Lambda}{\pi}(1-\gamma_s^2)\sin 2\varphi_0, \quad \kappa_i^2=\frac{\zeta_i^2}{4\pi\Delta T}=\frac{\sigma_0}{c^2}\frac{y_i^2}{\Delta t}. \tag{5.8}$$

The convolution function $S_i^{(2)}$ depends on both the time argument ΔT directly and a combination of the spatial coordinate and the time, κ_i [see (5.8)], which is characteristic of the behavior of a transverse electric field (transverse with respect to the propagation direction). [Here κ_i is the ratio of the coordinate to the effective skin depth $\delta^*=(c^2\Delta t/4\pi\sigma_0)^{1/2}$; the quantity Δt^{-1} plays the role of a frequency.] The behavior of $S_i^{(2)}$ is shown schematically in Fig. 3. The difference $S_2^{(2)}-S_1^{(2)}$ which figures in (5.8) is most substantial in the case in which a cluster reaches the surface within a contact (sign $\zeta_1=-$sign ζ_2): for $\Delta T\ll 1$, the amplitude of the spike is $\propto(\Delta T)^{-1/2}$. The asymptotic value of the amplitude is $\propto(\zeta_1+\zeta_2)(\Delta T)^{-2}$ (under the conditions $\Delta T\gg 1\gg\kappa_i$). If the cluster reaches the surface away from a contact (sign $\zeta_1=$sign ζ_2), then the original value (for $\Delta T\to 0$) of the spike is smaller ($\propto 1/\zeta_i$), and the asymptotic decrease remains a power law ($\propto(\zeta_2-\zeta_1)/(\Delta T)^2$ under the conditions $\Delta T\gg 1\gg\kappa_i$). In this case, the decay of the convolution function $S_2^{(2)}-S_1^{(2)}$ is typically smoother.

6. ESTIMATE AND CONCLUSION

Expressions were derived above for the effective potential excited at a contact by a moving dislocation cluster or twin. Certain approximations were made in the derivation. These approximations are based for the most part on a qualitative picture of localized dislocation fluxes, which change rapidly when they reach a surface. These approximations made it possible to simplify the convolution functions in the integral expression for $\bar{W}(T)=\bar{W}^{(1)}+\bar{W}^{(2)}$ [see (5.6) and (5.8)] which determines the time dependence of the potential which is excited. An explicit demonstration of this dependence will require specifying details about the source function $Y(T)$ in (5.3). For example, one might use a specific model for the dynamics of the cluster or twin corresponding to the situation under consideration, and one might describe the behavior of the flux J when it reaches a surface. A model study of that sort is a separate problem, which we are not taking up in this paper. Instead, we offer below some qualitative estimates of the nature of $\bar{W}(T)$ on the basis of a physical picture of plasticity processes. These estimates correspond to some extreme but still fairly realistic cases.

In Sec. 4 we used approximation (4.8), which is based on the assumption that the rate of change of the flux, $J(x,t)$, is large when the cluster comes to a halt in a surface region of thickness x_0. For the function $J(x,t)$ itself, as for $Y(t)$ $=\int_0^{x_0}dxJ(x,t)$, we would expect a nonmonotonic behavior here. The dislocation velocity $v_d(x,t)$ in the layer $(0,x_0)$ decreases to zero as time elapses, while the density $\rho_d(x,t)$, like the number of dislocations in the layer, $\int_0^{x_0}dx\rho_d(x,t)$ $=N(t)/a$, increases from zero to the value reached at the time of the halt. Accordingly, the flux $J(x,t)=v_d(x,t)\rho_d(x,t)$ initially increases (after the time at which the cluster enters the layer, t_a) and then falls off during the slowing and at the halt at the time t_s. We denote by $\Delta t_0=t_s-t_a=\Delta T_0/\beta s_t$, $\Delta T_0=T_s-T_a$ the characteristic duration of this process.

These extreme cases hold under one of the two following conditions:

1. $\Delta T_0\ll 1$. In other words, the function $Y(T)$ is sharper than the convolution functions $S^{(1)}$ and $S^{(2)}$, and we can use the approximation

$$Y(T)=\delta(T-T_0)\Delta T_0 Y_0, \quad T_0=\frac{T_a+T_s}{2}. \tag{6.1}$$

2. $\Delta T_0\gg 1$. This is the opposite limit, in which the source function can be replaced by a step function,

$$Y(T)=\theta(T-T_a)\theta(T_s-T)Y_0. \tag{6.2}$$

The amplitude Y_0 can be estimated from

$$Y_0\simeq\frac{\bar{v}_d N}{a}, \tag{6.3}$$

where $\bar{v}_d$ and N are the characteristic velocity and the number of dislocations in a cluster. The time interval ΔT_0 can be estimated by setting $\Delta t_0\simeq x_0/v_d\sim x_0/s$ (the velocity of the clusters associated with an abrupt deformation is large: $v_d\sim s$; Refs. 17 and 18).

Let us look at the cases in which (6.1) holds, i.e., the cases with $\Delta T_0<1$ or $x_0<\beta^{-1}\sim 10^{-2}$ cm (in estimating β we set $\tau\simeq 10^{-13}$ s) and $s\simeq 10^5$ cm/s. In this case the effective-voltage pulse reproduces the shape of the convolution functions $S_i^{(1,2)}$:

$$\bar{W}(T)=\bar{W}^{(1)}+\bar{W}^{(2)}=m_0 s_t^2\frac{\bar{v}_d b}{es_t d}N\Delta T_0\theta(T-T_0)$$
$$\times\{B_1[S_2^{(1)}(T-T_0)-S_1^{(1)}(T-T_0)]+B_2[S_2^{(2)}$$
$$\times(T-T_0)-S_1^{(2)}(T-T_0)]\}. \tag{6.5}$$

Initially, at the time $T=T_0$, a "skin" pulse $S^{(2)}$ arises. Later, at the times $T=T_0+|\zeta_i|/u_1$, "surface-wave" spikes $S_i^{(1)}$ are excited. As was mentioned in Sec. 5, the skin pulse is significant when a cluster reaches the surface within the area of a contact [under the condition $T-T_0 \ll 1$ the amplitude varies $\propto (T-T_0)^{-1/2}$]; it is weakened if it reaches the surface away from a contact ($\propto 1/|\zeta_i|$ under the conditions $T-T_0$, $|\zeta_i| \ll 1$). Accordingly, the potential difference between the two contacts may react to a skin perturbation with only one spike: when a cluster arrives at one of the contacts or near an edge of a contact. The surface-wave spikes each bring two responses to each of the contacts on the surface at which the cluster arrives. The amplitudes of the responses are identical at their peaks and are significant [see (5.7)]. There is a definite order in their signs and in the times of the spikes. To estimate an amplitude of this type, it is natural to cut off the pole divergence in (5.7) at ΔT_0. In (6.5) we then find $(\bar{W}^{(1)})_{\max} \sim 10^2 B_1 N(b/d)$ μV. The quantity $\bar{W}^{(1)}_{\max}$ can be reconciled with the experimental values of the amplitude ($|W| \sim 1$ μV), since the case $Nb/d \sim 10^{-2}$ is completely feasible. The amplitude $\bar{W}^{(2)}$ of the skin peak is estimated equivalently $\bar{W}^{(2)}_{\max}/\bar{W}^{(1)}_{\max} \sim (\Delta T_0)^{1/2}$ and $\sim \Delta T_0/|\zeta_i| \simeq x_0/|y_i|$, for the cases in which the cluster reaches the surface inside and outside a contact, respectively.

As was mentioned in Sec. 5, the spikes of the convolution functions S increase sharply, and they decay far more slowly after the passage of the peak. Among them, the pulses $S^{(1)}$ in (5.6) are more complicated: They increase sharply, change sign at the peak, and fall off smoothly. The change in sign is due to the pole singularity in (5.7). If $Y(T)$ is smeared out slightly, and we depart from the representation (6.1), then the pole singularity may be suppressed to a significant extent, but the logarithmic singularity and the ordinary asymmetric peak remain. In this case one can argue that the shape found for the pulses here is similar to that observed experimentally.[2–5].

Perhaps the most important result, however, is the qualitative result of version (6.1) regarding the duration (δt) of the voltage pulses at the contacts, which is determined by the behavior of the convolution functions $S(T)$, i.e., by the reaction of the electron system, rather than by the properties of the source function $Y(T)$, which depends on the course of the plasticity processes. We mentioned back in the Introduction that, for the pulse lengths specifically, experiments reveal a deviation from the universal laws for the distribution functions: $\Delta \sim 10^{-6}$ s, with a small scatter around this value.[3,6] For a numerical estimate we adopt $\Delta T \simeq 3$ for the length of the pulses of the convolution functions. This assumption is acceptable according to the discussion in Sec. 5. This gives us $\delta t = \Delta T/\beta s_t \sim 10^{-6}$–$10^{-7}$ s in real time, in fair agreement with experiment.

Finally, we note one more qualitative conclusion which can be drawn from the results derived in the present paper: The planning of experiments and the analysis of their results must be more systematic and more detailed. Here it is important to make use of what has been learned about the consequences of the relative arrangement, shape, and size of the contacts [the multiple pulses caused by the arrival of one cluster at the surface, the separation of these pulses in time, the polarity, the shape, the effects of the size of the contacts, e.g., the effects of a reduction of their size, down to the transverse dimensions of the clusters, in which case averaging over the area of the contact, (5.1), will be incorrect, new features of the spikes will be seen, etc.].

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APPENDIX

Here are the exact expressions for certain integrals containing the functions $K_{nm}(x)$, $M(x)$, and $\Gamma(x|y)$ which were used in the calculations of Sec. 3. The derivations of these equations, which we omit, are based on the use of (3.4), modifications of that equation, and definitions (2.22), (3.2), (3.3), (3.6), and (3.7).

$$\int_0^x dy M(-y) M(x-y) = \frac{\operatorname{sh} qx}{q},$$

$$\int_0^\infty dy M(-y) \exp(\pm qy) = \frac{l}{\sqrt{3}} \exp(\pm q z_0),$$

$$\int_0^\infty dy M(-y) = \frac{l}{\sqrt{3}\operatorname{ch} q z_0} \times \left[1 - \frac{q^2 l}{\sqrt{3}\operatorname{ch} q z_0} \int_0^\infty dy M_1(y)\right]^{-1},$$

$$\int_0^\infty dy M(y) K_{03}(x+y) = \frac{2}{ql}\left(\frac{l}{l'}\right)^2 - \int_x^\infty dy M(-y) \operatorname{sh} q(y-x)$$

$$\int_0^\infty dy M(y) K_{13}(x+y) = \frac{2l}{l'} \int_x^\infty dy M(-y) \operatorname{ch} q(y-x),$$

$$\int_0^\infty dy M(y) K_{03}(|x-y|) = \frac{2}{\sqrt{3}} \left(\frac{l}{l'}\right)^2 \frac{\operatorname{sh} q(x+z_0)}{q},$$

$$\int_0^\infty dy M(y) \operatorname{sign}(y-z) K_{13}(|x-y|) = \frac{2}{\sqrt{3}} \frac{l^2}{l'} \operatorname{ch} q(x+z_0),$$

$$\int_0^\infty dy \Gamma(x|y) K_{03}(|y-z|) = -K_{03}(|x-z|) + 2\left(\frac{l}{l'}\right)^2 \left\{\frac{M(x)}{\sqrt{3}} \times \exp[-q(z+z_0)] - \theta(x-z) \times \frac{\operatorname{sh} q(x-z)}{ql}\right\}$$

$$\int_0^\infty dy\Gamma(x|y)\mathrm{sign}(y-z)K_{13}(|y-z|)$$

$$=-\mathrm{sign}(x-z)K_{13}(|x-z|)-2\,\frac{l}{l'}$$

$$\times\left\{\frac{M(x)}{\sqrt{3}}\,ql\,\exp[-q(z+z_0)]\right.$$

$$\left.-\theta(x-z)\mathrm{ch}\,q(x-z)\right\}.$$

Here $x \geqslant 0$ and $q=|q|$.

[1)]Experimentally, one detects the potential difference between two contacts. If the second contact is also in the $x=0$ plane, then it is also necessary to consider the spikes similar to those which have been described and which are associated with the coordinates of the second contact. If the second contact lies on another surface, then its contribution decreases under the approximation of Secs. 4 and 5.

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