On the theory of a weakly collisional plasma in a magnetic field

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A new electron magnetization condition, which determines the suppression of transport across the magnetic field is obtained for the conditions under which the electron mean free path is large compared to the scale of the electron temperature variation along the field. For stationary conditions, an expression for the electron density perturbation due to a high-frequency heating field is obtained and the nonlocal, effective, electron thermal conductivity across the magnetic field, having an essential dependence on the transport along the field, is determined. The contribution from weakly collisional magnetized electrons to the longitudinal dielectric constant of the plasma is obtained.

1. In Refs. 1 and 2 some results of the analytical theory of transport in a completely ionized plasma with rare collisions are presented. For a plasma with a high degree of ionization, $Z=|e_i/e|\gg 1$, the conditions considered are such that λ, the nonuniformity scale of the plasma particle distribution, turns out to be less than l_{ei}, the mean free path of the thermal electrons with respect to their collisions with the ions,

$$\lambda<l_{ei}=(3/4\sqrt{2\pi})\kappa_{\mathrm{B}}^2T_e^2/e^4Z\, n_e\Lambda\equiv v_{\mathrm{Te}}/v_{ei}, \tag{1.1}$$

where e is the electronic charge and κ_{B} the Boltzmann constant; T_e and n_e are the electron temperature and number density, respectively; Λ is the Coulomb logarithm; $v_{\mathrm{Te}}=\sqrt{\kappa_{\mathrm{B}}T_e/m_e}$ is the electron thermal velocity, and ν_{ei}, the electron-ion collision frequency for the electrons whose velocities are of the order of the thermal. The electron-ion collision frequency for the electron velocity v may be described by the formula

$$\nu(v)=3(\pi/8)^{1/2}\nu_{ei}(v_{\mathrm{Te}}/v)^3. \tag{1.2}$$

This form of dependence of the collision frequency on the velocity suggests that even under the rare-collision condition (1.1) one can indicate small values of the electron velocity v such for which

$$\lambda>l(v)=v/\nu(v). \tag{1.3}$$

Therefore even when for the thermal electrons, with $v\sim v_{\mathrm{Te}}$, collisions are rare, there are slow (subthermal) electrons for which, according to Eq. (1.3), collisions are frequent. In Refs. 1 and 2 it is shown that conditions exist for which in the rare collision case (1.1) the main contribution to the nonequilibrium electron density perturbation comes from that part of the electron velocity distribution for which the condition (1.3) is met. As discussed in Refs. 3 and 4, the approach of Refs. 1 and 2 enables one to determine the nonlocal electron heat transport by using the notion of the integral relation between the electron heat flow density and the electron temperature perturbation[3]; as well as by invoking the electron heat transport inhibition concept.[4] The results of Refs. 1 and 2 were obtained for conditions for which

$$Zl_{ei}^2=l_{ee}l_{ei}\gg\lambda^2, \tag{1.4}$$

where $Zl_{ei}=l_{ee}$ is the mean free path of the thermal electrons with respect to their mutual collisions. It is in this case that the asymptotic solution of the kinetic equation revealed that the density perturbation is mainly due to the electrons with velocities much less than thermal. Here lies a qualitative distinction between the weakly collisional regime (1.4) and the usual highly collisional regime.[5]

The analytical result of Ref. 1 for the electron heat flow density in a nonmagnetized plasma (see also Refs. 3 and 4) may be written in the form of the following relation

$$\delta\mathbf{q}(\mathbf{k})=-\mathbf{i}\mathbf{k}\delta T_e(\mathbf{k})\kappa(\mathbf{k}) \tag{1.5}$$

between the Fourier transforms of the heat flow perturbation $\delta\mathbf{q}$ and the temperature perturbation δT_e. Here

$$\kappa(\mathbf{k})=\kappa_{SH}[1+(\alpha\lambda_e k)^\beta]^{-1} \tag{1.6}$$

is the Fourier transform of the nonlocal kernel of the electron thermal conductivity

$$K(\mathbf{r})=\int\frac{d\mathbf{k}}{(2\pi)^3}e^{i\mathbf{kr}}\kappa(\mathbf{k}). \tag{1.7}$$

In Eq. (1.6), written in the form suggested in Refs. 6 and 7, $\kappa_{SH}=C_{SH}n_ev_{\mathrm{Te}}\kappa_{\mathrm{B}}l_{ei}$ is the electron thermal conductivity of highly collisional, completely ionized plasma, the coefficient $C_{SH}=128/3\pi$ for $Z\gg1$, and $\lambda_e=(2l_{ei}l_{ee}/9\pi)^{1/2}$. The analytical theory of Ref. 1 yields $\alpha=21.1$ and $\beta=10/7$. These values are not inconsistent with approximate numerical results available from a large number of papers on nonlocal heat transport in a plasma. Note, however, that the values that have been reported are $\beta=2$[7], $\beta=1$[8], $\beta=(4/3)$[6,8,9], $\beta=1.44$[8,10], $\beta=1.148$[11]. Whereas the value from Ref. 7, corresponding to the Padé approximation, stands out somewhat among the others, these latter, while showing some degree of scatter, group around the analytical-theory value, $\beta=(10/7)$[1]. In the literature there is still no agreement on the value of the coefficient β in Eq. (1.6). However, it has been argued that only the values 4/3 and 1.44 are consistent with numerical electron heat transport studies for a plasma heated by high-frequency electromagnetic radiation via the inverse-bremsstrahlung absorption mechanism.[8,9,10] In this connection it should be stressed that the value $\beta=10/7=1.42857...$, obtained from the theory of the effect of a high-

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frequency plasma-heating field,[1] is very close to numerical simulation results. The argument of Ref. 12 that the theory of small perturbation may lead to $\beta=1$ is in conflict with the analytical theory of Ref. 1. This argument could be of relevance in a nonlinear analysis of the electronic inverse-bremsstrahlung absorption and heat transport, of the kind performed, for example, in Ref. 13. The theory of Ref. 1 is linear and neglects all the nonlinear effects of Ref. 13 (see also Ref. 14). In such a linear theory the main effect comes from the electrons with subthermal velocities, whereas nonlinear approaches have exhibited a considerable redistribution of the superthermal electrons. At the same time it should be stressed that while for stationary processes we know from Ref. 3 that Eq. (1.6), with $\beta=10/7$, does indeed describe heat transport that determines the temperature distribution for a nonuniform plasma heating, for nonstationary processes it does not. In fact, based on the results from the nonstationary theory of Ref. 2, it has been shown[15] that the use of Eq. (1.6) with $\beta=10/7$ leads to an incorrect expression for the damping factor of ion-acoustic waves in a weakly collisional plasma. This, in our view, is due to the insufficient understanding of precisely in what transport equations the use of the expression (1.6) is permissible. In the highly collisional limit the answer is obtained within the framework of the Hilbert–Chapman–Enskog method in the derivation of the hydrodynamics equations in the usual way. In the weakly collisional limit, Grad's equations of moments are often applied. It is the danger of using such system of equations which is the message of Ref. 15. Therefore it seems, in particular, that further work on problems in the kinetics of a weakly collisional plasma would be worthwhile. In this work, the theory of asymptotic expansions developed in Refs. 1 and 2 for fractional negative powers of the Knudsen number (λ_{ei}/λ) is applied to conditions different from those considered originally.

Specifically, the results describing the effect of a static magnetic field on the nonequilibrium electron density perturbation are presented. Also, new conditions determining the effect of a magnetic field on transport in a weakly collisional plasma are established.

2. For simplicity we will assume, as in Refs. 1 and 2, that the high-frequency electromagnetic field heating the plasma has a frequency ω_0 much greater than the electron gyroscopic frequency Ω_e. At the same time we assume that the magnetic field is sufficiently strong to exert an appreciable influence on plasma perturbations, which vary in time and space according to the law

$$\exp(-i\omega t+i\mathbf{kr}). \tag{2.1}$$

Assuming the electron distribution function $f=f_{\mathrm{M}}+\delta f$ to differ little from the Maxwellian f_{M}, for the perturbation δf we adopt, following Refs. 1 and 2, the expression

$$\delta f=f_{\mathrm{M}}\left[\left(\frac{v^2}{3v_{\mathrm{Te}}^2}-2\right)I-\frac{e\,\delta\varphi}{\kappa_{\mathrm{B}}T_e}\right]+\frac{1}{2}\sum f_{\mathrm{M}}+\delta f_c, \tag{2.2}$$

where

$$I=\frac{e^2|\mathbf{E}|^2}{4m_e^2\,\omega_0^2\,v_{\mathrm{Te}}^2}\,;\quad \sum=\frac{e^2(v_iv_j-\frac{1}{3}v^2\delta_{ij})}{4m_e^2\,\omega_0^2\,v_{\mathrm{Te}}^4}\times(E_i\,E_j^*+E_i^*E_j-\tfrac{2}{3}\,\delta_{ij}|\mathbf{E}|^2). \tag{2.3}$$

Here it is assumed that the nonequilibrium perturbation of the electron distribution is due to the high-frequency electromagnetic field and the nonequilibrium electric-field potential $\delta\varphi$. Then the electron kinetic equation for the function δf_c is

$$-i(\omega-\mathbf{kv})\,\delta f_c+\frac{e}{m_e c}\,[\mathbf{vB}]\frac{\partial\delta f_c}{\partial\mathbf{v}}-J_{ee}[\delta f_c]-J_{ei}[\delta f_c]=Y_0+Y_a, \tag{2.4}$$

where $\mathbf{B}$ is the static magnetic field and

$$Y_0=\nu_{ei}I\left[-(2\pi)^{1/2}v_{\mathrm{Te}}^3\,\frac{\partial}{\partial v}\left(\frac{v}{v^3}\,f_{\mathrm{M}}\right)+\frac{i\omega}{\nu_{ei}}\left(\frac{v^2}{3v_{\mathrm{Te}}^2}-2\right)f_{\mathrm{M}}\right]-\frac{i\omega e\,\delta\varphi}{\kappa_{\mathrm{B}}T_e}\,f_{\mathrm{M}}, \tag{2.5}$$

$$Y_a=\gamma_{ei}\sum\left[\left(3-\frac{v^2}{2v_{\mathrm{Te}}^2}\right)\frac{3\pi^{1/2}v_{Te}^5}{2^{1/2}v^5}+\frac{i\omega}{2\nu_{ei}}\right]f_{\mathrm{M}}, \tag{2.6}$$

$$\nu_{ei}=2^{5/2}\pi^{1/2}3^{-1}e^2e_i^2n_i\Lambda m_e^{-2}v_{\mathrm{Te}}^{-3}. \tag{2.7}$$

Let us express the perturbation δf_c in the form

$$\delta f_c=\delta f_0+\delta f_a, \tag{2.8}$$

where $\delta f_0=\langle\delta f_c\rangle$ is the isotropic part of the perturbation, obtained by averaging δf_c over the angles, and $\delta f_a=\delta f_c-\langle\delta f_c\rangle$ is the anisotropic part of the perturbation. Averaging (2.4) over the angles gives (cf. Ref. 2)

$$-i\omega\delta f_0+i\langle\mathbf{kv}\delta f_a\rangle-J_{ee}[\delta f_0]=Y_0. \tag{2.9}$$

The anisotropic part δf_a of the electron distribution perturbation obeys the equation

$$-i\omega\delta f_a+i\mathbf{kv}\delta f_0+i(\mathbf{kv}\delta f_a-\langle\mathbf{kv}\delta f_a\rangle)+\frac{e}{m_e c}[\mathbf{vB}]\,\frac{\partial\delta f_a}{\partial\mathbf{v}}-J_{ee}[\delta f_a]-J_{ei}[\delta f_a]=Y_a. \tag{2.10}$$

The last equation will be solved approximately. The degree of ionization is assumed to be high,

$$|e_i/e|=Z\gg 1. \tag{2.11}$$

This enables one to neglect the electron-electron collision integral J_{ee} in Eq. (2.10). In the electron-ion collision integral we neglect small terms of the order of the electron-to-ion mass ratio. Then

$$J_{ei}[\delta f]=\nu(v)\frac{\partial}{\partial v_2}\left[(v^2\delta_{rs}-v_r\,v_s)\frac{\partial\delta f}{\partial v_s}\right]. \tag{2.12}$$

Apart from the inequality (2.11) we also assume the condition

$$kv\ll|\nu(v)+i\omega|\,. \tag{2.13}$$

This permits to neglect the terms $i(\mathbf{kv}\delta f_a-\langle\mathbf{kv}\delta f_a\rangle)$ in Eq. (2.10). Then it is readily shown that retaining the right-hand side of Eq. (2.10) does not contribute to $\langle\mathbf{kv}\delta f_a\rangle$ and thus the following approximate equation can be used:

$$i\omega\delta f_a+\Omega_e[\mathbf{vb}]\frac{\partial\delta f_a}{\partial\mathbf{v}}+J_{ei}[\delta f_a]=i\mathbf{kv}\delta f_0. \tag{2.14}$$

The solution to this equation has the form

$$\delta f_a=i\delta f_0\left\{\frac{(\mathbf{kb})(\mathbf{vb})}{-i\omega+2\nu(v)}+\frac{\Omega_e(\mathbf{b}[\mathbf{kv}])+[-i\omega+2\nu(v)](\mathbf{k}[\mathbf{b}[\mathbf{vb}]])}{\Omega_e^2+[-i\omega+2\nu(v)]^2}\right\}. \tag{2.15}$$

On substituting (2.15) into (2.9) we obtain the following equation for the symmetric part δf_0 of the electron perturbation:

$$\left\{-i\omega+\frac{k_\parallel^2v^2}{3(2\nu(v)-i\omega)}+\frac{[2\nu(v)-i\omega]k_\perp^2v^2}{3(\Omega_e^2+[2\nu(v)-i\omega]^2)}\right\}\delta f_0-J_{ee}[\delta f_0]=Y_0. \tag{2.16}$$

From now on $k_\parallel$ everywhere denotes the absolute magnitude of the longitudinal wave vector component. Using

$$\delta f_0(v)=\frac{9\pi^{1/2}}{8k_\parallel^2l_{ei}^2}f_{\mathrm{M}}(c)F\left(\frac{v^2}{2v_{Te}^2}\right)\equiv\frac{Z}{2N_\parallel}f_{\mathrm{M}}F(X) \tag{2.17}$$

and

$$N_\parallel=\frac{4}{9\pi^{1/2}}Zk_\parallel^2l_{ei}{}^2 \tag{2.18}$$

Eq. (1.6) may be written in the form

$$\frac{1}{N_\parallel}\mathscr{L}[F(X)]-x^3RF(X)=S, \tag{2.19}$$

where (cf. Ref. 2)

$$R=\frac{1}{1-[i\omega/2\nu(v)]}-\frac{6i\omega\nu(v)}{k_\parallel^2v^2}+\frac{k_\perp^2(1-[i\omega/2\nu(v)/]}{k_\parallel^2([\Omega_e/2\nu(v)]^2+[1-(i\omega/2\nu(v))]^2}, \tag{2.20}$$

$$S=I\,\pi^{1/2}\left[\delta(X)-1+\frac{2i\omega X^{1/2}}{\pi^{1/2}\nu_{ei}}\left(1-\frac{X}{3}\right)+\frac{i\omega e\,\delta\varphi X^{1/2}}{\nu_{ei}\kappa_{\mathrm{B}}T_e}\right], \tag{2.21}$$

$$\mathscr{L}[F(X)]=F''(X)\frac{3}{2}\int_0^X dye^{-y}y^{1/2}+\left[\frac{3}{2}X^{1/2}X^{3/2}\right]\int_X^\infty dye^{-y}\times[F'(X)-F'(y)]-\int_0^x dye^{-y}y^{3/2}[F'(X)-F'(y)]. \tag{2.22}$$

In contrast to Refs. 1 and 2, the parameter $N_\parallel$ is only determined by the longitudinal (along the magnetic field) wave vector component, $k_\parallel$. The solution to Eq. (2.19) will be carried out in the asymptotic limit

$$N_\parallel\gg1 \tag{2.23}$$

when, as readily seen from Ref. 1 (see also Ref. 2), we can assume in determining the electron density perturbation that

$$\mathscr{L}[F(X)]=\frac{d}{dX}\left(X^{3/2}\frac{dF}{dX}\right). \tag{2.24}$$

3. We start by deriving an electron magnetization condition for a weakly collisional, completely ionized plasma. To this end consider Eq. (2.20) in the stationary case, when it can be written as

$$R=1+\frac{k_\perp^2}{k_\parallel^2}\frac{1}{1+(16\Omega_e^2/g\pi\nu_{ei}^2)X^3}. \tag{3.1}$$

The presence of the X^3 dependence is due to the dependence on the electron velocity v of the collision frequency (1.2). The theory of Refs. 1 and 2 predicts that the main contribution to the electron density perturbation comes from that part of the electron velocity distribution for which it turns out that

$$X=\frac{v^2}{2v_{\mathrm{Te}}^2}\sim\frac{1}{N_\parallel^{2/7}}. \tag{3.2}$$

Here, in contrast to Refs. 1 and 2, the quantity $N_\parallel$ is used which manifests, first of all, a formal difference of the theory of a magnetized plasma. A real difference arises when instead of the no magnetization ($\Omega_e=0$) limit of Eq. (3.1),

$$R_0=1+\frac{k_\perp^2}{k_\parallel^2}=\frac{k^2}{k_\parallel^2} \tag{3.3}$$

the opposite limit is realized, in which (3.1) differs little from unity. This is clearly possible in the strong field limit. In the ordinary theory of a highly collisional plasma this limit is determined by the inequality

$$\Omega_e^2\gg\nu_{ei}^2, \tag{3.4}$$

due to the fact that perturbations are usually formed by thermal or superthermal electrons. Our situation is qualitatively different in that the low velocity part of the electron distribution turns out to be by far predominant. In view of the relation (3.2), the strong field limit is found to be determined by the inequality

$$\Omega_e^2\gg\nu_{ei}^2N_\parallel^{6/7}(k_\perp^2/k_\parallel^2), \tag{3.5}$$

when Eq. (3.1) takes the form

$$R=1+\frac{9\pi\,\nu_{ei}^2k_\perp^2}{16\Omega_e^2k_\parallel^2X^3}\equiv1+\frac{C}{X^3}. \tag{3.6}$$

With the condition (3.5) fulfilled, the right-hand side of Eq. (3.6) differs little from unity at $X\sim N_\parallel^{-2/7}$, and it is this circumstance which manifests perturbation suppression across the magnetic field. Comparing the inequalities (3.4) and (3.5) it is readily seen that in the case of a weakly collisional plasma, much higher magnetic fields are required in order to suppress the transverse transport.

4. Consider the electron density perturbation in the stationary limit $\omega=0$, when

$$S=I\,\pi^{1/2}[\delta(X)-1] \tag{4.1}$$

and when also the electron magnetization condition (3.5) for a weakly collisional plasma is met. Then, using Eq. (2.24) and (3.6) we obtain from Eq. (2.19)

$$\frac{1}{N_\parallel}\left(X^{3/2}\frac{d^2F}{dX^2}+\frac{3}{2}X^{1/2}\frac{dF}{dX}\right)-(X^3+C)F=I\pi^{1/2}[\delta(X)-1]. \tag{4.2}$$

In Ref. 1 this equation is solved for $C=0$, which enables the following asymptotic solution for $N_\parallel \gg 1$ to be written:

$$F_0(X)=\tilde{F}_0(\xi)\cong -I\Gamma\left(\frac{6}{7}\right)\times\left(\frac{2}{7}\right)^{1/7}\frac{4N_\parallel^{8/7}}{\pi^{1/2}\xi^{1/4}}\sin\frac{\pi}{7}K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right), \tag{4.3}$$

where $\xi=XN^{2/7}$, $\Gamma(z)$ is the Γ function. and $K_{1/7}(z)$ is the Mcdonald function. Assuming a small effective influence of the constant C, its role can be accounted for in first-order perturbation theory. Then the approximate solution to Eq. (4.2) may be written in the form

$$F(X)=\tilde{F}_0(\xi)-C\,N^{6/7}\psi(\xi,[\tilde{F}_0(\xi)]), \tag{4.4}$$

where we have used the functional

$$\psi(\xi,[\varphi(\xi)])=\frac{4}{7}\xi^{-1/4}\left\{I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_\xi^\infty d\xi\xi^{-1/4}\varphi(\xi)\times K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)+K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_0^\xi d\xi\,\xi^{-1/4}\times\varphi(\xi)I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\right\}. \tag{4.5}$$

Equation (4.4), using Eqs. (2.2), (2.8), and (2.17), yields for the electron density perturbation

$$\delta n_e=-n_e\frac{e\delta\varphi}{\kappa_B T_e}-n_e I\left(1+1{,}17\frac{Z}{N_\parallel^{2/7}}-\frac{\beta_\perp Z^2}{N_\parallel^{3/7}}k_\perp^2\rho_e^2\right), \tag{4.6}$$

where $\rho_e=v_{Te}/\Omega_e$, and the numerical coefficient $\beta_\perp$ is determined by the relation

$$\beta_\perp=\frac{2}{\sqrt{\pi}}\left(\frac{2}{7}\right)^{8/7}\Gamma\left(\frac{6}{7}\right)\sin\frac{\pi}{7}\int_0^\infty dz.z^{1/4}\times\left\{I_{1/7}\left(\frac{4}{7}z^{7/4}\right)\int_z^\infty\frac{dy}{y^{1/2}}\left[K_{1/7}\left(\frac{4}{7}y^{7/4}\right)\right]^2+K_{1/7}\left(\frac{4}{7}z^{7/4}\right)\int_0^z\frac{dy}{y^{1/2}}I_{1/7}\times\left(\frac{4}{7}y^{7/4}\right)K_{1/7}\left(\frac{4}{7}y^{7/4}\right)=0.76. \tag{4.7}$$

The smallness of the last term on the right-hand side of Eq. (4.6) in comparison with the last but one term follows from the inequality (3.5).

5. We use Eq. (4.6) to determine the longitudinal and transverse components of the effective thermal conductivity. To this end we use the fact that the Ohmic collisional heating of the plasma by a high-frequency field in the stationary regime is balanced by the electron heat transport,

$$\text{div }\mathbf{q}=\frac{e^2n_e\nu_{ei}}{2\,m_e\,\omega_0^2}|\mathbf{E}|^2. \tag{5.1}$$

We assume, further, that the sum of the plasma pressure plus the Miller force pressure remains unchanged. This suggests that the density perturbation δn_e and the temperature perturbation δT_e are related by (cf. Ref. 3)

$$I\,n_e\kappa_B T_e+n_e\kappa_B\,\delta T_e+\kappa_B T_e\delta n_e=0. \tag{5.2}$$

This, using Eq. (4.6), yields for the electron temperature perturbation

$$\kappa_B\,\delta T_e=\left(1{,}17\frac{Z}{N_\parallel^{2/7}}-\frac{Z^2\beta_\perp}{N_\parallel^{3/7}}k_\perp^2\rho_e^2\right)\frac{e^2|\mathbf{E}|^2}{4m_e\,\omega_0{}^2}. \tag{5.3}$$

Here we have neglected the perturbation in the electric potential which, in the stationary regime being considered here, gives a relatively small contribution $\sim Z^{-1}$.

Turning now to the longitudinal and transverse components of the effective thermal conductivity, for their Fourier transforms Eq. (5.1) gives

$$(k_\parallel^2\kappa_\parallel+k_\perp^2\kappa_\perp)\delta T_e=\frac{e^2n_e\nu_{ei}}{2m_e\omega_0^2}|\mathbf{E}|^2. \tag{5.4}$$

Substituting the temperature perturbation (5.3) this yields

$$\kappa_\parallel+\frac{k_\perp^2}{k_\parallel^2}\kappa_\perp=\frac{\kappa_{SH}}{31{,}7N_\parallel^{5/7}-48\pi^{-1/2}\beta_\perp ZN_\parallel^{4/7}k_\alpha^2\rho_e^2}. \tag{5.5}$$

In the denominator on the right we shall add unity, which will allow us to obtain an interpolation formula for the longitudinal effective thermal conductivity. Noting the small value of $k_1^2\rho_e^2$ we find

$$\kappa_\parallel(k_\parallel)=\frac{\kappa_{SH}}{1+31{,}7N_\parallel^{5/7}}=\frac{\kappa_{SH}}{1+(21{,}1k_\parallel\lambda_e)^{10/7}}, \tag{5.6}$$

$$\kappa_\parallel(k_\parallel)=\frac{\kappa_{SH}108\beta_\perp N_\parallel^{11/7}}{[1+31{,}7N_\parallel^{5/7}]^2}\frac{\nu_{ei}{}^2}{\Omega_e^2}=\frac{\kappa_{SH}787\beta_\perp(k_\parallel\lambda_e)^{22/7}}{[1+(21{,}1k_\parallel\lambda_e)^{10/7}]^2}\frac{\nu_{ei}^2}{\Omega_e^2}\simeq\kappa_{SH}\frac{\nu_{ei}^2}{\Omega_e^2}7{,}6(k_\parallel\lambda_e)^{2/7}. \tag{5.7}$$

Equation (5.6) resembles Eq. (1.6). However, the difference of the expression (5.6) from the formula of the type (1.6) in Ref. 1 lies in the appearance of the longitudinal wave vector component $k_\parallel$–which is natural, even though for qualitatively new conditions given by Eq. (3.5). The expression (5.7) for the transverse component of the effective electron thermal conductivity in the weakly collisional limit (1.4) has not been published earlier. While according to Eq. (5.6) the nonlocal longitudinal heat transport under the conditions (2.23) is considerably smaller than in the theory of weakly collisional transport, from Eq. (5.7) the effective transverse electron heat conduction by far exceeds that from the weakly collisional theory. This enhancement is the larger the greater $k_{\|}$, indicative of the nonuniformity in the direction along the magnetic field.

6. We now turn to consideration of the density perturbation due to the the electric potential. For this perturbation the right-hand side of Eq. (2.19) is of the form

$$S_{\varphi}=\frac{i\omega e\,\delta\varphi}{\nu_{ei}\,\kappa_{B}T_{e}}\,X^{1/2}. \tag{6.1}$$

Taking the frequency ω to be small,

$$\omega\ll\nu_{ei}N_{\|}^{2/7}Z^{-1},\quad \omega\ll\gamma_{ei}N_{\|}^{1/7}Z^{-1/2},\quad \omega\ll\Omega_{e}, \tag{6.2}$$

and assuming also the strong magnetic field condition (3.5), we neglect the departure of (2.20) from unity. Then Eq. (2.19) gives

$$\frac{1}{N_{\|}}\left(X^{3/2}\frac{d^{2}F_{\varphi}}{dX^{2}}+\frac{3}{2}X^{1/2}\frac{dF_{\varphi}}{dX}\right)-X^{3}F_{\varphi}=\frac{i\omega e\delta\varphi}{\nu_{ei}\kappa_{B}T_{e}}X^{1/2}. \tag{6.3}$$

In accordance with Ref. 2, the solution to this equation can be written in the form

$$F_{\varphi}(X)=\frac{i\omega e\,\delta\varphi}{\nu_{ei}\kappa_{B}T_{e}}F_{1/2}(X), \tag{6.4}$$

where

$$F_{1/2}(X)=\tilde{F}_{1/2}(\xi)=-N_{\|}^{5/7}\psi(\xi,[\xi^{1/2}]). \tag{6.5}$$

It is natural that the departure from the treatment of Ref. 2 is due to the magnetic field and that it manifests itself in the appearance of the expression (2.18). The solution (6.4) yields the following formula for the electron density perturbation:

$$\delta n_{\varphi}=-n_{e}\frac{e\,\delta\varphi}{\kappa_{B}T_{e}}\left[1+\frac{i\omega\,1,5\,Z^{2/7}}{\nu_{ei}(k_{\|}l_{ei})^{10/7}}\right]. \tag{6.6}$$

This formula makes it possible, in particular, to write down the following expression for the electron contribution to the complex longitudinal dielectric constant of the plasma:

$$\delta\xi_{e}(\omega,k)=\frac{4\pi e^{2}n_{e}}{\kappa_{B}T_{e}k^{2}}\left[1+\frac{i\omega}{k_{\|}v_{Te}}\left(\sqrt{\frac{\pi}{2}}+\frac{3Z^{2/7}}{2k_{\|}^{3/7}l_{ei}^{3/7}}\right)\right]. \tag{6.7}$$

Here, for comparison, we have also included the contribution from the collisionless Cherenkov wave absorption due to electrons. It is evident that the collision effects dominate the electron dissipation for

$$Z^{2}>k_{\|}^{3}\,l_{li}^{3}. \tag{6.8}$$

Presenting the result (6.7) we wish also to emphasize that such an expression for the electronic contribution to the dielectric constant is not obtainable from the equations of two-liquid hydrodynamics (based on the Grad method, for example), these equations employing an electron heat flux density expression dependent on the effective thermal conductivity (5.6) alone. This is indicative of the unusual situation obtaining in the weakly collisional transport regime.

7. To summarize, by means of our theory of a high-Z plasma in the weakly collisional regime with

$$k_{\|}^{2}\,l_{ee}\,l_{ei}\gg 1$$

a new condition, Eq. (3.5), has been established, which predicts the value of the (strong) magnetic field securing the suppression of the transverse relative to longitudinal transport in a static magnetic field. Having solved the kinetic equation it proved possible to determine the static electron density perturbation due to the heating high-frequency field. When obtained from this perturbation, the Fourier transforms of the longitudinal and transverse components of the effective thermal conductivity are both found to depend on the modulus of the longitudinal wave vector component. While in the limit (7.1) the longitudinal conductivity turns out to be less than Spitzer–Harm thermal conductivity, for the transverse thermal conduction the reverse situation obtains. It is the nonlocality of the transport along the magnetic field which enhances the transverse thermal transport compared to the theory of a weakly collisional plasma.

The expression obtained for the electron contribution to the dielectric constant, Eq. (6.7), determines the wave vector region Eq. (6.8) in which collisions dominate. At the same time, Eq. (6.7) reveals the limited applicability of the concepts of effective thermal conductivities in a weakly collisional plasma.

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