

# Correlation of giant magnetoresistance and magnetization of metallic superlattices

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It is shown that by studying the correlation properties of electrical resistivity and magnetization in metallic superlattices, in addition to measuring the absolute magnitude of the magnetic susceptibility, the main parameters characterizing the various possible mechanisms for giant magnetoresistance in superlattices can be determined; one mechanism is due to the change in the intensity of “mixing” of the electronic trajectories of conduction electrons with different spins upon passage from layer to layer, while another involves a change in electron scattering by defects in the interface between layers as the mutual orientation of the magnetization of neighboring magnetic layers changes.

## 1. INTRODUCTION

The discovery in 1988 of the giant magnetoresistance effect (GMR) in Fe/Cr superlattices<sup>1</sup> was followed by the appearance of many articles devoted to the study of the magnetic and transport properties of multilayer structures of alternating ferromagnetic and normal metals (see the bibliography in the review<sup>2</sup> and the joint monograph<sup>3</sup>). Apart from the desire to establish its physical nature, the chief interest in systems exhibiting GMR is in using such materials to learn more about the magnetic properties of the charge carriers.

At present it is generally accepted that GMR results from the dependence of the effective mean free paths of electrons in different spin states on the type of magnetic ordering in the superlattice layers. However, which particular mechanisms lead to this dependence remains open to question. Is GMR a consequence of “mixing” of the trajectories of electrons from different spin sub-bands, characterized by different bulk mean free paths, occurring as an electron passes from layer to layer? Or does the change in the mutual orientation of the magnetizations of neighboring layers simply lead to an appreciable change in the intensity of scattering of charge carriers at interfaces? What is the relative contribution to the GMR effect of bulk and interface scattering?

We describe below a treatment of the GMR effect within the framework of a quasiclassical kinetic theory of transport phenomena on the basis of a relatively simple superlattice model; analytic expressions are obtained both for the magnitude of the GMR effect and for the correlated dependence of magnetoresistance and magnetization. We describe how the formulae obtained may be used in combination with experimentally observed dependences to answer the questions posed above. We also discuss analysis of experimental data<sup>1</sup> in terms of our model.

## 2. BASIC CONSIDERATIONS

We consider a superlattice of layers of ferromagnetic metal, each of which has thickness  $L$  and magnetic moment density  $\mathbf{M}_i$ . Since in real lattices which show GMR the thickness of the layers of nonmagnetic metal is, as a

rule, considerably less than  $L$ , we will henceforth neglect the finite thickness of the nonmagnetic layers. We consider that in the ground state (magnetic field  $H=0$ ), neighboring moments  $\mathbf{M}_i$  and  $\mathbf{M}_{i+1}$  are antiparallel (Fig. 1a) and lie along the easy magnetization axis  $Oy$  in the plane of the layers. The application of a magnetic field  $H$  in the plane of the film changes the orientation of the vectors  $\mathbf{M}_i$ , where their modulus is assumed to remain constant:  $|\mathbf{M}_i|=M_0$ . The relative orientation of neighboring moments will be characterized by the angle  $\theta=\theta(H)$  between  $\mathbf{M}_i$  and  $\mathbf{M}_{i+1}$ . In an arbitrary magnetic field (in Fig. 1b the disposition of  $\mathbf{M}_i$  is shown for specimen magnetization along  $Oz$ ), the mean over the  $x$  coordinate of the relative magnetization is  $\mu(H)=M(H)/M_s=\cos\theta/2$ . In fields  $H$  exceeding the saturation field  $H_s$ , the density  $M$  coincides with the saturation magnetization  $M_s=M_0$ .

The maximum value of the magnetoresistance  $\delta R(H)=R(H)-R(0)$  will be characterized by the parameter  $\Delta=[R(H_s)-R(0)]/R(0)$ , while the change in resistivity with field is described by the dimensionless function  $\delta(H)=[R(H)-R(0)]/[R(H_s)-R(0)]$ . By finding the values of  $\delta$  and  $\mu$  as functions of  $H$  on the basis of experimental  $R(H)$  and  $M(H)$  dependences, the correlated dependence of these quantities can then be established, eliminating the common variable  $H$ . As will be shown below, we can compare these curves with the theoretical dependences by expressing the experimental results in the form  $\delta=\delta(\mu^2)$  in order to estimate the microscopic parameters.

## 3. ELECTRICAL RESISTANCE OF A SUPERLATTICE

Let  $\varepsilon_\sigma(\mathbf{k})$  represent the spectrum of conduction electrons in the ferromagnetic metal, where  $\sigma$  is + or – corresponding to the two electron polarizations;  $\sigma=-$  represents those electrons with spin parallel to the magnetization  $\mathbf{M}$  and  $\sigma=+$  corresponds to those antiparallel.  $A_\pm$  are the areas of the corresponding Fermi surfaces specified in quasimomentum  $\mathbf{k}$  space by the expression  $\varepsilon_\pm(\mathbf{k})=\zeta$ , where  $\zeta$  is the chemical potential of the metal. The transport properties of a ferromagnet depend on the magnitude of the electron velocities  $\mathbf{v}_\pm=\partial\varepsilon_\pm(\mathbf{k})/\partial\mathbf{k}$  at the Fermi surfaces of the spin subbands, and also on the fre-

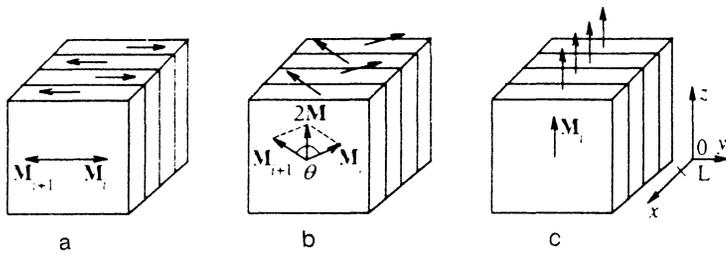


FIG. 1. Magnetic ordering of ferromagnetic layers in the present model of a metallic superlattice: a) initial state in the absence of an external magnetic field:  $\mathbf{M}_{i+1} = -\mathbf{M}_i$ ;  $|\mathbf{M}_i| = M_0$ ; mean magnetization of the superlattice  $M = 0$ ; b)  $0 < H < H_s$ ;  $M = M_0 \cos \theta/2$ ; c)  $H > H_s$ ;  $M = M_0$ .

quency of electron scattering by bulk defects  $\nu_{\pm}$  or the corresponding mean free path lengths  $l_{\pm} = v_{\pm}/\nu_{\pm}$ .

In a rigorous semiclassical description of the response of a superlattice to the application of an electric field  $E$ , it is necessary to write down the system of well-known kinetic equations for the nonequilibrium part of the distribution function giving the number of particles  $\phi_{\pm}^{(i)}(x, \mathbf{k})$  in each of the layers  $i$ , and to supplement it with boundary conditions that specify the distribution functions of electrons that start from each of the interfaces as a linear combination of the distribution functions of electrons incident upon a given interface from both adjacent layers. The scheme for deriving these conditions is analogous to that given earlier.<sup>4,5</sup>

To establish the boundary conditions we consider the quantities that characterize electron interaction with the interface between two ferromagnetic layers. Let  $R_{\sigma}$  be the probability of coherent (specular) reflection from an interlayer boundary of an electron with given spin projection  $\sigma$ , when its energy and tangential quasimomentum are conserved and the signs of the normal components of the quasimomentum  $k_{\sigma}^x$  and velocity  $v_{\sigma}^x$  are reversed (assuming mirror symmetry of the Fermi surface relative to the plane  $k_x = 0$ ). We also introduce the matrix  $T_{\sigma\sigma_1}$  of the coefficients of coherent transmission through the interface, where  $\sigma_1$  and  $\sigma_2$  are the spin projections in the initial and final states with quantization axes  $z_1$  and  $z_2$ , directed along  $\mathbf{M}_1$  and  $\mathbf{M}_2$  and rotated by an angle  $\theta$  relative to one another. Scattering processes at the interface will be characterized by diffusion coefficients  $S_{\sigma}$ . The quantities  $R$ ,  $T$  and  $S$  satisfy the normalization conditions

$$R_{\sigma} + T_{\sigma\sigma} + T_{\sigma(-\sigma)} + S_{\sigma} = 1. \quad (1)$$

The boundary condition for the distribution function  $\phi_{\sigma}^{(i)}(x_i, k_{\sigma}^x)$  at the boundary  $x = x_i$  for current flow along the layers is of the form

$$\begin{aligned} \phi_{\sigma}^{(i)}(x_i, k_{\sigma}^x) = & R_{\sigma} \phi_{\sigma}^{(i)}(x_i, -k_{\sigma}^x) + \sum_{\sigma_1} T_{\sigma\sigma_1} \phi_{\sigma_1}^{(i-1)} \\ & \times (x_i, k_{\sigma_1}^x). \end{aligned} \quad (2)$$

All the incident terms proportional to  $S$  on the right-hand side of Eq. (2) vanish because of the assumed scattering symmetry in the boundary plane and the odd parity of the distribution function  $\phi$  in  $k^y$  and  $k^z$ . However, the boundary conditions depend implicitly on  $S$  through the normalization condition of Eq. (1).

Since it is sufficient to know the mean current density  $j$  over the  $x$  coordinate to evaluate the effective conductivity of the superlattice  $\sigma$ , we can go from the known equa-

tions for  $\phi_{\sigma}^{(i)}(x, \mathbf{k})$  to equations for the distribution functions  $\phi_{\sigma}(\mathbf{k})$  averaged over the  $x$  coordinate, which in view of the equivalent kinetic properties of all layers does not depend on  $i$  number. The equations for  $\phi_{\sigma}(\mathbf{k})$  are obtained by integrating the original kinetic equations for  $\phi_{\sigma}^{(i)}(x, \mathbf{k})$  over  $x$  and subsequently using the boundary conditions to express all of the resulting  $\phi(x_i, \mathbf{k})$  in terms of the distribution function of electrons incident on the boundary. Since we are interested in the case in which the mean free path  $l_{\sigma}$  exceeds the layer thickness  $L$  and an electron transfers "information" from the boundaries  $x_i = x \pm L$  to the boundaries  $x = x_i$  without significant loss, the difference between the distribution functions of the incident electrons and the averaged functions can be neglected in the resulting expressions. As a result we obtain a closed system of two equations for the averaged distribution functions  $\phi_{\sigma}(\mathbf{k})$  of electrons belonging to different spin subbands:

$$\begin{aligned} \left[ v_{\sigma} + \frac{|v_{\sigma}^x|}{L} (1 - R_{\sigma} - T_{\sigma\sigma}) \right] \phi_{\sigma}(\mathbf{k}) - \frac{|v_{\sigma}^x|}{L} T_{\sigma(-\sigma)} \phi_{(-\sigma)}(\mathbf{k}) \\ = e v_{\sigma}^z \delta(\varepsilon_{\sigma}(\mathbf{k}) - \xi) E, \end{aligned} \quad (3)$$

where  $e$  is the electronic charge and  $\delta(\varepsilon)$  is the Dirac delta function. The system of equations (3) has a simple physical meaning. With the present approximations, the existence of interfaces leads to the appearance in the collision integral of, firstly, an additional "outgoing" term (the second term in square brackets can be interpreted as the effective electron drift velocity from a state with spin  $\sigma$  due to scattering at layer interface defects and transitions from a state with opposite spin upon coherent transmission through an interface) and secondly, an "incoming" term [the second term on the left-hand side of Eq. (3), proportional to  $\phi_{(-\sigma)}$ , describing the change in distribution function  $\phi_{\sigma}$  due to arrival of electrons from the  $(-\sigma)$  spin subband upon coherent transmission through the interface].

By solving Eq. (3), we can find the current density in the superlattice averaged over  $x$ ,

$$j = \frac{e}{(2\pi\hbar)^3} \sum_{\sigma} \int d^3\mathbf{k} v^z \phi_{\sigma}(\mathbf{k}) \quad (4)$$

and its conductivity  $\sigma = j/E$ , which can be represented in the form

$$\sigma = \sigma_{+} + \sigma_{-} + \sigma_T, \quad (5)$$

where

$$\sigma_{\pm} = \frac{e^2}{(2\pi\hbar)^3} \int dA_{\pm} (u_{\pm}^z)^2 L_{\pm}^s; \quad (6)$$

$$\sigma_T = -\frac{e^2 L}{(2\pi\hbar)^3} \int dA_- \times \frac{(\Lambda_-^s u_-^x - \Lambda_+^s u_+^x) [\Lambda_-^s (u_-^z)^2 - \Lambda_+^s (u_+^z)^2 u_-^x / u_+^x] T}{1 + T(\Lambda_-^s u_-^s + \Lambda_+^s u_+^s)}. \quad (7)$$

The integration in Eqs. (6) and (7) is carried out over the Fermi surfaces  $A_{\pm}$ , and we have introduced the effective mean free paths  $L_{\pm}^s = l_{\pm} / [1 + S_{\pm}(l_{\pm}/L)u_{\pm}^x]$  in the superlattice, which depend on the direction of the electron velocity relative to the interface ( $u_{\pm}^j = |v_{\pm}^j|/v_{\pm}$ ,  $j=x, z$ ). The quantities  $\Lambda_{\pm}^s = L_{\pm}^s/L$  are the relative path lengths. In deriving Eq. (7) for  $\sigma_T$ , the equality of off-diagonal components of the  $T$ -matrix was used, which is a consequence of the microscopic reversibility of the coherent transmission process:  $T_{+-} = T_{-+} \equiv T$ .

As follows from Eqs. (5)–(7), the presence of layer interfaces in the superlattice influences the conductivity in two ways. First, scattering processes at imperfect interfaces lead to a reduction in the effective mean free path of current carriers of both spin subbands from  $l_{\pm}$  to the value  $L_{\pm}^s$ . In the present approximation, the expressions for  $\sigma_{\pm}$  are identical for conduction electrons in the two spin subbands of a bulk ferromagnet, up to the  $L_{\pm}^s$  for  $l_{\pm}$ . Second, the existence of coherent electron transmission processes through an interface leads to a contribution  $\sigma_T$  to the conductivity whose magnitude depends on the coherent transmission coefficient  $T$  and the difference between the effective lengths  $L_+^s$  and  $L_-^s$ , and on the difference in electron velocities of different spin subbands. We note that the sign of  $\sigma_T$  can be either negative or positive for an arbitrary size ratio of the Fermi surfaces  $A_+$  and  $A_-$ , as can easily be seen from Eq. (7), and only in the special case of complete coincidence in the velocity distribution on the surfaces  $A_+$  and  $A_-$  can it be definitely stated that  $\sigma_T \leq 0$ . As a result, the magnetoresistance of superlattices, usually negative, can have the opposite sign in certain cases.

#### 4. ORIENTATION DEPENDENCE OF THE $S$ AND $T$ COEFFICIENTS

So far we have not made any assumptions about the form of the dependence of the diffusion and coherent transmission coefficients  $S$  and  $T$  on the angle  $\theta$  between the magnetic moments of adjacent layers. The explicit form of  $S$  and  $T$  can be determined for specific quantum-mechanical models of an interface. Here we shall, however, restrict ourselves to determining the overall functional dependence of  $S$  and  $T$  on  $\theta$  for an arbitrary scattering potential at the interface between magnetic layers.

Rotation of the coordinate system about the  $x$  axis by an angle  $\theta$  corresponds, as is well known, to a bilinear transformation in the  $E_2$  space of two-component spinors, given by the rotation operator  $\hat{V}_x^{\pm}(\theta) = \exp(i\hat{\sigma}_x\theta/2)$ , where  $\hat{\sigma}_x$  is the first Pauli spin matrix. Using the explicit form of these matrices and their commutation properties, the operator  $\hat{V}_x$  can also be represented in explicit form:

$$\hat{V}_x = \begin{pmatrix} \cos(\theta/2) & i \sin(\theta/2) \\ i \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}. \quad (8)$$

Writing out the action of the rotation operator on an eigenfunction of the operator  $\hat{\sigma}_z$ , it is easy to show that the probability of finding an electron with positive spin projection on the  $z_2$  axis, rotated by an angle  $\theta$  relative to  $z_1$ , is  $\eta = \cos^2(\theta/2)$  provided that this probability in the previous coordinate system was equal to unity. Knowing  $\eta$ , it is easy to express the coefficients  $S_{\pm}$  in explicit form as functions of  $\eta$  in terms of the values of these quantities at  $\eta=0$  and  $\eta=1$ , which we denote by  $s_{\pm}$  and  $s_{\pm} + \Delta s_{\pm}$  respectively:

$$S_{\pm}(\eta) = (1-\eta)s_{\pm} + \eta(s_{\pm} + \Delta s_{\pm}) = s_{\pm} + \Delta s_{\pm}\eta. \quad (9)$$

Analogously

$$T(\eta) = (1-\eta)t, \quad (10)$$

where  $t$  is the probability of coherent electron transmission through an interface between layers, when the latter are antiferromagnetically ordered from one surface ( $A_{\pm}$ ) to the next ( $A_{\mp}$ ). All the newly introduced quantities  $s_{\pm}$ ,  $\Delta s_{\pm}$ , and  $t$  are, in the general case, functions of  $\mathbf{k}$ , and at a given energy they depend on the angle of approach of an electron to the interface. As it is not our aim in the present work to study effects that depend on such a dependence, we henceforth consider these quantities to be numerical constants of the theory.

In this way, knowing the relations in Eqs. (9) and (10), the dependence of the conductivity  $\sigma$  on  $\eta$  can be calculated theoretically using Eqs. (5)–(7), and consequently so can the specific electrical resistance of the superlattice  $\rho(\eta) = \sigma^{-1}(\eta)$ . Finally, we have the possibility of directly comparing the experimental dependence  $\delta(\mu^2)$  with the corresponding calculated function  $\delta(\eta)$ .

#### 5. "OCTAHEDRAL" MODEL OF THE FERMI SURFACE

The hardest part of analyzing the GMR effect on the basis of Eqs. (5)–(7) is the need to integrate over the Fermi surfaces  $A_+$  and  $A_-$ , which have a rather complicated shape in real ferromagnetic metals. In striving to obtain analytic expressions for the  $\delta(\eta)$  dependence, one must choose a simple model of the electron spectrum  $\varepsilon_{\sigma}(\mathbf{k})$  for practical calculations. Below we use the dispersion relation

$$\varepsilon_{\pm}(\mathbf{k}) = v_{\pm} \sum_j |k^j| \mp \varepsilon_e, \quad (11)$$

in which the parameters are the electron velocity  $v_{\pm}$  and the spin splitting energy  $\varepsilon_e$ . The main feature of the model spectrum of Eq. (11) is the constancy of the electron velocity vector on different parts of the Fermi surfaces  $A_+$  and  $A_-$ , each of which is represented by a regular octahedron. In this model, the fact that the magnitude of the electron velocity component depends on the position of the wave vector on the Fermi surface can, in fact, be neglected: the quantities entering the expressions under the integral in Eqs. (6) and (7) are  $u_{\pm} \equiv 1/\sqrt{3}$ ; the integration over the

surface  $A_\sigma$  reduces to a simple summation and the expressions for the conductivity  $\sigma$  can be written in the form of explicit algebraic expressions:

$$\sigma_\pm = \frac{e^2}{8\pi^3 \sqrt{3}\hbar^3} \frac{A_\pm l_\pm}{1 + \Lambda_\pm S_\pm}, \quad (12)$$

$$\sigma_T = -\frac{e^2}{8\pi^3 \sqrt{3}\hbar^3} A_- L \frac{(\Lambda_-^s - \Lambda_+^s) 2T}{1 + (\Lambda_+^s + \Lambda_-^s) T}. \quad (13)$$

Since the dependences of  $S$  and  $T$  on  $\eta$  are given by Eqs. (9) and (10), Eqs. (12) and (13) enable  $\Delta$  to be calculated and the explicit functional  $\delta(\eta)$  dependence to be found, in which the quantities  $l_\pm$ ,  $s_\pm$ ,  $\Delta s_\pm$ , and  $t$  act as parameters. We make one more simplification, putting  $A_+ = A_- = A$ . Taking account of the difference between  $A_+$  and  $A_-$  in the cases considered below leads to a non-fundamental renormalization of some quantities, without changing the form of the functional  $\delta(\eta)$  dependence. The final expression for the specific electrical resistivity assumes the simple form

$$\rho(\eta) = \frac{8\pi^3 \sqrt{3}\hbar^3}{e^2} \frac{1}{AL} \frac{1 + (\Lambda_+^s + \Lambda_-^s)t(1-\eta)}{\Lambda_+^s + \Lambda_-^s + 4\Lambda_+^s \Lambda_-^s t(1-\eta)}, \quad (14)$$

where  $\Lambda_\pm^s = \Lambda_\pm^s(\eta) = \lambda_\pm / [1 + \lambda_\pm (s_\pm + \Delta s_\pm \eta)]$ .

It is now not difficult to find the form of the  $\delta(\eta)$  dependence,

$$\delta(\eta) = \frac{\alpha\eta + \beta\eta^2}{1 - (1 - \alpha - \beta)\eta}, \quad (15)$$

where the coefficients  $\alpha$  and  $\beta$  can be expressed in terms of  $t$ ,  $s_\pm$ ,  $\Delta s_\pm$ , and  $l_\pm$ . We will not give the general form of  $\alpha$ ,  $\beta$  and  $\Delta$  here because of the complexity of the expressions, but will discuss instead the more interesting limiting cases.

If the change in the diffusion coefficient  $\Delta s$  on passing from ferromagnetic to antiferromagnetic ordering is small compared with  $t$  and  $s_\pm$ , then  $\beta \ll \alpha$ , and the function

$$\delta(\eta) \simeq \frac{\alpha\eta}{1 - (1 - \alpha)\eta}, \quad (16)$$

where

$$\alpha = [1 + 4t\lambda_+^s \lambda_-^s / (\lambda_+^s + \lambda_-^s)]^{-1}, \quad (17)$$

while

$$\Delta = -t(\lambda_-^s - \lambda_+^s)^2 / (\lambda_+^s + \lambda_-^s) [1 + (\lambda_+^s + \lambda_-^s)t]. \quad (18)$$

Here  $\lambda_\pm^s = \lambda_\pm / (1 + \lambda_\pm s_\pm)$ . Therefore, if the experimental  $\delta(\mu^2)$  curve is parametrized by using Eq. (15), which determines the values of  $\alpha$  and  $\beta$ , and  $\beta \ll \alpha$ , then from Eqs. (17) and (18), considered as a system of equations referring to the unknown  $t\lambda_+^s$  and  $t\lambda_-^s$  for given  $\alpha$  and  $\Delta$ , the numerical values of the products  $t\lambda_-^s$  and  $t\lambda_+^s$  can be found, as can their ratio  $\gamma$ , which is equal to the ratio of the mean free paths:

$$t\lambda_\pm^s = \frac{1 - \alpha}{2} \frac{1}{\alpha} [1 \pm \sqrt{\Delta / (\alpha + \alpha\Delta - 1)}]^{-1}, \quad (19)$$

$$\gamma = \frac{l_-^s}{l_+^s} = \frac{1 + \sqrt{\Delta / (\alpha + \alpha\Delta - 1)}}{1 - \sqrt{\Delta / (\alpha + \alpha\Delta - 1)}}. \quad (20)$$

In the other limiting case, in which the transmission coefficient  $t$  is small compared with  $s_\pm$  and  $\Delta s_\pm$ , and the change in the diffusion coefficients is the main GMR mechanism under conditions  $l_\pm \gg L$ , the expressions for  $\Delta$ ,  $\alpha$  and  $\beta$  take the form

$$\Delta = \frac{\frac{\Delta s_+}{s_+} + \gamma \frac{\Delta s_-}{s_-} + (1 + \gamma) \frac{\Delta s_+ \Delta s_-}{s_+ s_-}}{1 + \gamma + \frac{\Delta s_-}{s_-} + \gamma \frac{\Delta s_+}{s_+}}, \quad (21)$$

$$\alpha = \frac{1}{\Delta} \left( \frac{1}{1 + \gamma} \frac{\Delta s_+}{s_+} + \frac{\gamma}{1 + \gamma} \frac{\Delta s_-}{s_-} \right), \quad \beta = \frac{1}{\Delta} \frac{\Delta s_+ \Delta s_-}{s_+ s_-}, \quad (22)$$

where  $\gamma = s_+ / s_-$ .

By parametrizing the experimental curves according to Eq. (15), the microscopic parameters  $\Delta s_+ / s_+$ ,  $\Delta s_- / s_-$ , and  $s_+ / s_-$  can be established using the values of  $\Delta$ ,  $\alpha$ , and  $\beta$  determined in this way:

$$\frac{\Delta s_+}{s_+} = \frac{(1 - \alpha - \beta)\gamma + \alpha\Delta}{1 - \gamma}; \quad (23)$$

$$\frac{\Delta s_-}{s_-} = -\frac{1 - \alpha - \beta + \alpha\gamma\Delta}{1 - \gamma}; \quad (24)$$

$$\frac{s_+}{s_-} = \gamma = \xi + (\xi^2 - 1)^{1/2}, \quad (25)$$

where

$$\xi = 1 - (1 - \alpha - \beta + \alpha\Delta)^2 / 2\Delta(\alpha + \beta)(1 - \alpha). \quad (26)$$

Thus, while the experimental  $R(H)$  and  $M(H)$  curves determine the correlation behavior  $\delta(\mu^2)$ , so that the function  $\delta(\eta)$  has finite derivatives  $d_0 \equiv d\delta/d\eta|_{\eta=0}$  and  $d_1 \equiv d\delta/d\eta|_{\eta=1}$ , the  $\delta(\eta)$  curve found experimentally can be approximated by the theoretical curve of Eq. (15) characterized by the same values of  $d_0$  and  $d_1$  and uniquely related to these parameters

$$\alpha = d_0, \quad \beta = (d_0 d_1 - 1) / (1 - d_1); \quad (27)$$

the experimental and theoretical curves then meet asymptotically for  $\eta=0$  and  $\eta=1$ .

If it turns out that  $|\beta| \ll \alpha$ , then one can try to extract information on one or the other set of parameters either according to Eqs. (19) and (20) or from Eqs. (23)–(25). If the parameters  $\alpha$  and  $\beta$  are of the same order of magnitude, then it can be asserted that either both of the GMR mechanisms indicated above provide a comparable contribution to the effect or that the influence of the ‘‘mixing’’ mechanism can be neglected. In the first case it is impossible to solve uniquely for their magnitude without drawing on independent data, because of the large number of unknown parameters. In the latter case an attempt can be made to verify the correctness of the proposal by using Eqs. (23)–(25) to determine  $\Delta s_\pm / s_\pm$  and  $s_+ / s_-$ .

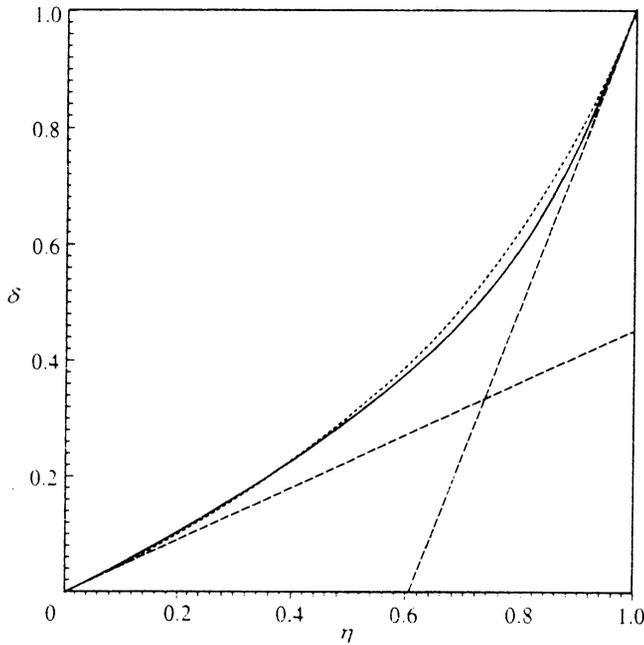


FIG. 2. Correlation behavior of the relative magnetoresistance  $\delta$  and the square of the relative magnetization  $\mu^2 = \eta$ : the solid curve is based on the experimental data,<sup>1</sup> the dashed curve on Eq. (15). The dashed lines determine values of the derivative of  $\delta(\eta)$  at  $\eta=0$  and  $\eta=1$ . Numerical values of the parameters are given in the text.

## 6. EXPERIMENTAL RESULTS

We illustrate the possibility of a practical implementation of the proposed procedure for analyzing experimental results with an example of its application to GMR in the superlattice (Fe 30 Å/Cr 12 Å)<sub>35</sub> (Ref. 1). The observed value of the effect is  $\Delta = -0.33$ . The plot of  $\delta(\mu^2)$  on the basis of the  $R(H)$  and  $M(H)$  curves given by Baibich *et al.*<sup>1</sup> is shown in Fig. 2 (solid curve). This function has finite derivatives  $d_0 = 0.45$  and  $d_1 = 2.5$  at  $\eta = 0$  and  $\eta = 1$ . For the theoretical curve (dashed) shown in Fig. 2 and constructed according to Eq. (15), the parameters  $\alpha$  and  $\beta$  are chosen according to Eq. (27):  $\alpha = 0.45$  and  $\beta = -0.08$ . If we assume the "mixing" mechanism to be dominant, then Eqs. (19) and (20) yield  $tl_+^s/L \approx 0.37$ ,  $tl_-^s/L \approx 1.95$ , and  $l_-^s/l_+^s \approx 5.39$ . The value obtained for the ratio of effective mean free paths is close to the known result for the ratio of scattering cross-sections of electrons with different spin on chromium impurities in a matrix of bulk iron.

If, however, we assume that the change in diffusion coefficient of interfaces is the main GMR mechanism, then analysis of the results using Eqs. (23)–(25) yields

$\Delta s_+/s_{0+} \approx -0.75$ ,  $\Delta s_-/s_{0-} \approx -0.04$ , and  $\gamma = s_+/s_- \approx 5.34$ . The result for  $\gamma$  is extremely close to that obtained above for the "mixing" mechanism. A choice in favor of one or the other mechanism can be made if there are values of the transmission coefficient  $t$  or absolute values of the mean free path  $l_{\pm}$  from independent measurements. It is important to stress that both forms of analysis of the experimental results give essentially identical results for the ratio  $\gamma$  without using any unsubstantiated assumptions about the relation between the coefficients of coherent transmission  $t$  and the scattering  $s_{\pm}$ .

## 7. CONCLUSIONS

Analysis of the giant magnetoresistance effect by the scheme described above, using a simple model of a magnetic superlattice, shows that a study of the correlation properties of magnetoresistance and magnetization can be an effective method for obtaining information about the parameters that determine the magnitude of GMR effects. In particular, its application to describing experimental measurement of the GMR effect in regular superlattice structures enables the changes observed to be interpreted in terms of the corresponding microscopic parameters. In view of its relative simplicity, the proposed scheme can be used to analyze features of other transport properties of giant magnetoresistance superlattices. The description, in a single model, of the behavior of different transport properties enables the contribution of different GMR mechanisms to be uniquely determined, and the corresponding microscopic parameters to be obtained.

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<sup>1</sup>M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas, *Phys. Rev. Lett.* **61**, 2472 (1988).

<sup>2</sup>R. E. Camley and R. L. Stamps, *J. Phys. Condens. Matter* **5**, 3727 (1993).

<sup>3</sup>*Ultrathin Magnetic Structure*, B. Heinrich and A. Bland (eds.), Springer-Verlag New York (1992).

<sup>4</sup>V. V. Ustinov, *Sov. Sci. Rev.: A. Phys. Rev.*, I. M. Khalatnikov (ed.), Harwood Academic, New York (1986), Vol. 7, 227.

<sup>5</sup>V. V. Ustinov and D. Z. Khusainov, *Phys. Status Solidi B134*, 313 (1986).

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