The interference effect in the scattering of an electronby a nucleus in the field of two plane electromagnetic waves

S. P. Roshchupkin

Institute of Applied Physics, Ukrainian Academy of Sciences, 244007 Sumy, Ukraine
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This paper presents a theoretical study of stimulated bremsstrahlung and absorption by an electron scattered by a nucleus in the field of two plane waves of arbitrary intensities and frequencies under conditions such that the Bunkin–Fedorov quantum parameters of multiphoton $\gamma_{1,2}$ are small and multiphoton processes are determined by the quantum parameter $\beta_{1,2}$ and the quantum interference parameter $\alpha_{\pm}$. This occurs when the electron is scattered in the plane perpendicular to the wave polarization vector (for the case of equal linear polarizations of the two waves) and in the plane of the initial electron momentum and the wave vector (for the case of elliptical polarizations of the two waves). The effect of wave interference is found to be great, which leads to correlated emission and absorption by the electron of equal numbers of photons of the two waves. When there is no interference ($\alpha_{\pm} \ll 1$), the electron independently emits and absorbs an even number of photons of the first and second waves in the process of being scattered by the nucleus. This paper obtains the cross sections for these processes and shows that these may considerably exceed the scattering cross section in any other geometry.

1. INTRODUCTION

Stimulated bremsstrahlung and absorption are well-known phenomena that manifest themselves in the scattering of an electron by a nucleus in the field of a single plane electromagnetic wave. The groundwork was laid by Bunkin and Fedorov[1] and by Denisov and Fedorov[2] (see also Refs. 3 and 4 and Fedorov's monograph[5]). Karapetyan and Fedorov[6] studied stimulated bremsstrahlung and absorption by electrons in the process of electron scattering by a potential in the presence of two plane electromagnetic waves in the nonrelativistic limit. Their analysis was done in the dipole approximation in the interaction of electrons with the electric fields of both waves. More recently stimulated bremsstrahlung and absorption was studied in the general relativistic case for an electron scattered by a nucleus in the field of two plane waves propagating in the same direction.[7] However, the study was restricted to the kinematic region where the main multiphoton parameter is the Bunkin–Fedorov quantum parameter $\gamma_{1,2}$ [see Eq. (17)].

This paper treats the given problem a kinematic region where the Bunkin–Fedorov quantum parameter is small and processes related to wave interference have a significant effect on the scattering process. As will shortly be seen, the main multiphoton parameters are the quantum parameters $\beta_{1,2}$ and $\alpha_{\pm}$ [see Eqs. (19) and (20)], and the probability of such a process can be considerably higher than that of the previous process. Below we use the relativistic system of units, $\hbar=c=1$.

2. PROBABILITY OF STIMULATED BREMSSTRAHLUNG AND ABSORPTION BY AN ELECTRON SCATTERED BY A NUCLEUS

We select the 4-vector potential of the external field in the form of the sum of two plane electromagnetic waves propagating parallel to the z axis:

$$A=A_1(\varphi_1)+A_2(\varphi_2), \tag{1}$$

where

$$A_j(\varphi_j)=\frac{F_i}{\omega_j}(e_{jx}\cos\varphi_j+\delta_j e_{jy}\sin\varphi_j). \tag{2}$$

Here δ_j is the ellipticity parameter, $e_{jx}=(0,\mathbf{e}_{jx})$ and $e_{jy}=(0,\mathbf{e}_{jy})$ are the 4-vectors of wave polarization, F_j and ω_j are the field strength and frequency of the first ($j=1$) and second ($j=2$) waves, and the argument φ_j has the form

$$\varphi_j=\omega_j(t-z),\quad j=1,2. \tag{3}$$

An expression for the probability of stimulated bremsstrahlung and absorption by an electron scattered by a nucleus Ze in the field of two plane electromagnetic waves propagating in the same direction [Eqs. (1) and (2)] for arbitrary intensities and frequencies was derived in Ref. 7. Here we write this expression in a form more convenient for analysis:

$$dW_{\mathrm{fi}}=\sum_{l,s=-\infty}^{\infty} dW_{\mathrm{fi}}^{(ls)}, \tag{4}$$

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where the partial probability of emission ($l,s>0$) or absorption ($l,s<0$) of $|l|$ photons of the first wave and $|s|$ photons of the second is specified as

$$dW_{\text{fi}}^{(ls)}=\frac{2(Ze^2)^2}{\widetilde{E}_{\text{i}}\widetilde{E}_{\text{f}}} H_{\text{fi}}^{(ls)}\delta(\widetilde{E}_{\text{f}}-\widetilde{E}_{\text{i}}+l\omega_1+s\omega_2)\frac{d^3p_{\text{f}}}{\mathbf{q}^4}. \tag{5}$$

Here

$$H_{\text{fi}}^{(ls)}=(m^2+E_{\text{i}}E_{\text{f}}+\mathbf{p}_{\text{i}}\mathbf{p}_{\text{f}})|I_{ls}|^2+\frac{m^4}{32\kappa_{\text{i}}\kappa_{\text{f}}}|B_{ls}|^2 +\frac{m^2}{4}f_1|\mathbf{D}_{ls}|^2+\frac{m^2}{2}\text{Re}\Big\{f_2I_{ls}^*B_{ls} +\frac{1}{\kappa_{\text{i}}\kappa_{\text{f}}}(\mathbf{p}_{\text{f}}\mathbf{D}_{ls})(\mathbf{p}_{\text{i}}\mathbf{D}_{ls}^*)-B_{ls}^*(\mathbf{f}_3\mathbf{D}_{ls}) -I_{ls}^*(\mathbf{f}_4\mathbf{D}_{ls})\Big\}. \tag{6}$$

The scalar functions f_1 and f_2, the vector functions $\mathbf{f}_3$ and $\mathbf{f}_4$, the 4-vector $D_{ls}=(0,\mathbf{D}_{ls})$ defined in terms of the 4-vectors of polarization of both waves, and the complex-valued functions B_{ls} have the following form:

$$f_1=\frac{1}{\kappa_{\text{i}}\kappa_{\text{f}}}(m^2+E_{\text{f}}\kappa_{\text{i}}-E_{\text{i}}\kappa_{\text{f}}-E_{\text{i}}E_{\text{f}}+\mathbf{p}_{\text{i}}\mathbf{p}_{\text{f}}) -\frac{1}{\kappa_{\text{i}}^2}\left[E_{\text{f}}\kappa_{\text{i}}-E_{\text{i}}\kappa_{\text{f}}+\frac{(\kappa_{\text{f}}-\kappa_{\text{i}})^2}{2\kappa_{\text{f}}}(E_{\text{i}}+\mathbf{np}_{\text{i}})\right], \tag{7}$$

$$f_2=\frac{1}{2\kappa_{\text{i}}\kappa_{\text{f}}}(m^2-E_{\text{i}}E_{\text{f}}+\mathbf{p}_{\text{i}}\mathbf{p}_{\text{f}}+E_{\text{i}}\kappa_{\text{f}}+E_{\text{f}}\kappa_{\text{i}}),$$

$$\mathbf{f}_3=\frac{m(\mathbf{p}_{\text{i}}+\mathbf{p}_{\text{f}})}{4\kappa_{\text{i}}\kappa_{\text{f}}}, \tag{8}$$

$$\mathbf{f}_4=\frac{\mathbf{p}_{\text{f}}}{m}\left[\frac{2E_{\text{i}}}{\kappa_{\text{i}}}+\left(\frac{1}{\kappa_{\text{f}}}-\frac{1}{\kappa_{\text{f}}}\right)(E_{\text{i}}+\mathbf{np}_{\text{i}})\right]+\frac{\mathbf{p}_{\text{i}}}{m}\left(1+\frac{2E_{\text{f}}-\kappa_{\text{f}}}{\kappa_{\text{i}}}\right), \tag{9}$$

$$\mathbf{D}_{ls}=\eta_1(e_1I_{l-1,s}+e_1^*I_{l+1,s})+\eta_2(e_2I_{l,\,s-1}+e_2^*I_{l,s+1}), \tag{10}$$

$$e_j=e_{jx}+i\delta_je_{jy},\ j=1,2, \tag{11}$$

$$B_{ls}=\eta_1^2[2(1+\delta_1^2)I_{ls}+(1-\delta_1^2)(I_{l+2,s}+I_{l-2,s})] +\eta_2^2[2(1+\delta_2^2)I_{ls}+(1-\delta_2^2)(I_{l,s+2}+I_{l,\,s-2})] +2\eta_1\eta_2(d_-I_{l-1,s-1}+d_-^*I_{l+1,s+1}+d_+I_{l-1,s+1} +d_+{}^*I_{l+1,s-1}), \tag{12}$$

$$d_\pm=(1\pm\delta_1\delta_2)\cos\Delta+i(\delta_1\pm\delta_2)\sin\Delta. \tag{13}$$

Here Δ is the angle between the polarization vectors $\mathbf{e}_{1x}$ and $\mathbf{e}_{2x}$ of the two waves, $n=(1,\mathbf{n})$ ($\mathbf{n}$ is the unit vector specifying the direction of propagation of the waves), $\kappa_j=E_j-\mathbf{np}_j$, η_1 and η_2 are the classical relativistic-invariant parameters of the first and second waves,

$$\eta_{1,2}=\frac{eF_{1,2}}{m\omega_{1,2}}, \tag{14}$$

and the functions I_{ls} depending on eight parameters (actually on ten, but two parameters, $\tau_\pm$, which affect only the phase factors in the argument, are not included) and expanded in a series of Bessel functions J_r of integral order have the form

$$I_{ls}(\chi_1,\gamma,\beta_1;\chi_2,\gamma_2,\beta_2;\alpha_+,\alpha_-)=\sum_{r,r'=-\infty}^{\infty}\exp[-i(r\tau_- +r'\tau_+)]J_r(\alpha_+)J_{r'}(\alpha_-) \times L_{l-r'-r}(\chi_1,\gamma_1,\beta_1)L_{s+r'-r}(\chi_2,\gamma_2,\beta_2), \tag{15}$$

with the functions $L_{l-r'-r}$ and $L_{s+r'-r}$ determined by the parameters of the first and second waves, respectively, and describing multiphoton processes in the field of one wave:

$$L_r(\chi,\gamma,\beta) =\exp(-ir\chi)\sum_{s'=-\infty}^{\infty}\exp(2is'\chi)J_{r-2s'}(\gamma)J_{s'}(\beta). \tag{16}$$

The parameters that determine the functions I_{ls} are

$$\gamma_j=\eta_j\frac{m}{\omega_j}\sqrt{(e_{jx}g_{\text{fi}})^2+\delta_j^2(e_{jy}g_{\text{fi}})^2}, \tag{17}$$

$$\tan\chi_j=\delta_j\frac{e_{jy}g_{\text{fi}}}{e_{jx}g_{\text{fi}}},\quad g_{\text{fi}}=\frac{p_{\text{f}}}{\kappa_{\text{f}}}-\frac{p_{\text{i}}}{\kappa_{\text{i}}}, \tag{18}$$

$$\beta_j=(1-\delta_j^2)\eta_j^2\frac{m^2}{8\omega_j}\left[\frac{1}{\kappa_{\text{f}}}-\frac{1}{\kappa_{\text{i}}}\right],\ j=1,2, \tag{19}$$

$$\alpha_\pm=\eta_1\eta_2\frac{m^2|d_\pm|}{2(\omega_1\pm\omega_2)}\left[\frac{1}{\kappa_{\text{f}}}-\frac{1}{\kappa_{\text{i}}}\right], \tag{20}$$

$$\tan\tau_\pm=\frac{\text{Im}\,d_\pm}{\text{Re}\,d_\pm}=\frac{\delta_1\pm\delta_2}{1\pm\delta_1\,\delta_2}\tan\,\Delta. \tag{21}$$

Note that $\gamma_{1,2}$ specified by Eq. (17) represents the well-known Bunkin–Fedorov quantum parameter of multiphoton. In Eq. (5) the 4-vector $q=(q_0,\mathbf{q})$ and the 4-momenta $\widetilde{p}_j=(\widetilde{E}_j,\widetilde{\mathbf{p}}_j)$ before ($j=\text{i}$) and after ($j=\text{f}$) scattering are given by the following formulas:

$$q=\widetilde{p}_{\text{f}}-\widetilde{p}_{\text{i}}+(l\omega_1+s\omega_2)n, \tag{22}$$

$$\widetilde{p}_j=p_j+\frac{m^2}{4\kappa_j}[(1+\delta_1^2)\eta_1^2+(1+\delta_2^2)\eta_2^2]n. \tag{23}$$

The expressions (4)–(5) for the probability are valid for arbitrary values of intensities and frequencies of both waves and for electron velocities $v_{\text{i,f}}\gg Z/137$. It can easily be demonstrated that if one wave is switched off (say, $F_2=0$), Eqs. (4)–(6) determine the probability of an electron being scattered by a nucleus in the field of one wave,[2] and if both waves are switched off ($F_1=F_2=0$), they transform into the ordinary Mott probability of an electron being scattered by a nucleus.[10]

Note that if the waves have equal frequencies ($\omega_1=\omega_2$) and the same polarizations ($\delta_1=\delta_2$), Eqs. (4)

and (5) must transform into the expressions for the probabilities of an electron being scattered by a nucleus in the field of one wave whose field strength F and polarization vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ are linked to the initial parameters of the waves through the following relations:

$$F=\sqrt{F_1^2+F_2^2+2F_1F_2\cos\Delta}, \tag{24}$$

$$\mathbf{e}_x=\frac{1}{F}(F_1\mathbf{e}_{1x}+F_2\mathbf{e}_{2x}),\quad \mathbf{e}_y=\frac{1}{F}(F_1\mathbf{e}_{1y}+F_2\mathbf{e}_{2y}). \tag{25}$$

Hence we assume that the frequencies of the waves differ by $\Delta\omega=\omega_2-\omega_2$. Then the energy conservation law [the argument of the delta function in Eq. (5)] assumes the form

$$\tilde{E}_f-\tilde{E}_i+\left[l+s\left(1+\frac{\Delta\omega}{\omega_1}\right)\right]\omega_1=0. \tag{26}$$

This shows that if

$$\frac{|\Delta\omega|}{\omega_1}\ll 1, \tag{27}$$

this term in (26) can be ignored. Then we can introduce a new photon number $l'=l+s$ and sum the probability specified by Eqs. (4) and (5) over all the values of s. In doing this we use the easily verifiable relations

$$\sum_{s=-\infty}^{\infty} L_{r-s}(\chi_1,\gamma_1,\beta_1)L_s(\chi_2,\gamma_2,\beta_2)=L_r(\chi,\gamma,\tilde{\beta}), \tag{28}$$

$$\sum_{s=-\infty}^{\infty} J_s(\alpha)L_{r-2s}(\chi,\gamma,\tilde{\beta})=L_r(\chi,\gamma,\tilde{\beta}+\alpha). \tag{29}$$

Here we have introduced the notation

$$\tan\chi=\frac{\gamma_1\sin\chi_1+\gamma_2\sin\chi_2}{\gamma_1\cos\chi_1+\gamma_2\cos\chi_2}, \tag{30}$$

$$\gamma=\sqrt{\gamma_1^2+\gamma_2^2+2\gamma_1\gamma_2\cos(\chi_1-\chi_2)},\quad \tilde{\beta}=\beta_1+\beta_2. \tag{31}$$

Allowing for all this, we can easily find that

$$\sum_{s=-\infty}^{\infty} I_{l'-s,s}=\exp(-i\alpha_-\sin\tau_+)L_{l'}(\chi,\gamma,\beta), \tag{32}$$

where

$$\beta=\beta_1+\beta_2+\alpha_+ =(1-\delta_1^2)\frac{m^2}{8\omega_1}\left(\frac{1}{\kappa_f}-\frac{1}{\kappa_i}\right)(\eta_1^2+\eta_2^2+2\eta_1\eta_2\cos\Delta). \tag{33}$$

Thus, in accordance with condition (27) we have the well-studied process of the scattering of an electron by a nucleus in the field of a plane wave.[2] Hence in what follows we assume that the frequencies of the waves are not close, that is, we assume the condition opposite to (27):

$$\frac{|\Delta\omega|}{\omega_1}\gtrsim 1. \tag{34}$$

In Eqs. (4)–(6) the I_{ls} are functions determining the multiphoton processes. Here, as Eq. (15) shows, for a given number of absorbed and emitted photons of the waves (for given values of l and s) there are virtual processes with correlated absorption and emission of an equal number of photons of both waves, $(\omega_1+\omega_2)r$ and $(\omega_1-\omega_2)r'$, with the intensity of the virtual processes determined by the quantum interference parameters $\alpha_\pm$ [Eq. (20)]. If $\alpha_\pm\gtrsim 1$ holds, the processes with correlated emission and absorption of one or several photons of both waves provide the same contribution (in order of magnitude) to the sum in Eq. (15). At the same time, when the interference parameters are small ($\alpha_\pm\ll 1$), such processes can be ignored ($r=r'=0$) and the functions I_{ls} [Eq. (15)] separate into products of functions determining the independent emission and absorption of photons of the first and second waves:

$$I_{ls}(\chi_1,\gamma_1,\beta_1;\chi_2,\gamma_2,\beta_2;0,0)=L_l(\chi_1,\gamma_1,\beta_1)L_s(\chi_2,\gamma_2,\beta_2). \tag{35}$$

The form of the functions I_{ls} strongly depends on the polarization of the waves. These functions are written below explicitly for circular and linear polarizations. When both waves are circularly polarized, $\delta_1^2=\delta_2^2=1$. In view of this we have $\beta_1=\beta_2=0$ [see Eq. (19)]. If we combine this result with (16), the functions I_{ls} of Eq. (15) assume the following form:

$$I_{ls}(\chi_1,\gamma_1,0;\chi_2,\gamma_2,0;\alpha_\pm) =\exp[-i(l\chi_1+s\chi_2)]\sum_{r=-\infty}^{\infty}\exp[i(\chi_1-\Delta \pm\chi_2)r]J_r(\alpha_\pm)J_{l-r}(\gamma_1)J_{s\mp r}(\gamma_2). \tag{36}$$

Here we have introduced the notation

$$\alpha_\pm=\eta_1\eta_2\frac{m^2}{\omega_1\pm\omega_2}\left(\frac{1}{\kappa_f}-\frac{1}{\kappa_i}\right), \tag{37}$$

$$\gamma_j=\eta_j\frac{m}{\omega_j}|\mathbf{g}_{fi\|}|,\quad \chi_j=\angle(\mathbf{e}_{jx},\mathbf{g}_{fi\|}),\quad j=1,2. \tag{38}$$

In Eqs. (36)–(38), $\mathbf{g}_{fi\|}$ is the component of $\mathbf{g}_{fi}$ (Eq. (18)) parallel to the polarization plane of the waves; the lower sign in the formulas corresponds to the case where electric field vectors rotate in the same direction (counterclockwise, $\delta_1=\delta_2=1$), and the upper sign to the case where they rotate in opposite directions ($\delta_1=1$ and $\delta_2=-1$). Equations (36)–(38) show that for a given number of emitted (absorbed) photons of both waves (for definite values of l and s) there are virtual processes with correlated absorption and emission of an equal number of photons of both waves, $(\omega_1+\omega_2)r$ or $(\omega_1-\omega_2)r$. For $\alpha_\pm\ll 1$ we arrive at (35), that is,

$$I_{ls}(\chi_1,\gamma_1,0;\chi_2,\gamma_2,0;0)=\exp[-i(l\chi_1 +s\chi_2)]J_l(\gamma_1)J_s(\gamma_2). \tag{39}$$

When both waves are linearly polarized, $\delta_1=\delta_2=0$. Hence $\chi_{1,2}=\tau_\pm=0$ and $d_\pm=\cos\Delta$ [see Eqs. (13), (17), and (20)]. If we allow for (16), Eq. (14) assumes the form

$$I_{ls}(0,\gamma_1,\beta_1;0,\gamma_2,\beta_2;\alpha_+,\alpha_-)=\sum_{r,r'=-\infty}^{\infty} J_r(\alpha_+)J_{r'}(\alpha_-)$$
$$\times J_{l-r'-r}(\gamma_1,\beta_1)J_{s+r'-r}(\gamma_2,\beta_2). \tag{40}$$

Here the $J_r(\gamma,\beta)$ are generalized Bessel functions, studied in detail by Reiss:[11]

$$J_r(\gamma,\beta)=L_r(0,\gamma,\beta)=\sum_{s'=-\infty}^{\infty} J_{r-2s'}(\gamma)J_{s'}(\beta), \tag{41}$$

$$\gamma_j=\eta_j\frac{m}{\omega_j}|\mathbf{e}_{jx}\mathbf{g}_{\mathrm{f}}|, \tag{42}$$

$$\beta_j=\eta_j^2\frac{m^2}{8\omega_j}\left(\frac{1}{\kappa_{\mathrm{f}}}-\frac{1}{\kappa_{\mathrm{i}}}\right),\quad j=1,2, \tag{43}$$

$$\alpha_\pm=\eta_1\eta_2\frac{|\cos\Delta|m^2}{2(\omega_1\pm\omega_2)}\left(\frac{1}{\kappa_{\mathrm{f}}}-\frac{1}{\kappa_{\mathrm{i}}}\right). \tag{44}$$

Note that the parameters $\alpha_\pm$ specified by Eq. (44) strongly depend on the angle Δ. In a narrow cone near $\Delta=\pi/2$, the interference parameters satisfy the condition $\alpha_\pm\ll1$, and for arbitrary wave intensities we can ignore virtual processes with correlated emission and absorption of an equal number of photons of both waves. Then the I_{ls} [Eq. (15)] assume the form (35), that is,

$$I_{ls}(0,\gamma_1,\beta_1;0,\gamma_2,\beta_2;0,0)=J_l(\gamma_1,\beta_1)J_s(\gamma_2,\beta_2). \tag{45}$$

Note also that the functions I_{ls} assume the form (45) for arbitrary angles Δ in the case of low intensities, when $\alpha_\pm\ll1$.

It would also be interesting to establish the regions in which the Bunkin–Fedorov quantum parameters are small and the quantum parameters $\beta_{1,2}$ (for elliptically polarized waves, excluding circularly polarized) and the quantum interference parameters $\alpha_\pm$ (for circularly polarized waves) serve as the multiphoton parameters. Clearly, such a situation occurs for the general case where both waves are elliptically polarized when as a result of scattering of the electron the vector $\mathbf{g}$ [Eq. (18)] is directed along the wave vector, that is, when

$$g_{\mathrm{fi}}\,e_j=0,\quad j=1,2. \tag{46}$$

Obviously, this is true only if the electron is scattered in the plane formed by the initial electron momentum and the wave vector. Note also that condition (46) is met only when both waves are linearly polarized and $\Delta=0$ in the scattering of electrons in the plane perpendicular to the polarization vector of the waves. In view of (46), $\chi_1=\chi_2=\gamma_1=\gamma_2=0$ [see Eqs. (17) and (18)]. By allowing for $L_\nu(0,0,\beta)=J_{\nu'}(\beta)\delta_{\nu,2\nu'}$ the functions I_{ls} [Eq. (15)] can be reduced to the following form:

$$J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)=I_{ls}(0,0,\beta_1;0,0,\beta_2;\alpha_+,\alpha_-)$$
$$=\sum_{r,r'=-\infty}^{\infty}\exp[-i(r\tau_-+r'\tau_+)]J_r(\alpha_+)J_{r'}(\alpha_-)J_{\nu'}(\beta_1)J_{\mu'}(\beta_2)\delta_{\nu,2\nu'}\delta_{\mu,2\mu'}. \tag{47}$$

Here $\nu=l-r'-r$, $\mu=s+r'-r$, and $\delta_{\nu,\mu}$ is the Kronecker symbol. The formula simplifies considerably when both fields are circularly polarized:

$$J_{l,\pm l}(0,0,\alpha_\pm)=\exp(-il\Delta)J_l(\alpha_\pm). \tag{48}$$

Here the lower and upper signs correspond to the same ($\delta_1=\delta_2=1$) or different circular polarizations of the waves ($\delta_1=-\delta_2=1$) and, in accordance with this, to correlated emission (absorption) of an equal number of photons of both waves: $(\omega_1-\omega_2)l$ and $(\omega_1+\omega_2)l$, respectively. Equations (47) and (48) show that the Bunkin–Fedorov quantum parameters $\gamma_{1,2}$ do not affect the scattering of electrons at all in the region specified by (46). Multiphoton stimulated bremsstrahlung and absorption by an electron scattered by a nucleus are determined in this case by the quantum parameters $\beta_{1,2}$ or the interference parameters $\alpha_\pm$ with correlated absorption and emission of an equal number of photons of both waves.

In what follows we assume that the wave frequencies obey the conditions $\omega_1>\omega_2$ and

$$\omega_{1,2}\ll\begin{cases} m, & E_{\mathrm{i}}\gg m,\\ mv_{\mathrm{i}}^2/2, & v_{\mathrm{i}}\ll1.\end{cases} \tag{49}$$

In view of what has been said, it is natural to isolate two kinematic regions of electron scattering: one where condition (46) is not met in scattering and which we call the Bunkin–Fedorov region, and the other where this condition is met. Clearly, in the Bunkin–Fedorov region the angle φ between the scattering plane and the polarization vector (when both waves are linearly polarized and $\Delta=0$) is not close to $\pi/2$ and the angle ψ between vector $\mathbf{g}$ (Eq. (18)) and the direction of propagation of both waves (for elliptically polarized waves) is not small:

$$\left|\varphi-\frac{\pi}{2}\right|\gtrsim\frac{\omega_{1,2}}{mv_{\mathrm{i}}^2}\frac{v_{\mathrm{i}}}{\eta_{1,2}},\quad \psi\gtrsim\frac{\omega_{1,2}}{mv_{\mathrm{i}}^2}\frac{v_{\mathrm{i}}}{\eta_{1,2}}, \tag{50}$$

(here $v_{\mathrm{i}}=p_{\mathrm{i}}/E_{\mathrm{i}}$). In the region where condition (46) is met we have the opposite inequalities:

$$\left|\varphi-\frac{\pi}{2}\right|\ll\frac{\omega_{1,2}}{mv_{\mathrm{i}}^2}\frac{v_{\mathrm{i}}}{\eta_{1,2}},\quad \psi\ll\frac{\omega_{1,2}}{mv_{\mathrm{i}}^2}\frac{v_{\mathrm{i}}}{\eta_{1,2}}. \tag{51}$$

Note that the expressions on the right-hand sides of (50) and (51) are small, since in the opposite case the Bunkin–Fedorov parameters are small. In the region (50) the Bunkin–Fedorov quantum parameters $\gamma_{1,2}$ are the main multiphoton parameters. Stimulated bremsstrahlung and absorption by an electron scattered by a nucleus in this region are studied in detail in Ref. 7. Here we consider stimulated bremsstrahlung and absorption by an electron scattered by a nucleus in the region where conditions (51) hold, with the quantum parameters $\beta_{1,2}$ and $\alpha_\pm$ being the multiphoton parameters.

In the scattering region (50) the relation between the Bunkin–Fedorov quantum parameters $\beta_{1,2}$ and $\alpha_\pm$ can be written as follows:

$$\gamma_{1,2}\sim\alpha_\pm\frac{E_i}{m}\frac{1}{\eta_{2,1}}=\begin{cases}\alpha_\pm/\eta_{2,1} & v_i\ll 1,\\ \alpha_\pm/\xi_{2,1}, & E_i\gg m,\end{cases}$$

$$\alpha_\pm\sim\beta_{1,2}\,(\eta_{2,1}/\eta_{1,2}), \tag{52}$$

where ξ is a parameter (see Sec. 3). This shows that for nonrelativistic and relativistic electron energies (when $\eta_{1,2}\ll 1$) and for ultrarelativistic electron energies [when $\xi_{1,2}\ll 1$; see Eq. (55)] we have

$$\gamma_{1,2}\gg\alpha_\pm\gtrsim 1,\quad \gamma_{1,2}\gg\beta_{1,2}\gtrsim 1. \tag{53}$$

Now we can easily show (see Ref.12) that

$$\frac{|I_{ls}(\chi_1,\gamma_1,\beta_1;\chi_2,\gamma_2,\beta_2;\alpha_+,\alpha_-)|^2}{|J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)|^2}\sim\begin{cases}(\gamma_1\gamma_2)^{-1}\ll 1 & l\ll\gamma_1,\quad s\ll\gamma_2,\\ (\gamma_1\gamma_2)^{-2/3}\ll 1, & l\sim\gamma_1,\quad s\sim\gamma_2.\end{cases} \tag{54}$$

Comparison of the probabilities of multiphoton stimulated bremsstrahlung and absorption in the electron-scattering regions (50) and (51) suggests that their ratio is equal, in order of magnitude, to the ratio of functions in (54) and, hence when conditions (53) are met, the probability of multiphoton stimulated bremsstrahlung and absorption by an electron scattered by a nucleus is much higher for scattering in region (51) than in the Bunkin–Fedorov region (50).

3. ELECTRON SCATTERING IN THE PLANE PERPENDICULAR TO THE POLARIZATION VECTOR

Let us assume that both waves are linearly polarized, with $\mathbf{e}_{1x}=\mathbf{e}_{2x}\equiv\mathbf{e}_x$, and that the electron is scattered by the nucleus in the plane perpendicular to the polarization vector (the first condition in (51) is met). Here the main processes are emission and absorption of $(l\lesssim\beta_1)$-photons of the first wave and $(s\lesssim\beta_2)$-photons of the second (the interference parameters $\alpha_\pm$ determine the virtual processes of correlated emission and absorption of an equal number of photons of both waves). Hence the fraction of the energy that the electron emits or absorbs from the first and second waves, respectively, is, in order of magnitude, $|l|\omega_1/E_i\lesssim\zeta_1$ and $|s|\omega_2/E_i\lesssim\zeta_2$ (in the nonrelativistic limit, $|l|\omega_1/mv_i^2\lesssim\zeta_1$ and $|s|\omega_2/mv_i^2\lesssim\zeta_2$), where

$$\zeta_{1,2}=\xi_{1,2}^2\frac{p_i}{E_i},\quad \xi_{1,2}=\eta_{1,2}\frac{m}{p_i}. \tag{55}$$

Here $\xi_{1,2}$ and $\zeta_{1,2}$ are the classical parameters determining the integral characteristics of the process in the kinematic regions (50) and (51), respectively.[7-9] When conditions (51) hold, the expression for $H_{fi}^{(ls)}$ of (6) in probability (5) simplifies considerably and assumes the following form:

$$H_{fi}^{(ls)}=(m^2+E_iE_f+\mathbf{p}_i\mathbf{p}_f)J_{ls}^2+\frac{m^4}{32\kappa_i\kappa_f}B_{ls}^2-\frac{m^2}{4}f_1\mathbf{D}_{ls}^2+\tfrac{1}{2}f_2m^2J_{ls}B_{ls}. \tag{56}$$

Here the arguments $\beta_{1,2}$ and $\alpha_\pm$ of the functions B_{ls} and J_{ls} are given by Eqs. (43) and (44) and the functions assume the form

$$B_{ls}=2(\eta_1^2+\eta_2^2)J_{ls}+\eta_1^2(J_{l+2,s}+J_{l-2,s})+\eta_2^2(J_{l,s+2}+J_{l,s-2})+2\eta_1\eta_2(J_{l-1,s-1}+J_{l+1,s+1}+J_{l-1,s+1}+J_{l+1,s-1}), \tag{57}$$

$$\mathbf{D}_{ls}=[\eta_1(J_{l-1,s}+J_{l+1,s})+\eta_2(J_{l,s-1}+J_{l,s+1})]\mathbf{e}_x. \tag{58}$$

Conditions (55) clearly show that in the general relativistic case for $\zeta_{1,2}\ll 1$, and also nonrelativistic electrons with $\zeta_{1,2}\gtrsim 1$, conditions (53) are met, that is, the probability of an electron being scattered by a nucleus in the plane perpendicular to the polarization vector of the wave is considerably higher than the probability of scattering in the Bunkin–Fedorov region. We therefore study these cases in greater detail.

Suppose that the external fields are moderately strong: $\zeta_{1,2}\ll 1$. Then the product of the intensities of the waves as a function of the electron energy satisfies the following conditions:

$$\eta_{1,2}^2\ll\begin{cases}v_i, & v_i\ll 1,\\ 1, & E_i\sim m,\\ (E_i/m)^2, & E_i\gg m.\end{cases} \tag{59}$$

In view of these conditions, we can ignore the photon energy in comparison to the electron energy: $|l|/\omega_1/E_i\ll 1$ and $|s|/\omega_2/E_i\ll 1$. Then $\tilde{p}_{i,f}\approx p_{i,f}$, and the law of energy conservation assumes the form $E_f=E_i$. After integrating with respect to the energy of the final electrons and dividing by the flux of the initial electrons, the partial cross section of stimulated bremsstrahlung and absorption assumes the following form:

$$\frac{d\sigma^{(ls)}}{d\Omega}=J_{ls}^2(\beta_1,\beta_2,\alpha_+,\alpha_-)\frac{d\sigma_M}{d\Omega}. \tag{60}$$

We see that the partial cross section can be written as the product of the Mott cross section by the probability of emission and absorption of photons of both waves. As noted earlier, in conditions (59) the cross section (60) considerably exceeds the cross section in the Bunkin–Fedorov region.[7] It must be emphasized that this result is also true for an electron scattered by a nucleus in the region (51) in the field of a single wave ($F_2=0$). Indeed, in this case Eq. (60) yields

$$\frac{d\sigma^{(l)}}{d\Omega}=J_{l'}^2(\beta_1)\frac{d\sigma_M}{d\Omega},\quad l=2l', \tag{61}$$

and for $\beta_1 \gtrsim 1$ the cross section (61) is much larger than the cross section in the Bunkin–Fedorov region.[12] Note that the scattering of an electron by a nucleus in the kinematic region (51) was not examined in Ref. 2. From (61) we see that in the case of only one wave the quantum parameter β_1 determines the emission and absorption of an even number of photons of the wave (the emission and absorption of an odd number of photons are suppressed). But if there are two waves, because of virtual interference processes of correlated absorption of an equal number of photons of both waves, the cross sections of emission (absorption) of an even and odd number of photons of the first and second waves are of the same order of magnitude (l and s in (60) can be either even or odd). Note also that the conditions (59) imposed on the field intensities are less stringent than similar conditions in the Bunkin–Fedorov region (see conditions (42) in Ref. 7). The partial cross section (60) can be summed over all possible processes of absorption and emission of photons of both waves. As a result all essentially quantum contributions originating in the quantum parameters $\beta_{1,2}$ and $\alpha_\pm$ cancel out and the cross section of the process is determined by the Mott cross section:

$$\frac{d\sigma}{d\Omega}=\frac{d\sigma_M}{d\Omega}\sum_{l,s=-\infty}^{\infty} J_{ls}^2(\cdots)=\frac{d\sigma_M}{d\Omega}. \tag{62}$$

Now let us examine the case of relativistic electron energies in the range of strong fields, where $\zeta_{1,2}\gtrsim 1$. Then the wave intensities satisfy the following condition:

$$\eta_{1,2}^2 \gtrsim v_{\rm i} \ll 1. \tag{63}$$

Here we also assume that

$$\eta_{1,2}^2 \ll 1. \tag{64}$$

Conditions (63) and (64) imply that the velocities with which an electron oscillates in the field of the two waves are large compared to the velocity of translation motion ($\eta_{1,2}\gg v_{\rm i}$). The law of energy conservation in this case assumes the form

$$\frac{\mathbf{p}_{\rm f}^2}{2m}-\frac{\mathbf{p}_{\rm i}^2}{2m}+\tfrac{1}{4}(\eta_1^2+\eta_2^2)(p_{\rm f}\cos\vartheta_{\rm f}-p_{\rm i}\cos\vartheta_{\rm i})-l\omega_1+s\omega_2=0, \tag{65}$$

where

$$\vartheta_{\rm i,f}=\angle(\mathbf{k},\mathbf{p}_{\rm i,f}). \tag{66}$$

The final electron velocity can be found from Eq. (65):

$$v_{\rm f}=\begin{cases} v_1\equiv\sqrt{b^2+a}-b, & a>0,\\ v_\pm\equiv|b|\pm\sqrt{b^2+a}, & a<0,\ \vartheta_{\rm f}>\pi/2,\end{cases} \tag{67}$$

where

$$a=v_{\rm i}^2+\frac{(\eta_1^2+\eta_2^2)v_{\rm i}\cos\vartheta_{\rm i}}{2}-\frac{2(l\omega_1+s\omega_2)}{m},\qquad b=\frac{(\eta_1^2+\eta_2^2)\cos\vartheta_{\rm f}}{4}. \tag{68}$$

From (67) and (68) it follows that $v_{\rm f}\sim v_{\rm i}$. With conditions (63) and (64) met, the partial probability integrated with respect to the final electron energy can easily be obtained from (5):

$$\frac{dW_{\rm fi}^{(ls)}}{d\Omega}=4Z^2r_e^2\frac{v_{\rm f}^2}{v_{\rm f}+b}\frac{J_{ls}^2(\beta_1,\beta_2,\alpha_+,\alpha_-)}{(\mathbf{v}_{\rm f}-\mathbf{v}_{\rm i})^4}, \tag{69}$$

where

$$\beta_j=\eta_j^2\frac{m}{8\omega_j}(v_{\rm f}\cos\vartheta_{\rm f}-v_{\rm i}\cos\vartheta_{\rm i}),\quad j=1,2, \tag{70}$$

$$\alpha_\pm=\eta_1\eta_2\frac{m}{2(\omega_1\pm\omega_2)}\{v_{\rm f}\cos\vartheta_{\rm f}-v_{\rm i}\cos\vartheta_{\rm i}\}, \tag{71}$$

and r_e is the classical electron radius. In Eqs. (69)–(71) $v_{\rm f}=v_{\rm i}$, and in the case of two values of the final electron velocity the partial probability is the sum of two expressions of the form (69): one with $v_{\rm f}=v_+$, the other with $v_{\rm f}=v_-$ [see Eq. (67)]. Equations (70) and (71) show that in the intensity range specified by condition (63) we have $\alpha_\pm\sim\beta_{1,2}\sim mv_{\rm i}^2/\omega_{1,2}\gg 1$. In view of this, the given intensity range is characterized primarily by multiphoton processes with absorption and emission of ($|l|\sim\beta_1$)-photons of the first wave and ($|s|\sim\beta_2$)-photons of the second. Although the given partial probabilities are low, they are considerably higher than the respective probabilities when electrons scatter in the Bunkin–Fedorov region.[7]

4. ELECTRON SCATTERING IN THE PLANE FORMED BY THE INITIAL ELECTRON MOMENTUM AND THE WAVE VECTOR

Let us examine the scattering of an electron by a nucleus in the field of two waves when the second condition in (51) is met. In such a case the 4-vector g of Eq. (18) satisfies the following equalities:

$$g^2=g_0^2-\mathbf{g}^2=0,\quad \mathbf{g}^2=(\mathbf{ng})^2. \tag{72}$$

Equations (18) and (72) immediately imply that scattering takes place in the plane formed by the initial electron momentum and the wave vector, with the azimuthal angles of the electron in the initial and final states being equal ($\varphi_{\rm i}=\varphi_{\rm f}$), and the polar angles and the energies are linked by the following relations:

$$\frac{|\mathbf{p}_{\rm f}|}{\kappa_{\rm f}}\sin\vartheta_{\rm f}=a_{\rm i},\quad a_{\rm i}\equiv\frac{|\mathbf{p}_{\rm i}|}{\kappa_{\rm i}}\sin\vartheta_{\rm i}. \tag{73}$$

Taking this into account, we can write an expression for vector $\mathbf{g}$ (Eq. (18)) that satisfies conditions (51):

$$\mathbf{g}=\frac{|\mathbf{p}_{\rm i}|}{\kappa_{\rm i}}\left(\frac{\sin\vartheta_{\rm i}}{\sin\vartheta_{\rm f}}\mathbf{n}_{\rm f}-\mathbf{n}_{\rm i}\right),\quad \mathbf{n}_{\rm i,f}=\frac{\mathbf{p}_{\rm i,f}}{|\mathbf{p}_{\rm i,f}|}. \tag{74}$$

Note that in what follows we exclude the polar angles $\vartheta_{\rm i,f}=0,\pi$ from our consideration (i.e., the cases of forward and backward scattering are not examined). From (74) it follows that the polar angles $\vartheta_{\rm i,f}$ and the scattering angle ϑ are linked by the following relations (see also Fig. 1):

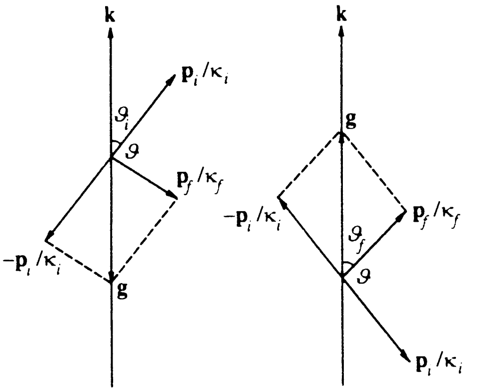

FIG. 1. The geometry of an electron scattered by a nucleus in the plane formed by the initial electron momentum and the wave vector.

$$\vartheta_f=\begin{cases}\vartheta_i+\vartheta, & \vartheta_i<\pi/2,\\ \vartheta_i-\vartheta, & \vartheta_i>\pi/2.\end{cases} \tag{75}$$

Relations between the final electron energy and momentum and an expression for κ_f can easily be obtained from (75):

$$v_f=\frac{|\mathbf{p}_f|}{E_f}=\frac{a_i}{\sin\vartheta_f+a_i\cos\vartheta_f}, \quad \kappa_f=\frac{\tan\vartheta_f}{a_i+\tan\vartheta_f}E_f. \tag{76}$$

If we allow for (76), the law of energy conservation (the argument of the delta function in (5)) simplifies considerably and assumes the form of a quadratic equation, rather than an equation of the fourth degree, in the final electron energy:

$$E_f^2-2b_iE_f+b_0m^2(1+a_i\cot\vartheta_f)=0. \tag{77}$$

Here we have used the notation

$$2b_i=E_i+\frac{b_0m^2}{\kappa_i}-l\omega_1-s\omega_2, \tag{78}$$

$$4b_0=(1+\delta_1^2)\eta_1^2+(1+\delta_2^2)\eta_2^2. \tag{79}$$

Equations (76) and (77) make it possible to determine the energy and the scattering angle of the final electron. Note that at $v_i\approx v_f\approx 1$ Eq. (73) breaks down. Hence, in what follows we consider relativistic and nonrelativistic energies of an electron in fields whose intensities obey the inequality

$$\eta_{1,2}\ll 1 \tag{80}$$

and conditions (53) are met, i.e., the probability of an electron being scattered in the plane formed by the initial electron momentum and the wave vector is much higher than that in any other geometry. Note also that when both waves are circularly polarized, the classical interference parameter

$$\xi_{\text{int}}=\xi_1\xi_2,\frac{p_i}{E_i} \tag{81}$$

is similar to $\zeta_{1,2}$ for the case of elliptically polarized fields (excluding circular polarization; see Eq. (55)).

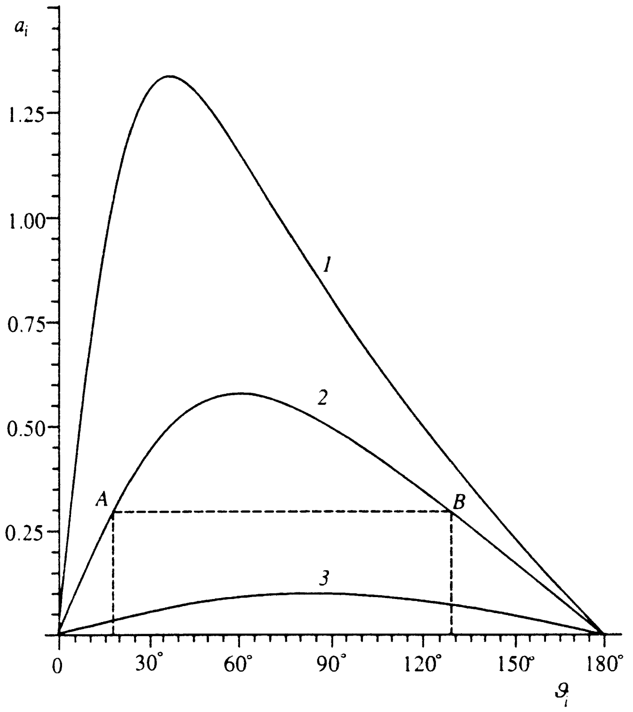

FIG. 2. The dependence of a_i defined in (73) on the polar angle ϑ_i of the initial electron momentum at different electron energies: $E_i\simeq 0.85$ MeV (curve *1*), $E_i\approx 0.59$ MeV (curve *2*), and $E_i\approx 2.55$ keV (curve *3*). Points *A* and *B* on curve *2* correspond to equal values of a_i at $\vartheta_i\simeq 18°$ and $\vartheta_i\simeq 130°$.

We begin with relativistic electron energies, $E_{i,f}\sim m$. Then, with condition (80) met, the law of energy conservation (77) implies $E_f\approx E_i$. In view of this, from (73) we can derive the following expression for the electron scattering angle:

$$\tan\frac{\vartheta}{2}=\begin{cases}(\cos\vartheta_i-v_i)/\sin\vartheta_i, & \vartheta_i<\pi/2,\\ (|\cos\vartheta_i|+v_i)/\sin\vartheta_i, & \vartheta_i>\pi/2.\end{cases} \tag{82}$$

We see that for $\vartheta_i<\pi/2$ conditions (73) are met only if $\cos\vartheta_i<v_i$(the case of $\cos\vartheta_i=v_i$ corresponds to forward scattering). The reason can easily be understood if we examine the variable a_i defined in (73) as a function of the polar angle of the initial electron (Fig. 2). The a_i vs ϑ_i curve exhibits a peak at $\cos\vartheta=v_i$. In the nonrelativistic limit of electron velocities the peak is symmetric with respect to polar-angle axis ($\vartheta_i\approx\pi/2$). But as the energy increases, the peak shifts toward the region of small angles, and in the ultrarelativistic case merges with the vertical axis ($\vartheta_i\approx 0$). Hence, excluding the ultrarelativistic case, we can always identify a range of angles ϑ_i in the first and second quadrants for which a_i has the same value at two distinct values of the polar angle. Since the left- and right-hand sides of (73) have the same structure in the given case, the a_i vs ϑ_i curve makes it possible to determine the outgoing electron angle ϑ_f knowing the electron angle of incidence ϑ_i. In Fig. 2 the intersections of the horizontal dotted line with curve *2* mark the points where a_i has the same value (points *A* and *B*). Hence, if an electron with an energy $E_i=0.59$ MeV and an angle of incidence $\vartheta_i^A\approx 18°$,

its outgoing angle is $\vartheta_f^B \approx 130°$, and vice versa, i.e., for $\vartheta_f^B \approx 130°$ we get $\vartheta_i^A \approx 18°$ (the latter is true only if point B lies in the range where $\vartheta_i > \pi/2$). This result, obviously, also follows from (82).

If we allow for (80), the scattering cross section for relativistic electrons integrated over the final electron energies assumes the form

$$\frac{d\sigma^{(ls)}}{d\Omega} = |J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)|^2 \frac{d\sigma_M}{d\Omega}. \tag{83}$$

Here $\beta_{1,2}$ and $\alpha_\pm$ are defined in Eqs. (19) and (20) at $E_f = E_i$ for the scattering angle specified by Eqs. (82) and (75). The above expression holds true when both waves are elliptically polarized. It must be emphasized that owing to virtual interference processes characterized by the parameters $\alpha_\pm$ there is emission and absorption of both an even and an odd number of photons of the first and second waves. Note that when both waves are linearly polarized, when the angle Δ between their polarization vectors is close to $\pi/2$, i.e.,

$$|\Delta - \frac{\pi}{2}| \ll \frac{\omega_{1,2}}{p_i}\left(\frac{E_i}{m}\right)^2 \frac{1}{\eta_1\eta_2} \lesssim 1, \tag{84}$$

the interference parameters $\alpha_\pm$ are much smaller than unity, and from (83) it follows that in the process of scattering the electron independently absorbs and emits an even number of photons of the first and second waves (emission and absorption of an odd number of photons of both waves is suppressed, as in the case of only one wave):

$$\frac{d\sigma^{(ls)}}{d\Omega} = J_{l'}^2(\beta_1) J_{s'}^2(\beta_2) \frac{d\sigma_M}{d\Omega}, \quad l=2l', \quad s=2s'. \tag{85}$$

When both waves are circularly polarized ($\delta_1 = 1$ and $\delta_2 = \mp 1$), Eqs. (83) and (48) yield the partial cross section for scattering of an electron by a nucleus with correlated emission and absorption of an equal number of photons of the first and second waves (here the processes with emission and absorption of unequal numbers of photons of the first and second waves are suppressed):

$$\frac{d\sigma^{(l)}}{d\Omega} = J_l^2(\alpha_\pm) \frac{d\sigma_M}{d\Omega}. \tag{86}$$

This shows that when both waves are circularly polarized and the electron is scattered in the plane formed by the initial electron momentum and the wave vector and the scattering angles are defined in (82), wave interference manifests itself and the quantum interference parameter $\alpha_\pm$ serves as the multiphoton parameter.

It must be emphasized that in view of conditions (53) the cross sections specified by Eqs. (83), (85), and (86) are considerably greater than the respective electron scattering cross sections in the Bunkin–Fedorov region.[7] Note also that this remains true for the scattering of an electron by a nucleus in the field of only one wave ($F_2 = 0$), with the exception of the case of a circularly polarized wave, for which, as Eq. (86) with $\alpha_\pm = 0$ implies, the process is accompanied by neither emission nor absorption of photons and the scattering cross section coincides with the Mott cross section. The cross sections specified by Eqs. (83), (85), and (86) can be summed over all processes of emission and absorption of photons of both waves, all the essentially quantum contributions that originate in the quantum parameters $\beta_{1,2}$ and $\alpha_\pm$ cancel out, and the scattering cross section is determined by the Mott cross section.

Now we consider the range of nonrelativistic electron energies, $v_{i,f} \ll 1$, assuming that condition (80) is met. After simple transformations the law of energy conservation (77) takes the form

$$\frac{\mathbf{p}_f^2}{2m} - \frac{\mathbf{p}_i^2}{2m} + l\omega_1 + s\omega_2 = 0. \tag{87}$$

Combining this with (73), we find that the outgoing polar angle of the final electron obeys the following relation:

$$\sin\vartheta_f = \frac{1}{\rho_{ls}} \sin\vartheta_i, \tag{88}$$

where

$$\rho_{ls} = \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} = \sqrt{1 - \frac{2(l\omega_1 + s\omega_2)}{mv_i^2}}. \tag{89}$$

Equation (88) shows that generally the scattering angle depends on the number of absorbed and emitted photons of the waves, with a possible restriction on the values of the initial electron polar angle (if $\rho_{ls} < 1$):

$$\sin\vartheta_i < \rho_{ls}. \tag{90}$$

Since for the case when both waves are elliptically polarized (excluding circular polarization) the quantum parameters $\beta_{1,2}$ serve as the multiphoton parameters, while for circularly polarized waves the quantum interference parameters $\alpha_\pm$ serve as such parameters, in what follows we distinguish between these two cases. Clearly, for fairly strong fields with $\eta_{1,2}^2 \gg v_i$ (for elliptically polarized waves) or $\eta_1\eta_2 \gg v_i$ (for circularly polarized waves) the second term on the right-hand side of (89) may be large, i.e., there are chiefly multiphoton processes of absorption of photons of both waves. As a result the final electron velocity can be considerably higher than the initial electron velocity, $v_f \gg v_i$, i.e., we find that $\rho_{ls} \gg 1$ and, in view of Eq. (88), $\vartheta_f \approx 0$ (forward and backward electron scattering). Hence, we restrict the intensities of the waves as follows:

$$\begin{aligned} &\eta_{1,2}^2 \lesssim v_i \quad \text{for elliptically polarized waves,} \\ &\eta_1\eta_2 \lesssim v_i \quad \text{for circularly polarized waves.} \end{aligned} \tag{91}$$

With such restrictions the scattering cross section for nonrelativistic electrons scattered in the plane formed by the initial electron momentum and the wave vector, with the outgoing angle ϑ_f satisfying Eq. (88), can easily be obtained from Eqs. (5) and (6):

$$\frac{d\sigma^{(ls)}}{d\Omega} = 4Z^2 r_e^2 \frac{m^4}{(\mathbf{p}_f - \mathbf{p}_i)^4} \rho_{ls} |J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)|^2, \tag{92}$$

where

$$\beta_j=(1-\delta_j^2)\eta_j^2\,\frac{mv_{\rm i}}{8\omega_j}\,(\rho_{ls}\cos\vartheta_{\rm f}-\cos\vartheta_{\rm i}),\quad j=1,2, \tag{93}$$

$$\alpha_\pm=\eta_1\eta_2|d_\mp|\,\frac{mv_{\rm i}}{2(\omega_1\pm\omega_2)}\,(\rho_{ls}\cos\vartheta_{\rm f}-\cos\vartheta_{\rm i}). \tag{94}$$

When both waves are linearly polarized and the angle Δ between their polarization vectors is close to $\pi/2$, i.e., condition (84) is met, Eq. (92) implies, as Eq. (85) does in the nonrelativistic case, that in the process of scattering the electron independently emits and absorbs an even number of photons of the first and second waves:

$$\frac{d\sigma^{(ls)}}{d\Omega}=4Z^2r_e^2\,\frac{m^4}{(\mathbf{p}_{\rm f}-\mathbf{p}_{\rm i})^4}\,\rho_{ls}J_{l'}^2(\beta_1)J_{s'}^2(\beta_2),$$

$$l=2l',\ s=2s'. \tag{95}$$

When both waves are linearly polarized ($\delta_1=1$ and $\delta_2=\mp1$), Eq. (92) yields the partial cross section with correlated emission and absorption of an equal number of photons of both waves (here the processes with emission and absorption of unequal numbers of photons of the first and second waves are suppressed):

$$\frac{d\sigma^{(ls)}}{d\Omega}=4Z^2r_e^2\,\frac{m^4}{(\mathbf{p}_{\rm f}-\mathbf{p}_{\rm i})^4}\,\rho_lJ_l^2(\alpha_\pm). \tag{96}$$

Here ρ_l is given by formula (89) with $s=\pm l$.

If the intensities of the fields meet the conditions

$$\eta_{1,2}^2\lesssim v_{\rm i}\ \text{for elliptically polarized waves,}$$

$$\eta_1\eta_2\lesssim v_{\rm i}\ \text{for circularly polarized waves,} \tag{97}$$

we can neglect the number of emitted (absorbed) photons of both waves, i.e., in these conditions $v_{\rm f}\approx v_{\rm i}$ and the electron scattering angle is (see curve *3* in Fig. 2)

$$\vartheta=\pi-2\vartheta_{\rm i}. \tag{98}$$

Consequently, the expression (92) for the scattering cross section assume the form

$$\frac{d\sigma^{(ls)}}{d\Omega}=\frac{Z^2r_e^2}{4v_i^4\cos^4\vartheta_{\rm i}}\,|J_{ls}(\beta_1,\beta_2,\alpha_+,\alpha_-)|^2, \tag{99}$$

where

$$\beta_j=(1-\delta_j^2)\eta_j^2\,\frac{mv_{\rm i}}{4\omega_j}\cos\vartheta_{\rm i},\quad j=1,2, \tag{100}$$

$$\alpha_\pm=\eta_1\eta_2|d_\mp|\,\frac{mv_{\rm i}}{\omega_1\pm\omega_2}\cos\vartheta_{\rm i}. \tag{101}$$

Note that if conditions (97) are met, Eqs. (95) and (96) assume the form of (99) with $|J_{ls}|^2=J_{l'}^2J_{s'}^2$ and $J_{ls}^2=J_l^2(\alpha_\pm)$, respectively. Note also that for $\beta_{1,2}\gtrsim1$ and $\alpha_\pm\gtrsim1$ the cross sections (92), (95), (96), and (99) are considerably larger than the respective scattering cross sections in the Bunkin–Fedorov region (this is also true in the field of a single ($F_2=0$) elliptically polarized wave, excluding the case of circular polarization). The cross sections (99) can be summed over all processes of emission and absorption of photons of both waves. As a result we get the Mott cross section.

5. THE MAIN CONCLUSIONS

The study of stimulated bremsstrahlung and absorption by a relativistic electron scattered by a nucleus in the field of two elliptically polarized waves propagating in the same direction shows that, depending on the intensities, polarizations, and frequencies of the two waves, the process of electron scattering takes place in different kinematic regions and is characterized by different multiphoton parameters. For instance, when the electron is scattered in the Bunkin–Fedorov region, the quantum parameter $\gamma_{1,2}$ serves as the multiphoton parameter and the classical parameter $\xi_{1,2}$ determines the integral characteristics of the process. But if the electron is scattered in the plane formed by the initial electron momentum and the wave vector (for elliptically polarized waves, with the exception of circular polarization) or in the plane perpendicular to the polarization vector of the waves (for waves with equal linear polarizations), the quantum parameter $\beta_{1,2}$ serves as the multiphoton parameter and the classical parameter $\zeta_{1,2}$ determines the integral characteristics of the process. A special case is when both waves are circularly polarized and the scattering of the electron takes place in the plane formed by the initial electron momentum and the wave vector. This is a direct manifestation of interference of waves, which leads to correlated emission and absorption of an equal number of photons of both waves (processes of emission and absorption of unequal numbers of photons are suppressed). Here the quantum interference parameter $\alpha_\pm$ serves as the multiphoton parameter and the classical interference parameter $\zeta_{\rm int}$ determines the integral characteristics of the process.

Note that when the intensities and frequencies of both waves are such that $\beta_{1,2}\ll1$ and $\alpha_\pm\ll1$ hold (at optical frequencies these conditions correspond to the following restrictions on the field strengths: $F_{1,2}\ll 10^7$–10^8 $V\,{\rm cm}^{-1}$ for relativistic electron energies, and $F_{1,2}\ll(10^7$–$10^8)v_i$ V cm^{-1} for nonrelativistic energies), stimulated bremsstrahlung and absorption by an electron scattered by a nucleus take place in the Bunkin–Fedorov region. However, if we have $\beta_{1,2}\gtrsim1$ and $\alpha_\pm\gtrsim1$ but $\eta_{1,2}\ll1$ (10^7–$10^8\lesssim F_{1,2}\ll10^{10}$–$10^{11}$ V cm^{-1} for relativistic electron energies, and $(10^7$–$10^8)v_{\rm i}\lesssim F_{1,2}\ll10^{10}$–$10^{11}$ V cm^{-1} for nonrelativistic energies), electron scattering occurs in the plane formed by the initial electron momentum and the wave vector with a fixed scattering angle (scattering in any other geometry is suppressed). Here, for linearly polarized waves, when the angle between the polarization vectors is close to $\pi/2$, in the process of scattering the electron independently emits and absorbs an even number of photons of the first and second waves (processes of emission and absorption of unequal numbers of photons are suppressed), and for circularly polarized waves the electron emits and absorbs an equal number of photons of both waves in a correlated manner.

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