Transient Hamiltonian chaos in the interaction of an electromagnetic field with two-level atoms

K. N. Alekseev and G. P. Berman

Kirenskii Institute of Physics, Siberian Division, Russian Academy of Sciences, 660036 Krasnoyarsk, Russia

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The nonlinear dynamics that arises when an ensemble of two-level atoms in a high Q Fabry–Perot cavity interacts with one mode of the self-consistent field and with an external amplitude-modulated resonance field is considered. It is shown that when the atoms are in exact resonance with the field and only the ground state of the atoms is populated initially, the Hamiltonian chaos that arises in the system is always transient. The differences in the chaotic dynamics for the cases in which the cavity geometries are of the Fabry–Perot and ring types are considered, and the conditions for experimental observation of transient Hamiltonian chaos are discussed.

I. INTRODUCTION

The Tavis–Cummings model,[1] which describes in the rotating-wave approximation[2] the interaction of an ensemble of two-level atoms with one mode of the electromagnetic field, is one of the principal models for quantum and nonlinear optics. The principal dynamical effects predicted for this model have recently become accessible to experimental verification (see Refs. 3 and 4 and the literature cited therein).

In the work of Belobrov *et al.*,[5] it was shown that in the framework of a semiclassical description (a classical field), rejection of the rotating-wave approximation leads to chaos (see also Refs. 6). However, for typical transitions in the optical and radiofrequency bands, violation of the conditions for applicability of the rotating-wave approximation requires extremely high atom densities, at which a description in the framework of a gas of noninteracting two-level atoms also becomes inadequate.

Recently, other very simple generalizations of the Tavis–Cummings model that admit Hamiltonian chaos in the framework of the rotating-wave approximation have appeared.[7–13] These models describe the interaction of three-level atoms with two cavity modes,[7–8] and of two-level atoms with one mode of the self-consistent field in the cavity and with an external field that is injected into the cavity and has a constant[1)] (Refs. 9–11) or modulated[12] amplitude. For the latter two cases, the interaction with the external field was taken into account in the standard "spatial mean field" approximation (mean-field model).[15]

A common feature of all the work mentioned above is the consideration of samples with characteristic dimensions shorter than a wavelength or of a ring cavity with a wave traveling in only one direction. For a Fabry–Perot cavity, allowance for the spatial variation of the field as a consequence of the interference of waves traveling in opposite directions can have an important influence on the nonlinear characteristics of the interaction of the atoms with the field. For example, the important role of spatial effects has long been known in the theory of optical bistability (see Refs. 16 and 17 and the literature cited therein).

In this paper, in the framework of a semiclassical description, we consider the influence of standing-wave effects on the transition to Hamiltonian chaos in the interaction of an electromagnetic field with two-level systems. We use the model of Ref. 12, but instead of the ring-cavity geometry we consider the case of a Fabry–Perot cavity.

It is shown that in the very simple case in which the two-level atoms are in exact resonance with the field and only their ground state is initially populated, the coupled system of Maxwell–Bloch equations reduces to a Hamiltonian system with 1.5 degrees of freedom—a periodically excited "Bessel pendulum." The conditions for the transition to chaos are found numerically. The chaos that arises in the system is transient: the chaotic oscillations of the polarization and of the difference of the populations are replaced after a certain time by regular oscillations. Unlike the other, previously known models (of physical systems) that admit transient Hamiltonian chaos (see, e.g., Refs. 18–23), in our case the effective potential of the Hamiltonian system with 1.5 degrees of freedom is a many-well potential. This causes the dynamics of the system to have a highly complicated character. In particular, the time during which chaos is possible in the system depends in a complicated way on the parameters of the system. For trajectories with a sufficiently long chaotic part, the character of the steady-state regular oscillations of the field in the cavity was observed to be sensitive to small changes of the initial conditions and of the parameters of the system.

The paper is organized as follows. In Sec. 2 we introduce the model describing the dynamics of a system consisting of two-level atoms plus field. Here too we shall consider an ansatz which, in the case of exact resonance and certain initial populations of the atoms, reduces the coupled Maxwell–Bloch system to one equation for a periodically excited "Bessel pendulum." In Sec. 3, after a brief survey of the literature on transient Hamiltonian chaos during escape from potential wells, we describe the nonlinear dynamics of our system. In Sec. 4 we compare distinctive features of the dynamics of the interaction of

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the atoms with the field in the geometries of Fabry–Perot and ring cavities, and also discuss the possibility of experimental observation of transient chaos.

We not turn to a detailed study of the problem.

2. THE MODEL

Our system consists of N identical two-level atoms with transition frequency ω_0 and dipole matrix element d, interacting with one mode of a radiation field with frequency $\omega \approx \omega_0$. The sample, of length L and volume V, completely fills a high-Q Fabry–Perot cavity $(-L \leqslant z \leqslant 0)$. An external amplitude-modulated field $E_{\rm ext}$ is injected into the cavity at the plane $z=0$. This field has the form

$$E_{\rm ext}=E_0F(t)\cos\omega t, \tag{1}$$

where $F(t)$ is a periodic function of time $(|F(t)|=1)$, slowly varying in comparison with the carrier frequency ω. In this paper we shall assume the very simple modulation law $F(t) = \sin(\Omega t)$, but we may expect that the principal results will also be applicable to a modulation $F(t)$ containing many harmonics. Following the classic work of Spencer and Lamb,[24] we shall regard the external field as a weak perturbation that preserves the spatial structure of the self-consistent field

$$E_{\rm sc}(z,t)=E(t)\sin(kz) \tag{2}$$

in the cavity; here, $k=n\pi L^{-1}$ is the wave number of the selected nth mode of the cavity. The dynamics of the atoms is described by the Bloch equation[2]

$$\begin{aligned}
\dot S_1(z,t)&=-\omega_0 S_2(z,t),\\
\dot S_2(z,t)&=\omega_0 S_1(z,t)+\frac{2d}{\hbar}E_{sc}(z,t)S_3(z,t),\\
\dot S_3(z,t)&=-\frac{2d}{\hbar}E_{sc}(z,t)S_2(z,t),
\end{aligned}\tag{3}$$

$$S_1^2+S_2^2+S_3^2=1, \tag{4}$$

where the pseudospin variables S_i $(i=1,2,3)$ are related as follows to the probability amplitudes a_j and b_j of population of the upper and lower levels, respectively, of the jth atom:[9,12,25]

$$\begin{aligned}
S_1(z,t)&=\frac{1}{N_s}\sum_{j\in\Delta V}^{N_s}(a_j^*b_j+a_jb_j^*),\\
S_2(z,t)&=-\frac{i}{N_s}\sum_{j\in\Delta V}^{N_s}(a_j^*b_j-a_jb_j^*),\\
S_3(z,t)&=\frac{1}{N}\sum_{j\in\Delta V}^{N_s}(|a_j|^2-|b_j|^2).
\end{aligned}\tag{5}$$

In (5), $\Delta V=(\Delta z)\pi r^2$ is the small volume over which the averaging is performed, z is the coordinate at the center of a layer of thickness $\Delta z \ll \lambda$ (λ is the wavelength of the radiation), r is the characteristic radius of the sample with the gas of atoms, and N_s is the number of atoms in the volume ΔV $(N_s \gg 1)$.

The behavior of the field inside the cavity is determined by the Maxwell equation that takes the influence of the external field into account:[24]

$$\begin{aligned}
\ddot E(t)+\omega^2E(t)&=-4\pi\ddot P_a(t)-\frac{2\omega c}{L}E_{\rm ext},\\
P_a(t)&=\frac{2}{L}\int_{-L}^{0}dzP(z,t)\sin(kz),
\end{aligned}\tag{6}$$

where $P(z,t)$ is the polarization of the two-level medium, and is related to the pseudospin variable $S_1(z,t)$:

$$P(z,t)=\frac{N}{V}dS_1(z,t). \tag{7}$$

The self-consistent Bloch–Maxwell system (3), (6), (7) completely describes the dynamics of the system over a time interval shorter than all the characteristic relaxation times of both the field and the atoms. These equations can be simplified substantially by separating the fast and slow motions. For this we transform to a rotating coordinate frame:

$$\begin{aligned}
S_1(z,t)&=u(z,t)\cos(\omega t)-v(z,t)\sin(\omega t),\\
S_2(z,t)&=u(z,t)\sin(\omega t)+v(z,t)\cos(\omega t),\\
S_3(z,t)&=w(z,t),
\end{aligned}\tag{8}$$

and introduce the envelope of the self-consistent field:

$$E(t)=E_1(t)\cos(\omega t)+E_2(t)\sin(\omega t). \tag{9}$$

Next, we shall assume that the envelopes are varying slowly in comparison with the carrier frequency ω:

$$\begin{aligned}
&|\dot u| \ll \omega|u|, \quad |\dot v| \ll \omega|v|, \quad |\dot w| \ll \omega|w|,\\
&|\dot E_{1,2}| \ll \omega|E_{1,2}|.
\end{aligned}\tag{10}$$

Then, substituting (1), (8), and (9) into (3), (6), (7) and using the rotating-wave approximation and the approximation of slowly varying amplitudes,[2] we obtain

$$\begin{aligned}
\dot u&=\Delta v+w\varepsilon_2\sin(kz),\\
\dot v&=-\Delta u+w\varepsilon_1\sin(kz),\\
\dot w&=-(u\varepsilon_2+v\varepsilon_1)\sin(kz),\\
\dot\varepsilon_1&=\omega_c^2\frac{2}{L}\int_{-L}^{0}dzv(z,t)\sin(kz)+GF(t),\\
\dot\varepsilon_2&=\omega_c^2\frac{2}{L}\int_{-L}^{0}dzu(z,t)\sin(kz).
\end{aligned}\tag{11}$$

In (11), $\Delta=\omega-\omega_0$ is the offset from optical resonance, $G=c\varepsilon_0/L$, $\varepsilon_j=dE_j/\hbar (j=0,1,2)$, and $\omega_c=(2\pi Nd^2\omega_0/\hbar V)^{1/2}$ is the so-called cooperative frequency[26] that characterizes the vibrational exchange of energy between the atoms and the field in the absence of the external field $(E_0=0)$.[2)] The system of Maxwell–Bloch equations admits conservation of the pseudospin length:

$$u^2(z,t)+v^2(z,t)+w^2(z,t)=1. \tag{12}$$

Substituting (8) into (7), we obtain an expression for the polarization in terms of the slow variables u and v:

$$P(z,t)=\frac{Nd}{V}[u(z,t)\cos(\omega t)-v(z,t)\sin(\omega t)]. \tag{13}$$

The conditions (10) for applicability of the rotating-wave and slowly-varying-amplitude approximations can be written in the form

$$\max\{G^{1/2},\omega_c,\Omega\}\ll\omega\sim\omega_0. \tag{14}$$

For typical optical systems, as a rule, this condition is fulfilled.

The system (11) now contains only slow variables. To solve it, it is necessary to specify the initial spatial distributions $u(z,t=0)\equiv u_0(z)$ and $v_0(z)$ of the polarization components, the population difference $w_0(z)$, and the initial values $\varepsilon_{1,2}(0)$ of the field envelopes. It does not appear to be possible to find solutions of the system of equations (11) in the general case, and we shall consider only particular solutions at exact resonance $\Delta=0$ and for specified initial conditions.

It is possible to show that at exact resonance ($\Delta=0$), if $u(z,t=0)=\varepsilon_2(t=0)=0$, $u(z,t)=\varepsilon_2(t)=0$ as well at any time t.[3)] In this case, it follows from (12) that the polarization $v(z,t)$ and population difference $w(z,t)$ can be parametrized by means of one angular variable $\varphi(z,t)$:

$$v(z,t)=-\sin\varphi(z,t),$$

$$w(z,t)=-\cos\varphi(z,t). \tag{15}$$

We introduce the following ansatz:

$$\varphi(z,t)=\vartheta(t)\sin(kz),\quad \vartheta(t)=\int_0^t dt'\varepsilon_1(t'). \tag{16}$$

Then it follows from (15) that

$$v(z,t)=-2\sum_{n=0}^{\infty}J_{2n+1}[\vartheta(t)]\sin[(2n+1)kz],$$

$$w(z,t)=-J_0[\vartheta(t)]-2\sum_{n=0}^{\infty}J_{2n}[\vartheta(t)]\cos(2nkz), \tag{17}$$

where $J_n(x)$ is the Bessel function of the first kind of order n. In these variables, the system of equations (11) can be reduced to the one equation of a periodically excited "Bessel pendulum":

$$\ddot{\vartheta}+2\omega_c^2J_1(\vartheta)=G\sin\Omega t,$$

$$\dot{\vartheta}=\varepsilon_1\equiv\varepsilon(t). \tag{18}$$

Taking into account that for small x we have $J_1(x)\approx x/2$, we find that the frequency of the linear oscillations of this pendulum (with $G=0$) is precisely the cooperative frequency ω_c.

The ansatz (16), (17), so far as we know, was first introduced in Ref. 27 in a study of the metastable states in a coupled atoms-plus-field system. In the same paper, the equation of a Bessel pendulum without external excitation ($G=0$) was also obtained.

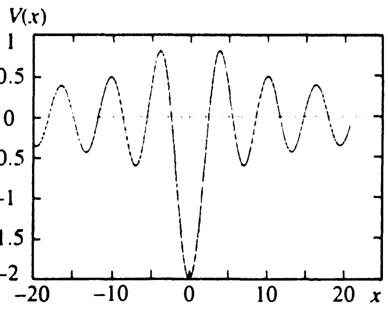

FIG. 1. Form of the potential for a Bessel pendulum: $V(x)=-2J_0(x)$.

We shall now discuss an important question that was not considered in Ref. 27—that of the limits of applicability of the above ansatz. The expansion (17) should be valid for arbitrary $v(z,t)$ and $w(z,t)$, including the initial distributions $v_o(z)$ and $w_0(z)$. This implies that there should exist a value $\vartheta(0)\equiv\vartheta_0$ such that the expansions

$$v_0(z,t)=-2\sum_{n=0}^{\infty}J_{2n+1}(\vartheta_0)\sin[(2n+1)kz],$$

$$w_0(z,t)=-J_0(\vartheta_0)-2\sum_{n=0}^{\infty}J_{2n}(\vartheta_0)\cos(2nkz) \tag{19}$$

are identities.[4)] However, far from all $v_0(z)$ and $w_0(z)$ satisfy this condition. Amongst the initial distributions $v_0(z)$ and $w_0(z)$ for which (19) is valid, it is possible to identify at least two physically interesting cases: 1) the case of a periodic δ-function polarization distribution and a population difference corresponding to $\vartheta_0\gg1$ and 2) the case of a spatially uniform weak initial excitation of the two-level medium: $w_0(z)\approx-1$ and $v_0(z)\approx0$. Such an initial distribution corresponds to $\vartheta_0\approx0$. We note that the ansatz (16), (17) is applicable to arbitrary initial field values $\varepsilon(0)$.

Below we shall confine ourselves to considering the physically most interesting case, in which all the atoms are initially in the ground sate [$w_0(z)=-1$ and $v_0(z)=0$] and the self-consistent field of the cavity is zero [$\varepsilon(0)=0$]. This implies that for the pendulum (18), the initial conditions are $\vartheta(0)=\dot{\vartheta}(0)=0$.

3. HAMILTONIAN DYNAMICS

Equation (18) can be written in the Hamiltonian form

$$\frac{d\vartheta}{d\tau}=\frac{\partial H}{\partial p},\quad \frac{dp}{d\tau}=-\frac{\partial H}{\partial\vartheta},$$

$$H=\frac{p^2}{2}+V(\vartheta)-\bar{G}\vartheta\sin(\bar{\Omega}\tau),\quad V(\vartheta)=-2J_0(\vartheta), \tag{20}$$

where $p\equiv\varepsilon/\omega_c$, $\tau=\omega_ct$, $\bar{G}=G/\omega_c^2$, and $\bar{\Omega}=\Omega/\omega_c$. The form of the potential $V(\vartheta)$ is depicted in Fig. 1.

We shall go over to an extended phase space[28] and introduce a pair of new canonically conjugate variables (ϕ,I) by the formulas

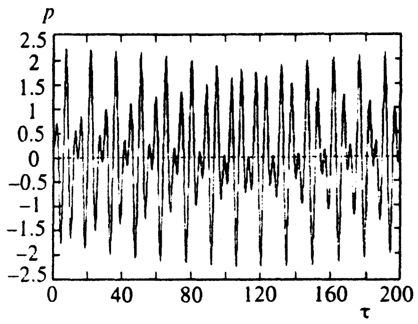

FIG. 2. Dependence of the self-consistent field $p=\varepsilon/\omega_c$ on time for regular motion inside the first potential well: $\bar{G}=0.8$, $\bar{\Omega}=1.3$.

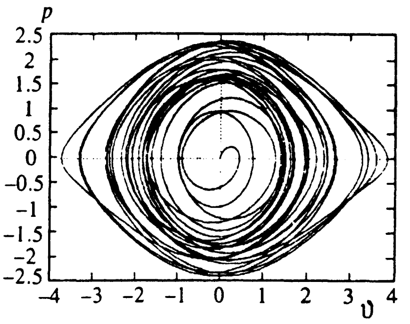

FIG. 3. A weakly unstable chaotic orbit of length $\tau=200$, localized in the first potential well: $\bar{G}=0.271$, $\bar{\Omega}=0.8$.

$$\frac{d\psi}{d\tau}=\frac{\partial\tilde{H}}{\partial I},\quad \frac{dI}{d\tau}=-\frac{\partial\tilde{H}}{\partial\psi},\quad \psi=\bar{\Omega}\tau,$$

$$\tilde{H}=H+\bar{\Omega}I. \tag{21}$$

The new Hamiltonian $\hat{H}$ is an integral of the motion in the extended phase space (ϑ,p,ψ,I). Equations (21) were used to monitor the errors in the numerical calculations.

Despite the rather simple form of Eq. (18), the dynamics of this Hamiltonian system with 1.5 degrees of freedom can be rather complicated. This is due to the fact that the potential $V(\vartheta)$ is not periodic and is a decreasing function of ϑ as $|\vartheta|\to\infty$. Under the action of a periodic perturbation both regular and chaotic motion are possible. Since $V(\vartheta)\to 0$ as $\vartheta\to\pm\infty$, asymptotically the behavior of the system is regular. Therefore, if Hamiltonian chaos is possible in the system, it is transient chaos.

In recent years great attention has been paid to the study of transient Hamiltonian chaos (see, e.g., the review articles Refs. 18 and 19 and the literature cited therein). A large part of the work has been devoted to the study of the scattering of particles by two-dimensional or multidimensional potentials.[18–20] Comparatively recently it has been realized[21–23] that the stochastic ionization of atoms and molecules, or, generally speaking, any escape of a chaotic trajectory from a potential well, is also an example of transient Hamiltonian chaos. Transient chaos in "stochastic ionization" has not only been much less well studied, but is also organized in a much more complicated way than chaos in potential scattering.[22] This is due to the fact that the phase space of typical Hamiltonian systems has a nonuniform structure: besides the stochastic seas there are numerous islands of stability, near which a chaotic trajectory can linger for a long time.

In Ref. 21, stochastic ionization of a Morse oscillator under the action of a periodic sequence of δ-function pulses was considered. It was shown that the time t_{esc} of chaotic escape from the well can be sensitive to small changes in the parameters of the system. The boundaries between regions with different times t_{esc} are fractals in the space of the parameters of the amplitude and period of the external perturbation.

In Ref. 23 the escape of chaotic trajectories from a two-dimensional potential was investigated. It was shown that for strong chaos, the direction and time of the escape from the potential can depend strongly on a small change in the initial conditions. The region in the space of initial conditions for which trajectories leave the potential well after a specified number of iterations is a fractal.

In the papers listed, chaotic escape from a two-dimensional potential (two degrees of freedom) or from a one-dimensional and one-well potential (1.5 degrees of freedom) was considered. The distinctive feature of our system with 1.5 degrees of freedom is the fact that the effective potential contains many wells (see Fig. 1).

We shall illustrate the principal types of nonlinear dynamics of the system. First, if the conditions for chaos are not fulfilled, the trajectory is always localize din the first well ($-3.83\leqslant\vartheta\leqslant 3.83$). The corresponding form of the field [the momentum in the effective system (20)] for regular motion is shown in Fig. 2. In the transition to chaos, more-varied behavior is possible:

(a) weakly unstable chaotic trajectories spending a long time inside the first well (Figs. 3 and 4). Such trajectories exist principally for parameter values lying near the boundary of the transition to chaos;

(b) for stronger local instability (a larger value of the Lyapunov exponent), the trajectory can rapidly leave the first well and then visit several more wells in a random manner; finally, asymptotically regular behavior is established (Fig. 5). Here, the signs of the coordinate ϑ and momentum p that are established as $t\to\infty$ are random. A small change in the initial conditions leads to an entirely

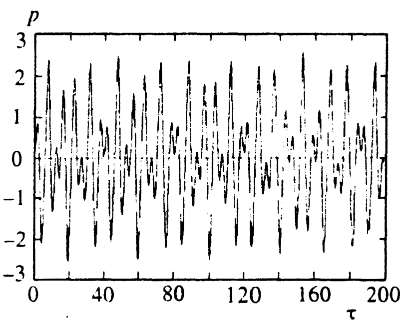

FIG. 4. Dependence of the field on time. The parameters have the same values as in Fig. 3.

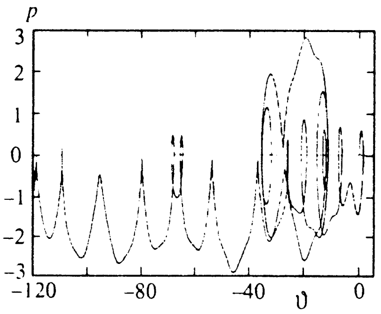

FIG. 5. Form of a trajectory of length $\tau=200$ for transient chaos ($\bar{G}=0.7$, $\bar{\Omega}=0.6$).

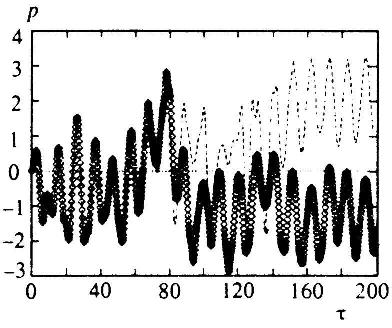

FIG. 7. Change in the asymptotic state of the field under a small change of the initial conditions. The process marked by squares correspond to the initial condition $p(0)=0$, and that not marked by squares corresponds to the initial condition $p(0)=10^{-4}$. The values of the parameters are the same as in Figs. 5 and 6.

different asymptotic state (compare Figs. 5 and 6). This leads to an interesting physical effect: a small fluctuation in the initial value of the field in the cavity induces a change of sign of the asymptotic state of the field (Fig. 7);

(c) an asymptotic regular state can be established after a long enough time. An example of such behavior is shown in Fig. 8. This trajectory leaves the first well comparatively rapidly ($\tau_{esc}\approx 10.8$), then visits several neighboring wells, and again returns to the first well. The corresponding chaotic oscillations of the field are shown in Fig. 9;

(d) on the other hand, the emergence into regular behavior can be very rapid (Figs. 10, 11).

Regions with regular and chaotic behavior in the space of the perturbation-amplitude parameter and modulation-frequency parameter are shown in Fig. 12. The boundary of chaos is denoted by asterisks, and the windows of regularity that lie near the boundary of chaos are indicated by squares. The smallest value of the external field amplitude ($\bar{G}\approx 0.14$) for which chaos is possible was observed at the frequency $\bar{\Omega}\approx 0.85$. In the region lying above the boundary of chaos there exist trajectories with chaotic phases of different lengths. For these trajectories, during the time interval of chaotic motion, the largest Lyapunov exponent is positive. Our preliminary numerical investigations have shown an extremely complicated dependence of the length of the chaotic phase on the parameters. However, it is possible to state with confidence that the trajectories with the longest chaotic phase ($\tau_{esc}\sim 10^2$) are found mainly in a layer of width ~ 0.1 near the boundary of chaos.

We also examined the sensitivity of the sign of the asymptotic state of the self-consistent field to a small change in the amplitude of the external field. The results are presented in Fig.13. The dimensionless parameter $\bar{G}$ of the perturbation was varied over the interval [0.89,0.9] with step 10^{-4}. When regular oscillations with $\varepsilon>0$ were established the value $+1$ was placed on the diagram, and in the opposite case, with $\varepsilon<0$, the value -1 was placed on the diagram. It can be seen from the figure that over a certain range of values of the perturbation parameter $\bar{G}$, the sign of the asymptotic state of the field is sensitive to small changes in $\bar{G}$. Such behavior is characteristics of trajectories with a sufficiently long chaotic phase ($\tau_{esc}\approx 40$–100).

To conclude this section, we shall consider the spatial structure of the polarization and population difference for regular and chaotic dynamics [see Eq. (17)]. A substantial contribution to the expansion (17) is made only by those spatial harmonics for which the corresponding Bessel function have index smaller than their argument. In regular dynamics, the argument ϑ of the Bessel functions is a smooth regular function of time. In this case, the spatial spectrum of v and w can contain many harmonics, but the pumping of energy between the different modes is regular. In contrast, in chaotic temporal dynamics ϑ is a random function of time. As a consequence, spatial chaos appears

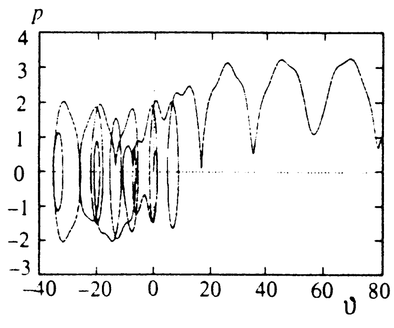

FIG. 6. The same as in Fig. 5, but with a slightly altered initial condition for the field: $p(0)=10^{-4}$.

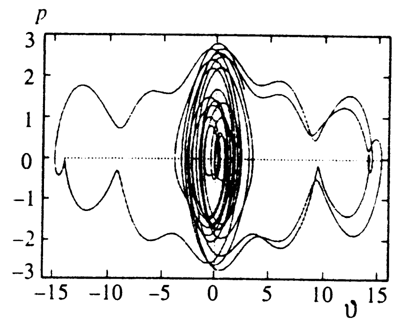

FIG. 8. A chaotic trajectory of length $\tau=200$ for $\bar{G}=0.6$ and $\bar{\Omega}=0.9$.

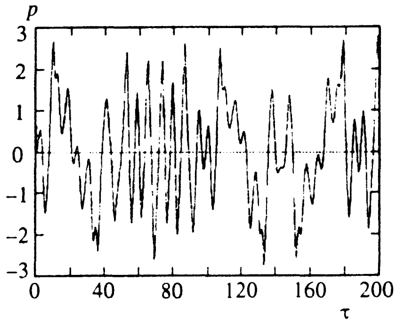

FIG. 9. Chaotic oscillations of the field for parameter values as in Fig. 8.

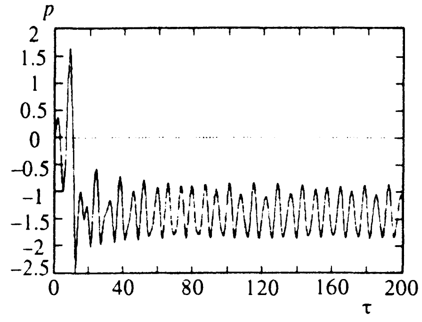

FIG. 11. Dependence of the field on time. The values of the parameters are the same as in Fig. 10.

in the distribution of the population difference and polarization.

4. DISCUSSION

It is interesting to compare the nonlinear dynamics of the system consisting of two-level atoms plus self-consistent field plus external amplitude-modulated field for different cavity geometries. For a Fabry–Perot cavity, the corresponding Maxwell–Bloch system reduces to the periodically excited "Bessel pendulum" (18). The Hamiltonian chaos in this system is transient. In contrast, in the case of a ring cavity, in the framework of the same approximations, the dynamics of the system is determined by the equation of a periodically excited physical pendulum:[12]

$$\ddot{\vartheta}+\omega_0^2 \sin\vartheta = GF(t),$$

where $\vartheta(t)$ is the Bloch angle. Since the corresponding potential $V(\vartheta)=-\omega_c^2\cos\vartheta$ is a periodic function of ϑ, with a periodic $F(t)$ the chaos is stationary. The onset of transient chaos is possible only for a special form of $F(t)$, wherein $F(t)$ contains the zeroth harmonic.[12] The effective potential then loses translational invariance.

Thus far, oscillatory exchange of energy between atoms and a field mode has been observed both in the microwave and in the optical range.[4] We shall give some estimates. For an allowed transition with $d\sim 1$ Debye and a density of 10^{14} atoms cm^{-3}, the value of the cooperative frequency $\omega_c\sim 10^{10}\ s^{-1}$ can substantially exceed the relaxation constants of the atoms and cavity. We believe that rapid progress in this field is making possible the experimental observation of transient Hamiltonian chaos as well.

We now summarize the results. We have considered the influence of the spatial structure of a field in the form of a standing wave on the chaotic dynamics in the interaction of two-level atoms with the field. We have shown that in this case, the Hamiltonian chaos is transient. For trajectories with a long chaotic phase, the asymptotic behavior of the field in the cavity is sensitive to a small change in the initial conditions or parameters. The temporal chaos manifests itself in the spatial behavior of the polarization and population difference.

In conclusion, we note some problems for further research. First, the nonlinear dynamics of the periodically excited "Bessel pendulum" merits more-detailed and deeper study in the context of the general problem of transient chaos. Second, the study of the influence of quantum effects on chaos in potential scattering is currently of great interest to researchers (see the review Ref. 29). However, the influence of quantum effects on other types of transient Hamiltonian chaos has essentially not been considered. In view of this, investigation of the influence of the quantization of the field in the cavity on transient chaos in the framework of the model discussed in this article is of interest.

We are grateful to Professor V. S. Egorov for providing a reprint of Ref. 27. One of us (K. N. A.) also thanks

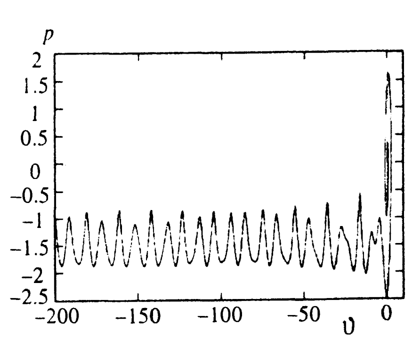

FIG. 10. Rapid escape from the first potential well ($\tau_{esc}=14.8$) and establishment of regular motion for $\bar{G}=0.4$ and $\bar{\Omega}=0.9$.

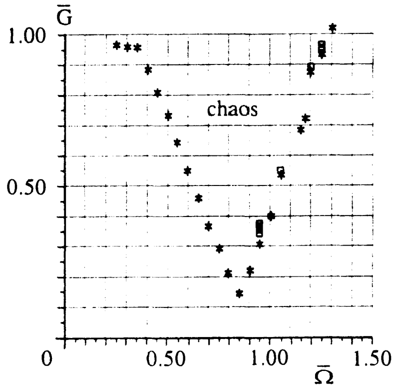

FIG. 12. Regions of regular and chaotic motion in the space of the amplitude parameter and frequency parameter.

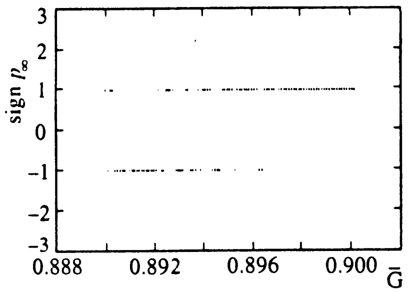

FIG. 13. Effect of variation in the amplitude of the perturbation on the sign of the asymptotic state of the field (for explanation, see text); $\bar{\Omega}=0.4$.

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1)We note that this model is also applicable to the description of the interaction of an impurity center in a crystal with light and phonons.[14] In this case the cavity is absent.

2)In the current literature the terms "collective Rabi frequency" and "vacuum-field Rabi frequency" are also used for this frequency. See the discussion of these terminological questions in Ref. 4.

3)Or, if $v(z,0)=\varepsilon_1(0)=0$, then $v(z,t)=\varepsilon_1(t)=0$ for any t.

4)This fact was drawn to our attention by Professor Wei-Mou Zheng.

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