Two-photon optics: diffraction, holography, and transformation of two-dimensional signals

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It is found that spontaneous two-photon emission generated by parametric downconversion should experience a distinctive type of diffraction, which can be observed by the photon correlation method. It is convenient to interpret the effect in terms of advanced waves. Two-photon analogs of other optical phenomena are also considered: holography, two-dimensional Fourier transforms, and image formation.

1. INTRODUCTION

In recent years a significant number of experimental and theoretical papers in quantum optics have been dedicated to the investigation of variants of the two-photon interference effect (see, e.g., Refs. 1–9). It is natural to ask whether two-photon analogs of other effects of ordinary (single-photon) linear optics exist, e.g., diffraction, holography, etc. By a two-photon analog we mean the presence of a characteristic dependence of the probability of coincidence of the photon counts P_c of two detectors (i.e., the correlation function of the intensity at two points in space) on the parameters of the optical system: the relative positions of the detectors and the light source, objectives, screens, transparencies, etc., which might resemble the corresponding dependence of the light intensity at one point (the probability of detecting one photon) on the parameters of a system of ordinary optical devices.

In what follows we will lay the groundwork for an intuitive treatment of two-photon effects within the framework of the formal concept of advanced waves (in more detail see Refs. 7, 10, and 11) and with its help propose some possible experimental schemes which are, in our opinion, of definite interest.

2. INTERPRETATION OF TWO-PHOTON CORRELATION WITH THE HELP OF ADVANCED WAVES

As is well known, the probability of detection of two photons at two points in space-time with coordinates $(\mathbf{r}_1,t_1)$ (point 1) and $(\mathbf{r}_2,t_2)$ (point 2) is proportional to the intensity correlation function G_{12} at these points:

$$P_c \propto G_{12} \equiv \langle E_1^{(-)} E_2^{(-)} E_2^{(+)} E_1^{(+)} \rangle. \qquad (1)$$

Here E_n are the Heisenberg operators of the field at the points $n=1,2$. It is assumed that the detection volume is much smaller than the field coherence volume, i.e., the detectors are point or single-mode detectors (they do not average the field over space or time).

We take the effective interaction Hamiltonian of the field with nonlinear matter in parametric downconversion to be of the form

$$H = -\frac{1}{2}\chi \int_V d\mathbf{r}_0 \mathscr{E}_0^{(+)} E_0^{(-)} E_0^{(-)} + \text{h.c.}, \qquad (2)$$

where χ is the quadratic nonlinearity of the material, $\mathscr{E}_0 = \mathscr{E}(\mathbf{r}_0,t_0)$ is the classical prescribed pump field at the point $\mathbf{r}_0$ inside the sample which occupies volume V.

To first order in perturbation theory the correlation function (1) factors:

$$G_{12} = |F_{12}|^2, \qquad (3)$$

$$F_{12} = F_{21} \equiv \langle E_1^{(+)} E_2^{(+)} \rangle = -i\hbar \int_V d\mathbf{r}_0 \chi \int_0^\tau d\tau \mathscr{E}_0^{(+)} D_{10} D_{20}. \qquad (4)$$

Here τ is the time during which the perturbation acts, with $t_{1,2} > \tau$ (in the theory it is convenient to assume that the pumping is of finite duration, much longer than other characteristic times of the model), $D_{n0} = D(\mathbf{r}_n,t_n;\mathbf{r}_0,t_0) = -D^*_{0n}$ is the propagator describing the propagation of the fields $E^\pm$ between the points n and 0, allowing for the action of filters, screens, etc.

Let the point 2 be fixed. Then, according to (3), the function $F_{12} \equiv E_{\text{eff}}(\mathbf{r}_1,t_1)$ plays the role of an effective field, whose intensity determines the probability of detecting a photon at the point-event 1 given that the other photon has been detected at the point 2. According to (4), this field can be found by assuming that an advanced pulse is emitted from point $\mathbf{r}_2$ at time t_2. It arrives at the point of the sample $(\mathbf{r}_0,t_0)$, interacts there with the pump and then, already converted, arrives at the point 1. The contributions of the points of the sample illuminated by the pump are then summed.

Thus, the factoring property (3) and the structure of the correlator (4) allow an intuitive treatment of two-photon correlation with the help of the concept of an effective field acting upon one of the two detectors and formed by parametric conversion of the advanced wave emitted by the second detector. The presence in (4) of the classical propagators D_{n0} means that by placing the various optical elements—lenses, screens, diffraction gratings, interferometers, polarizers—between the light source and the detectors, it should be possible to obtain two-photon analogs of linear effects of classical optics. Note that the theory of atomic sources of two-photon light can also be represented in analogous form.[11]

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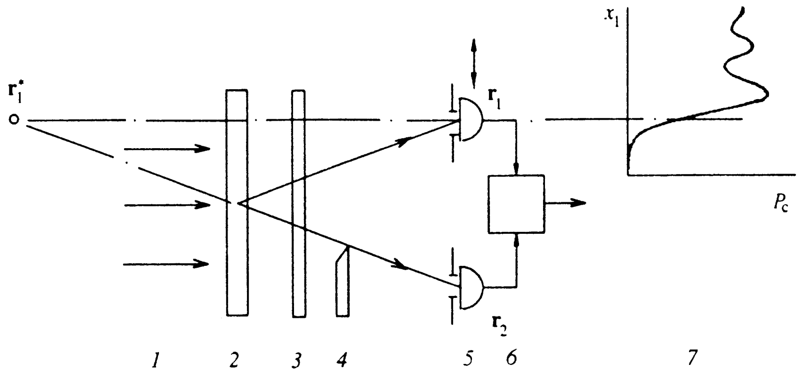

FIG. 1. Layout for observing two-photon diffraction: *1*—pump, *2*—nonlinear converter, emitting photon pairs, *3*—filter, *4*—screen, *5*—photon counter, *6*—coincidence detector, *7*—dependence of the probability of coincidence on the transverse coordinate of the upper detector.

It is interesting that, according to (4), the space-time structure of the field (the "biphoton") F_{12} can be controlled not only by the propagators D_{n0}, but also by varying the pump field $\mathscr{E}_0$ and/or the shape of the nonlinear region V. Thus, Ref. 12 considered the effect of focusing the pump radiation. In this case, the effect of parametric conversion on the advanced field is as if it were reflected from a spherical mirror. Reference 9 considered the case of two nonlinear crystals, placed in tandem, excited by a common coherent pump.

Below we will consider the peculiarities of formation of the transverse structure of a biphoton due to D_{n0}.

3. TWO-PHOTON DIFFRACTION

The layout of a possible experiment is shown in Fig. 1. A plane monochromatic pump wave (ω_0,k_0) excites a flat, birefringent, thin piezocrystal of thickness l, which emits a pair of correlated photons $(\omega,\mathbf{k}_1)$ and $(\omega,\mathbf{k}_2)$. The assumed uniformity of the model in the transverse plane (x,y) gives the conditions

$$k_{1x}+k_{2x}=0,\quad k_{1y}+k_{2y}=0, \tag{5}$$

which are satisfied with accuracy of order $1/a$, where a is the diameter of the pump beam.

A narrowband filter is tuned to the degenerate frequency $\omega=\omega_0/2$ with bandwidth $\Delta\omega$. It suppresses the pump radiation and ensures the necessary coherence length of the field thus formed. Two point photon-counters with coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ and a means of recording coincidences in some temporal window T serve to measure the intensity correlation function in the presence of a diffracting screen in the path of one of the photons.

We assume that the phase matching condition $k_1=k_2=k_0/2$ is fulfilled in the crystal for waves of frequency ω. Because of the finite thickness of the crystal l, the photons are emitted inside a cone at angles $\vartheta_1=\vartheta_2<\vartheta_{\max}$ from the vertex, where $\vartheta_{\max}\propto l^{-1}$.

We observe the dependence of the probability of coincidences of photon-counts P_c on the transverse coordinate of one of the detectors, e.g., x_1. In the given case formula (4) can be interpreted in the following way. The advanced wave emitted from the point $\mathbf{r}_2$ is frequency-filtered, diffracts from the boundary of the screen and falls upon the crystal where, interacting with the pump, it generates the effective field which acts on the detector at $\mathbf{r}_1$. Integrating over x and y in (4) leads to the condition of conservation of transverse momentum in the form (5), which for $\omega_1=\omega_2$ gives $\vartheta_1=\vartheta_2$. Thus, the advanced wave is effectively reflected by the wavefronts of the pump wave inside the crystal, i.e., the thin crystal serves as a mirror for the advanced wave. Consequently, the effective field in the region $\mathbf{r}_1^*$ (Fig. 1) has the usual diffraction structure formed by a monochromatic point source at $\mathbf{r}_2$ and the screen. Such a function $I(x_1)\propto P_c(x_1)$ would be observed if an ordinary point source were located at $\mathbf{r}_2$ instead of detector 2, and the detector were located at the point $\mathbf{r}_1^*$.

An analogous dependence $P_c'(x_2)$ should also be observed if detector 2 or the screen is moved (in the first case the effective source is located at $\mathbf{r}_1^*$).

In the geometric-optics approximation, the presence of a "shadow" region with $P_c\simeq 0$ and a sharp boundary follows at once from conditions (5) if we assume that the photons/particles are emitted in pairs from one point of the crystal at identical angles (Fig. 1). However, such a naive model is inconsistent with the diffraction oscillations of the function $P_c(x)$ in the transition region, which attest to the wave character of the process.

The effect considered here seems at first glance somewhat paradoxical—the screen along the path of one photon seems to influence the behavior of the second (compare with the phenomenon of "mutual focusing of photons," Refs. 10 and 13). We emphasize, however, that the only truly quantum paradox here is the low relative level of background ("random") coincidences in the "shadow" region, which attests to the inapplicability of the Cauchy–Schwarz inequality

$$|F_{12}|^2\leqslant\langle E_1^{(-)}E_1^{(+)}\rangle\langle E_2^{(-)}E_2^{(+)}\rangle, \tag{6}$$

which holds in the case of classical averaging.[7] The diffraction oscillations of $P_c(x)$ can be described with the help of a classical stochastic model with a "squeezed" Gaussian field and the semiclassical theory of photodetection.

An analogous situation arises in two-photon interference: the classical description differs from the quantum description only in the enhanced background level, i.e., lower visibility and contrast.

The close connection between the classical and quantum treatments for an arbitrary linear optical system, which can include parametric converters, was demonstrated in Ref. 8.

Figure 1 assumes that the diffraction pattern is recorded "point by point," that is, by scanning detector 1

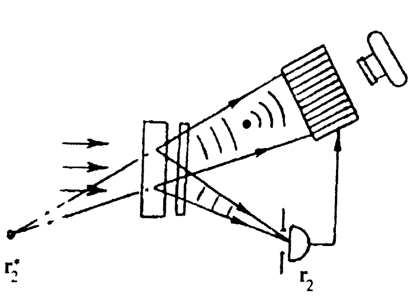

FIG. 2. Layout for obtaining a two-photon Gabor hologram. The hologram of the scattering point (the filled-in circle) is recorded by an electro-optical converter (the hatched rectangle) enabled for a short time by the output pulses of the point detector at $\mathbf{r}_2$. The mirror image of the latter at $\mathbf{r}_2^*$ serves as the source of the effective field.

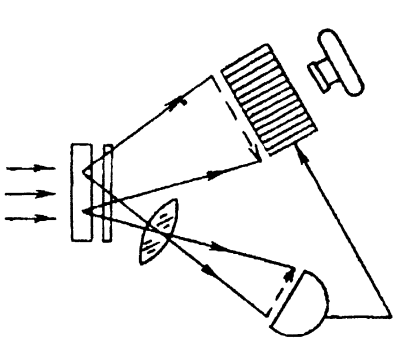

FIG. 3. Two-photon projector. The image τ' of the transparency τ is created as a result of angular correlation of the signal and idler photons emitted at equal angles as a consequence of the law of conservation of transverse momentum.

over the (x_1,y_1) plane. However, it is more convenient to use an electro-optical converter which is turned on only for a short time under the action of amplified pulses from detector 2: photography of the screen of the electro-optical converter for a sufficiently large exposure time will give the complete diffraction pattern in the (x_1,y_1) plane.

One more possibility is the use of a multielement mosaic (CCD) detector in conjunction with a multichannel analyzer (digital integrator), e.g., of the type OMA-SIT. Detector 2 in this case should provide temporal selection of the recorded signals.

It is possible to observe two-photon interference and diffraction by other objects—such as diffraction gratings, slits, and diaphragms—in a similar manner. It is natural to generalize our treatment to other types of transformation of two-dimensional optical signals, and it is to this that the following sections are dedicated.

4. TWO-PHOTON HOLOGRAPHY

Figure 2 illustrates the principle of obtaining two-photon holograms. For simplicity the simplest Gabor scheme with common-path object and reference beams is depicted.[14] Here a point on the object scatters secondary spherical waves, which interfere with the primary waves to produce a holographic "moiré" defined by the phase difference of the alternative paths. The two-dimensional hologram can be recorded by one of the aforementioned methods: with the help of an electro-optical converter, by scanning detector 1 or by means of a photodetector array. In the latter two approaches it is comparatively easy to apply the methods of digital holography with corresponding reconstruction of the hologram.

In the photographic recording of the hologram the regions of maximum darkening in space $\mathbf{r}_1$ correspond to the points with the maximum probability of coincidences $P_c(\mathbf{r}_1,\mathbf{r}_2)$ for the coordinates $\mathbf{r}_2$ of detector 2 fixed, with detector 2 serving as the source of the advanced field. Thus, the two-photon hologram coincides with the ordinary hologram obtained by replacing detector 2 with a point light source, and the crystal with a flat mirror. It can be reconstructed in one of two ways: by replacing the object in Fig. 2 with its hologram and recording anew the coincidences of photon-counts, or by the ordinary "single-photon" method with the help of a point light source. Of course, a hologram obtained in the conventional way can be reconstructed with two-photon light. In principle, two-photon volume holography is also possible, for example, by scanning detector 1 over three dimensions.

Interestingly enough, if the object in Fig. 2 is moved to the lower channel, the hologram will be created by the upper beam, even without the depicted object being in its path ("in absentia" holography). An analogous paradox also appears in the focusing of two-photon light.[10,13]

5. TWO-PHOTON TRANSFORMATION OF TWO-DIMENSIONAL IMAGES

We replace the point detector 2 by a detector with a large aperture, and we place a transparency with intensity transmittance $\tau(x,y)$ in front of it (Fig. 3). According to the interpretation of advanced waves, such a detector serves as an effective distributed source of incoherent light with intensity modulated according to $\tau(x,y)$. Recall that for formula (1) to be applicable it is necessary that during the resolving time of the electronic system T, the number of photon-count coincidences (which grows with increasing size of the apertures of the photodetectors) remain much smaller than unity.

The crystal serving as an effective mirror and the lens (Fig. 3) form a real inverted image of the transparency with appropriate magnification. This image can be recorded by the foregoing methods. As was already noted, if the lens is moved to the pump channel, the crystal is "converted" into a spherical mirror.[12]

The operation of a two-photon projector can be explained in terms of geometric optics and photons/particles emerging from the crystal at equal angles. If we use nondegenerate synchronism of the parametric converter, then image transfer will be accompanied by conversion from one spectral range to another. However, the transformation of the advanced wave in this case will not be equivalent to a simple reflection.

Generally speaking, in a two-photon projector it should be possible in principle to do without monochromatic filters by working, so to speak, with "white" light. The chromatic aberrations that arise in this case can be corrected by one or two optical wedges.

Let the transparency now be located right up against the lens, and a pinhole be placed again in front of detector 2. In this case in the image plane of the pinhole the probability of coincidences $P_c(x_1,y_1)$ in the upper channel will be proportional to the two-dimensional Fourier transform of the function $\tau(x_2,y_2)$.[14,15] Such a device can fulfill the function of a two-dimensional analog Fourier processor, analogous to a coherent spectrum analyzer.[15,16] Of course, as in the schemes in Figs. 1–3, it is necessary to use a narrowband filter tuned to the frequency ω.

6. CONCLUSION

Questions of the observation and practical use of the proposed effects, of course, require additional study. We have striven here only to interest the reader in some novel possibilities which have manifested themselves as a result of the development of effective sources of directed biphoton fluxes.[17] The necessary restrictions on the frequency bands $\Delta\omega$, the thickness of the nonlinear crystal l, the pump power, the apertures of the detectors, the resolution of the coincidence detector, etc. can be estimated following Ref. 12.

The following feature of two-photon devices is interesting to us at the applications level: if the illuminance in one beam (e.g., the upper one) is recorded without temporal selection, then it will be uniform and independent of the properties of the transparency in the other (lower) channel. Thus, the transformed information will be encoded, and the key to deciphering it is carried by the pulses from the lower photodetector, which can be transmitted by an independent communications channel, which enhances not only noise immunity, but also the degree of confidentiality of communication (compare with papers on quantum cryptography, e.g., Ref. 2 and the literature cited therein).

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